# VALUE AND DECISION

CARLOS FALLON, RCA Corporate Staff

NUMBER IN THE SERIES OF VALUE ANALYSIS PROFILES

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All the "keys to value" and all the "reasons for needless cost" hinge on good or bad decision patterns. The value specialist does not make decisions for others, but he must provide the kind of methods for scientific decision that can be actually used by the diverse specialists who control product value. This paper presents an overview of the aids to sound decision which have proved their merit in actual industrial use.

## THE DECISION PROCESS

### Option, Choice, Selection, and Decision

An *option* is defined as the right to exercise the power of choice. *Choice* implies the opportunity and privilege of choosing freely. It also imposes the requirement of giving up what is not chosen. The act of *selection* is defined as picking out one or more items from a larger number of similar or analogous items. This action is the same as rejecting the ones not selected. Choice and selection, therefore, imply a measure of conflict, but it is usually mild conflict *within* the decision maker. You pick out an apple and the other apples don't seem to mind.

*Decision*, on the other hand, settles external conflict of one sort or another. It *decides* issues in arbitration, in the prize-ring, and in court. It also resolves the less evident conflicts among design objectives, such as performance, reliability, low cost, and early delivery.

In the decision process, much of the conflict centers around the objectives and their relative importance. Once this has been settled, the task is reduced to comparing probabilities and information in order to make a simple choice or selection among the available courses of action. The "decision matrix", which will be described in this paper, identifies the elements of conflict by requiring the decision maker to define his objectives and assign to them numerical measures of relative importance. From that point on, the matrix serves only to compare choices which involve a lesser measure of conflict.

### The Element of Conflict

In manipulating the forces of nature, man has proved a master. He decides the conflict between soil-building and erosion by turning rampaging rivers into placid lakes to provide hydroelectric power. Out of the conflict between fire and water man gets the steam engine, but when it comes to conflict with other men, he has not made much progress; often he ceases to be rational. Yet, of all the decisions he makes, the most important ones have to do with this second type of conflict — conflict involving people.

Take the simple decision of what task to work on first when you sit down at your desk in the morning. That decision will settle, for better or worse, a conflict among *people*, among those persons, each of whom wants *his* work done first. Their goodwill or resentment will be your gain or loss. Unexpected new tasks are often assigned to us as follows: "Do this at once, but don't drop anything else." This imposes a queue discipline called "Last in, first out". When this queue discipline is imposed, and the queue must still be kept moving, an additional server is required. But we don't often get him. If we were machines, we would *have* to have him, but being men, our boss expects us to juggle our chores against time.

### The Element of Time

Faith, hope, imagination, and planning can initiate a cause by anticipating its effect, but the cause must nevertheless precede the effect in real time. This necessary one-directional sequence of cause and effect governs much of our activity in science and engineering — so much so that Critical Path Methods have been created to identify cause-and-effect relations and to predict the consequences. This sequential pattern is repeated throughout industry in many activities that really need not be sequential. One way to gain time is to identify these activities and carry them out concurrently.

Let us stop for a moment to think about the nature of time with respect to the sequence of events. In this context we perceive at once the relation of time to chance. Those events that have already taken place *before* the instant of observation are certain and inalterable, although they may be unknown. Those events expected to take place *after* the instant of observation are uncertain and may not happen at all. But events taking place at the very instant of observation can be controlled. They effect the outcome of future events and they constitute both *opportunity and risk.*

### The Element of Chance

Although we do not know what the future will bring, we *do know* that everything we do or fail to do affects the future. We also know what the future *might* bring, and we know this with different degrees of likelihood.

Like the command of early Roman armies which was shared by two consuls, command of our destiny is shared with us by *Chance* who controls the future. But *we* control the present. In the here and now we face *risk*, we face *opportunity*, and we commit *resources.* Then *Chance* steps in with the consequences. So we must:

- Recognize and appraise risk, relating it to the *consequences of failure.*
- Recognize and appraise opportunity, relating it to the *consequences of success.*

- Recognize and appraise our resources, committing them in proportion to the *importance of the expected outcome.*
- Relate risk, opportunity, and resources to time and space in order to achieve our ends through timely, decisive *action.*

Students of management science complain that industrial executives, when they let their hair down, admit that their own personal risk is the first thing they consider when making a decision. This is as it should be. Life on earth follows the strategy of survival and growth, and survival comes first. An executive cannot contribute much to his company's growth if he has been fired. A company cannot grow if it has been wiped out. For this reason the strategy outlined above must consider the risk first.

### The Elements of Strategy

About 400 B.C., when the great thinkers of ancient Greece had concluded that strategy could only be learned in a lifetime of fighting battles, a gifted Chinese soldier and thinker, who successfully commanded the armies of the King of Wu in the period of The Warring States, wrote a treatise on strategy. Sun Tzu's *Art of War* is as good today as it was then. Once we understand how the elements of conflict, time, and chance enter into the decision process, we become interested in Sun Tzu's words: "Now the elements of the art of war are: first, measurement of space; second, estimation of quantities; third, calculations; fourth, comparisons; fifth, probability of victory." Coming from a man whose book is admittedly the basis of Mao Tze Tung's, Chu Te's, and Ho Chin Ming's strategy, this is a pretty good case for the quantitative approach to decision making. On the merits of quantitative methods, Sun Tzu is explicit: "Quantities derive from measurement, numbers from quantities, comparisons from numbers, and victory from comparisons."

## THE DECISION MATRIX

This writer has found the "decision matrix" a most useful tool in the analysis of value. The particular algorithm that we use most often serves a different purpose at each stage of our work. First, it helps us analyze the function, task, or objective of a product by breaking it down into a number of requirements. Once these requirements have been determined, the algorithm aligns them with their measures of relative importance or weighting factors.

The identical algorithm can be used as a "sub-matrix" for combining the various benefits that constitute a complex requirement, and for noting their relative contribution towards meeting this requirement. Finally the algorithm provides a framework for evaluating various courses of action, or choices, in order to select:

- The best combination of desirable characteristics for a given level of resources.

or

- A given combination of desirable characteristics for the least cost in resources.

More important, putting numbers into this framework makes people think. The matrix serves as a two dimensional check-list which forces us to face, one at a time, the major factors leading to a decision. For instance, at this point we are obliged to study the choice between *the best combination for the given resources,* or the *least resources for a given combination.* Trying for both the best combination and the least resources is like trying to get both ends of a seesaw to go up at the same time. No technique of mathematical optimization can achieve this.

### The Difference Between Optimization and Outright Improvement

Mathematics can only work with what we have to begin with, but other sciences can be used to improve what we have. Applying physics, we can put a jack under the seesaw, and by adding some effort, we can then raise both ends of the seesaw at the same time. The "decision matrix", serving now as an algorithm for precise comparison, can point out those areas where the available effort can be spent to greatest advantage.

We set up our matrix with the requirements along the top and the various choices in a column down the left. Here is a hypothetical example: a transportable battlefield radar to be designed for sale to one of the larger South American nations. We chose this example because the writer is familiar with the potential customer's needs. These needs include high MOBILITY, low initial cost, which we will call INITIAL ECONOMY, low field cost, or FIELD ECONOMY, good PERFORMANCE, and a property which the customer calls COMBAT ENDURANCE. This latter property includes reliability, low repair time, negligible maintenance, the capacity to withstand the shock of near misses, and the capacity to withstand rugged transportation conditions.

### The Art and Science of Combination

Once the customer's requirements have been defined, they must be expressed numerically, and studied systematically, in order to generate a number of satisfactory combinations. The key concept here is that of a *balanced combination* rather than the most of the best of everything. This is the difference between well-allocated effort and maximum effort in all directions.

The most promising combinations of requirements are then compared by expressing, in numerical measures, the extent to which each element of each combination contributes to the particular requirement it is supposed to meet.

When we suggest numerical measures, people frown. They look at each other and someone says, "You can't compare peaches and pears!" But we *can.* It is no problem if we take one property at a time. A pound of peaches weighs exactly as much as a pound of pears. A carload of melons occupies the same number of cubic feet as a carload of potatoes. A dozen elephants is just *one* dozen, and it has the same number of units as a dozen ping-pong balls. At this point, someone will say, "Ha! Just what property do ENDURANCE, MOBILITY and COST have in common?"

### Commensurable Units and a Standard Scale

ENDURANCE, MOBILITY, and COST all contribute to the mission of the equipment. How does COST contribute to the mission? It contributes *numerically*. Cost is the inverse measure of the *number* of equipments you can get from the resources committed. Cost, size, and weight determine *how many* equipments can be provided in a given place at a given time. This contribution, this satisfaction of the customer's needs is the common property we must measure for all major characteristics. In school, reading, and riting, and rithmetic are measured numerically on a standard scale in which 70 is passing, 90 is very good, and 100 is perfect. This scale has the advantage that it is immediately meaningful to everyone — engineer, shop foreman, buyer.

We can use this standard scale, but we may not want to pay the full price of perfection implied by the grade 100. Perfection could cost too much and take too long. So the real end points of our scale are 70 and 90. Figure 1 shows how we can transform system weight into a standard scale. This figure is based on the assumption that the correlation is linear. Often, however, the relationship is not linear. In figure 2 one can see that as certain objects get lighter and lighter they do not get uniformly more mobile. At first you can't budge them, then you can barely budge them, and finally they begin to move. This nonlinear relationship in physical interaction is a counterpart to Bernoulli's utility function in economics[1]. Such nonlinearity should be suspected, searched for, and taken into account.

## Utility: The Concept and its Growing Importance

To us it seems easy enough to understand that an article can be worth more to one person than to another, or that it can be worth more for one particular application than for another. Economic utility, or worth to a particular customer for a particular application, becomes increasingly important as the number of free, individual customers increases. Daniel Bernoulli formulated a good mathematical approximation of utility as early as 1738[1]. He was followed by Herman Gossen, William Jevons, Maffeo Pantaleoni, and Alfred Marshall.

John Von Neumann and Oskar Morgenstern[2] combined the previous groundwork with their own innovations to synthesize a theory of utility applicable to present day economics and to the modern analysis of value. The customer's concept of value starts out with his personal utility. The product must *do* something for him. He has to want it before he even considers the cost. Then he compares what the product is worth to him and what it will cost him. A favorable relation between product worth and product cost is the sort of value we strive for in value analysis.

## Utility as a Measurement of Customer Satisfaction

We try to give the customer either a little more for his money or something every bit as good for less money. The techniques of innovation, simplification, up-dating, and better use of people sometimes make it possible for us to give the customer something *better* for *less* money. To do this we must improve efficiency by taking advantage of new discoveries in the properties of matter, the sources of power, *and* the customer's needs. We then relate these elements to each other by shorter, more direct paths. But first we must find out what the customer really wants, whether he is an infantry captain knee-deep in a jungle swamp, or a scientist looking at televised pictures of the moon. Today, this respect for what the customer wants, as opposed to what somebody else thinks he *should* have, has become *an all important aspect of competitive business.*

Adjusting for utility involves the delicate process of interpreting customer's needs and desires in terms of numbers[3]. This requires considerable training in eliciting information from the customer and converting it into the utility function on the graph. The danger here is that some utility functions coincide with certain well-known mathematical curves. This may lead one to assume that other unknown utility functions will fit neatly into mathematical patterns. Many do not. A most useful explanation of utility for our purposes can be found in Chernoff and Moses[4].

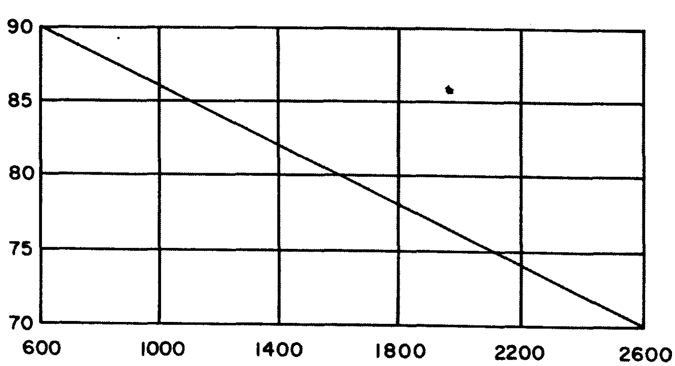



**Fig. 1 — Linear normalization of system weight: total system weight in pounds**

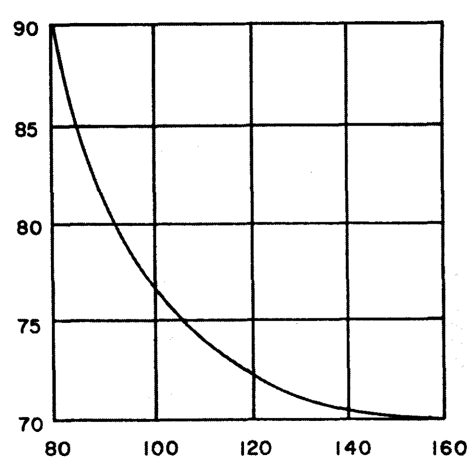



**Fig. 2 — Weight of heaviest case: the utility function**

## Identifying and Defining the Objective

Whether we are speaking of a lone decision maker or of one supported by a good staff, his first consideration is one of appraisal. Before he can consider the risk worth taking or the resources worth committing, he must appraise the value of the end he is trying to achieve. The Greek word for *end*, in the sense of an objective, is *teleios*, from *telos*, meaning end in the ordinary sense. This relationship added an element of completion to the concept of the objective. From the same root, the word *teleos* meant *perfect* in the sense of *well done*.

It was no accident that ancient Greece produced thinkers unsurpassed to this day. *Teleios*, the Greek notion of the objective, directs thought to the degree of completeness with which we must accomplish our ends to make them worthwhile, and it calls for an estimate of *how well* the task must be done in order to succeed. This requires numerical answers to such questions as how much, how well, how complete, how often, and for how long.

## Setting Upper and Lower Limits

The objective, more often than not, combines several requirements. Each of these requirements offers us a range of choice between a lower and an upper limit of demand. The threshold or lower limit is the "least favorable but adequate condition." This level must coincide with the equivalent condition of the other requirements and with the "passing grade" of the standard scale (usually 70). The upper limit is the "best practical condition". It must coincide with the equivalent condition of the other requirements and with the "very good" grade of the standard scale (usually 90). If 90 is "best practical", then 91 is "excessive" and 100 indicates the absolute maximum. These extra benefits can be accepted when they are not achieved at the expense of actual requirements within the working range.

Here is an example using two elements of MOBILITY VALUE in our battlefield radar — SYSTEM WEIGHT (Fig. 1), and WEIGHT OF LARGEST CASE (Fig. 2).

| LIMITS | SYSTEM WEIGHT | WEIGHT OF LARGEST CASE |
|---|---|---|
| The best practical | 600 lbs. | 80 lbs. |
| LESS | | |
| The least favorable but acceptable | 2600 lbs. | 160 lbs. |
| GIVES US | | |
| The range of choice | —2000 lbs. | —80 lbs. |

The minus sign indicates that the curve will slope down toward the right.

## Measures of Relative Importance

Assigning measures of relative importance to the requirements or benefits calls for the greatest possible information on the intended use of the equipment and on the objective of the decision maker. It also calls upon judgment in the highest degree.

The total value of the requirements or benefits is set equal to unity, which is then divided by the number of requirements, under the initial assumption that they are all equally important. This is seldom the case, but it is a good way to start. If there are five requirements, each weighted at 0.2, some will have to be raised and others lowered.

These measures of relative importance or weighting factors are placed under their corresponding requirements (Figure 3). The decision is really made when this is settled, but there is much backing and filling before it is settled. Once the relative importance of the objectives has been determined, all that remains is to find out which of the available choices best satisfies that decision.

## Efficiencies of the Various Choices

Each choice or course of action contributes to each of the requirements or desired benefits in varying degrees of efficiency. For example, SYSTEM WEIGHT and WEIGHT OF LARGEST CASE make a negative contribution to MOBILITY. We want a simple "additive"[5] matrix using non-negative numbers, so we must convert this contribution into its complement of "lightness" or *positive* contribution to MOBILITY, as shown on Figures 1 and 2.

## Use of Sub-Matrices

The various elements of mobility are combined in a sub-matrix (Figure 4) and weighted for relative importance. The efficiencies introduced from the graphs in Figures 1 and 2 are entered in the body of this sub-matrix, as are the other numbers arrived at in the same manner. These numbers are multiplied by the weighting factor at the top of their respective columns, and the results entered in the lower right-hand corner of each cell. These "weighted efficiencies" are added across to yield the right-hand column of "expected mobility value" which represents the efficiency with which each choice contributes to MOBILITY in the final matrix. Note that the numbers now appear on the upper left-hand corner of the cells in Figure 3.

## Racking-Up the Results

The efficiencies in the final matrix (Figure 3) are multiplied by their weighting factors, as before, and these weighted efficiencies are added across to yield the effectiveness, expected value, or relative merit of each of the choices.

Suppose that instead of selecting the best out of four alternatives, we were allowed to make changes resulting in different and perhaps better combinations. As a Persian mathematician once put it, "Could we but shatter this scheme of things entire, and make it nearer to the heart's desire!".

This calls for early decisions, decisions involving conflicting requirements, intricate and unsuspected rates of exchange, and the delightful discovery that the whole can be greater than the sum of its parts. Useful tools to accomplish this are linear and dynamic programming and the more general techniques of linear algebra and matrix theory.

ASSUMPTION: ABOVE THE "JUST ADEQUATE" THRESHOLD AND BELOW THE "EXCESSIVE" UPPER LIMIT, INCREMENTS OF THE BENEFITS ARE ADDITIVE.

| CHOICES | PERFORMANCE | ENDURANCE | MOBILITY | INITIAL ECONOMY | FIELD ECONOMY | VALUES |
|---|---|---|---|---|---|---|
| RELATIVE IMPORTANCE OF CONTRIBUTING BENEFITS | .20 | .18 | .22 | .23 | .17 | THE SUM OF THE WEIGHTING FACTORS = UNITY |
| BASIC UNIT | 70 / 14 | 73 / 13 | 72 / 16 | 70 / 16 | 75 / 13 | 72 |
| HIGH PERFORMANCE | 89 / 18 | 79 / 14 | 76 / 17 | 88 / 20 | 81 / 14 | 83 |
| HIGH ENDURANCE | 76 / 15 | 89 / 16 | 74 / 16 | 71 / 16 | 87 / 15 | 78 |
| HIGH MOBILITY | 70 / 14 | 72 / 13 | 88 / 19 | 75 / 17 | 70 / 12 | 75 |

Fig. 3 — An "additive" decision matrix

| CHOICES | TOTAL WEIGHT | HEAVIEST CASE | LARGEST CASE | FEWER UNITS | SET-UP TIME | VALUES |
|---|---|---|---|---|---|---|
| RELATIVE IMPORTANCE OF CONTRIBUTING FACTORS | .30 | .20 | .15 | .10 | .25 | THE SUM OF THE WEIGHTING FACTORS = UNITY |
| BASIC UNIT | 70 / 21 | 70 / 14 | 70 / 10 | 80 / 8 | 75 / 19 | 72 |
| HIGH PERFORMANCE | 73 / 22 | 80 / 16 | 74 / 11 | 82 / 8 | 75 / 19 | 76 |
| HIGH ENDURANCE | 80 / 24 | 74 / 15 | 70 / 10 | 74 / 7 | 71 / 18 | 74 |
| HIGH MOBILITY | 90 / 27 | 89 / 18 | 89 / 13 | 80 / 8 | 90 / 22 | 88 |

Fig. 4 — Elements of MOBILITY

# THE THEORY BEHIND IT

## General Case Under the Additivity Assumption

To every expected benefit $B_1$, $B_2$, ... $B_n$ governing the choice among available means to accomplish a given end, we assign a real, non-negative number $w_1$, $w_2$, ... $w_n$ which we obtain intially by dividing the number 1 by the number of benefits being considered ($1/B_n$), and which we then vary as the relative importance of each benefit varies from that of the other benefits.

A positive change in the relative importance of one benefit results in a corresponding negative change in the others. This is imposed by the constraint that *the sum of the measures of relative importance is equal to unity*

$$\sum_{j=1}^{n} w_j = 1 \quad \text{where } w_n \text{ is measured along an interval scale}$$

extending from 0 (minimum) to 1 (maximum) and where unity symbolizes the given limitation on resources.

Each of the available choices should meet, in varying degrees of probability or known capability, the requirements of each desired benefit. These expected efficiencies are then transformed from the original units of measure into an interval scale extending from a minimum defined as the least favorable condition *acceptable*, to a maximum defined as the best condition which is *practical*. Such a scale, extending from 70 (minimum) to 90 (maximum) in percentage points has proved to be immediately meaningful to the ordinary business man, accountant, shop foreman, and marketing man who must contribute to the economic aspects of a decision. It is equated by them with familiar school grades, where 70 is passing and 90 is very good.

This is important because the measure of utility — value to a particular user for a particular application — is frequently provided by mathematically unsophisticated customers.

Transforming the original units of measure into commensurable units on a standard scale requires a measurable common property. This common property is the efficiency with which each choice contributes to each of the desired benefits. Variations in efficiency, or in utility, are not always directly proportional to variations in the physical or economic units with which we measure the choices.

Utility or its physical counterpart, the nonlinearity between performance, on the one hand, and the effect of performance, on the other, can be taken into account by using a utility function as the means of transformation. We thus use a normalization technique which also adjusts for utility and which yields a matrix E of commensurable units, whose elements $e_{ij}$ are efficiency measures of the choices, or courses of action, which we are comparing.

To arrive at the expected value of the various choices, we multiply $e_{ij}$ by $w_j$ because $w_1$, $w_2$, ... $w_j$ constitute the measures of relative importance or weighting factors assigned to the desired benefits. This yields the system of equations:

THE WEIGHTED EFFICIENCIES ADD UP TO EXPECTED VALUE

| | | | | | VALUE |
|---|---|---|---|---|---|
| $e_{11}\,w_1$ | $e_{12}\,w_2$ | ... | $e_{1n}\,w_n$ | | $v_1$ |
| $e_{21}\,w_1$ | $e_{22}\,w_2$ | ... | $e_{2n}\,w_n$ | | $v_2$ |
| ... | ... | ... | ... | | ... |
| $e_{m1}\,w_1$ | $e_{m2}\,w_2$ | ... | $e_{mn}\,w_n$ | | $v_m$ |

WHICH IS THE PRODUCT OF THIS VECTOR

| | | | |
|---|---|---|---|
| $w_1$ | $w_2$ | ... | $w_n$ |

AND THIS MATRIX

| | | | |
|---|---|---|---|
| $e_{11}$ | $e_{12}$ | ... | $e_{1n}$ |
| $e_{21}$ | $e_{22}$ | ... | $e_{2n}$ |
| ... | ... | ... | ... |
| $e_{m1}$ | $e_{m2}$ | ... | $e_{mn}$ |

**Fig. 5 — Matrix theory behind the algorithm in figures 3, 4, and 6.**

$$\begin{aligned} e_{11}w_1 + e_{12}w_2 + \dots + e_{1n}w_n &= v_1 \\ e_{21}w_1 + e_{22}w_2 + \dots + e_{2n}w_n &= v_2 \\ \dots \quad \dots \quad \dots \quad \dots \quad &\dots \\ e_{m1}w_1 + e_{m2}w_2 + \dots + e_{mn}w_n &= v_m \end{aligned}$$

which is what actually appears on the algorithm of our decision matrix (shown on Figure 5) and which generates the information $v_m$ from which one selects the highest $v_i$ as the best choice.

The foregoing array may be represented by the matrix equation:

$$\begin{bmatrix} e_{11}e_{12} \dots e_{1n} \\ e_{21}e_{22} \dots e_{2n} \\ \cdot \quad \cdot \qquad \cdot \\ \cdot \quad \cdot \qquad \cdot \\ \cdot \quad \cdot \qquad \cdot \\ e_{m1}e_{m2} \dots e_{mn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_n \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ \cdot \\ v_m \end{bmatrix}$$

where e = efficiency, w = weighting factor or measure of relative importance, and v = expected value of the ith course of action; or in condensed form:

| | OBJECTIVES, DESIRED BENEFITS, OUTCOMES | | | | |
|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | . . . | $B_n$ | |
| RELATIVE IMPORTANCE OF OBJECTIVES, DESIRED BENEFITS, OR EXPECTED OUTCOMES | $w_1$ | $w_2$ | . . . | $w_n$ | SET THE SUM OF THE MEASURES OF RELATIVE IMPORTANCE EQUAL TO UNITY |
| $C_1$ | $e_{11}$ $e_{11}\ w_1$ | $e_{12}$ $e_{12}\ w_2$ | . . . | $e_{1n}$ $e_{1n}\ w_n$ | $v_1$ |
| $C_2$ | $e_{21}$ $e_{21}\ w_1$ | $e_{22}$ $e_{22}\ w_2$ | . . . | $e_{2n}$ $e_{2n}\ w_n$ | $v_2$ |
| . . . | . . . . . . | . . . . . . | . . . . . . | . . . . . . | . . . |
| $C_m$ | $e_{m1}$ $e_{m1}\ w_1$ | $e_{m2}$ $e_{m2}\ w_2$ | . . . | $e_{mn}$ $e_{mn}\ w_n$ | $v_m$ |
| CHOICES | | | | | VALUES |

**Fig. 6 — An algorithm for industrial decisions**

$$\sum_{j=1}^{n} e_{ij} w_j = v_i \qquad i = 1, 2, \ldots, m$$

## Sensitivity Studies: Perturbing the Weighting Factors

The most delicate, difficult and important phase of working with the decision matrix is the *assignment of measures of relative importance.* This calls for deep probing into the needs, desires, and values of the customer. How to elicit this information is not within the scope of this paper, but we can provide a technique for showing the customer the effect variations in his measures of relative importance, or weighting factors, have upon the expected value of the available choices.

This is done by trading back and forth among the weighting factors, or by perturbing these factors, one at a time, and studying the effect of this change on the system of equations. The first technique is simple enough, but the second calls for altering the numerical value of *one* weighting factor without affecting the relative proportions among the others. Here is a formula for it:

In the system of linear equations

$$e_{i1}w_1 + e_{i2}w_2 + \ldots + e_{ij}w_j + \ldots + e_{in}w_n = v_i$$

with $\sum_{j=1}^{n} w_j = 1$, we perturb $w_j$ by changing it to a new measure $u_j$. We then introduce $u_j$ into the equation in place of $w_j$ and multiply each of the remaining measures by an expression P made up of the difference between unity and the perturbed measure $(1 - u_j)$, divided by the sum of the remaining measures $(1 - w_j)$, thus:

$$e_{i1}u_1 + e_{i2}w_2P + \ldots e_{ij}w_jP = V_i$$

where $u_1$ is the perturbed measure, $V_i$ is the new value,

and $P = \dfrac{1 - u_1}{1 - w_1}$

# RELATING THEORY TO PRACTICE

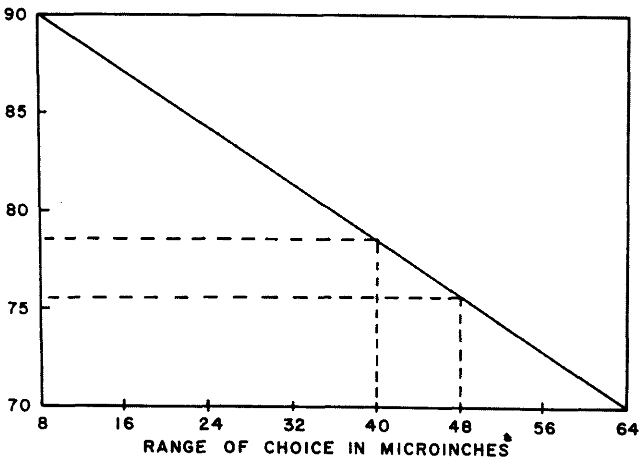
## Where To Begin

Whether we are studying a wafer-thin integrated circuit or a major missile launching system, we must start out by identifying and evaluating what the customer wants — the desired benefits. These are listed along the top of the matrix shown in Figure 6.

The letters and their subscripts are simple street numbers among the streets and avenues of the framework we use to align the information. We have to use them to identify

| | SMOOTHNESS OF SURFACE FINISH $B_1$ | ECONOMY OF PRODUCTION $B_2$ | DIMENSIONAL ACCURACY $B_3$ | LIGHTNESS OF WEIGHT $B_4$ | THE SUM OF THE WEIGHTING FACTORS = UNITY |
|---|---|---|---|---|---|
| WEIGHTING FACTORS | .2 | .2 | .5 | .1 | |
| $C_1$ STANDARD WAVE GUIDE COMPONENTS SPECIAL JOINTS | 78.60 / 15.72 | 70.00 / 14.00 | 74.00 / 37.00 | 72.70 / 7.27 | 73.99 |
| $C_2$ FORMED AND PUNCHED ALUM. SHEET DIP OR OVEN BRAZED | 75.65 / 15.13 | 90.00 / 18.00 | 70.00 / 35.00 | 90.00 / 9.00 | 77.13 |
| $C_3$ ELECTROFORMED ON FIXTURED MANDRELS | 90.00 / 18.00 | 78.80 / 15.76 | 90.00 / 45.00 | 88.80 / 8.88 | 87.64 |
| $C_4$ FORMED AND PUNCHED ALUM. SHEET, ELECTRON BEAM WELDED | 75.65 / 15.13 | 81.20 / 16.24 | 74.00 / 37.00 | 90.00 / 9.00 | 77.37 |
| $C_5$ INVESTMENT AND DIE-CAST COMPONENTS BRAZED TOGETHER | 75.65 / 15.13 | 71.90 / 14.38 | 74.00 / 37.00 | 70.00 / 7.00 | 73.51 |

Fig. 7 — Complex waveguide structure




| CHOICES | MICROINCHES | NORMALIZED |
|---|---|---|
| 1 | 40 | 78.60 |
| 2 | 48 | 75.65 |
| 3 | 8 | 90.00 |
| 4 | 48 | 75.65 |
| 5 | 48 | 75.65 |

Fig. 8 — Developing commensurable units: surface roughness into smoothness of surface finish

| BENEFIT | THE BEST PRACTICAL | less | THE WORST ACCEPTABLE | equals | THE RANGE OF CHOICE |
|---|---|---|---|---|---|
| $B_1$ SMOTHNESS OF SURFACE FINISH | 8 | – | 64 | = | 56 (–) |
| $B_2$ ECONOMY OF PRODUCTION | \$1,500 | – | \$5,500 | = | \$4,000 (–) |
| $B_3$ DIMENSIONAL ACCURACY | 0.0005" | – | 0.003" | = | 0.0025"(–) |
| $B_4$ LIGHTNESS OF WEIGHT | 150# | – | 190# | = | 40 # (–) |

Fig. 9 — Agreeing upon bench marks for establishing the standard scale

locations. And they do come in handy in understanding all sorts of arrays presented in this fashion.

In each of the boxes labeled $B_1$ to $B_n$ write each of the desired benefits. These are the ends you are trying to achieve. Put down only those which differ among the various choices you are considering. If all the choices had equal surface finish, you obviously would not need surface finish as a benefit.

## Weighting Technique

Under each of the boxes, at the points labeled $W_1$ to $W_n$ we put a number showing the importance of this particular benefit with respect to all of the others. We must emphasize again that the sum of the benefits must equal unity. This means we cannot add indiscriminately to one without adversely affecting the others.

If we are very familiar with the product *and the customer* we may be able to assign the numbers right off. Otherwise, we divide the number 1 by the number of benefits being considered ($1/B_n$), and then vary the resulting number as the relative importance of each benefit varies from that of the others — keeping the sum always equal to 1. Ideally, the customer himself, or someone thoroughly familiar with what the customer wants, should assign the weights.

## Efficiencies

The "e" in the other squares stands for efficiency. "Just how much can *this* choice do for the benefit above *this* column." If we are talking about $e_{12}$ (read "e one, two"), we write here a number showing how much the choice $C_1$ can do for the benefit $B_2$. Before we find this number let us remember that we will not have to bother with such notation as $C_1$, $B_2$, and $e_{12}$ in the actual computation. We are using it now just to show what goes where; for example, $e_{12}$ simply means *the efficiency measure to be written in the 1st row 2nd column.* If we want to talk about all the rows in the second column, that is, the efficiencies of all the choices $C_1$ to $C_n$ with respect to the benefit $B_2$, we say $e_{i2}$, using i for "the elements of all the rows that fall under column 2". If we want to talk about what one choice $C_1$ can do for the Benefits $B_1$ to $B_n$, we say $e_{1j}$, using j for "the elements of all the columns that line up along row 1". To speak of any typical single location, we say $e_{ij}$.

## Commensurable Units

As noted before we cannot put peaches and pears in our boxes and expect to add them up, unless we take one property at a time. The property we are interested in, here, is the efficiency with which each choice contributes to each of the benefits, or simply, how much of that benefit does it provide *measured along a standard scale.*

Now look at Figure 7. We have listed the requirements of this wave-guide structure as: smoothness of surface finish, economy of production, dimensional accuracy, and lightness of weight.

In conjunction with the customer, the program manager, and the systems and design engineers, we have arrived at the weights .2, .2, .5 and .1 respectively, all adding up to 1. Now we must introduce the efficiencies into the matrix.

Figure 8 shows how we convert peaches or pears into commensurable units.

## The Standard Scale

Fully as important as commensurable units is the standard scale. This scale extends between two bench marks agreed upon by the industrial team responsible for product value. These bench marks establish upper and lower limits on the significant factors affecting product worth. We must elaborate here on an aspect of the standard scale already touched upon in the section on *Setting Upper and Lower Limits.*

In simple language, the limits are between the worst condition acceptable, and the best condition practical (Figure 9). Raw data on product characteristics can only be related to product worth by identifying the points where these characteristics, (a) begin to improve the product and, (b) cease to improve the product. These are the points where the ends of the standard scale engage the scale of raw data. Why should 8 microinches have a full effectiveness of 90, and 64 microinches 70? Why not 16 and 125?

This is the other vital phase where the decision maker needs all the information, judgment, wisdom, and insight he can muster. The lower limit of the raw data we define as *the least we can live with,* and the upper limit as *the most we can effectively use* or *the most available,* whichever is less.

It is of the utmost importance that these *leasts* and *mosts* really *are* equivalent among the desired benefits. The best surface finish must be no better and no worse than the best price, the lightest weight, or the greatest dimensional accuracy. (See Figure 9.)

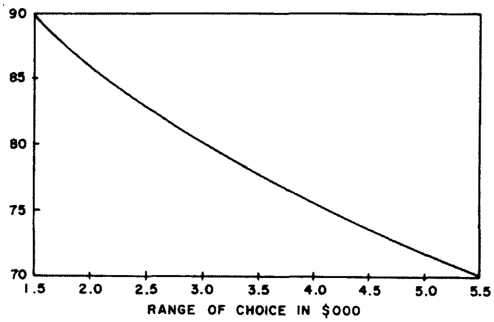
The straight line, in Figure 8, used to transform the 64 to 8 range of microinches into the 70 to 90 range of effectiveness, is called a normalizing function. All it does is convert variations along one scale to identical variations on a standard scale. In this case, we decided that the effectiveness of the interior finish varied inversely as the surface roughness, *but at the same rate.*

## Adjusting for Utility

As noted earlier, Daniel Bernoulli wrote in 1738 that the utility of money to a man declined in proportion to how much money he already had, and that this decline was not uniform but logarithmic. This is the law of marginal utility. For our purpose it simply means that the line on the graph which transforms raw data into commensurable units may not always be a straight line or a pat mathematical function.

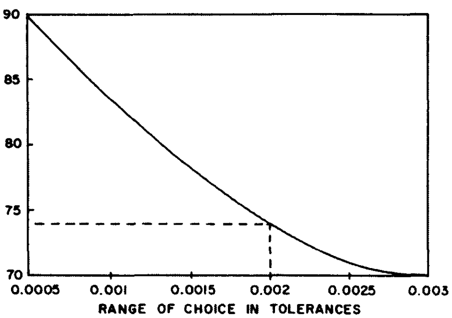
The fact that economy of production may decrease at a different rate than the increase in price is shown by the curvature of the line in Figure 10. This curve not only normalizes the dollar-cost of production into the 70 to 90 standard scale, but it also adjusts the dollar figures to take into account the "utility" of money to the customer.

It turns out, however, that the concept of utility goes beyond money. The efficiency of dimensional accuracy is not uniformly proportional to the actual change in tolerances (Figure 11), and the value-to-the-user of decreased



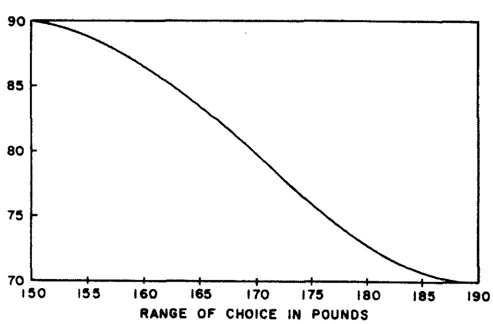

| CHOICES | DOLLARS | NORMALIZED |
|---|---|---|
| 1 | $5500 | 70.00 |
| 2 | 1500 | 90.00 |
| 3 | 3250 | 78.80 |
| 4 | 2750 | 81.20 |
| 5 | 5000 | 71.90 |

**Fig. 10 — Developing commensurable units: differences in cost into economy of production**




| CHOICES | TOLERANCES | NORMALIZED |
|---|---|---|
| 1 | 0.0020 | 74.00 |
| 2 | 0.0030 | 70.00 |
| 3 | 0.0005 | 90.00 |
| 4 | 0.0020 | 74.00 |
| 5 | 0.0020 | 74.00 |

**Fig. 11 — Developing commensurable units: tolerances normalized into dimensional accuracy**




| CHOICES | POUNDS | NORMALIZED |
|---|---|---|
| 1 | 180 | 72.7 |
| 2 | 150 | 90.0 |
| 3 | 155 | 88.8 |
| 4 | 150 | 90.0 |
| 5 | 190 | 70.0 |

**Fig. 12 — Developing commensurable units: difference in weight normalized into lightness of weight**

weight is not uniformly proportional to the change in weight (Figure 12).

What we have done, in Figures 9, 10, and 11 is to normalize, as well as adjust for utility, thus providing commensurable units which truly reflect both magnitude and utility. In Figure 8, we simply normalized because the improvement in surface finish and effectiveness was the same; there was no need to adjust for utility.

## The Final Choice

Looking at Figure 7, we see that electroforming provides the highest expected value *for this particular application.* And we see why. So do the representatives of the fabrication and electroforming shops. They also see that fabrication could have been the best choice, had cost been more important than dimensional accuracy.

## Make or Buy?

Let us say that we chose $C_3$ in Figure 7, that we have electroforming facilities in the prime division, that the plant can be put on three shifts if necessary, and that there are two good. electroforming vendors within a days travel from this plant.

The question is then, *make or buy?* This too calls for a matrix (Figure 13). The effect on factory load and on technical communications can be written in directly on the matrix. We know fairly well what they are, but the varying costs of making and buying (Figure 14), as well as the

| OBJECTIVES / WEIGHTINGS / CHOICES | FACTORY LOAD<br>0.1 | TECHNICAL COMMUNI-CATIONS<br>0.2 | PRICE ADVANTAGE<br>0.4 | PROMPT DELIVERY<br>0.3 | |
|---|---|---|---|---|---|
| $C_1$<br>MAKE ALL | 90.0<br>9.00 | 90.0<br>18.00 | 70.0<br>28.00 | 90.0<br>27.00 | 82.00 |
| $C_2$<br>MAKE 80 %<br>BUY 20 % | 86.0<br>8.60 | 70.0<br>14.00 | 72.0<br>28.80 | 87.0<br>26.10 | 77.50 |
| $C_3$<br>MAKE 60 %<br>BUY 40 % | 82.0<br>8.20 | 71.0<br>14.20 | 74.4<br>29.76 | 82.0<br>24.60 | 76.76 |
| $C_4$<br>MAKE 40 %<br>BUY 60 % | 78.0<br>7.80 | 73.0<br>14.60 | 78.0<br>31.20 | 74.9<br>22.47 | 76.07 |
| $C_5$<br>MAKE 20 %<br>BUY 80 % | 74.0<br>7.40 | 76.0<br>15.20 | 82.3<br>32.92 | 71.4<br>21.42 | 76.94 |
| $C_6$<br>BUY ALL | 70.0<br>7.00 | 80.0<br>16.00 | 90.0<br>36.00 | 70.0<br>21.00 | 80.00 |

**Fig. 13 — The make or buy decision: one particular case**

consequences of probable schedule slippages (Figure 15) must be both normalized *and* adjusted for utility.

Looking at the column of expected values in Figure 13, we see that the best choice is $C_1$, *make all*, followed by $C_6$, *buy all.*

Use of this method for relating means to ends provides valuable insights into the decision process itself. The reader will grasp this upon analyzing the reasons for the high expected value of $C_1$ and $C_6$ in comparison with all the compromise choices.

Habitual analysis of the worth of the ends sought. of the effectiveness of the means available. and of the relationship between worth and cost makes for better intuitive, as well as better formal decision-making. It reveals the advantage and the risk of the forthright decision. the price of compromise. and the cost of vacilation. Most important, it obliges us to exercise the faculty of judgment.

This example was worked out by an actual Value Task Group at the concept stage of design. The group included a systems engineer. electrical and mechanical design engineers. a buying manager. manufacturing engineers. a tool and die maker. a test methods engineer and the factory quality control manager.

The decision matrix, of course. does not make the decision. It simply lines up the information for the exercise of judgment by the man responsible. It has great value, however. in clarifying the customer's requirements, the goals of the design house. and the means available for meeting these goals.

Armed with the detailed design requirements. *and the reasons for them*, the Manufacturing Services of RCA's Corporate Staff searched for and found a method yielding even higher expected value than any of those discussed in this paper.

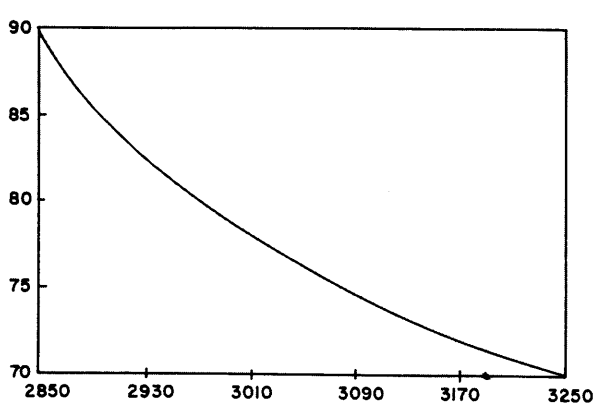
## THE MATHEMATICS OF RESOURCE USE

The first part of this paper considered some of the properties of linear algebra and simple matrix theory in order to provide the reader with a scientific yet simple method for comparing alternatives.

Because of its abundant literature and wide use in industry. the great usefulness of linear programming was simply pointed out without describing the technique. Two other techniques. however, deserve wider industrial use. These are queuing theory and the Monte Carlo method.

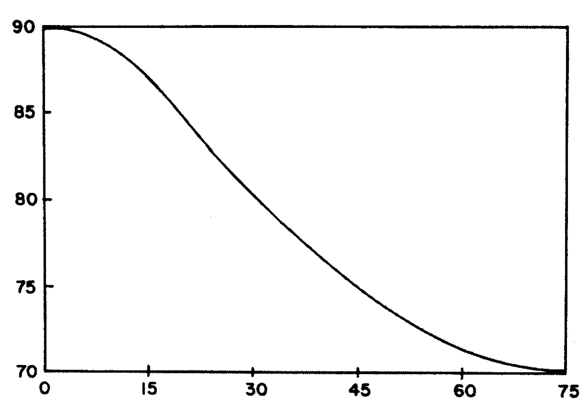
To give an idea of the various applications, we will start with the selection of a missile launching system for a small, coast patrol vessel. The ship is destined for close in-shore work either supporting or preventing landing operations.

In the "limited war" tasks for which it is being designed. the ship will have to have strong air-defense capability. While operating close to shore or in inland waters. it will be subject to attack by fighter bombers coming in at tree-top level over the jungle.




| CHOICES | DOLLARS | NORMALIZED |
|---|---|---|
| 1 | 3250 | 70.0 |
| 2 | 3170 | 72.0 |
| 3 | 3090 | 74.4 |
| 4 | 3010 | 78.0 |
| 5 | 2930 | 82.3 |
| 6 | 2850 | 90.0 |

**Fig. 14 — Developing commensurable units: differences in cost normalized into price advantage and adjusted for utility**




| CHOICES | DAYS | NORMALIZED |
|---|---|---|
| 1 | 00 | 90.0 |
| 2 | 15 | 87.0 |
| 3 | 30 | 82.0 |
| 4 | 45 | 74.9 |
| 5 | 60 | 71.4 |
| 6 | 75 | 70.0 |

**Fig. 15 — Developing commensurable units: differences in anticipated delay normalized into prompt delivery and adjusted for utility**

We will now study possible launching systems for this hypothetical vessel ordered by the mythical Republic of Freelandia. *All* our information comes from the *unclassified* naval experience of Freelandia. whose old coast defense cruisers and frigates were converted into missile vessels by the Peoples Republic of North Freedonia following a leftist coup in Freelandia. The coup failed. but Freelandia got stuck with the old launching systems. They want a better system on a smaller. faster vessel — something smaller than a destroyer-escort but having strong missile capability. See Figure 16.

### System Objective or Function: Destroy Targets

What targets? Enemy aircraft. What enemy aircraft? Experience shows that a single Freelandia vessel may be subject to sustained attacks by as many as twenty planes, the attacks lasting between one and three minutes. Mean rate of arrival of the planes is one every five seconds.

How well must we do the job? We would like to shoot them all down. This has not worked out in practice, but at least we should be able to launch one missile for every attacking plane.

How completely must we do the job to finish the task successfully? We must launch at least one missile at every attacking plane. It is up to us to decide what it takes to do this in terms of the launching system.

Now if the enemy planes came in exactly at five second intervals, we would have no problem, but Freelandia's enemies are not that predictable. The planes may arrive bunched up, and that could be bad. Before we can decide how many missiles we have to launch in any given interval, we must know something of how many planes we will have to contend with during that interval. We have to "queue-up the targets."

### Some Elements of Queuing Theory

A probabilistic study of telephone exchange problems in Copenhagen led A. K. Erlang[6] to a study of the consequences of delays and of the allocation of effort to minimize delays. These special studies developed into the theory of queues or waiting lines. A better name, perhaps, is Stochastic Service Systems.[7] If you stop for a moment to think of the relationship between causality and the flow of time, you grasp at once the vast importance of correct sequencing. Queuing theory can be of great value in optimizing certain types of scheduling decisions. It is used at oil loading and receiving terminals, in inventory control, in predicting demands for spares, in anticipating the calls for power by randomly activated experiments in spacecraft, in anticipating meteoric bombardment, and in many other activities.

Unfortunately, the examples usually cited are ridiculously elementary — customers at a hamburger stand, customers at a newsstand, cars in a car wash, etc. Unduly ponderous and complex mathematical tools are then brought to bear on these "problems" and the relationship between the formulae and their usefulness becomes more and more tenuous.

People who really need queuing theory, and who need it fast, use relatively simple formulae on *real* problems. And

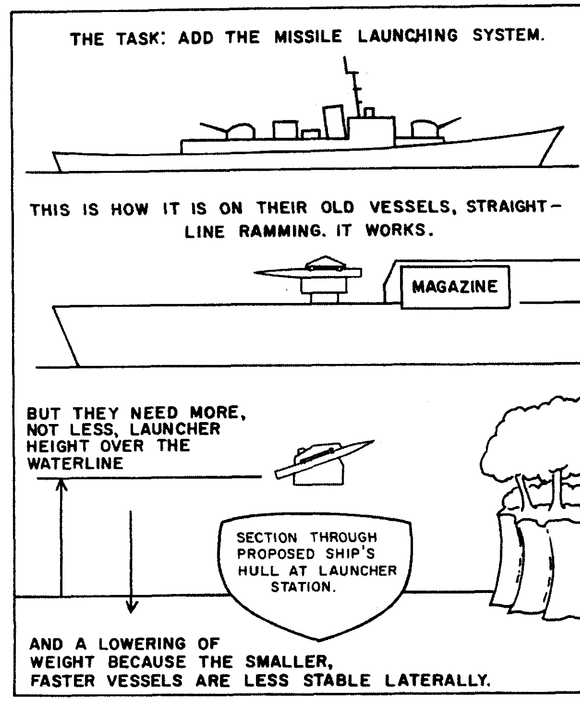



**Fig. 16 — Launching system for a coast patrol vessel**

we have a real problem here. Our customers are enemy planes. If we fail to "serve" them, they don't go away in disgust. they come right in, and *we* get served! The great risk here is congestion, or what the enemy would call *saturating the target.*

We may know how many planes the enemy has, and how many he is sending against us, but we don't know, and *they* may not know just how or when they will arrive. We may know the average rate of fall of meteors during the Cepheid Shower, but we do not know how they will be distributed on arrival unless we have studied the pattern of previous Cepheid Showers. Our basic datum is the *mean rate of arrival:* the total number of events expected or observed, divided by the total time during which they occur.

In ordinary probability we take the total number of events and divide it by the total number of possible occurrences. Not so in queuing theory. We cannot divide all the planes that attack by the total number of planes. There is an infinite number of planes that do not attack us. We cannot divide events by unevents; or can we? We can take a period of time, and when it is over, we can say, "No plane attacked us during the past five seconds. They usually take off at five second intervals. A couple of periods like this, and we had better get ready for a bunch of planes all at once."

This is intuitive queuing. What we want to do is to anticipate how many of these situations are likely to develop, in

ASSUMPTION: INTERACTION AMONG THE OBJECTIVES IS LIMITED BY THE UPPER END OF THE SCALE (90) WHICH EXCLUDES THE EXCESSIVE AND BY THE LOWER END WHICH EXCLUDES THE INADEQUATE

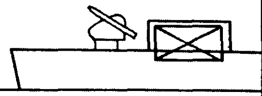
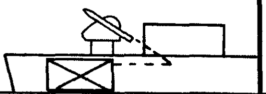
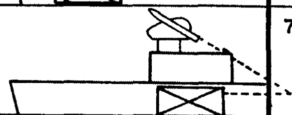
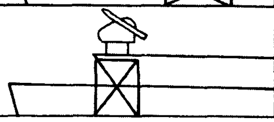
| OBJECTIVES / WEIGHTINGS / CHOICES | INCREASE TACTICAL EFFECTIVE-NESS | REDUCE POWER REQUIRE-MENTS | IMPROVE SHIP STABILITY | IMPROVE ECONOMIC EFFECTIVE-NESS | VALUES |
|---|---|---|---|---|---|
| | .5 | .1 | .2 | .2 | |
| $C_1$  | 77 / 38.5 | 81 / 8.1 | 70 / 14.0 | 76 / 15.2 | 75.8 |
| $C_2$  | 73 / 36.5 | 70 / 7.0 | 90 / 18.0 | 70 / 14.0 | 75.5 |
| $C_3$  | 77 / 38.5 | 70 / 7.0 | 82 / 16.4 | 73 / 14.6 | 76.5 |
| $C_4$  | 88 / 44.0 | 90 / 9.0 | 73 / 14.6 | 82 / 16.4 | 84.0 |

Fig. 17 — The decision matrix

order to acertain how we can meet them. All we have to work on is a number of events randomly spaced in time. Since we cannot relate events to unevents, we relate events to *intervals of time.* To do this, we need a probability law that not only accepts this relationship but also fits the facts reasonably well.

Such a law is the beautiful, practical, and surprisingly useful Poisson law. All it asks of us is the *mean rate of arrival* of the expected events, and a scale of time intervals. The *mean rate of arrival* is usually called *lambda, mu,* U, or m (which I will use here). The number of events that happen to fall within a particular time interval is usually called k, or x. I will use x.

To arrive at such intervals, you simply distribute the total time being considered into such regular periods as years, days, hours, or four-second periods, as we will do in the following example. The periods do not appear in the table. What the Poisson law reveals is the *probability that a certain number of events* will occur in any one time period, that is, it tells you if the arrival is likely to follow its mean rate, if you are likely to have no arrivals in that period, or if you are likely to face a congestion during the period.

We can say "The enemy can throw 20 planes at us, each arriving every 5 seconds. His attack should last $(20)(5) = 100$ seconds. During these 100 seconds the present system can only deliver 8 missiles against the 20 attacking planes. Maximum rate of fire is one missile every twelve seconds from two launching arms with a 24 second time cycle to load and fire. This inadequate response requires that the vessels travel in company for simple self-protection. No coastline can be effectively watched with the watchmen huddled together in little groups, hence the requirement for small, fast vessels capable of putting up an efficient single-ship defense.

Looking again at Figure 16, we see that in addition to improving the mechanical performance of the launching system, we have to modify its shipboard location. We must increase the height of the launcher while maintaining or lowering the system center of gravity. Now we are dealing with those opposing design requirements described under *The Element of Conflict* at the beginning of this paper.

One requirement calls for moving something up, and the other one calls either for moving everything down or at least for keeping the system center of gravity where it is. Now these requirements exist for entirely different reasons: (a) to improve tactical effectiveness for close in-shore operations; and (b) to maintain or improve the ship's stability on the high seas. Before we can make meaningful comparisons, we must understand the relative importance of such major design objectives as tactical effectiveness and ship stability. These objectives are listed across the top of the algorithm in Figure 17. The corresponding measures of relative importance, or weighting factors, appear immediately below them.

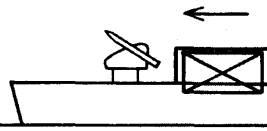
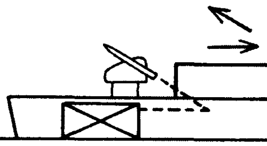
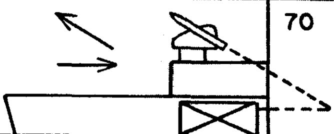
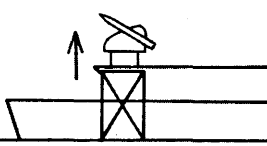
| OBJECTIVES / WEIGHTINGS / CHOICES | INCREASE CYCLING SPEED | INCREASE HEIGHT ABOVE WATERLINE | IMPROVE READINESS | INCREASE SECTOR COVERAGE | VALUES |
|---|---|---|---|---|---|
| | .5 | .3 | .1 | .1 | |
| $C_1$  | 76<br>38.0 | 70<br>21.0 | 90<br>9.0 | 90<br>9.0 | 77.0 |
| $C_2$  | 70<br>35.0 | 70<br>21.0 | 80<br>8.0 | 90<br>9.0 | 73.0 |
| $C_3$  | 70<br>35.0 | 90<br>27.0 | 80<br>8.0 | 70<br>7.0 | 77.0 |
| $C_4$  | 90<br>45.0 | 90<br>27.0 | 70<br>7.0 | 90<br>9.0 | 88.0 |

**Fig. 18 — Elements of tactical effectiveness (arrows show direction of ramming)**

These weighting factors tell us that increased tactical effectiveness is by far the most important requirement. The .5 means that this objective is as important as all the others put together, since they also add up to .5, the total sum being unity. Now let us see how we arrived at the efficiencies under the column headed *Increase Tactical Effectiveness*, that is, at the numbers 77, 73, 77, and 88 for the choices $C_1$, $C_2$, $C_3$, and $C_4$. This input of efficiencies, which appear in the upper left-hand corner of their respective cells, is the output column of the algorithm shown in Figure 18 — our old friend the combining sub-matrix. Note that at this stage they appear in the lower half of the cells of the extreme right-hand column.

## Putting the Sub-Matrix to Work

The sub-matrix in Figure 18 combines the elements of tactical effectiveness: cycling speed to load and fire, launcher height above waterline, readiness, and sector coverage *in proportion to what each contributes to tactical effectiveness.* The proportional measures of this contribution, 5, 3, 1, and 1, should be developed during sea trials and firing exercises aboard an experimental vessel. Far from being subjective measures of preference, these particular weighting factors are based on the physical interaction of the ship, the sea, and the targets.

Tabulated results of these tests assign the greatest importance to cycling speed to load and fire, and to launcher height above the waterline. They also generate a utility function by establishing the relationship between the raw data and the commensurable units we are using to compare efficiencies. For example, Figure 19 both normalizes (from the Latin *norma*, a carpenter's rule; to put into the same measure) and adjusts for utility. The tool for this adjustment is the utility function which models the effect of increased height on launcher efficiency. It reveals that if we were to raise the launcher another deck (for a total of 32 feet), we would run into trouble.

Figure 20, on the other hand, simply normalizes and does not bother with utility; one second at the beginning of a closed cycle is no better than one second at the end or in the middle, just as the spokes of a wheel all have the same utility.

## What the Decision Matrix Does for Us

Look at the algorithm in Figure 17. At the top we have our objectives and their respective measures of relative importance, here called *weightings.* This is not really part of the matrix, that is why I have been using the word algorithm. Here the algorithm is an arrangement which *includes* a matrix. The column of choices, on the left, is another

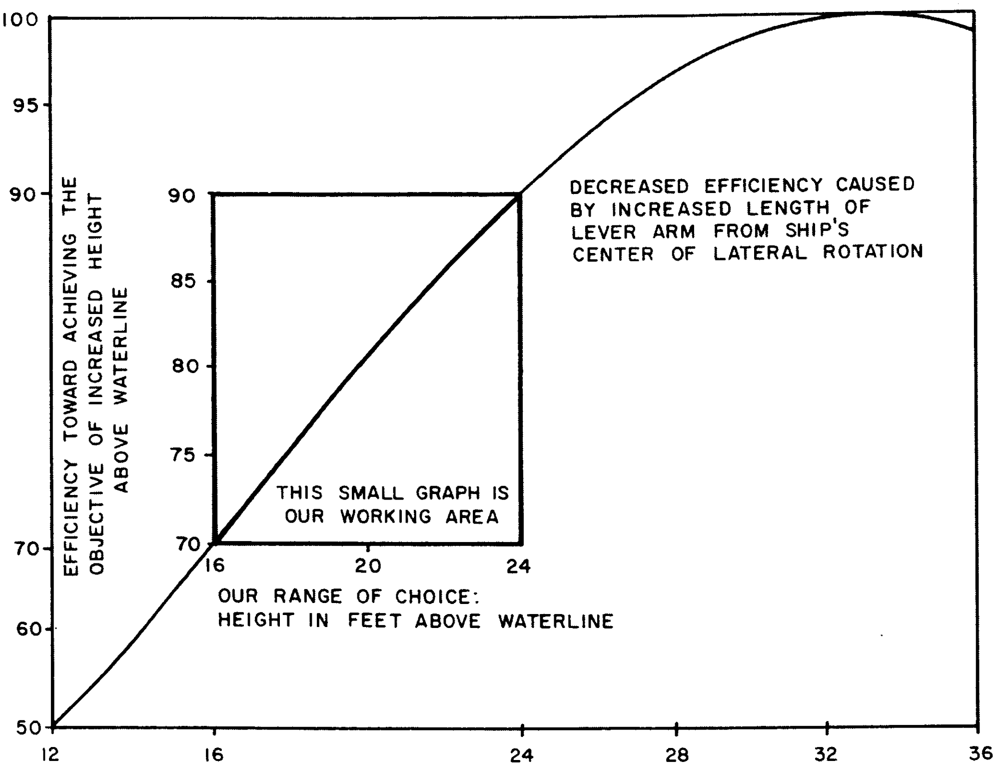



**Fig. 19 — Normalization and utility adjustment of height above waterline**

part of the algorithm but not of the matrix. The matrix, in this case, serves as a tool for relating the means, or choices, on its left: to the ends or objectives above it. The expected value of each choice appears in the output column at the right of the matrix.

The area enclosed by the heavier lines reveals what has been done to improve the original design ($C_1$). The first attempt had been a seat of the pants "improvement" suggested without the benefit of scientific comparison. If one of the objectives was to improve stability, one simply lowered the magazine to the deck below. The launcher remained where it was and there was no problem. Stability improved from 14 to 18 in the standard scale . . . BUT something else, *more important*, suffered. The added reverse-ramming operation plus the increased ramming distance reduced tactical effectiveness from 38.5 to 36.5 and reduced total expected value from 75.8 to 75.5.

## Precautions to Take with the Additivity Assumption

Let us note what the additivity assumption does to us when the increments in the benefits are not truly additive. When these increments are *completely* independent, they *are* truly additive: 100 Talents worth of gold, 100 of frankincense, and 100 of myrrh add up to 300 Talents, but the magic mirror to find the sick princess, the magic apple to cure her, and the magic carpet to deliver the apple, *do not add up* to a healthy princess. The *combination* of the three together does the trick, but the reward has *to be split three ways*, because without anyone of the three, the whole mission fails. This is not true of really additive benefits. The gold and the frankincense alone are still worth 200 Talents. The gold by itself is worth its original 100 Talents. But the magic apple, without a mirror to locate it, and without a magic carpet to deliver it, is not worth anything at all to the princess. If one of the three dependent items were reduced to zero, the value of the mission would disappear. $100 + 100 + 0 = 200$, but $100 \times 100 \times 0 = 0$.

Our 70 to 90 scale presupposes adequacy or minimum acceptability of all requirements, but when we "add" requirements that are partly dependent on each other, even though the low figures do not cause the result to disappear, we do get a distorted picture. Substantial differences in expected value are minimized. We still get a fair ranking, but the answers appear deceptively close together, such as the 75.8 and 75.5 in Figure 17. Unfortunately our objectives or requirements are seldom a true product or a true sum. When they are a true product we use a different algorithm, but by and large, the additivity assumption works well provided we are aware that small differences in expected value are quite meaningful.

## Getting on With the Design

The third choice ($C_3$) in Figure 17 also lowers the magazine, with the same disadvantages noted above, but it makes up for it by raising the launcher (see Figure 18), so that expected value is increased. Choice $C_4$ raises the launcher and lowers the magazine center of gravity slightly to compensate, but its big gains are in cycling time and general simplification. In the other designs, the ready missiles and the launcher arms have to meet at a fixed rendezvous and cannot meet at the midpoint between them. In the vertical system, while the launcher is slewing to fire, the magazine can be following it. When the launcher slews to load, the missile can meet its launcher arm at the nearest point. The magazine can index faster because a vertical magazine does not have to handle the weight of an unbalanced load. Now we believe we have a design approach worth working on. It improves expected value from 75.8 to 84.0, a very substantial improvement.

This choice could be improved further at the expense of economy. A lighter weight magazine could index even faster, for instance, but this would not improve total expected value. The main thing is that we have improved in the most important area, cycling time, and that this has been accomplished without seriously hurting the other requirements. The 90 under Increased Cycling Speed in Figure 18 indicates that we have achieved the most favorable condition we thought we needed in our range of choice. Cycling time has been reduced by a full 11 seconds, giving us an 8-second cycle and a system that can put a missile in the air every 4 seconds. We now divide the 100 seconds of the attack by 4 to give us 25 periods of 4 seconds each. Here are the numbers:

$$(20 \text{ planes})\ (\text{at 5 second intervals}) = 100 \text{ seconds}$$

$$\frac{(100 \text{ seconds of attack})}{(4 \text{ seconds to fire a missile})} = 25 \text{ periods}$$

$$\frac{(20 \text{ attacking planes})}{(25 \text{ periods})} = .8 \text{ mean rate of arrival per period} = \text{parameter } m.$$

In any given period we will have x number of arrivals, say in one period x will equal 0 arrivals, in another it should equal 1, and in certain periods it might equal 2 and even 3. What are the chances for 0, for 1, and for 3 arrivals?

If the planes follow the Poisson law under parameter m, the answer is given by the distribution function $e^{-m}(m^x/x!)$ where

x = number of events in any one period

m = the mean rate of arrival, in this case .8

The probability for x = 0 arrivals, x = 1 arrival, x = 2 arrivals, . . . , x = n arrivals can be found for limited values of m and x on Page 259 of Burlington and May[8] and on Page 444 of Parzen,[9] and for more extensive values, on Page 78 of the G.E. Poisson book.[10] Entering the tables with the number of estimated arrivals x and the mean rate of arrival .8, we get the results tabulated in Figure 21.

We note at once that there are 3.60 periods each suffering a two-plane attack.

## A Glimpse of the Monte Carlo Method

Gamblers who do not have the time to take down all the numbers that come up in the roulette over two or three months, and who want to develop a winning system, can buy a table of random numbers, work from that, and give up without losing their money. It turns out that anticipating random occurrences, distributing deliberate events in a random fashion, or selecting random samples, can give us valuable information on the present, and useful hints on the future. The Monte Carlo method serves to breakup deceiving or unwanted regularity, and it is useful in a surprising number of ways, such as the integration of irregular areas when the boundary can be broken up into known functions.

Perhaps the most frequent and useful application of Monte Carlo is in simulation of the type, "What would happen if . . . ?" which identifies *beforehand* the role likely to be played by chance in a hazardous situation.

For example, we are worried about those 3.6 periods in which our little ship will have to fight off two planes when it can only send up one missile.

We would like to know how closely the deadly 3 or 4 two-plane attacks are likely to bunch up; how closely the almost fatal three-plane attack will come to one of them; and how the more manageable attacks, and the slack time, will be distributed throughout the 25-second raid.

Somebody says, "Let's run a Monte Carlo on it."

### THE NATURE OF RANDOM NUMBERS

Utilizing the element of chance for our own ends is the essence of the Monte Carlo Method. To find out what can happen when chance takes over, we seek out the unpredictable and harness it. An early part of this harness was the martingale or checkrein, used to describe a betting system which can only be defeated by limits on the size of bets or the time of play. Other early elements were the ritual objects of the Goddess of Fortune, dice, cards, and the roulette wheel, hence the name Monte Carlo method.

Dice are probably the oldest random number generators. The RAND Corporation's *A Million Random Digits*[14] is one of the most modern sources. A pair of present day dice will give you random numbers according to its own peculiar distribution. The number seven, which is the easiest to make, will come up six times as often as either the numbers two or twelve. All the others fall in between. If we want to let chance distribute something in that particular pattern, we can use the dice (Figure 22). On the other hand, if we want a uniform pattern, we have to use only one of the two dice (Figure 23).

Because they provide insights into the very nature of uncertainty, gambling devices have always fascinated mathematicians. Long before Monte Carlo techniques were recognized as a mathematical method in this country, venturesome mathematicians and mathematically-inclined gam-

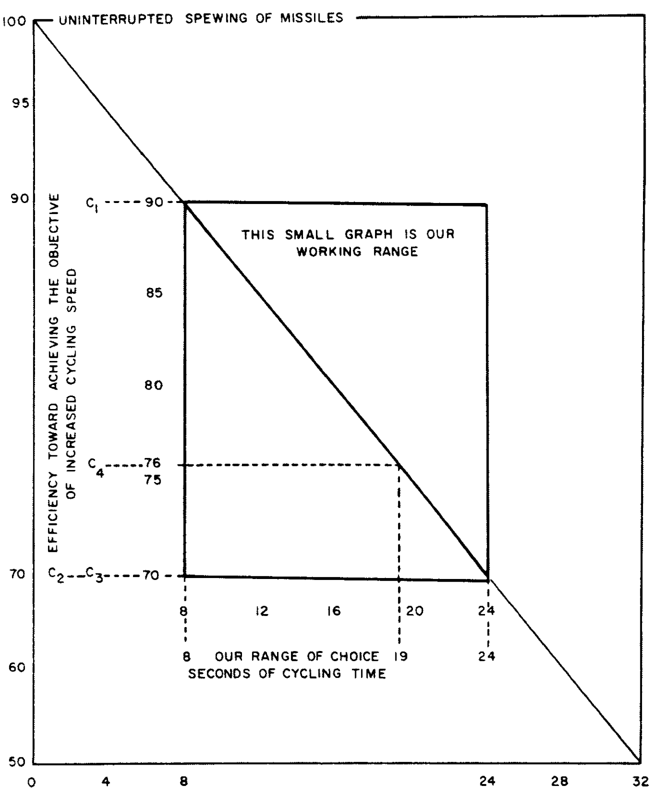



**Fig. 20 — Simple normalization of cycling speed**

T = Total number of four-second periods = 25

N = Total number of estimated arrivals = 20

N/T = 20/25 = .8 = estimated parameter m.

$e^{-m}(m^x/x!)$ is the probability that x arrivals will take place during any one four-second period.

| Estimated no. of arrivals | Prob. that x is a true estimate | Multi. by total no. of periods | Expected no. of periods with x arrivals each |
|---|---|---|---|
| x | $e^{-.8}\,\frac{.8^x}{x!}$ | | |
| 0 | .4493 | 25 | 11.25 |
| 1 | .3594 | 25 | 9.00 |
| 2 | .1437 | 25 | 3.60 |
| 3 | .0383 | 25 | .96 |
| 4 | .0076 | 25 | .19 |
| 5 | .0012 | 25 | .03 |
| 6 | .0001 | 25 | .00 |
| | | | 25.02 |

**Fig. 21 — Queuing the targets**

blers had developed tables of random numbers ranging from 0 to 36 in order to simulate the behavior of the roulette wheel. These tables of uniformly distributed random numbers were then used to simulate what would happen during a dice game despite the fact that a pair of dice generates numbers having an entirely different distribution (Figure 22). The gambler's transformation from roulette to dice is a revealing example of how one table of random numbers can be made to yield a different distribution.

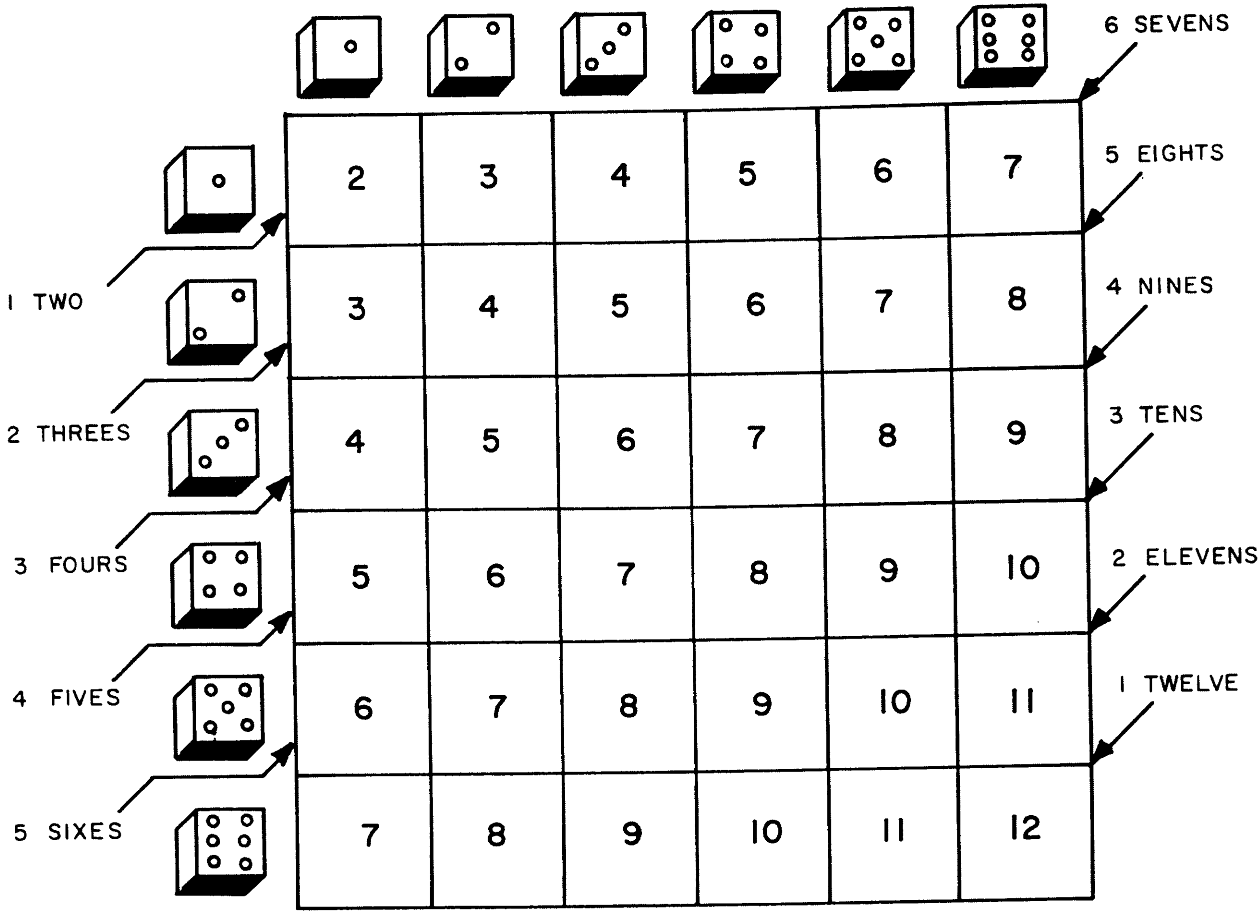
For example, to use a table of uniformly distributed random numbers to simulate events subject to the Poisson distribution, we follow a similar procedure. First, let us see how a table of uniformly distributed roulette numbers can simulate a dice game. In the roulette wheel the zero exists for the benefit of the house. It pays off bets placed on the zero itself, but it does not pay off on the dozens, the columns, high and low, or red and black. Most wheels in North and South America have both a zero and a double zero, giving the house a greater advantage.

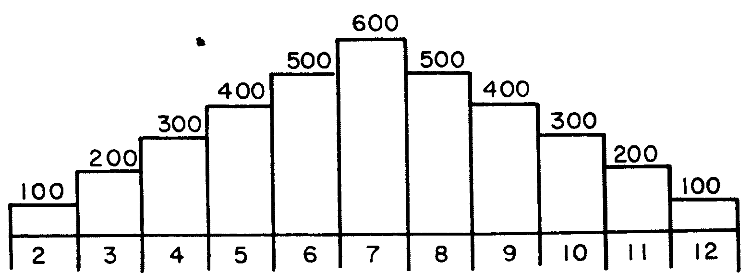
Since we are gambling only experimentally and do not have to pay overhead to the house, we will do away with the zero and use a table of random numbers ranging from 1 to 36, and having a uniform distribution resembling the one shown in Figure 23. We now partition this series of numbers to simulate the distribution generated by the dice. To do this we have to stack the bars on the histogram in Figure 22, one on top of the other, in order to form a continuous series of cells corresponding to all the different ways in which the various plays of the dice can be made (Figure 24).

The numbers which we will later take at random are here distributed in exact proportion to the way in which the various possibilities are distributed among the expected outcomes. For example, the outcomes 2 and 12, for each of which there is only one possibility, have only one number assigned to each of them. On the other hand, the outcome 7, which can be made in six different ways, has six numbers assigned to it. This illustrates how the numbers of the uniform distribution will fall into the cells constituting the elements of another discrete distribution, to yield a different set of numbers.

This is actually what gamblers do when they simulate a dice game with their tables of random roulette numbers. They simply leave out the zero and the double zero. But whenever we take predetermined digits out of a table of random numbers, we are tampering with its "randomness". We may well get away with it, but we should test the resulting new table to make certain it is truly random. The *Symposium on Monte Carlo Methods*[12] (pages 323 to 337), enumerates the many tests available and cites the relevant literature.

To complete our example we need a table of random numbers from 1 through 36. We can abstract such a table from the 14000 Random Digits starting on page 238 of Reference 13, using the first two digits of the first six columns when they fall above 0 and below 37. Then we check the new table for randomness by applying the Chi Square Test, first to our whole new table and then to the




IN 3600 THROWS, THE COMBINATIONS WOULD APPROACH THIS DISTRIBUTION. YOU WOULD GET ABOUT 100 TWOS, 100 TWELVES, 600 SEVENS, ETC.

**Fig. 22 — The faces of a pair of dice and the distribution of their 36 combinations**

portion we will use in the simulation. The Chi Square test measures the goodness of fit of our random numbers against the hypothetical rectangular distribution to which they are supposed to conform.

A most lucid description of this test appears on pages 150 and 151 of Reference 15. Methods of generating normally and exponentially distributed random numbers from a uniform table are found in Reference 16, page 180.

## TECHNIQUE OF SIMULATION

We will assume that you have the dice and you place a $2 center bet (Figure 25). Random numbers r will simulate the outcomes x. In the first play, r = 10 and x = 5. (Reading up the third column of Figure 24 you will find that the random numbers 7, 8, 9, and 10 all simulate a 5 on the dice.) Five is your point. Your opponent makes a side bet of 3 to 2 that you "dont' make it" (Figure 25, third line down). These are the usual odds for either 5 or 9.

This is not the place to explain the rules of the game, but you cannot help but note the persistent recurrence of 5. These are the occasions when people whisper, "The dice are hot — they're hot on 5!"

Whether sleeping quietly inside a table of random numbers, or galloping with two real dice, Fortune can create the unexpected and the unpredictable. So we use simulation to find out what she is up to and to get an inventory of the tricks she can use against us. That way we can at least prepare for the "runs" that we did not believe could happen.

## SIMULATING AN AIR RAID

You will note on Figure 21, Column 2, that we already have the Poisson probability distribution for the number of planes expected to attack during any given four-second period. This was obtained from Reference 10, Page 78 under U = .8, Column P(X). The column to the right of that, C(X), gives us the cumulative Poisson values added in the same way that we added the values of the distribution on Figure 24. This cumulative distribution provides us with intervals representing the distribution we want (Figure 26).

Now we need to assign random digits that will fall into these intervals. By far the best table to use is found in Reference 14. In this example, however, we will continue to use the excellent short table from the CRC Standard Mathematical Tables[13], because these tables are readily available to every engineer.

Now, to simulate the range of probability from 0 to 1, with four-decimal accuracy, we need 10,000 numbers *in four digits*. We have this in the series 0000 through 9999 because the entry 0000 plus the entries 0001 through 9999 together add up to 10,000. The stand-in for 10,000 will be the entry 0000.

In Figure 26 the horizontal axis represents x which takes the values of the number of attacking planes in any one four-second period. The vertical axis represents P(x), or the chances that x is correct.

The chances that no (0) planes will attack during a given four-second period are .4493, so we assign all numbers greater than 0 and equal to or less than 4493 to represent 0 (no planes attacking). Using r to represent any random entry from the series 0000 through 9999, and x to represent the number of planes that can attack in any one period, we can describe the procedure for the benefit of the computer.

The chances of single plane arrivals are .3594 as we know from Figure 21. In the cumulative distribution of Figure 26 this probability is represented by the interval between 4493 and 8087. We assign, therefore, all numbers greater than

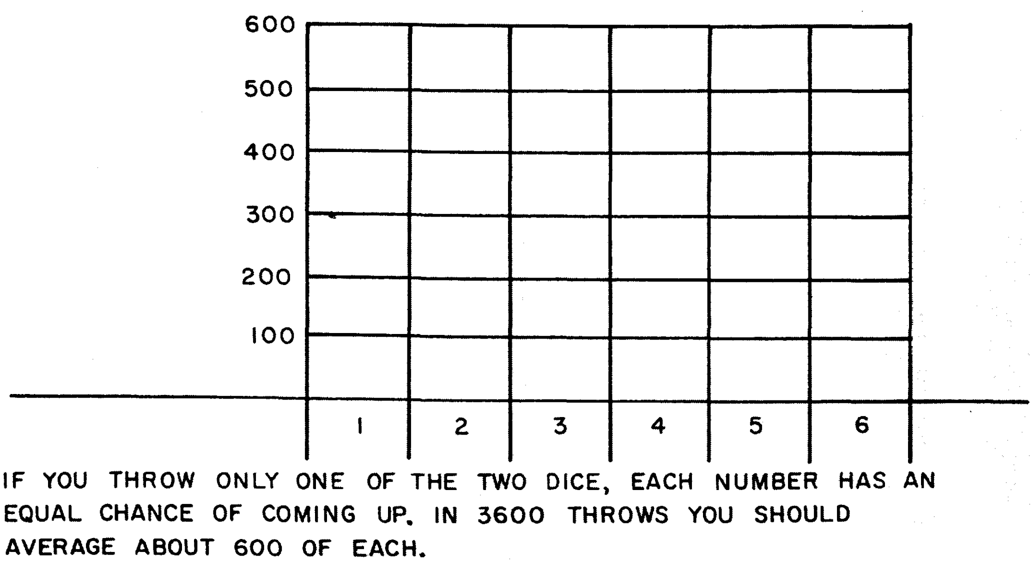



**Fig. 23 — The uniform of rectangular distribution of a single die**

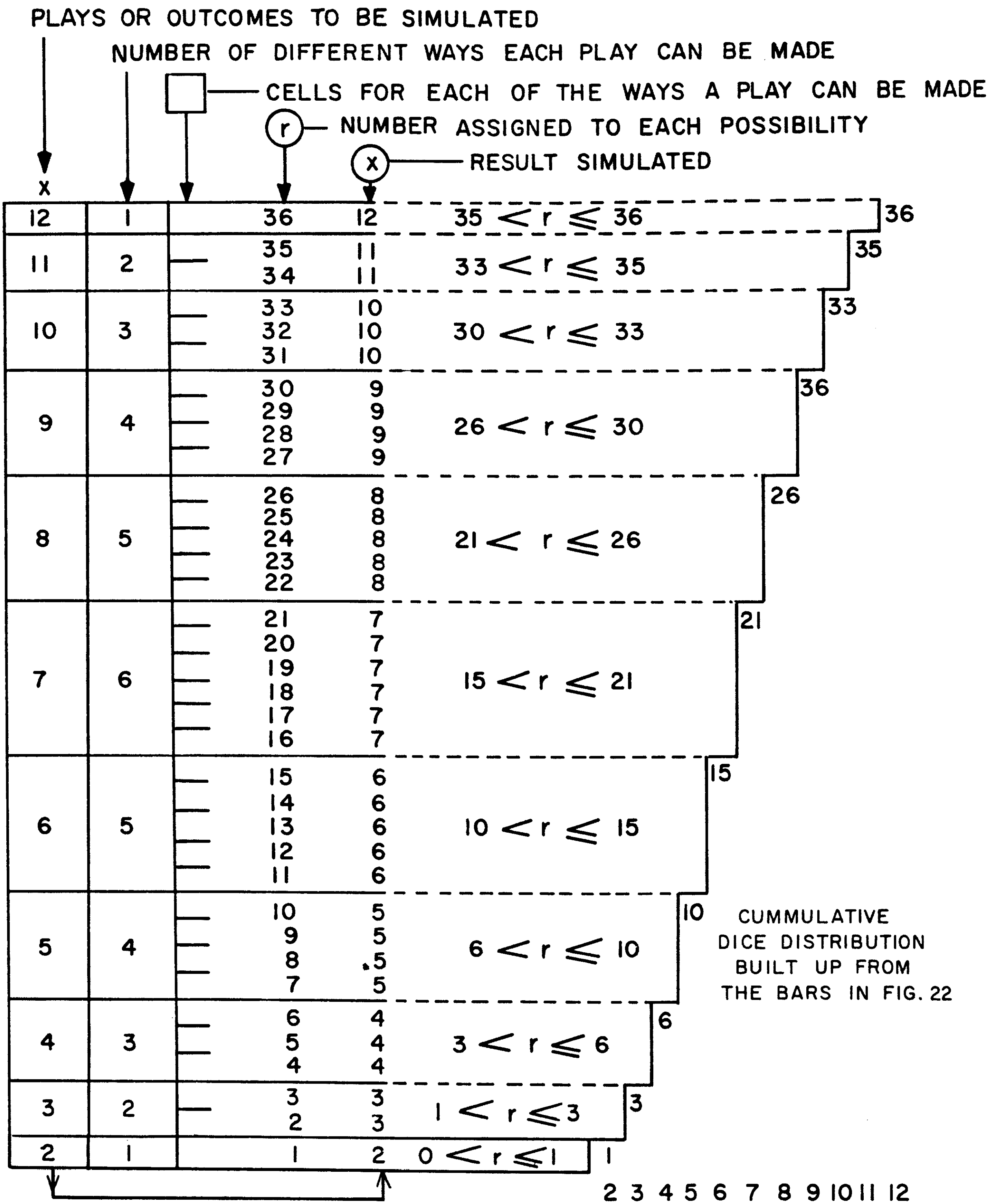



Fig. 24 — How roulette numbers were used to simulate dice games: using one distribution to simulate another

| r | E | | You Bet | He Bets | You Win | You Lose |
|---|---|---|---|---|---|---|
| | | You have the dice | $2 | $2 | | |
| 10 | 5 | Your point is 5, He says, | | | | |
| | | "3 to 2 you don't make it!" | $2 | $3 | | |
| 22 | 8 | No action | | | | |
| 24 | 8 | No action | | | | |
| 28 | 9 | No action | | | | |
| 09 | 5 | You made your point | | | $5 | |
| | | You still have the dice | $2 | $2 | | |
| 10 | 5 | 5 is your point again! And he | | | | |
| | | is 3 to 2 sure it won't repeat | $2 | $3 | | |
| 07 | 5 | You made your point again! | | | $5 | |
| | | You still have the dice | $2 | $2 | | |
| 02 | 3 | You lose | | | | $2 |
| | | He has the dice | | $2 | | |
| 01 | 2 | He loses on the first shot | | | $2 | |
| | | You have the dice | $2 | $2 | | |
| 07 | 5 | Your point is 5 and he says, | | | | |
| | | "3 to 2 you don't." | $2 | $3 | | |
| 32 | 10 | No action | | | | |
| 29 | 9 | No action | | | | |
| 02 | 3 | And he is right; you lose | | | | $4 |
| | | He has the dice | $2 | $2 | | |
| 29 | 9 | His point is 9, you say | | | | |
| | | "3 to 2* you don't make it!" | $3 | $2 | | |
| 05 | 4 | No action | | | | |
| 25 | 8 | No action | | | | |
| 09 | 5 | No action | | | | |
| 17 | 7 | He sevens out | | | $4 | |

Fig. 25 — Monte Carlo simulation of a game of dice

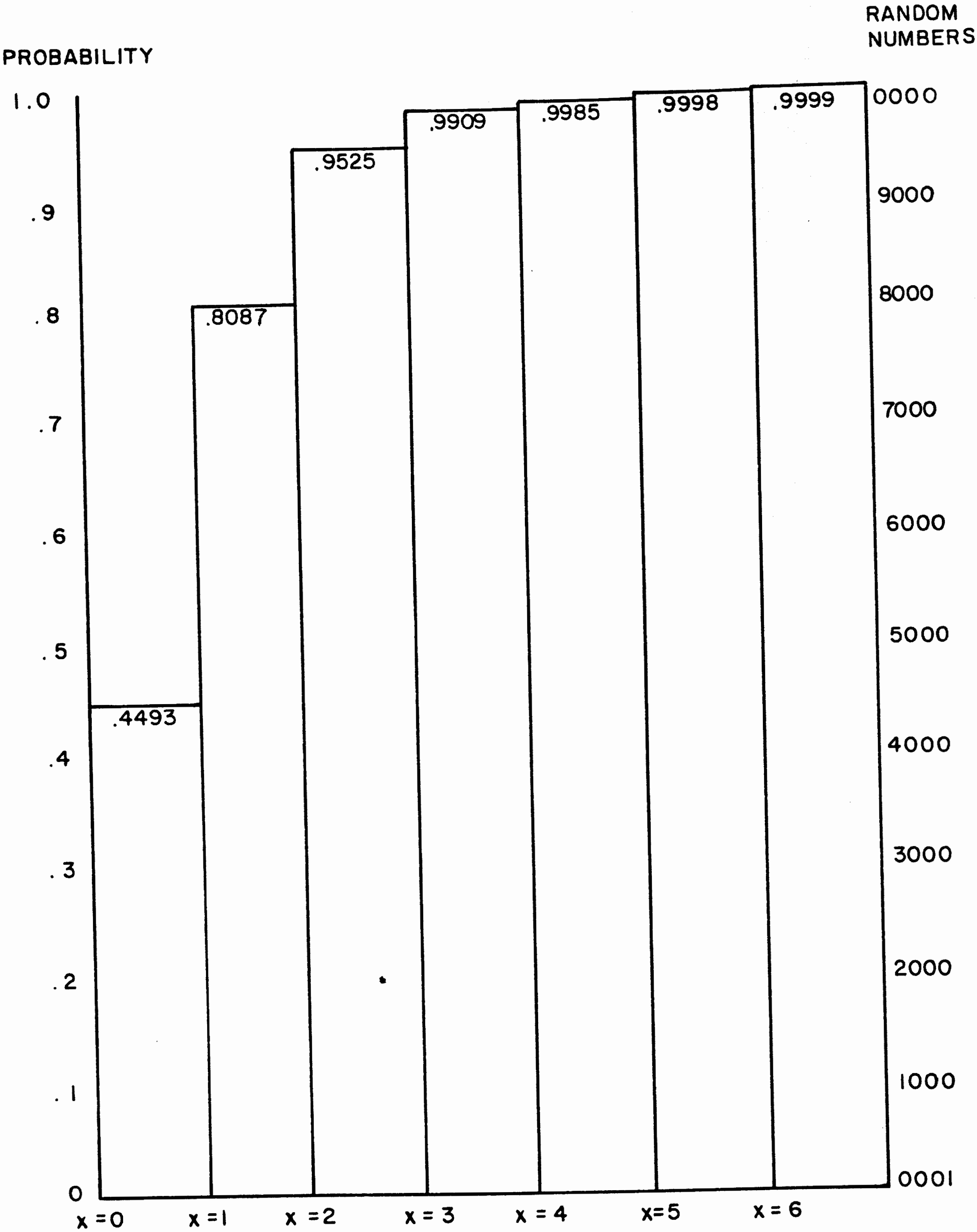



Fig. 26 — Cumulative Poisson distribution with a mean of .8 and x ranging from 1 to 6

4493 and equal to or less than 8087 to simulate 1 (a single plane), and so on up to the numbers greater than 9985, and equal to or less than 9998, which simulate the very remote probability that four planes will attack during a four-second period. The entry 0000 stands for 10,000 according to our agreement to keep the series down to four digits. The random number 10,000 simulates the probability .9999 that not more than six planes will converge on the little ship in any four-second period.

### How to Transform One Discrete Distribution Into Another

On page 78 of Reference 10, under U (the mean) = 0.8000000, we find values for the expected events x and for the cumulative Poisson distribution C(x). Into this mold we must press our series of uniform random numbers so that they fill the intervals as follows:

| x | C(x) | Random Numbers Simulating | Event x | |
|---|---|---|---|---|
| 0 | 4493 | $0 < r \leq 4493$ | print | 0 |
| 1 | 8087 | $4493 < r \leq 8087$ | print | 1 |
| 2 | 9525 | $8087 < r \leq 9525$ | print | 2 |
| 3 | 9909 | $9525 < r \leq 9909$ | print | 3 |
| 4 | 9985 | $9909 < r \leq 9985$ | print | 4 |
| 5 | 9998 | $9985 < r \leq 9998$ | print | 5 |
| 6* | 9999 | $9998 < r \leq$ 0000* | print | 6 |

*The entry 0000 stands for 10,000 in the random table (by agreement to keep the series down to four digits).

### The Computation is Quite Simple

Draw a random number r from the series 0000 to 9999 inclusive. Subtract from this random number the four digits corresponding to the shortest bar in the histogram of the Cumulative Poisson Distribution (Figure 26). If the random number is greater [denoted by "if(+)"], try the next higher bar, and so on until the result is negative. The event denoted by the value of x under this bar is the new value of the random number. All random numbers falling in the interval between the top of one bar and the top of the next higher bar acquire the value of the higher bar.

r — 0000, if (+), take the step shown below; if (0) print 6+
r — 4493, if (+), take the step shown below; if (0 or —) print 0
r — 8087, if (+), take the step shown below; if (0 or —) print 1
r — 9525, if (+), take the step shown below; if (0 or —) print 2
r — 9909, if (+), take the step shown below; if (0 or —) print 3
r — 9985, if (+), take the step shown below; if (0 or —) print 4
r — 9998, if (+), take the step shown below; if (0 or —) print 5
r — 9999, if (0) [r = 0000, r > 9998, so r = 9999] print 6

Note that this routine is applied to a single random number in order to determine the partition in the Poisson model to which it corresponds. The first step is designed to identify the 0000 entry which corresponds to 10,000 or .0001 probability that more than 6 planes will attack during a four-second interval. The "print" order is followed by an order to draw another random number. This one works up through the lower partitions until it finds its slot and prints out its Poisson value of x. Then another random number is drawn, and so on.

### Pattern of an Air Raid: Poisson Distribution with Mean = .8

0 = no attack during the given four-second period. 1 = one plane attacking during the period, 2 = two planes attacking, and so on. Each digit corresponds to one four-second period.

We begin our simulation by selecting a column at random from page 238 of Reference 13, say column 7, starting at the top to make it simple. The first four digits of the number are 6917, this is greater than 4493 but less than 8087, so we write 1, that is to say, the raid begins with a single plane attack during the first four seconds. This is how the enemy's twenty planes could come in: 1 0 0 0 1 0 1 2 1 0 1 0 0 1 1 1 1 0 2 1 2 1 2 1 1.

In a real simulation, of course, we would do many more iterations. This is the sort of thing where computer time vastly increase the reliability of the Monte Carlo Method. It is remarkable though that in this minimal sample, the random numbers simulated the total number of enemy planes with 95% accuracy. None of the two-plane attacks come together, and there is not one single three-plane attack. Such a brief simulation could lead to disaster. Let the reader continue the simulation for a number of 25-period raids. He will note that the third raid begins with a deadly cluster:

8303 simulates 2 planes
9766 simulates 3 planes!
8882 simulates 2 planes

and during this seven-plane attack the ship could only launch three missiles!

Now we understand the function. Destroy 7 targets in 12 seconds. This is the difference between seat-of-the-pants value analysis and value analysis with numbers. The missile launcher we are analyzing can only launch 3 missiles in 12 seconds. This is not as bad as it sounds because the function of destroying 7 targets in 12 seconds applies to the whole air defense system, including guns. This function must be factored to take into account the fact that some of the attacking planes will miss the ship altogether. Still, we must launch at least 5 missiles, in 12 seconds, to provide the function of the launching system itself. We can really pour in the money and reduce the cycling time — sophisticated bearings in the massive magazine; light metals and aircraft-type structures to reduce inertia; more power for indexing the magazine, for transferring missile to loading rail, for operating spanner rails and blast doors, for ramming and then withdrawing the ram-chain and pawl, and for slewing to fire and slewing to load.

Or we can stop to think.

If we maintain the present level of performance but minimize cost through simplification, innovation, and updating, we can have *two* launchers, one forward and one aft. These two simple, and rugged systems will deliver *six* missiles in 12 seconds, *for the same money*, as a single more sophisticated system tuned to the quick. The two simple systems will do the job more reliably, and they can be delivered sooner.

This is the aspect of value analysis and cost/effectiveness[17, 18] that so many people misunderstand. The objective is really greater value and greater overall effectiveness. This is achieved by improving the ratio of benefits to cost in order to give the customer more for his money.

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