

NETWORK GAMES AND OPTIMIZATION
WITH APPLICATIONS IN SUPPLY CHAIN OPERATIONS, CRISES,
AND POLICIES

by

Xiaowei Hu

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ABSTRACT

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Under the Supervision of Professors Wilkistar Otieno and Xiaohang Yue

In the current era of a growing population, systemic changes, and rising risk of crises, humanity is facing an unprecedented challenge of resource scarcity. In a broader sense, the world's shortage problem since 2019 has ranged from resources such as lumber, metals, and labor, to the end-products such as medical supplies, grocery items, and household cleaning products. In particular, the world's consumption of semiconductor chips, for instance, has surged due to societal changes such as the growing focus on climate-conscious carbon neutrality and remote working styles as a result of the COVID-19 pandemic. However, the supply of rare earth elements, which are critical for producing chips and a range of other essential applications, is under enormous stress. Overall, we are experiencing a shortage economy.

Resource scarcity and shortage can be induced by supply, demand, and structural factors. Stressors that contribute to resource scarcity include innate scarcity of the product, competition, demand surge, and supply-side policies. Additionally, certain modern inventory management practices, such as Just-in-Time inventory, can be associated with shortage risks in supply chains at the firm level. As the munificence of scarce resources is akin to the sustainability and growth of individual firms, societies, and the flourishing of humanity, meeting the the demands for scarce resources is a substantive economic, commercial, and societal matter for decision-makers.

Historically, supply-side fiscal policies are commonly used by governments as strategic instruments to address resource scarcity. The provision of fiscal policies to a resource sector in scarcity, directly or indirectly, can have an extreme relief and stimulus effect on the management of the targeted sector and the broader economy. Thus, it bears merit to seek an improved socio-economic response to the supply-demand dynamics by integrating policies and environmental risks along with conservation and competition from a supply chain perspective.

This dissertation encompasses multidisciplinary research on the economics of supply chains under the context of resource scarcity. It contributes to the equilibrium and dynamics in the modeling and analysis of scarce resource supply chain networks under the scenarios of competition, ownership, interplay, risks, policies, welfare, business practice, and supply chain design.

The dissertation begins by outlining the critical issues that are the cause and effect of resource scarcity in Chapter 1. Then a review of the state-of-the-art studies that serve as the conceptual and theoretical foundation of this work is conducted in the same chapter. The technical part of this dissertation, Chapter 2, includes a review of economics concept and operations research methods, upon which, Chapters 3, 4, and 5 are established.

Chapter 3 presents a two-period inventory model in a simple competitive supply chain setting. I strive to uncover the interplay between the firms, with a focus on inventory management at an operational level. In Chapter 4, I develop a general decentralized cross-sector supply chain network model that captures the unique features of scarce resources under a unifying fiscal policy scheme. In Chapter 5, I extend the model by incorporating a closed-loop structure and some uncertainty elements. As such, I develop a robust closed-loop supply chain network equilibrium model that investigates a critical material and the semiconductor chip's supply, production, recycling activities, and the uncertainties that arise from the policy shifts.

Finally, in Chapter 6, I point out the linkages between Chapters 3, 4, and 5, summarize the major contributions of the dissertation, identify the current research trends, and envision the post hoc research opportunities. The conceptual framework and methodologies discussed in this dissertation can also be applied to a host of extended supply chain design problems and many other areas.

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To
Kate, Nikos, Julie, Dan,
and
my Riverwest family.

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Chapter 1

INTRODUCTION

1.1 Background and Motivation

Humanity's survival depends on a diverse range of essential resources, including natural resources such as agricultural products, fisheries, wildlife, forests, petroleum, metals, minerals, air, soil, and water, which are not only critical for human flourishing but also indispensable for modern business operations ([Rosenberg, 1973](#); [Wagner and Light, 2002](#); [Krautkraemer, 2005](#)). The scarcity of resources has emerged as a prominent theme in futuristic opinions and critiques, fueling concerns about potential societal collapse and humanity's trajectory ([Friedman, 2009a,b](#); [Gilding, 2012](#)). The recent semiconductor chip shortage has further highlighted the need for effective supply chain strategies ([Economist, 2021a](#)). As a result, addressing resource demands has become a significant economic, commercial, and societal challenge for decision-makers. In this chapter, I delve into the complexities of managing scarcity in the modern economy, analyzing the multifaceted nature of resource shortages, business operation challenges, and supply-side legislature. Drawing on this interdisciplinary approach, I explore the latest research, theories, empirical evidence, and public interests to provide a comprehensive and in-depth understanding of these interconnected issues and their implications for supply chains.

In the remainder of this section, I will review these existential issues that are considered to be the cause and effect of global resource scarcity, which motivated the conception of this dissertation.

1.1.1 Scarce Resources and the “Shortage Economy”

Resource scarcity has been a persistent challenge throughout modern history, characterized by increasing magnitude and frequency of shortages. In the 1970s, the Western world faced oil crises that had lasting impacts on the United States, extending into the farm crisis of the 1980s with consequences such as low crop prices, foreclosed farmlands, and reduced farm income (Corbett, 2013; FDIC, 1997). More recently, since the 2000s, many regions around the world, including North and South Africa, South and Central Asia, and parts of the U.S., have been grappling with significant water stress. For example, California declared its driest year in 2013, and again in 2021 (Worland, 2014; Jones, 2021). Moreover, since 2019, induced by the global COVID-19 pandemic, the world has faced shortages across a wide range of commodities, from raw materials like lumber and precious metals, to end-products such as medical supplies, semiconductor chips, groceries, and household cleaning products (Gasparro et al., 2020; Chou, 2020; Valinsky, 2021). As aptly summarized in an article by Economist (2021b), we are now in a shortage economy.

The Population Reference Bureau highlights that resource scarcity can be induced by factors such as supply, demand, and structural dynamics (UN, 2012). With the world’s population projected to reach 8.5 billion by 2030 (UN, 2019), the demand for scarce resources continues to escalate at an alarming rate. The paradigm has shifted due to industrialization in the past two centuries, ending the era of “ever-cheaper” commodity resources and ushering in a new norm of fierce competition and rising prices for “ever-scarcer” resources (Hotelling, 1931; Krautkraemer, 1998). Additionally, crises such as the COVID-19 pandemic can exacerbate resource scarcity, as witnessed in the surge in demand for personal protective equipment (PPE) and food, stretching the capacity of global supply chains (Murray, 2020; Ranney et al., 2020; Xie et al., 2020; Cheung et al., 2020; WHO, 2020).

Adapting to resource scarcity requires new approaches by system designers. In situations of existential resource scarcity where demand is not limited to a single resource type, governments must identify optimal grand strategies. However, policies and regulations alone can unintentionally create sub-optimal signals for economic or environmental concerns. In response to the need for expanding and informing political dialogues, there has been a growing recognition of the cross-sectoral considerations, such as the well-known water-energy-food (WEF) nexus concept (Hoff,

2011) (see also [Bazilian et al. \(2011\)](#) and the references therein), emerging in the past decade. Similarly, the cross-sectoral construct considers critical issues in each commodity sector’s security through a holistic lens to achieve long-term sustainability (cf. [Biggs et al. \(2015\)](#)). For instance, discussions on integrated water resources management have emphasized issues through analyses of inter-sectoral competition for surface freshwater resources, integration of water management at farm, system, and basin scales, or balancing trade-offs with electricity generation and fuel supply ([Kurian, 2017](#); [Zhang and Vesselinov, 2016](#)). To date, the cross-sectoral approach has tended towards technical assessments to enable knowledge and information sharing, support inter-sectoral cooperation, and enhance productivity and synergies ([Biggs et al., 2015](#); [Olawuyi, 2020](#)).

1.1.2 Inventory Planning

In the early 1990s, the concept of operations management gained traction among business organizations, with a focus on designing and controlling production and business operations ([Stevenson and Hojati, 2007](#)). Effective supply chain management has been recognized as a key factor in gaining competitive advantage and improving performance ([Li et al., 2006](#)). In the face of increasing resource scarcity, meeting resource demands has become a driving force for product-design initiatives and a strategic challenge for businesses ([Duclos et al., 2010](#); [Wagner and Light, 2002](#)). Inventory planning is an important aspect of business resource management practice, and studies in supply chain management emphasize the need to cut down inventory costs through supply chain coordination, considering inventory as a “mismatch” between demand and supply ([Chopra and Meindl, 2016](#)).

The widespread implementation of Just-in-Time (JIT) systems worldwide, aimed at improving inventory management efficiency, has resulted in a reduction in national stocks, which contradicts the idea of pandemic planning and stockpiling. At the firm level, this has led to insufficient product stocks to meet the surge in demand for items such as personal protective equipment (PPE) during disease outbreaks ([Gereffi, 2020](#); [Park et al., 2020](#); [Zhu et al., 2020](#)). Therefore, a strategic reconsideration of inventory management may be warranted to maintain a demand-side balance in supply chains.

Inventory is typically carried in a supply chain setting for various reasons, including improving service levels, reducing logistics costs, managing uncertainty in customer demand and lead times,

ensuring availability of seasonal products throughout the year, hedging against price increases, and overcoming inefficiencies in logistics systems (Ghiani et al., 2004). Additionally, inventory may be carried as a strategic tool to influence decisions of market counterparts, which is referred to as strategic inventory.

1.1.3 Rare Earth Elements and Semiconductor Chip Shortage

The global semiconductor chip shortage has had a significant impact on various industries, particularly the automotive and electronics sectors. In May 2020, Toyota became the first automaker to cut production due to the shortage, followed by 16 other auto firms including Ford, General Motors, and Jaguar Land Rover in North America and Europe (Attinasi et al., 2021; Krolikowski and Naggert, 2021). It is projected that the auto industry will experience a loss of \$110 billion in sales for the fiscal year of 2021 due to the shortage (AlixPartners, 2021). Even companies with independent production capabilities, such as Apple Inc., have been affected, with revenue falls attributed to supply chain disruptions (Higgins, 2021). The shortage of semiconductor chips, which are essential components for many applications, has become a bottleneck in the global supply chains of technologies in the 21st century, with a trillion chips being produced annually and over 3,000 chips being used in electric cars alone (Economist, 2021a).

The chip shortage can be attributed to both demand and supply irregularities. On the demand side, one of the major causes is the increased demand for green and sustainable technologies, driven by climate change concerns (Ballinger et al., 2020; Maani et al., 2021). The popularization of electric vehicle (EV) has been a significant aspect of this shift, with 10 million electric cars on the roads by the end of 2020, a 41% increase from the previous year, and projections of over 18 million EVs on US roads by 2030 (Gül, 2021; Cooper and Schefter, 2018). The high demand for chips in the EV industry, along with increased IT spending by the remote workforce during the COVID-19 pandemic, has contributed to the demand pressure on semiconductor chips (Economist, 2021a). On the supply side, the scarcity of rare earth element (REE), which are critical materials for semiconductor chip production, has been a significant factor in the chip shortage.

REEs are a group of 17 chemical elements, including yttrium (Y), scandium (Sc), and 15 lanthanides. All these elements have similar physical and chemical properties, providing exceptional characteristics for a variety of modern applications, yet usually cannot be substituted one for

another (Golev et al., 2014). For example, Neodymium (Nd) and Dysprosium (Dy) are used in rare earth permanent magnet (REPM) that are now indispensable in emerging clean technologies such as wind turbines and EVs (Maani et al., 2021). Other applications consuming REEs include battery alloys, polishing powders, liquid crystal displays, and light-emitting diodes (Mancheri et al., 2019), as well as military applications such as guidance and control, targeting and weapon systems, and communication platforms (USDOE, 2011; European Commission, 2017). In the past two decades, those technologies have effervesced with the most rapid growth in REPMs, phosphors, catalysts, and batteries, accounting for over 60% of the REE demand (Mancheri et al., 2019).

China holds a dominant position in the global supply chain of REEs, being the largest producer, consumer, and exporter of these critical minerals (Golev et al., 2014). Although China's share of global mining has decreased from 95% in 2007 to 65% in 2018 (Maani et al., 2021), its expertise in concentrated REEs extraction remains unparalleled. REEs are scarce natural resources with limited supply sources and lack comparable and reliable substitutes (Skirrow et al., 2013). China's control over the mining and production of key intermediate products, such as magnets, which are critical inputs for high-growth industries, has raised concerns over the supply chain risks of REEs from other economic regions, given China's increasing global economic power (Mancheri et al., 2013; Whalen, 2021). The historically contentious foreign policies between China and the US have further escalated the significance of REEs as leverage in the conflict between the two world powers. A disruption in the supply of REEs could threaten the status quo of a sustainable economy (Schmid, 2019). As a result, REEs have been recognized as strategic resources for national security in countries including the USA, Japan, South Korea, the UK, and the EU (Ballinger et al., 2020).

To enhance the resilience of the REE supply chain and reduce supply uncertainties from China, recycling REEs from end-of-life (EoL) products has gained significant interest (Maani et al., 2021). Major Japanese firms such as Honda, Toyota, Hitachi, and Mitsubishi have initiated REE recycling initiatives, with some aiming to cover up to 10% of their REEs needs (Golev et al., 2014). The US Department of Energy has set goals for near and long-term EoL reuse and critical materials recycling at scale, with the aim of establishing a competitive supply chain spanning from consumer electronics to national defense (Granholm, 2021). Economic factors and environmental concerns about natural resource depletion are driving the motivation for recycling and reuse of commodities, including REEs. Despite being ecologically costly (Weber and Reisman, 2012; Tanaka et al., 2013),

REE recycling presents an opportunity for reprocessing the most desirable elements from products with high concentrations of valuable minerals, compared to the labor-intensive process of extracting from virgin ores (Binnemans et al., 2013a). Recycling can offset some of the primary supply of REEs, for example, through urban mining which recovers REEs from manufacturing waste, landfills, and EoL products, diversifying the supply of certain valuable minerals and complementing primary production (Golev et al., 2014). Advancements are being made in sustainable, effective, and selective recycling technologies, particularly in the area of REPMS (Dupont and Binnemans, 2015). For instance, Belgian chemical company Solvay has developed a process for extracting REEs from on-site monazite/xenotime processing tailings to reduce its reliance on Chinese feedstock (Binnemans et al., 2013b).

1.1.4 Supply-side Policies and Uncertainties

Supply-side fiscal policies are commonly employed by governments as strategic instruments to alleviate resource shortages, stimulate growth, and influence economic outcomes, particularly during critical times (Keynes, 1936). For instance, during the energy crisis of the 1970s, the U.S. Senate introduced supply-side fiscal incentives, including investment credits for domestic exploratory drilling and research and development on alternative energy sources, to mitigate the effects of the energy problems (Long, 1973). Similarly, in response to the global financial crisis of 2007-2008, the U.S. Congress passed the \$787 billion American Recovery and Reinvestment Act in 2009, following the \$125 billion provided by the Economic Stimulus Act of 2008 (Davig and Leeper, 2011). Moreover, during the COVID-19 pandemic of 2020, governments worldwide implemented fiscal, monetary, and financial policy measures as part of economic packages to mitigate the negative impact of the public health crisis on the economy and safeguard public welfare (Gourinchas, 2020). To date, the estimated global stimulus effort has reached \$10.4 trillion, with \$5.2 trillion from the U.S. (Economist, 2021b; Romer and Romer, 2021). Notably, monitoring the global fiscal policy in the aftermath of the COVID-19 pandemic, the IMF (2021) observes that the fiscal stimulus in many advanced economies, such as the European Union and the U.S., is beginning to make them more productive, equitable, and sustainable.

From the perspective of a scarce resource supply chain in an industrial setting, supply-side fiscal policies, such as production incentives, tax credits, or expansionary funds, can serve as means of re-

ducing costs and increasing revenues for firms in the short term, as well as promoting entrepreneurial processes in the long term (Arestis and Sawyer, 2003; Braunerhjelm, 2022). In particular, the provision of fiscal policies to a resource sector facing scarcity can have a significant preservative effect on the management of the targeted sector and the broader economy (Young, 2015). For instance, in response to groundwater depletion in Indian states like Punjab and Gujarat, farmers are provided with free access to electricity to pump groundwater, compensating for the inadequate investment in water-saving technologies. Such benefits can have a profound impact on the resilience of the supply chain as well. The recent resource scarcity challenges exposed by the COVID-19 pandemic have highlighted the vulnerability of supply chains and underscored the need for repair. Alongside the urgent call for fiscal stimulation (Blinder, 2021; Winegarden, 2021; Smith, 2021), it is worthwhile to investigate how supply-side fiscal policies can alleviate shortages and stimulate social outcomes.

China, as the world's second-largest economy and a major player in international trade, has implemented a series of economic policy reforms that have made it increasingly susceptible to shocks and long-term implications for its macro-economy and business cycles (Huang and Luk, 2020). These economic policies, which encompass trade agreements, foreign investments, and domestic environmental regulations, can be found in previous works by Wübbeke (2013), Mancheri et al. (2013, 2019), and the reference therein. These policies play a crucial role in the global supply chains of nearly all REE-related products. While supply-side policies are known to have relief and stimulus effects on economies, including addressing aggregate demand imbalances to some extent (see also Hepburn et al. (2020), Hu and Li (2022), *inter alia*), they may also pose short-term risks and adverse consequences for economies (IMF, 2021). A study on China's economic policy uncertainty by Huang and Luk (2020) showed that an increase in economic policy uncertainty can depress real economic activities. Mancheri et al. (2019) analyzed various policies that place price as a central feedback loop, with a focus on how price responds to resilience-influencing policies such as supply diversity, regulatory frameworks, and stockpiling. They argued that industrial and trade policies, specifically politics, have a larger influence on trade flows than physical disruptions. They found that policies such as international standards and Chinese imports of REEs weaken resilience in global supply chains due to the Chinese influence (Mancheri et al., 2019).

1.2 Literature Review

This section comprises a comprehensive analysis of relevant literature, focusing on both issue- and methodology-oriented reviews. These reviews aim to identify the latest research trends and scholarly emphases in the field. Furthermore, they provide a contextual placement for the current dissertation in relation to prior works, establishing its significance and contribution to the existing body of knowledge.

1.2.1 Scarce Resources: Integrative Studies

The broader studies on resource scarcity lie primarily in the fields of economics and management, each of which gives a self-containing viewpoint toward the characteristics of scarce resources, with both fields sharing a focus on fundamental concerns related to competencies, competition, productivity, wealth, and economic growth. On one hand, the equilibrium and monopoly theories of Adam Smith, John Stuart Mill, and John Nash, as well as the rent and location theories of Alfred Marshall and Alfred Weber, tie the scarcity to the location and quality of resources needed for industrial and agricultural purposes (Weber and Friedrich, 1929). On the other hand, among the rising organizational theories, the *Resource Dependence Theory* (Pfeffer and Salancik, 2003) claim the importance of resources by which the owners, producers, and suppliers are connected and interdependent (cf. Caniëls and Gelderman (2007)), whereas the *Resource Advantage Theory* (Hunt, 2000) posits resource's features, including their demand heterogeneity and roles, in a firm setting. Overall, from an economic perspective, scarce resources can be characterized as of oligopolistic supply-side power, heterogeneous intra-sector demand, and low price-elasticity of products. From a management perspective, the possession of scarce resources is indicative of a firm's competitive advantage and negotiation power. The coalescence of these two avenues of studies in resource scarcity has yet to occur in literature.

The scholarly interests in the interconnectedness of scarce resources started to agglomerate since the release of the World Economic Forum report introduced the WEF nexus as a novel concept (Hoff, 2011). Nearly all WEF-related literature to-date has sought integrated solutions or suggested the interconnection of resources. For instance, Bai et al. (2016) investigated the interplay between agricultural and fuel products; Zhang and Vesselinov (2016) studied the interaction between water

and energies; [Bakker et al. \(2018\)](#) exemplified a regional economy with water and agriculture in focus; [Mun et al. \(2021\)](#) provided an expansionary solution to the hydro networks for energy, irrigation and flood control for developing countries. It is worth remarking that almost all of these works, at the same time, amalgamate with supply chain management, in which resource scarcity was associated with a range of societal and economic issues. I will discuss such literature in the following subsections.

In the tumultuous times of critical resource scarcity, especially in the wake of the COVID-19 pandemic, a wealth of studies have emerged to demonstrate the utility of integrated quantitative models for the urgency of healthcare, agricultural, logistical, and humanitarian challenges. For example, [Kazaz et al. \(2016\)](#) developed a malaria medicine supply chain model to improve supply and reduce price volatility via interventions. [Arifoğlu and Tang \(2022\)](#) devised a decentralized system to combat the frequent supply and demand mismatches of vaccines. Both [Chen and Tang \(2015\)](#) and [Liao et al. \(2019\)](#) used stylized models to examine the impact of information provision policies on farmers in developing countries. [Yu et al. \(2020\)](#) constructed three-echelon supply chain models to study donors' optimal subsidy strategies for the development of under-developed areas. [Chintapalli and Tang \(2021\)](#) modeled, analyzed, and evaluated the impact of a credit-based scheme on the welfare of small-scale, risk-averse farmers. [Shen and Sun \(2021\)](#) explained the impact of the pandemic on supply chain resilience, identified the challenges that retail supply chains had experienced in China, and showcased the practical response of JD.com throughout the pandemic. Some other studies, at the same time, emerged to be the trailblazers to real-world solutions, as they all demonstrate the practicality of the integrated models in “fixing the broken supply chains”, an urgent need faced by humanity. For example, [Keilhacker and Minner \(2017\)](#) investigated the mitigation strategies caused by exogenous shocks to the rare earth elements supply chains. [Fontes et al. \(2018\)](#) identified the trends related to the supply chain of renewable energy by showing the importance of system-dynamics modeling. [Mehrotra et al. \(2020\)](#) presented a stochastic optimization model for allocating and sharing a critical resource in the case of a pandemic.

1.2.2 Supply Chain Management under Competitions

Research in resource supply chain management (SCM) has focused on green strategies and sustainability (e.g. [Seuring and Müller \(2008\)](#); [Pagell and Wu \(2009\)](#); [Piercy and Rich \(2015\)](#); [Abdul-](#)

Rashid et al. (2017), inter alia), as well as meeting demand crises (Jüttner and Maklan, 2011; Mehrotra et al., 2020; Rowan and Laffey, 2020). The term supply chain management is attributed to Oliver and Webber (1982), but the original focus tended to be on a firm acting independently with an emphasis on inventory management. Whereas most of the literature in SCM focus on either general products or a specific type, it remains largely barren on the inter-dependence of firms on specific scarce resources (cf. Bell et al. (2012)). A systematic literature review by Matopoulos et al. (2015) has underscored a need for further research on understanding the implications of resource scarcity for supply chain relationships and also its impact on supply chain configurations.

Under the broader scope of SCM, more related to this study are the supply chain network models with multi-agent decision-making, in which case, game theory is often used. Among the wealth of existing literature, the supply chain network equilibrium (SCNE) models, inaugurated by Nagurney et al. (2002), permit one to represent the interactions between decision-makers in the supply chain for general commodities in terms of connections, flows, and prices. Since then, this sub-domain has proliferated to a wide range of applications with timely interests. In conjunction with resource scarcity and shortages, I note, for instance, Wu et al. (2006) and Matsypura et al. (2007) modeled power transmissions. Masoumi et al. (2012), Dutta and Nagurney (2019) each modeled the supply and demand of a unique medical item; Besik and Nagurney (2017) and Wu et al. (2018) modeled the logistics of fresh produce in the scope of their qualities and preservation. In conjunction with supply-side fiscal policies, on the other hand, I am aware that Yu et al. (2018), Wu et al. (2019), and Yang et al. (2021, 2022), for example, captured the design of an environmental tax or subsidy policy in a green supply chain. Nagurney et al. (2019a,b) studied how global trade policies could impact the suppliers and markets. In a technical sense, almost all of the aforementioned models rely on the variational inequality framework (more in §2.2).

To date, albeit the research in SCNE models has been well developed, new studies continue to proliferate. In surveying the SCNE studies since 2020, I now identify a few trends of its application. First, the Chinese e-commerce industry has evolved rapidly with explosive consumer traffic in online shopping, payment, marketing, and services, as the works of Zhang et al. (2020), Guoyi et al. (2020), and Chen et al. (2020), all affiliated with Chinese institutions, demonstrated the opportunities and utilities of such model in this industry. Second, the studies in socially responsible practices, led by firms' environmentally conscientious strategies, have reflected the zeitgeist of our renewed societal

values. In particular, [Chen et al. \(2020\)](#), [Zhang et al. \(2021\)](#), and [Yang et al. \(2021, 2022\)](#) all showcased that corporate-level efforts like de-carbonization have embodied today’s global supply chain practitioners’ response. Last but not least, the COVID-19 pandemic has created a host of complex problems in the logistics of medical supplies and labor, among which, for example, are the works by [Nagurney \(2021a,b\)](#), inter alia.

1.2.3 Modeling and Design of Policy Instruments

The quantitative research on the modeling of policy instruments is also related to this work. Under such discipline, the design of policies often becomes a delicate and complex matter when presented with a decentralized system of multiple decision-making agents (cf. [Arrow and Kruz \(2013\)](#)). Generally, microeconomic approaches are often adopted to assess the interplay of incentives and behaviors (see [Frey and Jegen \(2001\)](#), [Huck et al. \(2012\)](#), inter alia). With such approaches, incentives may be linked to the network topology and its design (cf. [Belhaj and Deroïan \(2013\)](#), [Jackson and Zenou \(2015\)](#)). For instance, [Calvo-Armengol and Jackson \(2004\)](#) show that education subsidies and other labor market regulation policies display locally increasing returns due to the network structure.

In relation to SCM, a growing number of recent studies in supply chain policy design are dedicated to developing e-commerce, promoting new energies, and sustaining green production and consumption (see also [Waltho et al. \(2019\)](#)). For instance, [Zuo et al. \(2019\)](#) proposed a generic technology R&D subsidy object selection model to help the policy-makers take advantage of the limited public resource with greater rationality and sustainability. The model was devised to advocate the government-sponsored national adoption of new energy vehicles in China. [Fan et al. \(2020\)](#) proposed a pricing model with a new government-subsidy sharing scheme that is based on vehicle mileage to reduce energy consumption and thus greenhouse gas emissions while maintaining the service demand.

1.2.4 Closed-loop Supply Chain Models and Recycling

Akin to this dissertation is the concept of recycling and closed-loop supply chains. The significance of critical material recycling is articulated extensively in many review papers. I refer the readers to [Binnemans et al. \(2013a\)](#), [Yang et al. \(2017\)](#), and [Balaram \(2019\)](#). Since a seminal work by

[Savaskan et al. \(2004\)](#), closed-loop supply chain (CLSC) has emerged to be a prominent configuration to model recycling-like processes in the research areas of operations research and management science. From a system perspective, the closed-loop system contains inputs, processes, structure, and outputs, integrates forward and reverse logistics networks, allowing for modeling versatility. From an economics and management perspective, CLSC provides a framework for another growing concept, circular economy, which studies the efficiency of resource usage (see [Geissdoerfer et al. \(2017\)](#)). In the last decade, CLSC has attracted a deep interest due to its applications in modeling environmental concerns, needs for legislation, and awareness of global resource scarcity. Various strategic and operational aspects of CLSCs, such as forecasting, production/inventory planning, and control, have been investigated. For a comprehensive review of recently published CLSC literature, I refer the readers to [Govindan et al. \(2015\)](#) and [Guan et al. \(2020\)](#).

In the broader supply chain management literature, game theory often serves as the backbone of multi-agent decision-making. Sensibly, because the commodities that require recycling are often inherently scarce, the planning and operation decisions made by the entities within the CLSCs are usually strategical. Such a feature of CLSC provides rich soil for game theories to thrive as a modeling practice. As is pointed out by [Guan et al. \(2020\)](#) from a survey of 1155 CLSC articles published during 2001-2020, game theory is used by most scholars to study such topics. Even a dedicated literature survey on the closed-loop game-theoretic models ([De Giovanni and Zaccour, 2019](#)) has been conducted. Among those articles referenced therein, the supply chain network equilibrium model is a particularly proliferating sub-domain. Inaugurated by [Nagurney et al. \(2002\)](#), SCNE models rest upon a variational inequality framework known for its ability to link optimization problems with equilibria. [Nagurney and Toyasaki \(2005\)](#), [Hammond and Beullens \(2007\)](#); [Yang et al. \(2009\)](#) extended the SCNE model to the oligopolistic recycling, which includes manufacturers who are involved in the production of a homogeneous commodity from a raw-material and reusable material source. [Qiang et al. \(2013\)](#) then further expanded the above models with an addition of stochastic parameters. With the flourishing of green concept and recycling in recent years, the latest closed-loop supply chain network equilibrium (CLSCNE) literature also included modeling components in dynamics, policy intervention, and environmental concerns ([Tao et al., 2015](#); [Allevi et al., 2018](#); [Wu et al., 2019](#)), inter alia.

1.2.5 Modeling Strategic Inventories

Research on strategic inventory (SI) has a well-established history predating the rise of SCM in the 1990s. Early quantitative studies in this area focused on examining competitive dynamics among firms. For instance, [Kirman and Sobel \(1974\)](#) introduced an intertemporal-dependence decision, such as inventory, to the classical static model, resulting in a dynamic model where a firm's decisions, such as selling quantities or prices, vary over time. This dynamicity is introduced through carrying inventory from one period to the next, which distinguishes it from static models. Similarly, [Arvan and Moses \(1982\)](#) compared static and dynamic models, where dynamic models involved running production at maximum capacity in one period and shutting down in the following period until inventory is depleted. The findings indicated that consumers are better off in the dynamic case when the marginal revenue curve intersects the decreasing portion of the marginal cost curve.

SI can be employed as a deterrent to potential market entrants. [Arvan \(1985\)](#) examined the existence of a subgame-perfect equilibrium for two competing firms that produce and sell in the first period and carry unsold output as inventory for the second period. The study revealed that even in cases where revenues are symmetric, the equilibrium, if it exists, is not symmetric, and one firm must act as a leader in the second period. The author further demonstrated that a firm facing a threat from potential entrants can create an equilibrium where it accumulates a large inventory to deter their entry. [Ware \(1985\)](#) developed a model with an incumbent and a potential entrant, where the incumbent has the option of carrying inventory. The study showed that the threat of carrying inventory can effectively deter market entry. Additionally, the author argued that combining capacity investment with inventory investment would have a greater impact on entry deterrence.

SI has been recognized as a crucial tool for firms to enhance their sales in subsequent periods ([Mollgaard et al., 2000](#)). The literature suggests that the effectiveness of SI depends on several factors, such as the convexity of the cost function, the slope of the demand function, and the cost of storage. Empirical evidence indicates that firms with more market power tend to hold higher levels of inventory, as it helps in mitigating market price volatility during supply and demand shocks ([Amihud and Medenelson, 1989](#)). Additionally, SI can also influence firms' competitive behavior. For instance, [Mittraille and Moreaux \(2013\)](#) found that firms that carry inventory tend to act as

leaders in a multiple Cournot firm competition, thereby gaining a larger market share.

Furthermore, SI can also induce cooperation among firms in various settings. For example, [Matsumura \(1999\)](#) found that in a more-than-two-period model, firms tend to behave collaboratively, and deviations from collusion are deterred by the non-deviating firms acting as Stackelberg leaders. Additionally, [Rotemberg and Saloner \(1989\)](#) demonstrated that inventory can be used as a strategic tool to discourage firms from deviating from a collusive agreement, particularly when demand is high.

Despite its significance, SI has received limited attention in supply chain studies until ([Anand et al., 2008](#)). They study the interaction between an upstream supplier and a downstream retailer. In this seminal work, a supplier attempts to maximize his own profit by choosing the wholesale price, whereas a retailer in return attempts to maximize his own profit by choosing the purchasing quantity. The models allow the retailer to hold some of the quantity purchased in one period to be sold in the next period. The authors show that a retailer's SI mitigates the impact of the supplier's position as a leader. Their work became a platform of much literature which emerge thereafter.

Numerous studies have provided further evidence and expanded upon the findings of [Anand et al. \(2008\)](#). For example, [Desai et al. \(2010\)](#) introduced competition into the supply chain model and investigates how retailers engage in forward buying and holding inventory at a cost within more complex competitive channel structures, focusing on marketing variables such as merchandising support and trade promotions. The authors revealed that in competitive environments, retailers may engage in forward buying due to competitive pressures, resulting in a prisoner's-dilemma situation. [Hartwig et al. \(2015\)](#) conducted empirical research on the impact of SI on a supply chain, demonstrating that inventory can incentivize the supplier to increase their first-period wholesale price. Additionally, the study found that SI may enhance the overall performance of the supply chain. [Viswanathan \(2016\)](#) examined downstream competition between retailers, assuming that the inventory levels of both retailers are the same. The author compared the effects of downstream competition under two different contracts: dynamic and commitment contracts. The analysis revealed that under the dynamic contract, downstream competition leads to decreased profits for the manufacturer, retailers, and consumer surplus. However, under the commitment contract, only the manufacturer benefits from downstream competition.

Despite these findings, few studies have explored the circumstances in which carrying stocks

would be motivated or beneficial in a supply chain. Additionally, previous research has not explicitly linked SI to its holding cost or to the market power of the subjects that may invoke SI in a competitive setting. Further investigation in these areas is needed to gain a comprehensive understanding of the strategic use of inventory in SCM.

1.2.6 Supply Chain Models with Uncertainties

As an emerging optimization technique, robust optimization (RO) has been widely used in the supply chain management context. For technical details of RO, see §2.3. Some of the pioneer applications of RO in supply chain networks include [Bertsimas and Thiele \(2004\)](#), [Ben-Tal et al. \(2005\)](#), and [Atamtürk and Zhang \(2007\)](#), inter alia. The literature survey of this subsection primarily but not exclusively focuses on the CLSC models intersecting with game theory, uncertainties, or policy interventions. For instance, [Bertsimas and Thiele \(2004\)](#) analyzed inventory management problems under demand and cost uncertainties. [Pishvaei et al. \(2011\)](#) dealt with a market-to-market, closed-loop supply chain network under uncertain demands, returns, and transportation costs. They are the first to adopt robust modeling techniques in closed-loop supply chain networks. [Baghalian et al. \(2013\)](#) developed a multi-supply chain network model considering demand uncertainties and disruption risks. [Hirano and Narushima \(2019\)](#) considered a three-tier vertical supply chain network with competition. In their model, the strategies of the competing agents are unknown to one another and therefore are constructed as an uncertainty.

Concerning the uncertainties incorporated in those models, it is observed that the most popular source of uncertainty is demand (cf. [Amin et al. \(2020\)](#)). Several models including [Ghomi-Avili et al. \(2021\)](#), [Baghalian et al. \(2013\)](#), [Zeballos et al. \(2013\)](#), [Hasani and Khosrojerdi \(2016\)](#), and [Jabbarzadeh et al. \(2018\)](#) encountered supply-side uncertainties, but only for costs. [Hirano and Narushima \(2019\)](#) characterized a unique uncertainty due to the competing supplier's strategic information asymmetry.

1.3 Dissertation Overview

The aim of this Ph.D. research is to develop modeling frameworks for supply chain network decision-making that incorporate decentralized systems, legislation, and risks at both the system level and

spatial network scale. This integrated framework provides guidance for supply chain practitioners and policymakers to develop sustainable and resilient global supply chain economies. By expanding the understanding of competition, uncertainties, and potentially disruptive consequences in a broader social-economic ecosystem, this study contributes to the advancement of supply chain management and sustainable development practices.

In Chapter 2, the fundamental economics concepts and operations research methods are reviewed. In particular, the technical relationships among game theory, variational inequality, deterministic convex optimization, robust optimization, complementarity problems, and cone-related programming are discussed. These analytical tools facilitate quantitative solutions to the integral studies presented in the subsequent chapters. Additionally, this chapter offers methodological roadmaps for the entirety of the dissertation.

In Chapter 3, I present a two-period game-theoretic supply chain model, in which a singular manufacturer supplies products to a pair of identical Cournot duopolistic retailers. I show that the strategic inventory carried by the retailers under dynamic contract is Pareto-dominating for the manufacturer, retailers, consumers, the channel, and society as well. It is found that the retailer's strategic inventory, however, can be eliminated when the manufacturer commits to a wholesale contract or the inventory holding cost is too high. In comparing the cases with and without downstream competition, I also show that the downstream Cournot duopoly undermines the profits for the retailers, but benefits all others.

In Chapter 4, I develop the first general decentralized cross-sector supply chain network model that captures the unique features of scarce resources under a unifying fiscal policy scheme. I formulate the problem as a network equilibrium model with finite-dimensional variational inequality theories. I then characterize the network equilibrium with a set of classic theoretical properties, as well as with a set of properties that are novel to the network games application literature, namely, the lowest eigenvalue of the game Jacobian. Lastly, I provide a series of illustrative examples, including a medical glove supply network, to showcase how this model can be used to improve the efficacy of the imposed policies in relieving supply chain distress and stimulating welfare. The instilled managerial insights inform and expand the political dialogues on fiscal policy design, public resource legislation, social welfare redistribution, and supply chain practice toward sustainability.

In Chapter 5, I develop a robust closed-loop supply chain network equilibrium model that

investigates the critical material and semiconductor chip’s supply, production, recycling activities, and the uncertainties that arise. I formulate the model as a robust generalized Nash equilibrium problem and construct a set of theoretical properties of the network equilibrium using a host of advanced operations research methods. Lastly, I point out an algorithmic pathway to the solution of the model and potential extensions to this work.

1.3.1 Contributions to Literature

In Chapter 2, a novel characterization of a qualitative property of the network equilibrium is established. This work is a refinement of the theoretical study of network games among supply chain literature.

In Chapter 3, a closed-form solution of a two-period supply chain model with strategic inventories is provided. An inventory-cost threshold that governs the retailer’s inventory-carrying behaviors concerning the contracts is identified. A Lerner-index representation of the supply chain firms’ market power under the model settings is established.

In Chapter 4, the first decentralized 4-tier multi-product scarce resource SCNE model with supply-side fiscal instruments is constructed. The model could handle not only the supply chains of scarce resources but those of general resources that pertain to the aforementioned characteristics of scarcity. This is also the first work to examine a general supply-side fiscal policy that encompasses both economic incentives and taxes in one model, with the social outcome as the calibration. A rigorous variational inequality formulation of the modeled general Nash equilibrium problem is proffered for the consideration of both competition and resource capacities. The managerial insights derived from the numerical examples provide governments and supply chain practitioners valuable advice for handling scarce resource management, fiscal policy design, shortage relief, and sustainable development.

In Chapter 5, a quantitative study is conducted to examine the issues of semiconductor chips, their raw material supplies, and recycling. A closed-loop game-theoretic supply chain network model, in which both the resource capacity and the supply-side policies are uncertain, is constructed. The model is formulated as a variational inequality problem, and is the first robust generalized Nash equilibrium model in the literature. Among the SCNE literature, this work is also the foremost to investigate the uncertainty caused by supply-side policies and resource capacities. Furthermore,

this work is the first to use an equilibrium model to tackle policy uncertainties and the shortage issue of rare earth elements. Finally, an algorithmic procedure is proposed to solve the problem via a multi-phase reformulation.

While the studies presented in this dissertation solely address the supply chain network, the methodologies proposed can be broadly applied to other analogous problems. For instance, these approaches can be leveraged in transportation planning, designing city traffic, environmental economics with a focus on green initiatives, revenue management in e-commerce, and other network flow predicaments. Moreover, practitioners can modify these models to encompass emerging industries and competitive effects that intersect with existing supply chains.

Chapter 2

METHODOLOGIES

2.1 Introduction

This chapter provides an overview of fundamental theories from the existing literature, including convex optimization, variational inequality (VI), equilibrium problems, network game theory, and affine robust optimization.

Equilibrium concepts are central to the study of economic interactions. In literature, a range of methodologies have been developed for formulating, analyzing, and computing economic equilibria, such as optimization, complementarity theory, and systems of equations (Nagurney, 1999). In this dissertation, all of the proposed models in Chapter 3, 4, and 5 strive to establish and analyze the equilibria of competitive supply chain networks. Thus, as a preamble, I review the well-known solution concept of the equilibrium of a non-cooperative game, i.e., Nash equilibrium, in the landscape of mathematical optimization.

VI models represent a system of inequalities, which are solved by finding a solution that satisfies all the constraints and minimizes a specified objective function. The VI approach is suitable for modeling supply chain networks as it captures the complex interactions and nonlinear relationships among the various entities involved in the network. It can handle both continuous and discrete decision variables, making it flexible enough to model a wide range of supply chain problems. Furthermore, VIs can be used to model the impact of disruptions on supply chain networks and find solutions that are robust to disruptions. Therefore, VIs have become a popular tool for solving supply chain network problems, with nearly all the SCNE models exemplified in §1.2.2

adopting the VI framework. Along with network games, this dissertation highlights VIs as powerful methodological frameworks for multi-agent decision-making problems due to their connections to optimization problems, which can be applied to competitive network economic models.

Moreover, the theoretical properties associated with Nash equilibria that are expressed by VIs, as illustrated later in Chapter 4 and 5, are reviewed at length subsequently. These theoretical properties, pertaining to the existence and uniqueness of the solutions obtained by employing the VI theory, are discussed with a focus on classic literature. This dissertation also contribute to this part of literature by characterizing these properties in a novel way.

The other methodological framework for this dissertation is robust optimization, an optimization problem under uncertainty, with unknown distribution information. I review primarily a subset of this broad framework—the affine problems with benign uncertainty sets. Emphasis is placed on the tractability of the problem through various reformulations. Within the context, this chapter also reviews a class of complementarity problems that are constrained by “Lorentz cones”, namely, mixed nonlinear second-order cone complementarity problem, for its recent advancement in the algorithmic procedures.

To tie the aforementioned conceptual frameworks together, I discuss the links between these classes of problems, i.e., convex optimization and VIs, game theory and VIs, and robust optimization and game theory. These technicalities are crucial to this dissertation, and will be embodied in the later chapters as necessary avenues toward the respective solutions. At the end of this chapter, a roadmap for these frameworks will be provided. In addition, a schematic for the key reformulation-procedure performed in the more technical part of this dissertation, Chapter 5, will also be provided.

Due to the wealth of literature on the topics covered in this limited dissertation, the overview of theories conducted in this chapter cannot nearly be comprehensive. Hence, I provide the reference of important extensions and proofs of theorems for interested readers.

2.2 Variational Inequality Problem (VIP)

A variational inequality model is a mathematical optimization model that represents a system of inequalities, which is solved by finding a solution that satisfies all the constraints and minimizes a given objective function. The finite-dimensional VI framework is a versatile tool for modeling

and analyzing optimization and equilibrium problems in the mathematical sciences. Variational inequality theory will be utilized broadly in this dissertation as a fundamental method. The theories reviewed in this section are primarily based on [Nagurney \(1999\)](#) and [Facchinei and Pang \(2003\)](#), unless noted otherwise. For more details on finite-dimensional variational inequality theories, refer to the stated literature and the reference therein.

Definition 1 (Inner Product)

Let V be a vector space over Euclidean number of F . Then an inner product on V is a real-valued mapping $\langle \cdot, \cdot \rangle : V \times V \rightarrow F$ satisfying the following condition:

(a) [Linearity in the first variable] $\langle v + v', w \rangle = \langle v, w \rangle + \langle v', w \rangle$ and $\langle cv, w \rangle = c\langle v, w \rangle$ for all $v, v', w \in V$ and for all $c \in F$.

(b) [Hermitian symmetry] $\langle v, w \rangle = \overline{\langle w, v \rangle}$ for all $v, w \in V$.

(c) [Positive definiteness] If $v \in V, v \neq 0, \langle v, v \rangle$ is a positive real number.

Definition 2 (Finite-dimensional Variational Inequality Problem)

The finite-dimensional variational inequality problem, $VI(F, K)$, is to determine a vector $x^* \in K \subset R^n$, such that

$$F(X^*)^T \cdot (X - X^*) \geq 0, \quad \forall X \in K, \tag{2.1}$$

or more often,

$$\langle F(X^*), (X - X^*) \rangle \geq 0, \quad \forall X \in K, \tag{2.2}$$

where F is a given continuous function from K to R^n , K is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in n -dimensional Euclidean space.

In condition (2.2), $F(X) := (F_1(X), \dots, F_n(X))^T$, and $X := (X_1, \dots, X_n)^T$. $F(X)$ and X are both column vectors. Recall that for two arbitrary vectors $u, v \in R_n$, the inner product $\langle u, v \rangle = \|u\| \|v\| \cos\theta$, where θ is the angle between the vectors u and v , and (2.2) is equivalent to

$$\sum_{i=1}^n F_i(X^*) \times (X_i - X_i^*) \geq 0, \quad \forall x \in K. \tag{2.3}$$

The above problem $VI(F, K)$ states that $F(X^*)^T$ is “orthogonal” to the feasible set K at the point X^* . See Figure 2.1.

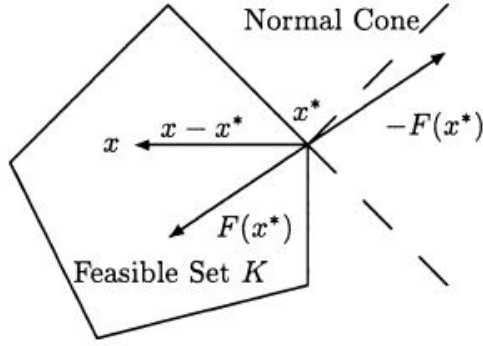


Figure 2.1: Geometric interpretation of VI (Nagurney, 1999)

The VIP is a general problem formulation that encompasses a host of mathematical problems, including nonlinear equations, optimization problems, complementarity problems, and fixed point problems. Later, I will show that both constrained and unconstrained optimization problems can be formulated as VIPs.

2.2.1 Variational Inequalities and Convex Optimization Problems

The variational inequality problem and convex optimization problem are known to be linked. The earliest landmark works establishing the equivalence between these two classes of problems date back to Eaves (1971) and Gabay and Moulin (1980). More recent expansions of the subject can be found in Bazaraa et al. (1993). I begin in this subsection by defining the convex optimization problem.

Definition 3 (Convex optimization (Boyd and Vandenberghe, 2004))

A convex optimization problem is one of the form:

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & f_i(x) \leq b_i, \quad i = 1, \dots, m \end{aligned} \tag{2.4}$$

where the functions $f, f_1, \dots, f_m : R^n \rightarrow R$ are convex, i.e., satisfy

$$f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y), \tag{2.5}$$

for all $x, y \in R^n$ and all $\alpha, \beta \in R$ with $\alpha + \beta = 1, \alpha \geq 0, \beta \geq 0$.

Some examples of convex optimization problems include linear programming problem, least square problem, and quadratic optimization problem (see Figure 2.2). The feasible set \mathcal{P} , which is a polyhedron, is shown shaded. The contour lines of the objective function, which is convex quadratic, are shown as dashed curves. The point x^* is optimal.

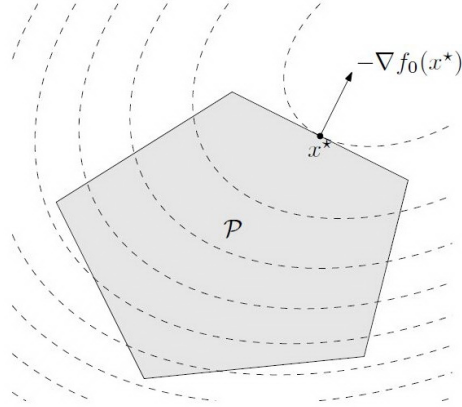


Figure 2.2: Geometric illustration of quadratic optimization as an example of constrained optimization (Boyd and Vandenberghe, 2004).

Proposition 1 (Variational Inequality and Optimization Problem)

Let X^* be a solution to the optimization problem (2.4), where f is continuously differentiable and K is closed and convex. Then x^* is a solution of the VI problem:

$$\langle \nabla f(X^*), (X - X^*) \rangle \geq 0, \quad \forall X \in K, \tag{2.6}$$

where, $\nabla f(X)$ is the gradient vector of f with respect to X , i.e., $\nabla f(X) := \left(\frac{\partial f(X)}{\partial X_1}, \dots, \frac{\partial f(X)}{\partial X_n} \right)^T$.

Proof: Let $\phi(t) = f(x^* + t(x - x^*))$, for $t \in [0, 1]$. Since $\phi(t)$ achieves its minimum at $t = 0$, $0 \leq \phi'(0) = \nabla f(x^*)^T \cdot (x - x^*)$, that is, x^* is a solution to (2.6).

Proposition 2

If $f(X)$ is a convex function and X is a solution to $VI(\nabla f, K)$, then X is a solution to the optimization problem 2.4. In the case that the feasible set $K = R^n$, then the unconstrained optimization problem is also a variational inequality problem.

The variational inequality problem can be reformulated as an optimization problem under certain symmetry conditions. The definitions of positive-semi definiteness, positive-definiteness, and

strong positive-definiteness are recalled next, followed by a theorem presenting the above relationship.

Theorem 1 (Jacobian Matrix)

Assume that $F(X)$ is continuously differentiable on K and that the Jacobian matrix, defined as

$$J(X) := \nabla F(X) = \begin{bmatrix} \frac{\partial F_1(X)}{\partial X_1} & \cdots & \frac{\partial F_1(X)}{\partial X_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n(X)}{\partial X_1} & \cdots & \frac{\partial F_n(X)}{\partial X_n} \end{bmatrix} \tag{2.7}$$

is symmetric and positive-semi definite. Then there is a real-valued convex function $f : K \mapsto R^n$ satisfying

$$\nabla f(X) = F(X),$$

with X^* the solution of $VI(F, K)$ also being the solution of the mathematical optimization problem:

$$\begin{aligned} \min \quad & f(X) \\ \text{s.t.} \quad & X \in K, \end{aligned}$$

where $f(X) = \int F(X)^T dx$, and where \int is a line integral.

Thus, variational inequality is a more general problem formulation than optimization problem formulation, since it can also handle a general function $F(X)$, as long as it has an asymmetric Jacobian (Nagurney, 1999). In the technical literature of network games, the Jacobian matrix, also commonly known as the *game Jacobian*, is akin to the topology, equilibrium analyses, strategic interaction, and comparative statics of the network. For more details, see Bramoullé et al. (2014), Jackson and Zenou (2015), Parise and Ozdaglar (2019), Melo (2018), and the reference therein.

2.2.2 Variational Inequalities and Game Theory

In this subsection, the relation between variational inequalities and game theory is briefly discussed.

John Nash (1950, 1951), in his seminal work, established non-cooperative game theory, involving multiple players, each of whom acts in its own interest. In particular, consider a game with m

players, each player i having a strategy vector $X_i = \{X_{i1}, \dots, X_{in}\}$ selected from a closed, convex set $K^i \subset R^n$. Each player i seeks to maximize its own utility function, $u_i : K \rightarrow R$, where $K = K^1 \times K^2 \times \dots \times K^m \subset R^{mn}$. Player i 's utility, u_i , depends not only on its own strategy vector, X_i , but also on the strategy vectors of all the other players, $(X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_m)$. A Nash equilibrium (NE) is achieved if no one can increase its utility by unilaterally altering the value of its strategy vector. The formal definition of the NE is recalled as follows.

Theorem 2 (Nash Equilibrium)

A Nash equilibrium is a strategy vector

$$X^* = (X_1^*, \dots, X_m^*) \in K,$$

such that,

$$u_i(X_i^*, \hat{X}_i^*) \geq u_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K, \quad \forall i, \tag{2.8}$$

where, $\hat{X}^ = (X_1^*, \dots, X_{i-1}^*, X_{i+1}^*, \dots, X_m^*)$*

[Hartman and Stampacchia \(1966\)](#), [Gabay and Moulin \(1980\)](#) have shown that, given continuously differentiable and concave utility functions, u_i , the Nash equilibrium problem can be formulated as a variational inequality problem on K . See [Theorem 3](#).

Theorem 3 (Variational Inequality Formulation of Nash Equilibrium)

Under the assumption that each utility function u_i is continuously differentiable and concave, X^ is a Nash equilibrium if and only if $X^* \in K$ is a solution to the variational inequality*

$$F(X^*)^T \cdot (X - X^*) \geq 0, \quad \forall X \in K, \tag{2.9}$$

where, $F(X) := (-\nabla_{X_1} u_1(X), \dots, -\nabla_{X_m} u_m(X))^T$, and $\nabla_{X_1} u_i(X) := (\frac{\partial u_i(X)}{\partial X_{i1}}, \dots, \frac{\partial u_i(X)}{\partial X_{in}})$.

$F(X)$ is commonly regarded as the *entry function* to the VI [\(2.9\)](#), and is also known as the *payoff gradient* of the game.

2.2.3 Qualitative Properties: Existence and Uniqueness

Next, the theoretical properties of variational inequality problems, specifically, the conditions for existence and uniqueness of a solution, are reviewed. The theories discussed in this subsection are mainly based upon [Harker and Pang \(1990a\)](#), [Nagurney \(1999\)](#), and [Melo \(2018\)](#), unless noted otherwise. Due to the complexity of the results in literature, it is impossible to review them all. For more details, I refer the readers to the aforementioned literature and the references therein.

* Existence and Uniqueness of VIP Solution

Theorem 4 is the most basic result on the existence of a solution to $VI(F, K)$ requires the set K to be compact and convex, and the mapping F to be continuous. From this basic result, many corollaries can be derived by replacing the compactness of K with additional conditions on F ([Harker and Pang, 1990a](#)).

Theorem 4 (Existence of a Solution)

If K is a compact convex set and $F(X)$ is continuous on K , then the variational inequality problem admits at least one solution X^ .*

Theorem 5 (Existence under Unboundedness)

If the feasible set K is unbounded, then $VI(F, K)$ admits a solution if and only if there exists an $\mathcal{R} > 0$ and a solution of $VI(F, S)$, $X_{\mathcal{R}}$, such that $\|X_{\mathcal{R}}^\| < \mathcal{R}$, where $S = \{X : \|X\| \leq \mathcal{R}\}$.*

Theorem 6 (Existence under Coercivity)

Suppose that $F(X)$ satisfies the coercivity condition

$$\frac{\langle F(X) - F(X_0), X - X_0 \rangle}{\|X - X_0\|} \rightarrow \infty \quad (2.10)$$

as $\|X\| \rightarrow \infty$ for $X \in K$, and for some $X_0 \in K$. Then $VI(F, K)$ always has a solution.

According to Theorem 6, the existence condition of a solution to a variational inequality problem can be guaranteed by the coercivity condition.

In addition to the coercivity condition discussed above, monotonicity conditions can also be utilized to characterize the qualitative properties of existence and uniqueness. I begin by reviewing several basic definitions of monotonicity.

Definition 4 (Monotonicity)

A mapping $F : K \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^n$ is said to be

(a) *monotone on K* if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in K. \quad (2.11)$$

(b) *strictly monotone on K* , if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in K, \quad X^1 \neq X^2. \quad (2.12)$$

(c) *strongly monotone on K* , if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > \alpha \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in K, \quad (2.13)$$

where $\alpha > 0$.

Definition 5 (Lipschitz Continuity)

$F(X)$ is *Lipschitz Continuous on K* , if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq L \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in K,$$

where L is the Lipschitz constant.

Theorem 7 (Uniqueness under Strict Monotonicity)

Suppose that $F(X)$ is strictly monotone on K . Then the solution to the $VI(F, K)$ problem is unique, if one exists.

Theorem 8 (Uniqueness under Strong Monotonicity)

Suppose that $F(X)$ is strongly monotone on K . Then there exists precisely one solution X to $VI(F, K)$.

In summary of Theorems 4, 7, and 8, strong monotonicity of the function F guarantees both existence and uniqueness, in the case of an unbounded feasible set K . If the feasible set K is

compact, that is, closed and bounded, the continuity of F guarantees the existence of a solution. The strict monotonicity of F is then sufficient to guarantee its uniqueness, provided its existence.

※ Existence and Uniqueness of NE

I now introduce the conditions for existence and uniqueness of a Nash equilibrium, established by Theorem 3. Theorem 9 states the existence under the assumptions, first presented by Rosen (1965), that K is compact and each u_i is continuously differentiable.

Theorem 9 (Existence of NE under Compactness and Continuous Differentiability)

Suppose that the feasible set K is compact and each u_i is continuously differentiable. Then existence of a Nash equilibrium is guaranteed.

Gabay and Moulin (1980), on the other hand, relaxed the assumption of the compactness of K , and proved existence of a Nash equilibrium after imposing a coercivity condition on $F(X)$.

Theorem 10 (Existence of NE under Coercivity)

Suppose that $F(X)$, as given in Theorem 3, satisfies the coercivity condition 2.10. Then there always exists a Nash equilibrium.

Furthermore, Karamardian (1969) demonstrated the existence and uniqueness of a Nash equilibrium under the strong monotonicity assumption. See Theorem 11.

Theorem 11 (Existence and Uniqueness of NE under Strong Monotonicity)

Assume that $F(X)$, as given in Theorem 3, is strongly monotone on K . Then there exists precisely one Nash equilibrium X .

Additionally, according to Theorem 7, the uniqueness of a Nash equilibrium can be guaranteed under the assumptions that $F(X)$ is strictly monotone and an equilibrium exists. See Theorem 12.

Theorem 12 (Uniqueness of NE under Strict Monotonicity)

Suppose that $F(X)$, as given in Theorem 2, is strictly monotone on K . Then the Nash equilibrium, X^ , is unique, if it exists.*

※ **A Novel Uniqueness Result**

To this end, I have discussed the characterization of the uniqueness of an NE using compactness/continuous differentiability, coercivity, and monotonicity conditions. I now discuss a novel approach to characterize the uniqueness of a network equilibrium by using the eigenvalues of the game Jacobian matrix. To proceed, I shall adopt an additional set of notations. For a Nash game defined by Theorem 3, I first declare the following supplemental assumptions, following Melo (2018).

Assumption 1 (Modified from Melo (2018))

For all player $i \in N$, assume the following:

- (i) The payoff function u_i is continuously differentiable and strictly concave on all X_i .
- (ii) The cross partial derivative of its payoff function

$$\frac{\partial^2 u_i(X)}{\partial X_i \partial X_j}$$

is well defined for all X_i and X_j , $\forall i \neq j$.

Note, that the condition (i) of Assumption 1 is stricter than what Theorem 9 requires, with an additional concavity requirement.

Re-express the payoff function $u(X)$'s game Jacobian matrix, $J(X)$, of (2.7), by redefining its entries as:

$$\mathcal{J}_{ii}(X) := -\frac{\partial^2 u_i(X)}{\partial X_i^2}, \quad \forall i \in N, \quad (2.14)$$

and

$$\mathcal{J}_{ij}(X) := -\frac{\partial^2 u_i(X)}{\partial X_i \partial X_j}, \quad \forall i, j \in N \text{ and } i \neq j. \quad (2.15)$$

Thus, $J(X)$ can be re-expressed as

$$J(X) := [\mathcal{J}_{ij}(X)]_{i,j}, \quad \forall X \in R_+^n. \quad (2.16)$$

Decompose the Jacobian matrix in terms of its *diagonal* and *off-diagonal* elements, yielding

$$J(X) = D(X) + N(X), \quad (2.17)$$

where, $D(X)$ is a diagonal matrix, defined as

$$D(X) := \begin{bmatrix} \mathcal{J}_{11}(X) & \dots & 0 \\ \vdots & \mathcal{J}_{ii}(X) & \vdots \\ 0 & \dots & \mathcal{J}_{nn}(X) \end{bmatrix}, \quad (2.18)$$

and $N(X)$ is an off-diagonal matrix, whose elements are $\mathcal{J}_{ij}(X)$, $i \neq j$. $N(X)$ is defined as

$$N(X) := \begin{bmatrix} 0 & \dots & \mathcal{J}_{1n}(X) \\ \vdots & 0 & \vdots \\ \mathcal{J}_{n1}(X) & \dots & 0 \end{bmatrix}. \quad (2.19)$$

According to [Melo \(2018\)](#), the matrix $N(X)$ is a transformation of the original players' network. In economic terms, the terms $\mathcal{J}_{ij}(X)$, $i \neq j$, capture the *nature* and *intensity* of strategic interaction among players.

Finally, I denote the lowest eigenvalue of a square matrix as $\lambda_{\min}(\cdot)$. Clearly, both $D(X)$ and $N(X)$ have real-numbered eigenvalues because they are symmetric, and thus, square matrices.

Proposition 3 (Strict Monotonicity under Eigenvalue Condition ([Melo, 2018](#)))

Let assumption 1 hold. If the condition

$$|\lambda_{\min}(N(X))| < \lambda_{\min}(D(X)) \quad (2.20)$$

holds for all $X \in R_+^n$, then $F(X)$ is strictly monotone.

With Proposition 3, I now present the new uniqueness result as the following.

Theorem 13 (Uniqueness under Eigenvalue Condition)

The solution X^ to VI (2.9) is unique, if*

(i) the feasible set K is compact, and

(ii) each payoff function u_i is continuously twice differentiable, strictly concave on all X , and

has well-defined cross partial derivative, and

(iii) the following condition holds

$$|\lambda_{\min}(N(X))| < \lambda_{\min}(D(X)), \quad \forall X. \quad (2.21)$$

Proof: First, with condition (ii) and (iii) by virtue of Proposition 3, it follows that $F(X)$ is strictly monotone on K . Then by combining Theorem 9 and 12, the uniqueness of X^* is proved. \square

Theorem 13 is new to the series of literature in network game applications, pioneered by Nagurney et al. (2002). Such characterization of the NE’s uniqueness by the eigenvalues is made available, due primarily to the additional concavity assumption of the payoff function. This novel characterization first appeared in the work by Hu and Li (2022).

More broadly, to characterize the uniqueness of the equilibrium (2.9), one could also establish similar sufficient conditions by the semidefiniteness of $J(X)$ as it pertains to the analogous monotonicity features (cf. Parise and Ozdaglar (2019); Melo (2018)). For more information on how the lowest eigenvalue of a game Jacobian allows us to elicit insights on the interplay, neighboring influences, and aggregate effect, see Bramoullé et al. (2014).

2.2.4 Variational Inequalities and Complementarity Problems

The variational inequality problem also contains the complementarity problem (CP) as a special case. For more details, I refer the readers to the well-known books by Nagurney (1999) and Facchinei and Pang (2003). Complementarity problems are defined on the non-negative orthant. Let R_+^n denote the non-negative orthant in R^n , and let $F : R^n \mapsto R^n$. The nonlinear complementarity problem (NCP) over R_+^n is a system of equations and inequalities stated as:

Definition 6 (Complementarity Problem)

Find $x^* \geq 0$, such that

$$F(x^*) \geq 0 \quad \text{and} \quad \langle F(x^*)^T, x^* \rangle = 0. \quad (2.22)$$

In the literature, the terminology “complementarity problem” is often used interchangeably with “complementary problem”. The above NCP can be denoted as $\text{NCP}(F)$. The relationship between the complementarity problem and the VI problem is as follows.

Proposition 4 (Variational Inequality and Complementarity Problem)

$VI(F, R_+^n)$ and (2.22) have precisely the same solution, if any.

Furthermore, if F is affine, $F(x) = Mx + q$, where $M \in R^{n \times n}$ (is a positive semi-definite matrix) and $q \in R^n$, then the Definition 2 is an affine variational inequality (AVI) problem and the problem (2.22) is a linear complementarity problem (LCP), denoted by $LCP(q, M)$.

The variational inequality problem is akin to many classes of equilibrium problem classes. Figure 2.3 summarizes the various classes of problems and their connections. The notation “ $P \Rightarrow Q$ ” means that one can derive problem Q by specializing problem P ; the notation “ $P \Leftrightarrow Q$ ” means that the two problems P and Q are “equivalent”. The notations used by [Facchinei and Pang \(2003\)](#) are mildly different from what is defined in this dissertation. For example, $VI(F, K)$ in this dissertation is written as $VI(K, F)$ in Figure 2.3. Nonetheless, it is believed that such a discrepancy will not lead to a misunderstanding under the given context.

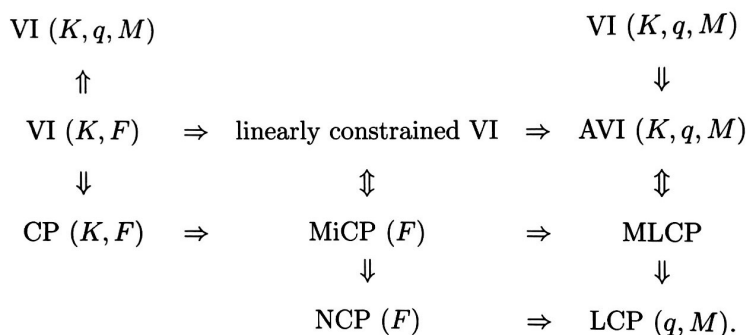


Figure 2.3: Relation between problem classes ([Facchinei and Pang, 2003](#))

Besides the defined acronyms in this subsection, CP, MiCP, and MLCP in Figure 2.3 stand for complementarity problem, mixed complementarity problem, and mixed linear complementarity problem, respectively.

2.3 Robust Optimization (RO)

The robust optimization is a modeling approach to handle parameter (i.e., data) uncertainties ([Ben-Tal and Nemirovski, 1998](#)). In general, when the parameters of a model are uncertain with no reliable knowledge of the probability distribution, one may adopt RO to seek a solution that is feasible for all possible realizations of such uncertain parameters. In the absence of empirical data,

the RO approach can be considered an alternative to risk mitigation. For more details, I refer to [Ben-Tal and Nemirovski \(1998, 2000\)](#); [Ben-Tal et al. \(2005, 2009\)](#).

Sought in this dissertation is one of the computationally tractable RO problems, i.e., affine robust optimization whose measures of robustness are in the form of a box or ellipsoidal set. These problems can be represented as equivalent deterministic counterparts in the value of the parameters. An affine robust optimization problem can be expressed as the following ([Ben-Tal et al., 2009](#)).

2.3.1 Affine Robust Optimization with Box Uncertainties

Consider the following deterministic linear optimization model.

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \end{aligned}$$

The corresponding uncertain linear optimization problem can be defined as

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & A, b, c \in \mathcal{U} \end{aligned} \tag{2.23}$$

In this problem, the parameter A, b, c vary in a given uncertainty set \mathcal{U} . A vector x is a feasible solution to problem (2.23) if it satisfies all realizations of the constraints from the uncertainty set \mathcal{U} . [Ben-Tal and Nemirovski \(1999\)](#) define the robust counterpart (RC) of problem (2.23) as the following

$$\min_x \left\{ \sup_{(c,A,b) \in \mathcal{U}} c^T x : Ax \leq b, \forall (c, A, b) \in \mathcal{U} \right\} \tag{2.24}$$

To illustrate the box uncertainty set, let us simplify the problem by fixing the parameters A and c . b is known to vary in a given set \mathcal{U} . Then problem (2.24) becomes

$$\max \quad c^T x \tag{2.25}$$

$$\text{s.t.} \quad Ax \leq b, \quad \forall b \in \mathcal{U}^{box} \tag{2.26}$$

Assume that the uncertain data b_t resides within a certain interval around a “nominal” value \bar{b}_t . As such, the box uncertainty set of the unknown parameter b can be expressed as

$$\mathcal{U}^{box} := \left\{ b \in R^T : |b_t - \bar{b}_t| \leq \rho G_t, \quad \forall t = 1, \dots, T \right\}. \quad (2.27)$$

where $t = 1, \dots, T$ is the index of the data point, the positive numbers G_t represent “uncertainty scale”, and $\rho > 0$ is the “uncertainty level.” A particular case of interest is $G_t = b_t$, which corresponds to a simple case, where \mathcal{U}^{box} contains the parameter b whose relative deviation from the nominal value \bar{b}_t is of size up to ρ . An uncertainty set of type \mathcal{U}^{box} may be used to model situations of independent parameter b .

[Ben-Tal et al. \(2005\)](#) show that the RC can be converted to a tractable equivalent model where \mathcal{U}^{box} is replaced by a finite set consisting of the extreme points of \mathcal{U}^{box} . Specifically, the constraint (2.26) can be converted to

$$Ax \leq b, \quad \forall b \in \mathcal{U}^{box} := \left\{ b \in R^T : |b_t - \bar{b}_t| \leq \rho G_t, \quad \forall t = 1, \dots, T \right\}. \quad (2.28)$$

The right-hand side of (2.28) contains the vector of uncertain parameters, while all parameters of the left-hand side are certain. Thus, the tractable form of the above inequality can be re-expressed as

$$Ax \leq \bar{b}_t - \rho G_t, \quad \forall t = 1, \dots, T. \quad (2.29)$$

Finally, the problem (2.25) - (2.26) can be expressed as the following deterministic linear optimization problem.

$$\max \quad c^T x \quad (2.30)$$

$$\text{s.t.} \quad Ax \leq \bar{b}_t - \rho G_t, \quad \forall t = 1, \dots, T. \quad (2.31)$$

2.3.2 Affine Robust Optimization with Ellipsoidal Uncertainties

In contrast, when parameters have correlated random variables, the ellipsoidal uncertainty set can be an alternative form of modeling. A robust linear program with an ellipsoidal uncertainty set can

be transformed into a second-order cone program (SOCP). Follow [Lobo et al. \(1998\)](#), [Boyd and Vandenberghe \(2004\)](#) and consider the following linear program

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & A_i^T x \leq b_i, \quad i = 1, \dots, m, \end{aligned}$$

where c and b_i are fixed, and a_i are known to lie in given ellipsoids \mathcal{U}_i^{ell} :

$$A_i \in \mathcal{U}_i^{ell} = \{\bar{A}_i + P_i \Delta \mid \|\Delta\|_2 \leq 1\},$$

where $P_i \in R^{n \times n}$ is a symmetric positive definite matrix and $\|\Delta\|_2 \leq 1$ is a Euclidean ball. Then it leads us to the following robust linear program

$$\max \quad c^T x \tag{2.32}$$

$$\text{s.t.} \quad A_i^T x \leq b_i, \quad \forall A_i \in \mathcal{U}_i^{ell}, \quad i = 1, \dots, m. \tag{2.33}$$

The robust linear constraint (2.33) can be expressed as

$$\sup\{A_i^T x \mid A_i \in \mathcal{U}_i^{ell} \leq b_i\},$$

the lefthand side of which can be expressed as

$$\begin{aligned} \sup\{A_i^T x \mid A_i \in \mathcal{U}_i^{ell} \leq b_i\} &= A_i^T x + \sup\{u^T P_i^T x \mid \|\Delta\|_2 \leq 1\} \\ &= A_i^T x + \left\| P_i^T x \right\|_2. \end{aligned}$$

Thus, the robust linear constraint can be expressed as

$$A_i^T x + \left\| P_i^T x \right\|_2 \leq b_i,$$

which is a second-order cone constraint. Hence, the robust linear program (2.32 - 2.33) can be

expressed as the the following SOCP, a deterministic convex optimization.

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & A_i^T x + \left\| P_i^T x \right\|_2 \leq b_i, \quad i = 1, \dots, m. \end{aligned}$$

2.3.3 Robust Optimization and Game Theory

In (2.24), when \mathcal{U} is a convex set, the “sup” operator can be interchanged with “max”. Thus, (2.24) is equivalent to

$$\min_x \left\{ \max_{(c,A,b) \in \mathcal{U}} c^T x : Ax \leq b, \forall (c, A, b) \in \mathcal{U} \right\}. \quad (2.34)$$

In a general n -player zero-sum game theory, the maximin value, denoted by v_i , is the highest value that the player can be sure to get without knowing the actions of the other players; equivalently, it is the lowest value the other players can force the player to receive when they know the player’s action (Maschler et al., 2020).

Definition 7 (n-player zero-sum game)

$$u_i = \max_{x_i} \min_{x_{-i}} u_i(x_i, x_{-i}) = \min_{x_{-i}} \max_{x_i} u_i(x_i, x_{-i}). \quad (2.35)$$

where: i is the index of player, $-i$ is all other players except player i , x_i is the action taken by player i , x_{-i} is the actions taken by all other players, and $u_i(\cdot, \cdot)$ is the payoff function of player i .

The above definition is akin to the well-known Minimax theorem, as its landmark proof by von Neumann (1928) opened the era of game-theoretic studies in modern economics. Furthermore, the computation of a player’s *maximin* value is done in a worst-case approach von Neumann et al. (2007), same as that of a robust optimization problem.

Thus, an RO problem (2.34) can be interpreted in terms of game theory. On one hand, the decision-maker attempts to minimize its disutility by deciding x , given all possible outcomes that can be influenced by \mathcal{U} . On the other hand, nature, though not consciously, attempts to find the worst-case scenario by varying the parameter in \mathcal{U} , given all possible strategies by the decision-maker.

2.4 Mixed Nonlinear Second-order Cone Complementarity Problem

A mixed nonlinear second-order cone complementarity problem (MNSOCCP) has the following form (Hayashi, 2018):

$$\begin{aligned}
 & \text{Find } (x, y, p) \in R^n \times R^n \times R^l \\
 \text{(MNSOCCP)} \quad & \text{s.t. } x \in \mathcal{K}, \quad x^T y = 0, \\
 & y = F_1(x, p), \quad F_2(x, p) = 0.
 \end{aligned} \tag{2.36}$$

where $F_1 : R^n \times R^l \rightarrow R^n$ and $F_2 : R^n \times R^l \rightarrow R^l$ are given continuously differentiable functions, and \mathcal{K} is a Cartesian product of several second-order cones, i.e.,

$$\mathcal{K} := \mathcal{K}^{n_1} \times \mathcal{K}^{n_2} \times \dots \times \mathcal{K}^{n_m}, \tag{2.37}$$

with $n_1 + n_2 + \dots + n_m = n$ and

$$\mathcal{K}^{n_i} := \begin{cases} \{z \in R \mid z \geq 0\} & (n_i = 1) \\ \{z \in R^{n_i} \mid z_1 \geq \sqrt{z_2^2 + \dots + z_{n_i}^2}\} & (n_i \geq 2). \end{cases} \tag{2.38}$$

Therefore, one obtains

$$R_+^n = \mathcal{K}^1 \times \mathcal{K}^1 \times \dots \times \mathcal{K}^1, \tag{2.39}$$

where R_+^n denotes the non-negative orthant of a Euclidean space.

MNSOCCP involves many kinds of problems as special cases. For the nonlinear second-order cone program (NSOCP):

$$\begin{aligned}
 & \min \quad f(z) \\
 & \text{s.t. } \quad G(z) \in \mathcal{K}, \quad H(z) = 0,
 \end{aligned} \tag{2.40}$$

where, $f : R^{l_1} \rightarrow R$, $G : R^{l_1} \rightarrow R^n$, and $H : R^{l_1} \rightarrow R^{l_2}$, its Karush–Kuhn–Tucker (KKT) conditions

are given as follows:

$$\begin{aligned}
 & \text{Find } (x, y, z, w) \in R^n \times R^n \times R^{l_1} \times R^{l_2} \\
 \text{(SOCP-KKT)} \quad & \text{s.t. } x \in \mathcal{K}, y \in \mathcal{K}, x^T y = 0, \\
 & y = G(z), H(z) = 0, \\
 & \nabla f(z) - \nabla G(z)x - \nabla H(z)w = 0.
 \end{aligned} \tag{2.41}$$

The above takes the form of MNSOCCP (2.36).

2.5 Chapter Summary

In this chapter, I review the established theories of convex optimization, game theory, variational inequality, robust optimization, and second-order cone programming. In particular, I discuss the theoretical properties of the solutions to a class of equilibrium problems. I now outline the relationship between these conceptual frameworks in less technical terms as follows.

1. A convex optimization problem has an equivalent expression in VIs.
2. VIs can be used to express the governing condition of a Nash equilibrium.
3. The VIP unifies a class of other equilibrium problems, such as complementarity problem.
4. An RO problem has a similar mathematical expression and interpretation to an n-player zero-sum game.
5. An affine RO problem with either a box or ellipsoidal uncertainty set may be reformulated as a deterministic problem (e.g., SOCP).

In Figure 2.4, I summarize the above insights by inheriting similar arrow notations as Figure 2.3.

$$\begin{array}{ccccccc}
 \text{Convex} & & \text{Variational Inequality} & & \text{Non-cooperative} & & \text{Robust} \\
 \text{Optimization} & \Leftrightarrow & \text{Theory} & \Leftrightarrow & \text{Game Theory} & \Rightarrow & \text{Optimization}
 \end{array}$$

Figure 2.4: Relation between conceptual frameworks: a technical roadmap of the dissertation

2.5.1 Some Technical Notes of The Chapters

In Chapter 3, a two-period dynamic model of a Cournot-Nash network is presented. Each decision agent aims to maximize a convex payoff function. The solution concept is based on Theorem 2, and the backward induction method is adopted as the solution procedure, followed by the derivation of a closed-form equilibrium solution (§3.2.1).

In Chapter 4, a static competitive general network model is proposed, featuring a combination of oligopolistic and perfectly competitive decision agents. The profit-maximizing behaviors of these agents are modeled as a series of convex optimization problems with benign mathematical properties on the payoff functions, collectively formulated as a VIP, as depicted by Theorem 3. The qualitative properties of the network Nash equilibrium, i.e., the existence and uniqueness, are established according to the theoretical discussions in §2.2.3. The problem is solved by the modified projection method, a Newton-type algorithm (§4.5).

In Chapter 5, I extend the network model presented in Chapter 4 by adding a closed-loop structure to the topology and incorporating uncertain parameters with unknown distribution information at the top tier of the network. These uncertainties are modeled as an affine Robust Optimization (RO) problem (§2.3.1 and §2.3.2), which is converted into a tractable, deterministic, and convex SOCP. This problem is further reformulated as a second-order cone constrained variational inequality (SOCCVI) problem, and then cast into a MNSOCCP by virtue of condition (2.41). The roadmap of such a procedure is summarized in Figure 2.5. Finally, a regularized smoothing Newton algorithm is proposed to solve this problem (§5.4).

$$\text{RO} \Rightarrow \text{SOCP} \Rightarrow \text{SOCCVI} \Rightarrow \text{MNSOCCP}$$

Figure 2.5: Reformulation procedure of Chapter 5

2.6 Glossary of Notation

Functions

$c_{mn}, c_n, c_{ki}, \hat{c}_{nk}$	The transaction costs of supplier m , producer n , and recycle center i , and market k , respectively.
$c_{tk}^{js} = c_{tk}^{js}(x_{tk}^{js})$	The transaction cost incurred by supplier (j, s) for market k via transportation mode t .
$c_s^{jm} = c_s^{jm}(x_s^{jm})$	The transaction cost incurred by producer (j, m) with supplier (j, s) .
$c_{jm}^{in} = c_{jm}^{in}(x_{jm}^{in})$	The transaction cost incurred by resource owner (i, n) with producer (j, m) .
$\hat{c}_{tk}^{js} = \hat{c}_{tk}^{js}(x_{tk}^{js})$	The transaction cost incurred by market k with supplier (j, s) via transportation mode t .
$D(\cdot)$	The diagonal matrix of game Jacobian $J(\cdot)$.
d_k	The demand quantity at market k .
$F(\cdot)$	A continuous function, or payoff gradient of the game.
$f(\cdot)$	A continuous function, $f(\cdot) = \int F(\cdot)^T dx$.
f_m, f_n, f_i	The operating costs of supplier m , producer n , and recycle center i , respectively.
$f^{in} = f^{in}(x^{in})$	The operating cost incurred by resource owner (i, n) in terms of outgoing shipments.
$f^{jm} = f^{jm}(x_{jm})$	The operating cost incurred by producer (j, m) in terms of incoming shipments.
$f^{js} = f^{js}(x^{js})$	The operating cost incurred by resource supplier (j, s) in terms of outgoing shipments.
$G(\cdot)$	An SOC-constrained function.
$H(\cdot)$	An equality constrain function.
$J(\cdot)$	The Jacobian matrix or game Jacobian function of $F(\cdot)$.
$\mathcal{J}_{ij}(\cdot)$	The entry of game Jacobian matrix $J(\cdot)$.
$N(\cdot)$	The off-diagonal matrix of game Jacobian $J(\cdot)$.
$p_{3k}^j = p_{3k}^j(d_{jk})$	The price-demand function of resource j at the market k .
$u(\cdot)$	A utility or payoff function.
$\alpha_g^i(\cdot)$	The incentive payment to resource owners in bracket g .
α_k	The consumer aversion at market k .
$\beta_g^j(\cdot)$	The incentive payment to resource producers in bracket g .

Φ_m The policy adversity for REE supplier m .

$\tilde{\pi}_m, \tilde{\pi}_n, \tilde{\pi}_i$ The profit of raw materials from supplier m , producer n , and recycle center i , respectively.

$\tilde{\psi}_m$ The disutility of supplier m .

Indices

k The index of a physical market. $k \in \{1, \dots, K\}$.

g The index of an incentive payment bracket. $g \in \{1, \dots, G\}$.

Matrices

H_{ms} The matrix of data point s , containing historical correlation information of the policy strengths on supplier m .

P_i A projection matrix, often containing correlational information of data.

Operators

$\langle \cdot, \cdot \rangle$ Inner product.

\times Cartesian product.

$\|\cdot\|, \|\cdot\|_2$ Euclidean norm.

sup Supremum.

Parameters .

A_g^i The g 'th incentive payment bracket of resource owners (i, n) .

B_g^j The g 'th incentive payment bracket of resource producers (j, m) .

G_t The uncertainty scale of Z_t .

U_i The total amount of resource i . $U \equiv \{U_1, U_2, \dots, U_I\}$ is a column vector.

w_{ts} The weight on market demand over supplier s via transportation mode t

w_m The strength of policy adversity on supplier m . The column vector $w_{m,s} \equiv (w_{m1,s}, \dots, w_{mn,s}, \dots, w_{mN,s})$ is a data point of w_m , indexed by $s \in \{1, \dots, S_m\}, \forall m$, with a nominal value \bar{w}_m .

Z The capacity of an REE mineral, with each data point indexed as Z_t and nominal value being \bar{Z}_t .

β^r The conversion rate from virgin REE material to finished chips. $\beta^r \in [0, 1]$.

β^u The conversion rate from recycled REE material to finished chips. $\beta^u \in [0, 1]$.

λ_{min}	The lowest eigenvalue of game Jacobian $J(\cdot)$.
ψ_{jm}^{in}	The non-negative conversion rate at resource producers (j, m) from owner (i, n) .
ρ	The uncertainty level of Z . $\rho \in (0, 1]$.
Sets	
K, \mathcal{P}	A closed, convex set.
$\mathcal{K}, \mathcal{K}^{n_i}$	The set confined by a second-order cone.
\mathcal{U}	An uncertainty set of a parameter.
\mathcal{U}^{box}	A box uncertainty set of a parameter.
\mathcal{U}^{ell}	An ellipsoid uncertainty set of a parameter.
Space	
V	A vector space.
Variables	
CS_{jk}	The consumer surplus of resource product j at market k .
d_{jk}	Demand of resource j at market k .
p_{0jm}^{in}	The transaction price between resource owner (i, n) to producer (j, m) .
p_{1s}^{jm}	The transaction price between resource producer (j, m) to supplier (j, s) .
p_{2tk}^{js}	The sales price by supplier (j, s) for market k via transportation mode t .
p_{3tk}^{js}	The transaction price at market k from supplier (j, s) via transportation mode t .
p_{mn}	The sales price of virgin REE materials from supplier m to chip producer n .
p_{nk}	The sales price of chips from producer n to demand market k .
p_{ki}	The buy-back price of used products from market k to recycle center i .
p_{in}	The sales price of recycled REE materials from recycle center i to chip producer n .
p_k	The demand price of new chips at market k , grouped by the column vector p .
SW	Social welfare of the entire supply chain network.
x_{jm}^{in}	The flow between resource owner (i, n) and producer (j, m) .
x_s^{jm}	The flow between resource producer (j, m) and supplier (j, s) .

x_{tk}^{js}	The flow between resource supplier (j, s) and market k via transportation mode t .
x^{in}, x_{jm}, x^{js}	The column vectors that collect $x_{jm}^{in}, x_s^{jm}, x_{tk}^{js}$, respectively.
x_{mn}	The non-negative flow of REE raw materials from supplier m to chip producer n , grouped by column vectors $Q^1 \in \mathbb{R}_+^{MN}$, $x_m \equiv (x_{m1}, \dots, x_{mn}, \dots, x_{mN}) \in \mathbb{R}^N$, or $x^n \equiv (x_{1n}, \dots, x_{mn}, \dots, x_{Mn}) \in \mathbb{R}^M$.
x_{nk}	The non-negative flow of semiconductor chips from producer n to demand market k , grouped by column vector $Q^2 \in \mathbb{R}_+^{NK}$.
x_{ki}	The non-negative flow of used chips from market k to recycle center i , grouped by column vectors $Q^4 \in \mathbb{R}_+^{KI}$, or $x_i \equiv (x_{1i}, \dots, x_{ki}, \dots, x_{Ki}) \in \mathbb{R}^K$.
x_{in}	The non-negative flow of recycled REE materials from recycle center i to chip producer n , grouped by column vectors $Q^3 \in \mathbb{R}_+^{IN}$, or $x_n \equiv (x_{1n}, \dots, x_{in}, \dots, x_{In}) \in \mathbb{R}^I$.
v_{ms}	The auxiliary variable corresponding to the conic constraint (5.13), grouped by column vector v .
\bar{v}_m	The auxiliary variable corresponding to the constraint (5.12), grouped by column vector \bar{v} .
δ_g^{in}	Resource owner (i, n) 's excess of output quantity to bracket g .
δ_g^{jm}	Resource producer (j, m) 's excess of output quantity to bracket g .
λ_i^0	The Lagrange multipliers associated with constraint (4.9).
λ_{jm}^1	The Lagrange multipliers associated with constraint (4.18).
λ_{js}^2	The Lagrange multiplier associated with constraints (4.24).
λ_t^0	The Lagrange multipliers associated with constraint (5.11), grouped by column vector $\lambda^0 \in \mathbb{R}_+^T$.
λ_{ms}^1	The Lagrange multipliers associated with constraint (5.12), grouped by column vector $\lambda^1 \in \mathbb{R}_+^{MSM}$.
λ_n^2	The Lagrange multiplier associated with constraints (5.15), grouped by column vector $\lambda^2 \in \mathbb{R}_+^N$.

λ_i^3	The Lagrange multipliers associated with constraint (5.17), grouped by column vector $\lambda^3 \in \mathbb{R}_+^I$.
λ_k^4	The Lagrange multipliers associated with constraint (5.23), grouped by column vector $\lambda^4 \in \mathbb{R}_+^K$.
μ_{ing}^0	The Lagrange multipliers associated with constraint (4.10).
μ_{jmg}^1	The Lagrange multipliers associated with constraint (4.19).
$\pi_{in}, \pi_{jm}, \pi_{js}$	The profit of resource owner (i, n) , producer (j, m) , and supplier (j, s) , respectively.

Chapter 3

STRATEGIC INVENTORIES IN A SUPPLY CHAIN WITH DOWNSTREAM COMPETITION: A TWO-PERIOD DYNAMIC GAME

3.1 Introduction

In business operations at the firm level, certain modern inventory management practices, e.g., Just-in-Time inventory, are associated with the shortage risks in supply chains (Gereffi, 2020; Park et al., 2020; Zhu et al., 2020). In this chapter, I consider a quantitative vertical supply chain model, in which a singular manufacturer supplies products to a pair of identical retailers in Cournot-duopolistic competition. The focus is placed on the game-theoretic aspect of the behaviors of the manufacturer and suppliers. This chapter is derived from the paper by Hu et al. (2021)¹.

The central insight from this chapter is that allowing inventory remains an effective strategy in a retailer duopoly for not only the retailers, but also the manufacturer, consumers, and the entire society; it benefits all parties' welfare. However, the effectiveness of strategic inventory appears to be vulnerable in two aspects. First, too high of a holding cost will discourage the retailers to carry

¹I thank Drs. Jaejin Jang, Nabeel Hamoud, and Amirsaman H. Bajgiran, who co-authored this paper, and in turn, contributed to this chapter.

inventories. Second, the manufacturer’s employment of a commitment wholesale contract could eliminate the retailer’s strategic inventory, although such an outcome would result in all parties in the supply chain being worse-off. I explain the retailer’s vulnerability by means of the Lerner Index in a static market. Moreover, it is uncovered that compared to a retailer monopoly, the Cournot-like competition between the downstream retailers undermines the aggregated profit of retailers even when they opt to carry inventory, despite that such competition benefits all other parties, the channel, and the society.

The rest of this chapter is organized as follows. Section 3.2 describes the problem and presents the models. Section 3.3 discusses the result of the equilibrium analysis. Section 3.4 highlights the key findings and concludes the chapter.

3.2 The Two-period Dynamic Model

I begin by constructing a supply chain model, in which a singular manufacturer attempts to sell his products to a market via two retailers. In the model, the manufacturer determines the wholesale price of the product and possesses complete bargaining power over the retailers. The two retailers compete with one another as Cournot duopolists, i.e., their choice of output quantities influences the market price. I stipulate the manufacturer to be upstream and the retailers to be downstream in this vertical supply chain. Two time periods are considered. I allow the retailers to store goods from one period to the next in the form of inventories, and assume the goods are non-perishable. I study the effect of inventories by examining the economic measures of the manufacturer, retailers, consumers, the supply chain, and society.

In the initial model, the economic activities manifest as follows. At the beginning of period 1, the manufacturer announces a linear contract with a wholesale price of w_1 per unit to the retailers, assuming all production cost is normalized to 0. Upon observing this information, retailer j ($j = 1, 2$) determines and purchases the product of quantity $q_{1j} + I_j$ from the manufacturer, selling q_{1j} to the market, and carrying the remainder I_j as inventory to period two, incurring an aggregated unit inventory holding cost, h per unit per period. I assume such cost is only incurred by the retailers in the period when the inventory is purchased (i.e., period 1). In period two, the manufacturer again announces another linear contract of the identical goods with wholesale price

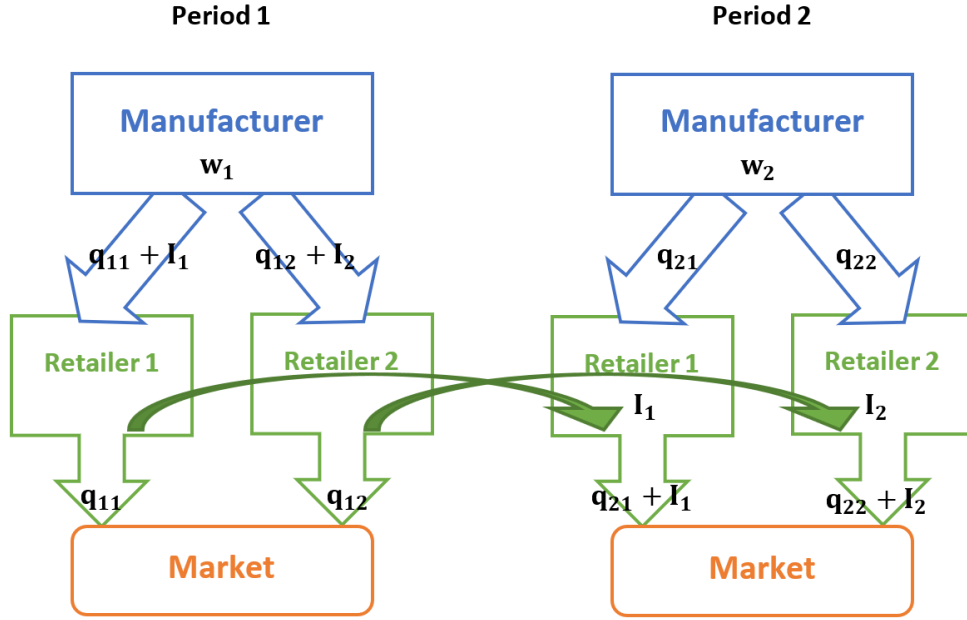


Figure 3.1: The two-period supply chain model with strategic inventories

w_2 to the retailers. Retailer j orders a quantity of q_{2j} , and sells $q_{2j} + I_j$ to the market in entirety. The sequence of events is depicted in Figure 3.1.

The retail price of each period is determined by a linear inverse price-demand function. That is, the unit price of a product sold in the market is $p(Q) = a - bQ$, where “ a ” is the reservation price, an exogenous and strictly positive value. I assume the market demand for the product to be sufficiently large, two retailers to be identical, the interest rate and the depreciation rate of the product in inventory to be reflected in the aggregated unit inventory holding cost rate, h .

Both the manufacturer and the retailers attempt to maximize the profit of their own over the entire time horizon: the first and second periods. In each period, the manufacturer’s decision is his wholesale pricing strategy, whereas each retailer’s decision includes her procurement and inventory quantities. Intuitively, for the manufacturer, a higher wholesale price will directly lead to higher revenue, but will also discourage retailers’ procurement via suppressing consumer demand. For the retailers, a larger procurement may benefit the total revenues and will certainly benefit the manufacturer, but could also lower the market price and hurt the retailers’ own profit. Moreover, an appropriate inventory will serve in the retailer’s favor if the wholesale price rises, but may also undercut the market price, leading to a decreased profit. Carrying inventory will certainly hurt the manufacturer in the event when the manufacturer intends to raise the wholesale price in the next

period. On balance, each firm's maximum profit would not be solely driven by its own decisions, but also by any other firms' action comprehensively.

Obviously, the inventory holding cost h is a drain of profit for retailers; if h is too high, holding inventory may hurt the retailer's profitability and undermine the retailer's motivation to adopt such a strategy. Therefore, I first base my main findings on the assumption of holding cost being $0 < h < a/4$. In §3.3.4, more discussion is provided on this assumption and explore its impact on each firm's decisions.

In the following two subsections, a game-theoretical model with two different types of manufacturer wholesale contracts is constructed. In §3.2.1, I consider an initial model, in which case, the manufacturer employs a dynamic wholesale contract. Under this contract, the wholesale price is revealed only at the beginning of each respective period. In §3.2.2, the initial model is altered by considering a commitment wholesale contract, in which case, the manufacturer announces the wholesale prices for both subsequent time periods at the beginning of period 1. The questions I attempt to answer are: what might be each firm's best practice? How sensitive are strategic inventories with respect to the cost of carrying them? Compared to monopoly, how does the Cournot duopoly in downstream impact the decisions and profit of the manufacturer and retailers? Do the entire supply chain and society do better or worse with different wholesale contracts by the manufacturer and the corresponding inventory strategies by the retailers?

3.2.1 Dynamic Wholesale Contract

This subsection examines a dynamic contract case, in which the manufacturer reveals the wholesale contract of each period to the retailers only at the beginning of such period. When the retailers observe the contractual information (i.e., the wholesale price), they proceed to determine their purchase and sales quantities to maximize their profits.

Since all goods will be sold out to the market by the end of period 2, retailer j 's profit function in period 2 is

$$\pi R_{2j}(q_{2j}) = [a - b(q_{21} + q_{22} + I_1 + I_2)](q_{2j} + I_j) - w_2 q_{2j}, \quad (3.1)$$

and her problem becomes

$$\max_{q_{2j} \geq 0} : \pi R_{2j} \quad (3.2)$$

At the beginning of period 2, the manufacturer's profit is

$$\pi M_2(w_2) = w_2(q_{21} + q_{22}) \quad (3.3)$$

and his problem becomes

$$\max_{w_2 \geq 0} : \pi M_2 \quad (3.4)$$

In period 1, retailer j's profit is,

$$\pi R_{1j}(q_{1j}, I_j) = [a - b(q_{11} + q_{12})]q_{1j} - w_1(q_{1j} + I_j) - hI_j \quad (3.5)$$

and her problem in period 1 becomes

$$\max_{q_{1j} \geq 0, I_j \geq 0} : \pi R_{1j} + \pi R_{2j} \quad (3.6)$$

At the beginning of period 1, the manufacturer's profit is

$$\pi M_1(w_1) = w_1(q_{11} + q_{12} + I_1 + I_2)\pi R_{1j} + \pi R_{2j} \quad (3.7)$$

Similarly, the manufacturer's decision problem is

$$\max_{w_2 \geq 0} : \pi M_1 + \pi M_2 \quad (3.8)$$

I solve the problem by backward induction reasoning, acknowledging the equilibrium obtained for the entire stage game is subgame-perfect (Selten, 1975).

Lemma 1 establishes the unique subgame-perfect Nash equilibrium (SPNE) under the dynamic contract setting.

Lemma 1 *When the inventory holding cost is high ($\frac{a}{12} \leq h < \frac{a}{4}$), the unique SPNE in the manu-*

facturer's pricing and retailers' quantity is²

$$\begin{aligned} \{^D\hat{w}_1^*, ^D\hat{w}_2^*\} &= \left\{ \frac{a}{2}, \frac{a}{2} \right\}, \\ \{^D\hat{q}_{11}^*, ^D\hat{q}_{12}^*, ^D\hat{q}_{21}^*, ^D\hat{q}_{22}^*, ^D\hat{I}_1^*, ^D\hat{I}_2^*\} &= \left\{ \frac{a}{6b}, \frac{a}{6b}, \frac{a}{6b}, \frac{a}{6b}, 0, 0 \right\}. \end{aligned}$$

When the inventory holding cost is low ($h < a/12$), the unique SPNE is

$$\begin{aligned} \{^Dw_1^*, ^Dw_2^*\} &= \left\{ \frac{3}{124(21a - 4h)}, \frac{1}{124(55a + 84h)} \right\}, \\ &\quad \left\{ ^Dq_{11}^*, ^Dq_{12}^*, ^Dq_{21}^*, ^Dq_{22}^*, ^DI_1^*, ^DI_2^* \right\} \\ &= \left\{ \frac{61a + 12h}{372b}, \frac{61a + 12h}{372b}, \frac{55a + 84h}{372b}, \frac{55a + 84h}{372b}, \frac{7(a - 12h)}{186b}, \frac{7(a - 12h)}{186b} \right\}. \end{aligned}$$

The derivation is included in the paper by [Hu et al. \(2021\)](#). Lemma 1 characterizes that the SPNE essentially depends upon the retailer's holding cost. I observe that $\{^Dw_1^*, ^Dw_2^*\}$ and $\{^Dq_{11}^*, ^Dq_{12}^*, ^Dq_{21}^*, ^Dq_{22}^*, ^DI_1^*, ^DI_2^*\}$ collapse into $\{^D\hat{w}_1^*, ^D\hat{w}_2^*\}$ and $\{^D\hat{q}_{11}^*, ^D\hat{q}_{12}^*, ^D\hat{q}_{21}^*, ^D\hat{q}_{22}^*, ^D\hat{I}_1^*, ^D\hat{I}_2^*\}$, respectively, when $h = a/12$. I also observe $^Dw_1^*$ is strictly greater than $^Dw_2^*$ when $h < a/12$; that is, when holding cost is low enough, each retailer will still carry inventory even with an anticipated manufacturer's wholesale price declining between periods—a phenomenon of strategic inventory case portrayed by previous studies ([Anand et al., 2008](#); [Arya and Mittendorf, 2013](#)).

The intuition primarily lies in the action taken by the market-power-dominating manufacturer on the retailers' response. Let us consider two inventory holding cost levels. When the holding cost is high, the retailer's best practice is to avoid carrying inventory between periods, because the holding cost is a drain of the retailer's profit. Anticipating this action, the manufacturer sets up a uniform wholesale price between two periods, since an unequal inter-period wholesale price will eliminate sales in the high-price period.

When the holding cost is low, clearly each retailer is able to build up inventories needed with less cut to her profit. The retailer is, in fact, motivated to do so, because carrying inventories from period 1 could coerce the manufacturer to price down in period 2 by virtually introducing a "competitor". Thus, a delicate decision left for the manufacturer here is his period-1 wholesale

²In this chapter, I use left-superscript D and C to denote dynamic and commitment contract, right-superscript Mon and Duo to denote retailer monopoly and duopoly, and $\hat{\cdot}$ to denote high inventory holding cost, hereafter.

price. Such price needs to be high enough to preclude the retailers from carrying too much inventory, but not too high for themselves to become unprofitable. In other words, the rationality for the manufacturer's declining wholesale pricing strategy is that such strategy is the manufacturer's attempt to discourage the retailers' motivation to build up stocks from period 1. In §3.3, I justify my claims with respect to the firms' market power and sensitivity to holding cost by analytical evidence.

Conversely, consider the market-power-dominated retailers' response to the manufacturer's price offerings. If the manufacturer offers a uniform or increasing inter-period contract, each retailer in response would relinquish inventory, achieving JIT. If the manufacturer offers a declining inter-period contract, each retailer would then choose to hold inventory as a threat to mitigate the market power of the monopolistic manufacturer in period 2. Clearly, such a threat cannot be credible unless holding inventory is not too costly. [Desai et al. \(2010\)](#) observe the similar inventory-carrying behavior from the retailers with respect to the holding cost.

3.2.2 Commitment Wholesale Contract

In this subsection, I explore another scenario, in which the manufacturer employs a commitment contract, i.e., at the onset of the first period, the manufacturer announces the wholesale prices for both subsequent periods. Here, the commitment contract resembles a future contract, a type of contract used by wholesalers in order to hedge the volatility of future market prices. Similar to the previous model, retailer j 's profit in period 2 is given by equation (3.1), and her procurement problem is (3.2). In period 1, retailer j 's profit is given by equation (3.5), and her procurement and inventory problem becomes (3.6). The manufacturer's profit in each period is given by equation (3.7) and (3.3), respectively. At the beginning of period 1, the manufacturer shall determine both w_1 and w_2 . Hence his production problem becomes

$$\max_{w_1 \geq 0, w_2 \geq 0} : \pi M_1 + \pi M_2 \quad (3.9)$$

I again solve the problem by backward induction. Lemma 2 establishes the unique SPNE under the commitment contract.

Lemma 2 *When the manufacturer exercises a commitment wholesale contract, the unique SPNE in the manufacturer's pricing and retailers' quantity is*

$$\begin{aligned} \{^C w_1^*, ^C w_1^*\} &= \left\{ \frac{a}{2}, \frac{a}{2} \right\}, \\ \{^C q_{11}^*, ^C q_{12}^*, ^C q_{21}^*, ^C q_{22}^*, ^C I_1^*, ^C I_2^*\} &= \left\{ \frac{a}{6b}, \frac{a}{6b}, \frac{a}{6b}, \frac{a}{6b}, 0, 0 \right\}. \end{aligned}$$

The derivation is included in [Hu et al. \(2021\)](#). Lemma 2 characterizes that the manufacturer's commitment contract eliminates retailers' necessity of carrying inventory regardless of the inventory carrying cost, h . Such a contract ascertains the retailer's response to be JIT in the sense of putting forward pricing information in exchange for consistent periodic procurements.

3.3 Results and Findings

In this section, I compare the dynamic and commitment contract under a retailer duopoly; I also compare each party's behaviors under such setting against those under retailer monopoly. I focus on performance measures of each firm in the supply chain, and also on the total profit of the supply chain, consumer surplus, and social welfare.

The consumer surplus (CS) of period i ($i=1,2$) is given by

$$CS_i = \frac{1}{2}(a - p_i)q_i, \quad (3.10)$$

where q_i and p_i represent the market quantity and price, respectively.

The channel profit of each period is the sum of the manufacturer and retailers' profits, given by

$$\pi_i = \pi M_i + \pi R_{i1} + \pi R_{i2}. \quad (3.11)$$

Also, I denote social welfare (SW) as the accrual of CS and channel profit in each period.

$$SW_i = CS_i + \pi_i. \quad (3.12)$$

3.3.1 Dynamic vs. Commitment Contracts

In this subsection, I compare the impact of the manufacturer's wholesale contractual strategies on the supply chain.

Proposition 5 establishes a pair of necessary-and-sufficient conditions in order for a given retailer's strategic inventory to be invoked.

Proposition 5 *The retailers' strategic inventory exists when and only when: 1) the manufacturer employs a dynamic wholesale contract, and 2) the retailer's inventory carrying cost is less than $a/12$.*

The first condition stipulates the existence of strategic inventory to the manufacturer's contractual strategy, an exogenous criterion of the retailers, whereas the second condition requires the maximum threshold of the cost in order for holding the inventories to be feasible, an endogenous criterion of the retailers. The proof of Proposition 5 is immediate from Lemma 1 and Lemma 2.

Moreover, because the manufacturer's strategy-set in the presented framework is between the two types of contracts, an important implication from the first condition is that the manufacturer's commitment contract eliminates the retailer's strategic inventory. Clearly, the commitment contract in this setting is essentially a layout of complete information concerning the wholesale price across the entire time horizon. Such information leads to a uniform wholesale price between periods and obsoletes the inventories carried by the retailers, because carrying inventory undoubtedly would result in additional costs and harm the retailers' profitability. Next, I discuss how strategic inventories impact each supply chain agent's performance. The most interesting case is when the retailers' strategic inventories are triggered. Acknowledging Proposition 5 and Lemma 1, I shall be able to carry out the desired comparison by setting the inventory holding cost to be low, i.e., $0 < h < a/12$. Then the presence of strategic inventories will solely depend on the wholesale contractual strategies adopted by the manufacturer. Proposition 6 establishes the strategic inventory's advantages delineated in Table 3.1 in a conclusive manner. Its proof is immediate.

Proposition 6 *In a Cournot duopoly of retailers, the manufacturer, retailers, and consumers are all better off under a dynamic contract.*

Table 3.1 displays the selected performance comparisons and outcomes between the dynamic

contract and commitment contract. The complete results from a more generalized setting are shown in Table A.1 of Appendix A.

Table 3.1: Retailer duopoly: commitment vs. dynamic contract under low holding cost

		Dynamic		Commitment
Wholesale Price	w_{avg}	$\frac{125a^2 - 24ah + 144h^2}{4(65a - 36h)}$	<	$\frac{a}{2}$
Retail Price	p_{avg}	$\frac{7849a^2 - 1632ah - 3600h^2}{186(65a - 36h)}$	<	$\frac{2a}{3}$
Consumer Purchase Qty	Q_{total}	$\frac{65a - 36h}{93b}$	>	$\frac{2a}{3b}$
Inventory	I	$\frac{7(a - 12h)}{186b}$	>	0
Retailer (ea.) Sales	q_{total}	$\frac{65a - 36h}{186b}$	>	$\frac{a}{3b}$
Retailer (ea.) Profit	πR_{total}	$\frac{4073a^2 - 3636ah + 10656h^2}{69192b}$	>	$\frac{a^2}{18b}$
Manufacturer Profit	πM_{total}	$\frac{125a^2 - 24ah + 144h^2}{372b}$	>	$\frac{a^2}{3b}$
Supply Chain Profit	π_{total}	$\frac{7849a^2 - 2934ah + 12024h^2}{17298b}$	>	$\frac{4a^2}{9b}$
Consumer Surplus	CS_{total}	$\frac{4241a^2 - 5064ah + 3600h^2}{34596b}$	>	$\frac{a^2}{9b}$
Social Welfare	SW_{total}	$\frac{19939a^2 - 10932ah + 27648h^2}{34596b}$	>	$\frac{5a^2}{9b}$

Several observations can be made. First, one notices that with the dynamic contract, the first (second) period wholesale price is higher (lower) than the respective prices with the commitment contract; i.e., $^D w_1 > ^C w_1 = ^C w_2 > ^D w_2$. The intuition for the decreasing wholesale price trend under dynamic contract is that, in period 1 the manufacturer is capable of capturing as much profit as possible by pricing the products high, knowing that during this sales season no inventory from the prior period exists. The manufacturer also anticipates with certainty that the retailers will carry inventory for the next period since the wholesale prices are not below the inventory-eliminating level $a/12$ for both periods. Hence, in order to discourage the retailer's credible threat to the manufacturer's monopoly power by stocking up products for the next sale period, the manufacturer must increase the price in period 1 and decrease the price in period 2.

Second, one can observe that the average retail price with the dynamic contract is lower than with the commitment contract, i.e., $^D p_{avg} < ^C p_{avg}$, where the average prices are weighted by the total output (i.e., consumer purchase) quantity in each period. Because the manufacturer and each of the retailers is a self-interest agent in a single vertical supply chain system, the product's price has to experience markup twice before reaching the market, resulting in a higher sale price and eroding the consumer surplus. Such phenomenon is called double marginalization (Spengler, 1950). Nonetheless, in this retailer duopoly framework, the dynamic contract and its resultant strategic inventories could ameliorate the losses due to double marginalization. I hence claim that strategic

inventory reduces the double-marginalization effect.

Third, as is evident, one can recognize that a dynamic contract is superior to a commitment contract for both the manufacturer and retailers, as well as the entire channel, and society. This finding is consistent with the work of [Desai et al. \(2010\)](#), when the retailers engage a general level of competition with one another. The rationality is that the wholesale price committed in advance by the manufacturer undercuts the retailers' market power and hence distorts each retailer's best response for a given time horizon, in that commitment contracts eliminate strategic inventories. Only a dynamic contract will trigger the retailer's strategic inventory that restores the fare loss caused by the committed wholesale prices.

To summarize this subsection, for each individual supply chain party to achieve a higher profit by invoking retailers' strategic inventories, both the manufacturer and retailers must collectively comply with the necessary conditions established by [Proposition 5](#). Furthermore, [Proposition 5](#) in part, also underscores how the manufacturer as a dominant player is capable of exploiting his counterparts, i.e., the retailers, because of his strictly dominant market power. In other words, the manufacturer's choice of contract could single-handedly dictate the retailers' payoff. However, it is imperative to recognize that the manufacturer's ability to eliminate the effective inventory strategy of his counterparts does not necessarily suggest the manufacturer's motivation to do so. Rather, the outcome of such a course of action would result in both parties and the entire system worse off, according to [Proposition 6](#).

It is also crucial to note that the previous study conducted by [Anand et al. \(2008\)](#) has demonstrated that, in a vertical supply chain as a whole of a singular manufacturer and a singular retailer, both firms individually and the system will do strictly better with a dynamic contract than with commitment contract, if holding inventory is not too costly. My findings show strong consistency in terms of the dynamic contract and strategic inventory's economic and social advantages. Therefore, in greater generality, I conclude that the dynamic contract and strategic inventory Pareto-dominates the commitment contract for all supply chain parties, the channel, and the society.

3.3.2 Downstream Monopoly vs. Duopoly

In this subsection, I focus on the performances of each supply chain entity and the whole system for Monopolistic and Duopolistic retailer cases. The results from the retailer Duopoly are analogous

to those of retailer Monopoly by [Anand et al. \(2008\)](#).

Table [A.2](#) first compares the results of a supply chain with a Monopolistic retailer against a pair of Duopolistic retailers when the inventory holding cost is sufficiently low, i.e., $h < a/12$, in which case, strategic inventory will be triggered according to Proposition [5](#). First, I find the downstream Duopoly suppresses the double marginalization effect, i.e. ${}^D p_{avg}^{Duo} < {}^D p_{avg}^{Mon}$. Second, noticeably I find the manufacturer's total profit is increased by the Duopolistic retailers, i.e. ${}^D \pi M_{avg}^{Duo} > {}^D \pi M_{avg}^{Mon}$. The most pronounced result is the transfer of the retailer's profit to the manufacturer, as I notice ${}^D \pi R_{total}^{Duo} < 2{}^D \pi R_{total}^{Duo} < {}^D \pi R_{total}^{Mon}$; that is, when an incumbent Monopolistic retailer is joined by her peer to form a Duopoly, not only would her profit be drastically shrinking, but also the total downstream profit of the Duopolists.

Why is the downstream competition culpable for the retailers? The rationale can be three-fold. Consider first the Cournot competition between the two retailers. The Cournot model stipulates that each competitor's only strategy is to choose her outputs during the selling season. Consequently, neither retailer could cut the output in order to capture more consumer surplus, as doing so will benefit her counterpart while marginalizing her own gain.

Consider next each retailer's market-power shift from the Monopoly to the Duopoly formed. Because of the presence of one additional player in the system, a shrinkage of the incumbent retailer's market power is the only plausible result, which will be further illustrated in the next subsection. In particular, to counteract the manufacturer, a Duopolistic retailer has a narrower range of output choice, since the output cut that would have been effective for her in Monopoly is now reprehensive. Hence, as each Duopolist's market power has weakened, the dynamic contract becomes too powerful for retailers to counteract. In terms of a retailer's profit, the formation of Duopoly diminishes the benefit of the strategic inventory.

Lastly, one shall not neglect the retailer's alternative strategy. In other words, could any retailer in Duopoly have done differently to ameliorate her profit? The answer is no, in that the subgame-perfect equilibrium, in this case, is also Pareto-dominating; any deviated strategy for the retailer, i.e., carrying no inventory would be rendered worse off. (See Table 3 and Table 4 in [Hu et al. \(2021\)](#).)

Further observations from Table [A.2](#) indicate that despite the shrinking profit of the Duopolistic retailers compared to that of the Monopolist, the channel profit can still benefit from the down-

stream competition, since ${}^D\pi_{total}^{Duo} > {}^D\pi_{total}^{Mon}$. This result further suggests that the manufacturer is the major gainer from such competition and that such gain can be viewed as retailers' loss of surplus transferred from downstream to upstream. Furthermore, I find that the social welfare is also improved by the downstream competition, as ${}^D SW_{total}^{Duo} > {}^D SW_{total}^{Mon}$.

Table A.3 next compares the result of a supply chain with a retailer Monopoly and Duopoly when the inventory holding cost is high, i.e., $a/12 < h < a/4$. Notice, even within different h range, the measures in Table A.3 are identical to those in Table A.2 for Monopoly model. Table A.4 compares the outcomes from a Monopolistic retailer against a pair of Duopolistic retailers under a commitment contract. Such a strategy eliminates the downstream retailer's inventory according to Proposition 5. The comparisons in Table A.3 and Table A.4 indicate a strong similarity to my findings from Table A.2. Hence, I conclude in broader generality that the downstream retailer's competition (under both contract types) reduces the average product price by weakening the double-marginalization effect, improves the manufacturer's profit, undermines the retailers' profit, and benefits the entire supply chain's profit and the social welfare.

3.3.3 Retailer's Market Power

In this subsection, I discuss how carrying strategic inventory affects each player's market power in the supply chain model presented. A key measure to a firm's market power in a static market is Lerner Index (Lerner, 1934), defined as the margin between the price and the marginal cost, i.e., $L = (P - MC)/P$, where P is the price of a product and MC is its marginal cost.

In this chapter, all Lerner Indices are computed by using the weighted average of retail and wholesale prices from each given setting. It is straightforward to show that the manufacturer's monopoly power is 1, a constant that is independent of strategic inventory carried by the retailers, indicating the manufacturer's unwavering dominance when downstream retailers exercise inventory strategies. The more delicate result is the market power change of each retailer due to the change of inventory holding cost, h , shown in Table 3.2. Particularly, when $h \geq a/12$ the indices collapse into those of the commitment contract, according to Lemma 2.

First, I compare the market power of a single retailer of duopoly against that of monopoly (Anand et al., 2008) under each wholesale contract. For simplicity, I limit the holding cost to $0 < h < a/12$ in a dynamic contract situation, despite that a monopolistic retailer's strategic

Table 3.2: The retailer's Lerner index ($h \leq a$)

	Dynamic (D)	Commitment (C)
Duopoly (Duo)	$\frac{4073a^2-1032ah-20592h^2}{2(7849a^2-1632ah-3600h^2)}$	1/4
Monopoly (Mon)	$\frac{155a^2+52ah-376h^2}{461a^2-84ah-104h^2}$	1/3

inventory will still be triggered when $a/12 < h < 4/a$. Not surprisingly, I find each retailer's market power diminishes as the number of identical retailers increases, i.e., $^DL^{Mon} > ^DL^{Duo}$ and $^CL^{Mon} > ^CL^{Duo}$, notwithstanding the upstream manufacturer's choice of contract.

Next, I compare each retailer's market power under the dynamic contract against the commitment contract in each type of downstream competition. Recall Proposition 5, to invoke strategic inventory, the retailers must maintain the holding cost sufficiently low, i.e., $0 < h < a/12$. Since $^DL^{Duo} > ^CL^{Duo}$, and $^DL^{Mon} > ^CL^{Mon}$, I conclude that the dynamic contract and its resultant strategic inventory strengthen each retailer's market power.

Furthermore, one observes that each retailer has a different inventory holding cost threshold in monopolistic and duopolistic downstream competition; any holding cost of $h \geq a/4$ would eliminate strategic inventory for the monopolistic retailer, whereas any holding cost $h \geq a/12$ would do so for a duopolistic retailer, as was also established by Lemma 1. Hence, I rewrite the Lerner Index for each competition under the dynamic contract in the following function with respect to the holding cost ratio, h/a .

$$^DL^{Duo}(x) = \begin{cases} \frac{4073-1032x-20592x^2}{2(7849-1632x-3600x^2)}, & 0 < x < \frac{1}{12} \\ \frac{1}{4}, & x \geq \frac{1}{12} \end{cases} \quad (3.13)$$

where, $x = \frac{h}{a}$.

For both monopoly and duopoly, the Lerner Index of a retailer at equilibrium reaches the minima and sustains such levels under one of the following two conditions: (a) when holding inventory is too costly, or (b) when the upstream manufacturer adopts a commitment wholesale contract, as is evident in Proposition 5. I graph the Lerner Indices in both cases in Figure 3.2.

3.3.4 Inventory Holding Cost

In the previous subsection, I have compared and demonstrated the superiority of dynamic contract and strategic inventory in bolstering the supply chain's welfare. In this subsection, I will discuss

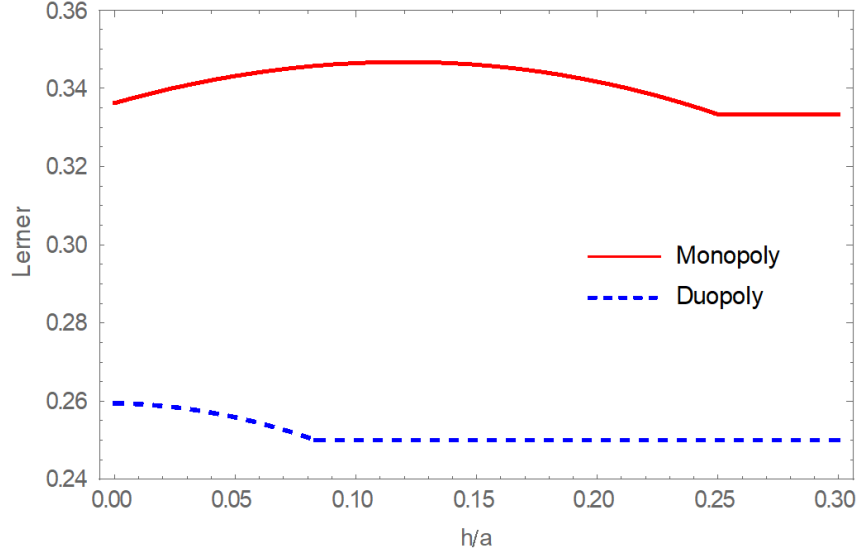


Figure 3.2: The retailer’s market power under strategic inventories

the impact of inventory holding cost rate, h , to both theoretical and practical extent.

Both Proposition 5 and Anand et al. (2008) indicate that the inventory holding cost rate is important to the system’s SPNE. Although Anand et al. (2008) and Arya and Mittendorf (2013) consider the value of h in the assumed range of $0 < h < a/4$, little literature so far has proffered analytical insights for the choice of this holding cost range. A further exploration of Anand et al. (2008) shows that when such an assumption is relaxed, a similar pattern of equilibrium solution resembling Lemma 1 emerges in the retailer’s monopoly model. (See Appendix A.) When $h \geq a/4$, the SPNE under dynamic contracts becomes identical to what is under commitment contracts. In other words, the elimination of the inventory becomes a self-enforcing agreement. I conjecture that in any given vertical supply chain model with one manufacturer and any number of retailers, there exists a critical value, c_0 , a *Costliness Threshold*, such that when $h/a \geq c_0$, a downstream retailer carries no inventory. The ratio h/a , which is also frequently referred to in this study, can be interpreted as the costliness threshold of holding inventories.

In §3.3.3, I have shown that the costliness threshold dictates the market power of a retailer, as is depicted in Figure 3.2. In a retailer monopoly setting, c_0 holds at $1/4$, whereas in a retailer duopoly setting, c_0 holds at $1/12$. Furthermore, similar to Table 3.1 in §3.3.2, Table A.1 in Appendix A can also be reproduced by setting up the problem with a dynamic contract case under different levels of costliness threshold (i.e., $h/a < 12$ vs. $h/a \geq 12$). Clearly, the costliness threshold of holding

inventories is a key variable that reins in the equilibrium solution.

In practice, when it comes to supply chain planning, businesses merely rely on heuristics to determine appropriate inventory holding cost range. The norm often links the holding cost to the value of inventory with a ratio. Anecdotal evidence suggests that a 25% of inventory value can be used in most industries as a rule-of-thumb (REM Associates). To evaluate if [Anand et al. \(2008\)](#)'s assumption can be coarsely validated by such evidence, one simply needs to examine the equilibrium solutions. In the supply chain models, the wholesale price w_{avg} appropriately represents the inventory value. Clearly, in the JIT cases, $w_{avg} = a/2$ holds true in both retailer monopoly and duopoly. Thus, 25% w_{avg} yields $a/8$, falling well within the range of $0 < h < a/4$. In a non-JIT (i.e., dynamic contract) case, because the value of w_{avg} and range of holding cost is codependent, gaining insights with respect to the wholesale price and holding cost from these quantities is difficult. Nonetheless, one could still observe that $w_1 > a/2 > w_2$ holds with $h < a/12$ in duopoly, and $h < a/4$ in monopoly.

3.4 Chapter Summary

The principal objective of this chapter is to examine how a retailer's behavior in utilizing inventory as a strategy is affected by and impacts all other parties in a supply chain. My analysis relies on a game-theoretic model, in which a supply chain consists of a singular upstream manufacturer and a pair of identical downstream retailers. I also broaden the scope of the retailer monopoly framework constructed by previous researchers by expanding the retailer's inventory holding cost range and the downstream competition type.

Overall, two perspectives emerge regarding the strategic inventory in the presented retailer duopoly framework. The first perspective concerns the effect of strategic inventories and the conditions under which such inventories exist. I find that the dynamic contract and strategic inventory remains an effective course of action for not only the retailers, but also the manufacturer, consumers, the channel, and the whole society since it Pareto-dominates the commitment contract and JIT operation. However, the existence of strategic inventory requires a pair of necessary-and-sufficient conditions: a relatively low costliness of holding inventory by the retailers, and a dynamic wholesale contract employed by the upstream manufacturer.

The second perspective concerns how the type of downstream competition affects each firm in the supply chain. The most significant finding in such perspective is that the Cournot-like duopoly between the downstream retailers will benefit each individual supply chain party and society as a whole but the retailers themselves when strategic inventories are carried. In essence, the downstream competition “transfers” the profits of retailers to the upstream manufacturer.

Chapter 4

RELIEF AND STIMULUS IN A SCARCE RESOURCE SUPPLY CHAIN NETWORK

4.1 Introduction

In this chapter, I construct a general supply chain network equilibrium model to better understand the scarce resources vis-à-vis conservation, competition, transportation, and policy intervention, by consolidating a resource's innate scarcity and the supply-demand caused shortage (more in §1.2.1). This chapter is derived from the paper by [Hu and Li \(2022\)](#)³.

In this model, the supply chain network incorporates the interdependency of the scarce resources by allowing for cross-resource sector links. With the inclusion of multiple resource products that are differentiated by markets, such a network emulates the scarce resource's heterogeneity feature in a similar approach by [Zhang \(2006\)](#). The interdependency and heterogeneity are two features inspired by [Pfeffer and Salancik \(2003\)](#), and [Hunt \(2000\)](#). Furthermore, in the model, a unifying supply-side fiscal policy scheme in relation to the scarce-resource ownership and production is installed. The efficacy of utilizing incentives and taxes in addressing the supply chain shortage is assessed from an industrial organization's viewpoint. Also, I incorporate a non-cooperative behavior of decision-

³I thank Dr. Peng Li, who co-authored this paper, and in turn, contributed to this chapter.

makers, considering the unique ownership of the resources and the nature of quantity competition under the context of scarcity. It is believed that such behavior is best represented by a decentralized network, a construct known for its generality, efficiency, and ability to synthesize local information in supply chains (see also, [Lee and Billington \(1993\)](#), inter alia). Finally, in illustration, a series of numerical examples, including one that concerns medical PPE supplies, are furnished to illustrate the utility of this model.

4.1.1 Research Gap and Relevance to Literature

While most of the literature in SCM focuses on either a category of general products or one specific type, it remains largely sparse on the interdependence of firms and the specific resource commodities they produce in a cross-sector framework (cf. [Bell et al. \(2012\)](#)). Such a gap can be exemplified by the papers referenced in §1.2.1, as they mainly emphasize one or two specific products. To the best of my knowledge, there is no known study in SCM that explores the characteristics of resource scarcity and the shortage conditions from both economic and management perspectives. A systematic review by [Matopoulos et al. \(2015\)](#) has underscored a need for further research on understanding the implications of resource scarcity for supply chain relationships and their impact on supply chain configurations.

Moreover, the fiscal policy scheme I implement in this model is inclusive of both incentives and taxes. I also set forth the scope of policy calibration as the broader social outcome and firm's collective profits, instead of specific performance measures (e.g., prices and quantities), as what appeared in the referenced papers in §1.2.3. The coalescence of these two aspects has been limited in the theoretical literature on supply chain networks. Such a topic would have provided invaluable managerial insights and should, therefore, merit attention. In illustration, I use [Table 4.1](#) to compare and differentiate this work with other closely related SCNE models mentioned in this section.

The remainder of this chapter is organized as follows. [Section 4.2](#) presents the scarce resource supply chain network model with fiscal policies and its mathematical formulation. In [Section 4.3](#), I show the solution concept of the model. [Section 4.4](#) establishes the theoretical properties of the equilibrium in the context of both classic and novel network game studies. [Section 4.5](#) proposes an algorithmic procedure to find the network equilibrium. [Section 4.6](#) demonstrates the utility

Table 4.1: A comparison of closely related SCNE articles

Article	Multi-product	Oligopolistic Competition	Capacitated	Policy Instrument	
				Single	Unifying
Masoumi et al. (2012)		•			
Besik and Nagurney (2017)		•			
Yu et al. (2018)		•		•	
Yang et al. (2021)		•		•	
Yang et al. (2022)		•		•	
Zhang (2006)	•				
Nagurney and Dutta (2021)	•				
Wu et al. (2006)				•	
Nagurney et al. (2019b)				•	
Wu et al. (2019)				•	
Hu and Qiang (2013)			•		
Nagurney et al. (2017)			•		
This chapter	•	•	•		•

of the model with small-scale illustrative examples. Section 4.7 provides the decision-makers and practitioners with real-world managerial insights. Lastly, Section 4.8 is a highlight of the current study and foresight to its extension.

4.2 The Cross-Sector Multi-Product Scarce Resource Supply Chain Network Model

In the network, there are I types of scarce resources, denoted as $i = 1, \dots, I$. I assign i 's alias j in order to use it in different tiers. Naturally, $j = 1, \dots, I$. Each type of resource is affiliated with N_i owners, M_j producers, and S_j suppliers. The unfinished or end-product throughout the entire supply chain network is assumed to be homogeneous by the types of resources. See Figure 4.1 for the network topology. Later, when referring to a certain type of resource, I will omit the phrase “type” or “type of”.

The first tier of nodes in the network represents the resource owner. For instance, a soybean-producing farmer is an owner of a resource type “soybean products”. As the stakeholder of a scarce resource, an owner may supply such a resource to any producer. A typical resource owner is denoted as (i, n) , where $n = 1, \dots, N_i$. All resource owner of the same type share a common pool of resource with a total amount of U_i .

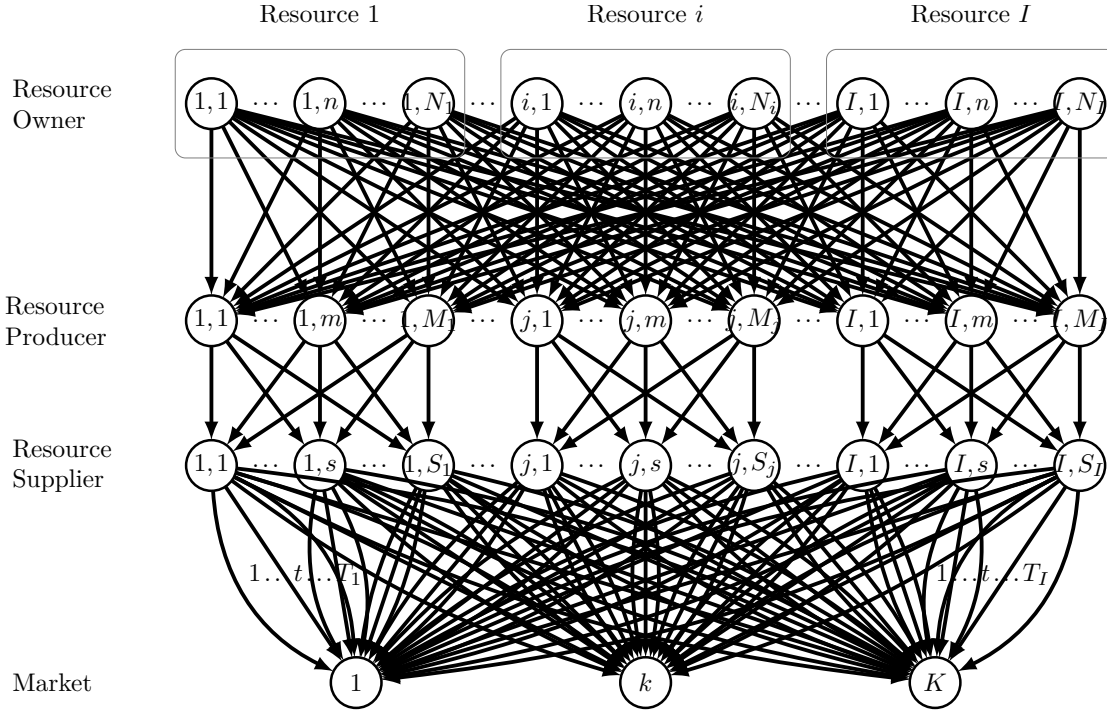


Figure 4.1: The cross-sector scarce resource supply chain network model

The second tier of nodes represents the resource producer. An example of a resource producer can be a food manufacturer who uses the soybeans purchased from the farmers as raw material and produces soybean-related foods. A resource producer is able to obtain any type of resource from any owner, as stated above, but is only able to sell and ship its products to the suppliers who are specialized in the same type of resource. A typical resource producer is denoted as (j, m) , where $m = 1, \dots, M_j$.

The third tier of nodes represents the resource supplier. A resource supplier can only obtain and supply one type of resource to all markets. A soybean supplier, for instance, does not supply any other resources. A typical resource supplier is denoted as (j, s) , where $s = 1, \dots, S_j$.

The fourth tier of nodes represents the physical markets. A typical physical market, consuming some resources from its suppliers, is denoted as k , where $k = 1, \dots, K$. Each supplier has t transportation modes, where $t = 1, \dots, T_j$, to ship a resource product to any physical market. To emulate the heterogeneity of the scarce resources characterized by [Hunt \(2000\)](#), I assume that each market has its own perception of the same resource, but is unable to distinguish the same resource from different suppliers or transportation modes. I assume also all markets to be competitive and, thereby,

price-takers. Figure 4.2 illustrates the proposed model as a two-resource cross-sector supply chain network.

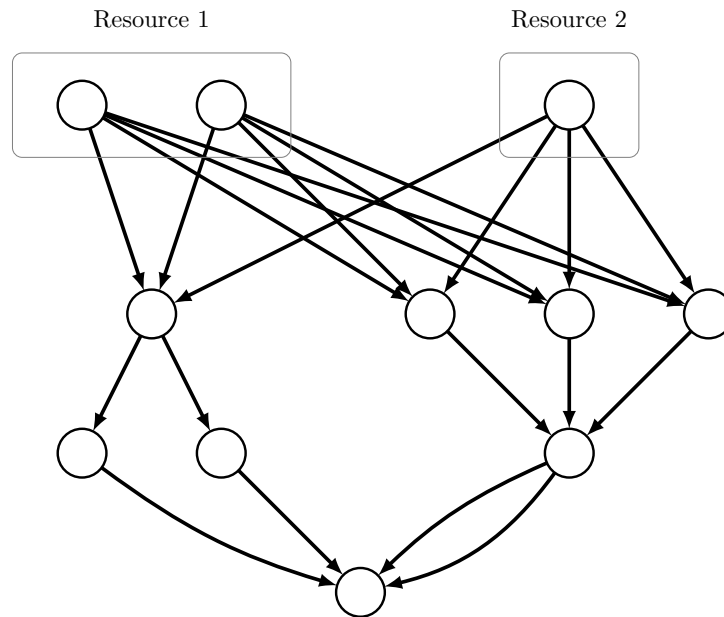


Figure 4.2: An example of the network with $I = 2$, $N_1 = 2$, $N_2 = 1$, $M_1 = 1$, $M_2 = 3$, $S_1 = 2$, $S_2 = 1$, $T_1 = 1$, $T_2 = 2$, $K = 1$

The network is considered to be decentralized, i.e., each firm of a given tier competes with all others in a non-cooperative fashion to maximize its own profit by determining its optimal production quantity and shipments, given the simultaneous best response of all competitors. All of the notations regarding the model are listed in §2.6.

4.2.1 The Fiscal Policy Scheme

For the ease of conceptualization, in this model, I apply a supply-side fiscal incentive to resource owners and producers. I design a piece-wise scheme to represent some of the most commonly used governmental incentive policies. The incentive scheme devised here can easily be expanded to a general fiscal policy to include taxation.

I now define the incentive scheme. Let A_g^i denote the cutoff bracket of the incentive on resource i , with g enumerating between $1, \dots, G$. Let δ_g^{in} denote the resource owner (i, n) 's output quantity excess over the bracket A_g^i . I assume the incentives given to each entity to be a series of linear functions, only affected by its output quantity, as most commonly used governmental economic

policies are (cf. [Sadka \(1976\)](#); [Huck et al. \(2012\)](#); [Yu et al. \(2018\)](#), inter alia). As such, I define the incentive payment function as

$$\alpha_0^i(\cdot) + \sum_{g=1}^G \alpha_g^i(\cdot), \quad (4.1)$$

where $\alpha_0^i(\cdot)$ is usually non-negative, both $\alpha_0^i(\cdot)$ and $\alpha_g^i(\cdot)$ are linear. For an arbitrary output quantity of x , the payment scheme for the resource owners is piece-wise linear in shape and can be expressed as:

If $0 \leq x < A_1^i$, then the incentive payment is $\alpha_0^i(x)$;

If $A_1^i \leq x < A_2^i$, then $\delta_1^{in} = x - A_1^i$, and the incentive payment is $\alpha_0^i(x) + \alpha_1^i(\delta_1^{in})$;

If $A_2^i \leq x < A_3^i$, then $\delta_2^{in} = x - A_2^i$, and the incentive payment is $\alpha_0^i(x) + \alpha_1^i(\delta_1^{in}) + \alpha_2^i(\delta_2^{in})$;

If $A_g^i \leq x < A_{g+1}^i$, then $\delta_g^{in} = x - A_g^i$, and the incentive payment is $\alpha_0^i(x) + \alpha_1^i(\delta_1^{in}) + \alpha_2^i(\delta_2^{in}) + \dots + \alpha_g^i(\delta_g^{in})$;

If $x \geq A_G^i$, then the incentive payment is $\alpha_0^i(x) + \alpha_1^i(\delta_1^{in}) + \alpha_2^i(\delta_2^{in}) + \dots + \alpha_G^i(\delta_G^{in}) = \alpha_0^i(x) + \sum_{g=1}^G \alpha_g^i(\delta_g^{in})$.

Finally, the general incentive payment function of the resource owners in terms of δ_g^{in} can be written as

$$\alpha_0^i(x) + \sum_{g=1}^G \alpha_g^i(\delta_g^{in}), \quad i = 1, \dots, I, \quad n = 1, \dots, N, \quad (4.2)$$

where,

$$\delta_g^{in} = \max\{x - A_g^i, 0\}, \quad g = 1, \dots, G. \quad (4.3)$$

In particular, when $\alpha_0^i(\cdot)$ is set to be a constant and $\alpha_g^i(\cdot)$ to be negative, expression (4.2) represents a regressive incentive.⁴ An example of this incentive scheme is the United States Economic Impact Payments of COVID-19.⁵ Naturally, with the properly chosen parameter values, expression (4.2)-(4.3) can also represent a general tax scheme. I will use this scheme to express the fiscal policies for the rest of the paper. Specifically, in §4.2.3, I will invariably define the incentives administered to the resource producers.

⁴A regressive incentive is one whose marginal rate decreases (cf. [Sadka \(1976\)](#)).

⁵See: <https://www.irs.gov/coronavirus-tax-relief-and-economic-impact-payments>.

4.2.2 Resource Owner's Problem

A typical resource i has a total capacity of U_i , shared by all N_i owners of the same resource. A resource owner (i, n) strategically determines its shipment quantity x_{jm}^{in} to a resource producer (j, m) . I use Q^0 , a column vector with $Q^0 \in R_+^{\sum_{i=1}^I \sum_{j=1}^I N_i M_j}$, to group all x_{jm}^{in} . I assume an owner incurs an operating cost of

$$f^{in} = f^{in}(x^{in}), \quad \forall i, n, \quad (4.4)$$

where $x^{in} \equiv (x_{11}^{in}, \dots, x_{jm}^{in}, \dots, x_{IM_j}^{in})$, and a transaction cost of

$$c_{jm}^{in} = c_{jm}^{in}(x_{jm}^{in}), \quad \forall i, n, j, m. \quad (4.5)$$

The incentive payment received by the resource owner (i, n) is

$$\alpha_0^i \left(\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in} \right) + \sum_{g=1}^G \alpha_g^i (\delta_g^{in}), \quad \forall i, n, \quad (4.6)$$

where, the quantity excess

$$\delta_g^{in} = \max \left\{ \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in} - A_g^i, 0 \right\}, \quad \forall g = 1, \dots, G. \quad (4.7)$$

I group all δ_g^{in} into a column vector \mathfrak{d}^0 , with $\mathfrak{d}^0 \in R_+^{\sum_{i=1}^I GN_i}$, and assume all of the cost functions above, i.e., f^{in} , c_{jm}^{in} , to be continuously differentiable.

The price of products from resource owner (i, n) to producer (j, m) is denoted by p_{0jm}^{in} . I further denote the equilibrium of such a price as p_{0jm}^{in*} . Hence, the total revenue of each owner is $\sum_{j=1}^I \sum_{m=1}^{M_j} p_{0jm}^{in} x_{jm}^{in}$.

As a profit-maximizer, each typical owner (i, n) faces the following problem.

$$\begin{aligned} \text{Maximize: } \pi_{in} = & \sum_{j=1}^I \sum_{m=1}^{M_j} p_{0jm}^{in} x_{jm}^{in} - f^{in}(x^{in}) - \sum_{j=1}^I \sum_{m=1}^{M_j} c_{jm}^{in}(x_{jm}^{in}) \\ & + \alpha_0^i \left(\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in} \right) + \sum_{g=1}^G \alpha_g^i (\delta_g^{in}) \end{aligned} \quad (4.8)$$

$$\text{Subject to: } \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in} \leq U_i, \quad (\lambda_i^0) \quad (4.9)$$

$$\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in} - \delta_g^{in} \leq A_g^i, \quad \forall g, \quad (\mu_{ing}^0) \quad (4.10)$$

$$x_{jm}^{in} \geq 0, \quad \forall j, m, \quad \delta_g^{in} \geq 0, \quad \forall g.$$

The constraint (4.9) is the resource capacity. The constraint (4.10), along with $\delta_g^{in} \geq 0$, is the equivalence of expression (4.7). λ_i^0 and μ_{ing}^0 are the Lagrange multipliers⁶ associated with inequality constraints (4.9) and (4.10), respectively. I subsequently group all λ_i^0 and μ_{ing}^0 into a column vector λ^0 and μ^0 , with $\lambda^0 \in R_+^I$ and $\mu^0 \in R_+^{\sum_{i=1}^I GN_i}$, respectively.

In this decentralized supply chain, the resource owners compete in a non-cooperative fashion, a behavior of classic Cournot-Nash (Cournot, 1838; Nash, 1950, 1951) stating that each owner determines the optimal output, given the simultaneously optimal ones of all competitors. As such, the optimal behavior of all resource owners can be expressed as a variational inequality: determine $(Q^{0*}, \mathfrak{d}^{0*}, \lambda^{0*}, \mu^{0*}) \in \mathcal{K}^1$, satisfying:

$$\begin{aligned} & \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\frac{\partial f^{in}(x_{jm}^{in*})}{\partial x_{jm}^{in}} + \frac{\partial c_{jm}^{in}(x_{jm}^{in*})}{\partial x_{jm}^{in}} - p_{0jm}^{in*} - \frac{\partial \alpha_0^i(x_{jm}^{in*})}{\partial x_{jm}^{in}} + \lambda_i^{0*} + \sum_{g=1}^G \mu_{ing}^{0*} \right] \times (x_{jm}^{in} - x_{jm}^{in*}) \\ & + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[-\frac{\partial \alpha_g^i(\delta_g^{in*})}{\partial \delta_g^{in}} - \mu_{ing}^{0*} \right] \times (\delta_g^{in} - \delta_g^{in*}) + \sum_{i=1}^I \left[U_i - \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in*} \right] \times (\lambda_i^0 - \lambda_i^{0*}) \end{aligned} \quad (4.11)$$

$$+ \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[A_g^i - \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in*} + \delta_g^{in*} \right] \times (\mu_{ing}^0 - \mu_{ing}^{0*}) \geq 0,$$

$$\forall (Q^0, \mathfrak{d}^0, \lambda^0, \mu^0) \in \mathcal{K}^1,$$

where, $\mathcal{K}^1 \equiv \{(Q^0, \mathfrak{d}^0, \lambda^0, \mu^0) | (Q^0, \mathfrak{d}^0, \lambda^0, \mu^0) \in R_+^{\sum_{i=1}^I (\sum_{j=1}^I N_i M_j + 2GN_i) + I}\}$.

Due to constraint (4.9), the solution to the optimality condition (4.11) is a variational equilibrium (Kulkarni and Shanbhag, 2012), a refinement of the general Nash equilibrium (GNE)⁷. In a variational equilibrium, all of the Lagrange multipliers associated with the shared constraints are

⁶For the details of such reformulation technique, see Bertsekas and Tsitsiklis (1989).

⁷Formally introduced by Debreu (1952), GNE problem has been frequently used in common-pool resource studies as a mathematical-economic tool. For more information on general Nash equilibrium problem (GNEP), see Harker (1991), Facchinei and Kanzow (2007).

equal. By formulating a GNE in VI, I can resort to a set of well-established theoretical properties⁸ (more in §4.4) and algorithmic schemes than with a formulation by quasi-inequality, a formalism for GNE problems (Facchinei and Kanzow, 2007).

In addition, condition (4.11) proffers a readily interpretable mathematical form from an economic perspective. According to the first summand, a resource owner will ship a positive amount of the product to a resource producer if the price that the producer is willing to pay for the product is exactly equal to, the owner's marginal cost less the marginal incentive benefit. If the owner's marginal cost, less the marginal incentive benefit, exceeds what the retailer is willing to pay for the product, then there will not be any shipments from the owner to the producer.

4.2.3 Resource Producer's Problem

A resource producer (j, m) strategically determines its shipment quantity x_s^{jm} to a resource supplier (j, s) . I group all x_s^{jm} into a column vector Q^1 , with $Q^1 \in R_{+}^{\sum_{j=1}^I M_j S_j}$. I introduce a non-negative conversion rate of production ψ_{jm}^{in} . I assume that the producer's operating cost is associated with its total raw material quantity receivables from the owners and that it can be written as

$$f^{jm} = f^{jm}(x_{jm}), \quad \forall j, m, \quad (4.12)$$

where, $x_{jm} \equiv (x_{jm}^{11}, \dots, x_{jm}^{in}, \dots, x_{jm}^{IN_i})$.

The producer's transaction cost is associated with the flow between each pair of producer and supplier

$$c_s^{jm} = c_s^{jm}(x_s^{jm}), \quad \forall j, m, s. \quad (4.13)$$

Similar to the incentive scheme for the resource owners, I let B_g^j denote the cutoff bracket of the incentives, with g enumerating between $1, \dots, G$. Let δ_g^{jm} denote the excess of output quantity within bracket B_g^j . As such, I define the resource producer's incentive payment function as

$$\beta_0^j(\cdot) + \sum_{g=1}^G \beta_g^j(\cdot), \quad (4.14)$$

⁸To the best of my knowledge, other than existence and local uniqueness theory, the theoretical study of GNE problem remains partial (cf. Facchinei and Kanzow (2007), Dreves (2016)).

Specifically, the incentive payment received by the resource producer (j, m) is a function of its total output quantity, i.e.,

$$\beta_0^j \left(\sum_{s=1}^{S_j} x_s^{jm} \right) + \sum_{g=1}^G \beta_g^j (\delta_g^{jm}), \quad \forall j, m, \quad (4.15)$$

where, the quantity excess

$$\delta_g^{jm} = \max \left\{ \sum_{s=1}^{S_j} x_s^{jm} - B_g^j, 0 \right\}, \quad \forall g = 1, \dots, G. \quad (4.16)$$

I group all δ_g^{jm} into a column vector \mathbf{d}^1 , and assume f^{jm} , c_s^{jm} to be continuously differentiable, β_0^j , and β_g^j to be linear.

The price of products shipped from resource producer (j, m) to supplier (j, s) is denoted by p_{1s}^{jm} . I further denote the equilibrium of such a price as p_{1s}^{jm*} . Hence, the total revenue of a producer is $\sum_{s=1}^{S_j} p_{1s}^{jm} x_s^{jm}$.

As a profit-maximizer, each typical producer (j, m) faces the following problem.

$$\begin{aligned} \text{Maximize: } \pi_{jm} = & \sum_{s=1}^{S_j} p_{1s}^{jm*} x_s^{jm} - \sum_{j=1}^I \sum_{n=1}^{N_i} p_{0jm}^{in*} x_{jm}^{in} - f^{jm}(x_{jm}) - \sum_{s=1}^{S_j} c_s^{jm}(x_s^{jm}) \\ & + \beta_0^j \left(\sum_{s=1}^{S_j} x_s^{jm} \right) + \sum_{g=1}^G \beta_g^j (\delta_g^{jm}) \end{aligned} \quad (4.17)$$

$$\text{Subject to: } \sum_{i=1}^I \sum_{n=1}^{N_i} x_{jm}^{in} \cdot \psi_{jm}^{in} \geq \sum_{s=1}^{S_j} x_s^{jm}, \quad (\lambda_{jm}^1) \quad (4.18)$$

$$\sum_{s=1}^{S_j} x_s^{jm} - \delta_g^{jm} \leq B_g^j, \quad \forall g, \quad (\mu_{jmg}^1) \quad (4.19)$$

$$x_s^{jm} \geq 0, \quad \forall s, \quad \delta_g^{jm} \geq 0, \quad \forall g.$$

The constraint (4.18) is the flow conservation in light of the inter-resource conversion. The constraint (4.19), along with $\delta_g^{jm} \geq 0$, is the equivalence of expression (4.16). λ_{jm}^1 and μ_{jmg}^1 are the Lagrange multipliers associated with inequality constraints (4.18) and (4.19), respectively. I subsequently group all λ_{jm}^1 and μ_{jmg}^1 into a column vector λ^1 and μ^1 , with $\lambda^1 \in R_+^{\sum_{j=1}^I M_j}$ and $\mu^1 \in R_+^{\sum_{j=1}^I GM_j}$, respectively. The optimal behavior of all resource producers can be expressed as

a VI: determine $(Q^{0*}, Q^{1*}, \mathfrak{d}^{1*}, \lambda^{1*}, \mu^{1*}) \in \mathcal{X}^2$, satisfying:

$$\begin{aligned}
& \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\frac{\partial f^{jm}(x_{jm}^*)}{\partial x_{jm}^{in}} + p_{0jm}^{in*} - \psi_{jm}^{in} \lambda_{jm}^{1*} \right] \times (x_{jm}^{in} - x_{jm}^{in*}) \\
& + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{s=1}^{S_j} \left[\frac{\partial c_s^{jm}(x_s^{jm*})}{\partial x_s^{jm}} - p_{1s}^{jm*} - \frac{\partial \beta_0^j(x_s^{jm*})}{\partial x_s^{jm}} + \lambda_{jm}^{1*} + \sum_{g=1}^G \mu_{jmg}^{1*} \right] \times (x_s^{jm} - x_s^{jm*}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[-\frac{\partial \beta_g^j(\delta_g^{jm*})}{\partial \delta_g^{jm}} - \mu_{jmg}^{1*} \right] \times (\delta_g^{jm} - \delta_g^{jm*}) \quad (4.20) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[B_g^j - \sum_{s=1}^{S_j} x_s^{jm*} + \delta_g^{jm*} \right] \times (\mu_{jmg}^1 - \mu_{jmg}^{1*}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\sum_{i=1}^I \sum_{n=1}^{N_i} x_{jm}^{in*} \cdot \psi_{jm}^{in} - \sum_{s=1}^{S_j} x_s^{jm*} \right] \times (\lambda_{jm}^1 - \lambda_{jm}^{1*}) \geq 0, \\
& \quad \forall (Q^0, Q^1, \mathfrak{d}^1, \lambda^1, \mu^1) \in \mathcal{X}^2,
\end{aligned}$$

where, $\mathcal{X}^2 \equiv \{(Q^0, Q^1, \mathfrak{d}^1, \lambda^1, \mu^1) | (Q^0, Q^1, \mathfrak{d}^1, \lambda^1, \mu^1) \in R_+^{\sum_{j=1}^I (\sum_{i=1}^I N_i M_j + M_j S_j + 2GM_j + M_j)}\}$.

4.2.4 Resource Supplier's Problem

A typical resource supplier is denoted as (j, s) . Each supplier has t modes to transport the resources to Market k . The supplier's strategic variable is x_{tk}^{js} , its shipment quantity to market k with transportation mode t . I group all x_{tk}^{js} into a column vector Q^2 , with $Q^2 \in R_+^{\sum_{j=1}^I K S_j T_j}$. I assume the supplier incurs an operating cost that is associated with the resource product flow from the producers, i.e.,

$$f^{js} = f^{js}(x^{js}), \quad (4.21)$$

where, $x^{js} \equiv (x_s^{j1}, \dots, x_s^{jm}, \dots, x_{s_j}^{jM_j})$, $\forall j, s$. The supplier also incurs a transaction cost with each outgoing shipment to the markets:

$$c_{tk}^{js} = c_{tk}^{js}(x_{tk}^{js}). \quad (4.22)$$

I assume f^{js} and c_{tk}^{js} to be continuously differentiable.

The price of goods from resource supplier (j, s) to market k via transportation mode t is denoted by p_{2tk}^{js} . I further denote the equilibrium of such a price as p_{2tk}^{js*} . Hence, the total revenue of each supplier is $\sum_{t=1}^{T_j} \sum_{k=1}^K p_{2tk}^{js*} x_{tk}^{js}$.

As a profit-maximizer, each typical supplier (j, s) faces the following problem.

$$\text{Maximize: } \pi_{js} = \sum_{t=1}^{T_j} \sum_{k=1}^K p_{2tk}^{js*} x_{tk}^{js} - \sum_{m=1}^{M_j} p_{1s}^{jm*} x_s^{jm} - f^{js}(x^{js}) - \sum_{t=1}^{T_j} \sum_{k=1}^K c_{tk}^{js}(x_{tk}^{js}) \quad (4.23)$$

$$\text{Subject to: } \sum_{t=1}^{T_j} \sum_{k=1}^K x_{tk}^{js} \leq \sum_{m=1}^{M_j} x_s^{jm}, \quad (\lambda_{js}^2) \quad (4.24)$$

$$x_{tk}^{js} \geq 0, \quad \forall t, k.$$

The constraint (4.24) is the flow conservation of the finished resource commodities at each supplier, with λ_{js}^2 being the associated Lagrange multipliers. I subsequently group all λ_{js}^2 into a column vector λ^2 with $\lambda^2 \in R_+^{\sum_{j=1}^I S_j}$. The optimal behavior of all resource suppliers can be expressed as a VI: determine $(Q^1, Q^2, \lambda^2) \in \mathcal{H}^3$, satisfying:

$$\begin{aligned} & \sum_{j=1}^I \sum_{s=1}^{S_j} \sum_{t=1}^{T_j} \sum_{k=1}^K \left[\frac{\partial c_{tk}^{js}(x_{tk}^{js*})}{\partial x_{tk}^{js}} - p_{2tk}^{js*} + \lambda_{js}^{2*} \right] \times (x_{tk}^{js} - x_{tk}^{js*}) \\ & + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{s=1}^{S_j} \left[\frac{\partial f^{js}(x_s^{js*})}{\partial x_s^{jm}} + p_{1s}^{jm*} - \lambda_{js}^{2*} \right] \times (x_s^{jm} - x_s^{jm*}) \\ & + \sum_{j=1}^I \sum_{s=1}^{S_j} \left[\sum_{m=1}^{M_j} x_s^{jm*} - \sum_{k=1}^K \sum_{t=1}^{T_j} x_{tk}^{js*} \right] \times (\lambda_{js}^2 - \lambda_{js}^{2*}) \geq 0, \end{aligned} \quad (4.25)$$

$$\forall (Q^1, Q^2, \lambda^2) \in \mathcal{H}^3,$$

where, $\mathcal{H}^3 \equiv \{(Q^1, Q^2, \lambda^2) | (Q^1, Q^2, \lambda^2) \in R_+^{\sum_{j=1}^I (KS_j T_j + M_j S_j + S_j)}\}$.

4.2.5 Demand Market Equilibrium Conditions

Scarce resources, such as clean water, energies, foods, industrial materials, are consumed at demand markets with unique needs. A market decides the purchase quantity from suppliers based on its willingness to pay. As such, I adopt the classic spatial price equilibrium model⁹ to represent the markets.

For an arbitrary market k ($k = 1, \dots, K$), without loss of generality, I assign a set of ex ante weight w_{ts} as its demand for resource j ($j = 1, \dots, I$). Hence, the demand for resource j at market

⁹Spatial Price Equilibrium (SPE) model appeared in the early literature of [Enke \(1951\)](#); [Samuelson \(1952\)](#); [Takayama and Judge \(1964\)](#), inter alia, and was thereafter extended with VI theories by [Dafermos and Nagurney \(1984\)](#).

k is

$$d_{jk} = \sum_{t=1}^{T_j} \sum_{s=1}^{S_j} w_{ts} \cdot x_{tk}^{js}, \quad \forall j, k. \quad (4.26)$$

I group all d_{jk} from (4.26) into a column vector d , i.e., $d \equiv (d_{11}, \dots, d_{jk}, \dots, d_{JK})$, where $d \in R_+^{IK}$. I further denote the equilibrium of such demands as d_{jk}^* and d^* , respectively. Because each scarce-resource product in each market has a unique demand function, as is in (4.26), it corresponds with a market price p_{3k}^j in the form of price-demand function

$$p_{3k}^j = p_{3k}^j(d). \quad (4.27)$$

Market k incurs a transaction cost from supplier (j, s) via transportation mode t

$$\hat{c}_{tk}^{js} = \hat{c}_{tk}^{js}(x_{tk}^{js}), \quad \forall j, s, k, t. \quad (4.28)$$

I assume \hat{c}_{tk}^{js} to be continuously differentiable.

The equilibrium condition is given by

$$p_{2tk}^{js*} + \hat{c}_{tk}^{js*}(x_{tk}^{js*}) \begin{cases} = p_{3k}^j(d^*) & \text{if } x_{tk}^{js*} > 0 \\ \geq p_{3k}^j(d^*) & \text{if } x_{tk}^{js*} = 0 \end{cases} \quad \forall j, s, t, k. \quad (4.29)$$

The intuition of equation (4.29) is, that the consumption of resource at the markets will remain positive if the supplier' purchase price of such a resource plus all associated cost is equal to the price consumers are willing to pay; however, if the supplier' purchase price plus cost turns out to be higher than what consumers are willing to pay, then there will be no shipments between the supplier and the market.

As such, the optimal behaviors of all demand markets as a spatial price equilibrium can be expressed in a VI form: determine $(Q^{2*}, d^*) \in \mathcal{H}^4$, satisfying:

$$\sum_{j=1}^I \sum_{s=1}^{S_j} \sum_{t=1}^{T_j} \sum_{k=1}^K [p_{2tk}^{js*} + \hat{c}_{tk}^{js*}(x_{tk}^{js*})] \times (x_{tk}^{js} - x_{tk}^{js*}) - \sum_{j=1}^I \sum_{k=1}^K p_{3k}^j(d^*) \times (d_{jk} - d_{jk}^*) \geq 0, \quad (4.30)$$

$$\forall (Q^2, d) \in \mathcal{H}^4,$$

where, $\mathcal{K}^4 \equiv \{(Q^2, d) | Q^2 \in R_+^{\sum_{j=1}^I K S_j T_j}, d \in R_+^{IK}\}$.

4.2.6 Welfare Estimates

To assess the social impact of the model elements in the presented supply chain network, I provide an estimate of consumer surplus and social welfare. The consumer surplus, termed to measure the aggregate of consumers' benefits upon purchasing a product (cf. Willig (1976)) of resource j at demand market k is given by:

$$CS_{jk} = \int_0^{d_{jk}^*} p_{3k}^j(z) dz - p_{3k}^{j*} d_{jk}^*, \quad \forall j, k. \quad (4.31)$$

I also denote social welfare of the network to be the aggregation of all firm profits and consumer surplus, i.e.,

$$SW = \sum_{i=1}^I \sum_{n=1}^{N_i} \pi_{in} + \sum_{j=1}^I \sum_{m=1}^{M_j} \pi_{jm} + \sum_{j=1}^I \sum_{s=1}^{S_j} \pi_{js} + \sum_{j=1}^I \sum_{k=1}^K CS_{jk}, \quad \forall i, j, n, m, s, k. \quad (4.32)$$

4.3 Network Equilibrium

Definition 8 (The scarce resource supply chain network equilibrium with fiscal policy)

A product flow pattern $(Q^{0*}, Q^{1*}, Q^{2*}, \mathfrak{d}^{0*}, \mathfrak{d}^{1*}, d^*, \lambda^{0*}, \lambda^{1*}, \lambda^{2*}, \mu^{0*}, \mu^{1*}) \in \mathcal{K}$ is said to constitute a cross-sector multi-product scarce resource supply chain network equilibrium with fiscal policy if it satisfies the sum of the optimality conditions (4.11), (4.20), (4.25), and (4.30).

Theorem 14 (Variational inequality formulation)

The equilibrium conditions governing the cross-sector multi-product scarce resource supply chain network, according to Definition 1, are equivalent to the solution to the following VIs:

Determine $(Q^{0*}, Q^{1*}, Q^{2*}, \mathfrak{d}^{0*}, \mathfrak{d}^{1*}, d, \lambda^{0*}, \lambda^{1*}, \lambda^{2*}, \mu^{0*}, \mu^{1*}) \in \mathcal{K}$, satisfying

$$\sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\frac{\partial f^{in}(x^{in*})}{\partial x_{jm}^{in}} + \frac{\partial f^{jm}(x_{jm}^*)}{\partial x_{jm}^{in}} + \frac{\partial c_{jm}^{in}(x_{jm}^{in*})}{\partial x_{jm}^{in}} - \frac{\partial \alpha_0^i(x_{jm}^{in*})}{\partial x_{jm}^{in}} \right. \\ \left. + \lambda_i^{0*} - \psi_{jm}^{in} \lambda_{jm}^{1*} + \sum_{g=1}^G \mu_{ing}^{0*} \right] \times (x_{jm}^{in} - x_{jm}^{in*})$$

$$\begin{aligned}
& + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{s=1}^{S_j} \left[\frac{\partial f^{js}(x^{js*})}{\partial x_s^{jm}} + \frac{\partial c_s^{jm}(x_s^{jm*})}{\partial x_s^{jm}} - \frac{\partial \beta_0^j(x_s^{jm*})}{\partial x_s^{jm}} \right. \\
& \qquad \qquad \qquad \left. + \lambda_{jm}^{1*} - \lambda_{js}^{2*} + \sum_{g=1}^G \mu_{jmg}^{1*} \right] \times (x_s^{jm} - x_s^{jm*}) \\
& + \sum_{j=I}^I \sum_{s=1}^{S_j} \sum_{t=1}^{T_j} \sum_{k=1}^K \left[\frac{\partial c_{tk}^{js}(x_{tk}^{js*})}{\partial x_{tk}^{js}} + \hat{c}_{tk}^{js}(x_{tk}^{js*}) + \lambda_{js}^{2*} \right] \times (x_{tk}^{js} - x_{tk}^{js*}) \\
& \qquad \qquad \qquad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[-\frac{\partial \alpha_g^i(\delta_g^{in*})}{\partial \delta_g^{in}} - \mu_{ing}^{0*} \right] \times (\delta_g^{in} - \delta_g^{in*}) \\
& \qquad \qquad \qquad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[-\frac{\partial \beta_g^j(\delta_g^{jm*})}{\partial \delta_g^{jm}} - \mu_{jmg}^{1*} \right] \times (\delta_g^{jm} - \delta_g^{jm*}) \quad (4.33) \\
& - \sum_{j=1}^I \sum_{k=1}^K p_{3k}^j(d^*) \times (d_{jk} - d_{jk}^*) + \sum_{i=1}^I \left[U_i - \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in*} \right] \times (\lambda_i^0 - \lambda_i^{0*}) \\
& \qquad \qquad \qquad + \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\sum_{i=1}^I \sum_{n=1}^{N_i} x_{jm}^{in*} \cdot \psi_{jm}^{in} - \sum_{s=1}^{S_j} x_s^{jm*} \right] \times (\lambda_{jm}^1 - \lambda_{jm}^{1*}) \\
& \qquad \qquad \qquad + \sum_{j=I}^I \sum_{s=1}^{S_j} \left[\sum_{m=1}^{M_j} x_s^{jm*} - \sum_{k=1}^K \sum_{t=1}^{T_j} x_{tk}^{js*} \right] \times (\lambda_{js}^2 - \lambda_{js}^{2*}) \\
& \qquad \qquad \qquad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[A_g^i - \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in*} + \delta_g^{in*} \right] \times (\mu_{ing}^0 - \mu_{ing}^{0*}) \\
& \qquad \qquad \qquad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[B_g^j - \sum_{s=1}^{S_j} x_s^{jm*} + \delta_g^{jm*} \right] \times (\mu_{jmg}^1 - \mu_{jmg}^{1*}) \geq 0, \\
& \forall (Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1) \in \mathcal{H},
\end{aligned}$$

where, $\mathcal{H} \equiv \mathcal{H}^1 \times \mathcal{H}^2 \times \mathcal{H}^3 \times \mathcal{H}^4 = \{(Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1) | (Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1) \in R_+^{\sum_{i=1}^I (\sum_{j=1}^I 2N_i M_j + 2GN_i) + \sum_{j=1}^I (2M_j S_j + 2KS_j T_j + 2GM_j + S_j + M_j) + IK + I}\}$.

Proof: See Appendix B.

Retrieving the price variables

It should be noted that variable p_{0jm}^{in*} , p_{1s}^{jm*} , and p_{2tk}^{js*} do not appear within the formulation of Theorem 1. They are endogenous to the model and can be retrieved once the solution is obtained. To retrieve p_{2tk}^{js*} , for all j, s, t, k , recall equilibrium condition (4.29). Since $p_{3k}^j(d^*)$ is readily available

from (4.27), if $x_{tk}^{js*} > 0$ for some j, s, t, k , then p_{2tk}^{js*} can be obtained by the equality

$$p_{2tk}^{js*} = p_{3k}^{j*}(a^*) - \hat{c}_{tk}^{js}(x_{tk}^{js*}). \quad (4.34)$$

Invariably, if $x_s^{jm*} > 0$ for some j, m, s , then from the second summand in (4.25), one may immediately obtain

$$p_{1s}^{jm*} = \lambda_{js}^{2*} - \frac{\partial f^{js}(x_{tk}^{js*})}{\partial x_s^{jm}}. \quad (4.35)$$

And, if $x_{jm}^{in*} > 0$ for some i, n, j, m , then from the first summand of (4.20),

$$p_{0jm}^{in*} = \psi_{jm}^{in} \lambda_{jm}^{1*} - \frac{\partial f^{jm}(x_{jm}^{*})}{\partial x_{jm}^{in}}. \quad (4.36)$$

4.4 Theoretical Properties

I provide a few classic theoretical properties of the solution to VI (4.33), based on [Gabay and Moulin \(1980\)](#), [Nagurney \(1999\)](#), and [Melo \(2018\)](#), inter alia. In particular, I derive existence and uniqueness results by incorporating strategic measure of network games from the latest theoretical advancement.

To facilitate the development in this section, I rewrite VI problem (4.33) in standard form as follows: determine $X^* \in \mathcal{K}$ satisfying

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X^* \in \mathcal{K}, \quad (4.37)$$

where, $X \equiv (Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1)$, with an indulgence of notation, $F(X) \equiv (F_{Q^0}, F_{Q^1}, F_{Q^2}, F_{\mathfrak{d}^0}, F_{\mathfrak{d}^1}, F_d, F_{\lambda^0}, F_{\lambda^1}, F_{\lambda^2}, F_{\mu^0}, F_{\mu^1})$, in which each component of F is given by each respective summand expression in (4.33), and, $\mathcal{K} \equiv \mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \times \mathcal{K}^4$. The notation $\langle \cdot, \cdot \rangle$ represents the inner product in a Euclidean space. Both F and X are \mathcal{N} -dimensional column vectors, where $\mathcal{N} = \sum_{i=1}^I (\sum_{j=1}^I 2N_i M_j + 2GN_i) + \sum_{j=1}^I (2M_j S_j + 2K S_j T_j + 2GM_j + S_j + M_j) + IK + I$.

First, I provide the existence properties. While F in (4.37) is continuous, the feasible set \mathcal{K} is not compact. This causes the existence condition of (4.33) not readily available. But one can impose a weak condition to guarantee the existence of a solution pattern, as in [Nagurney et al.](#)

(2002). Let

$$\mathcal{K}_b \equiv \{(Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1) \mid 0 \leq Q^0 \leq b_1, 0 \leq Q^1 \leq b_2, 0 \leq Q^2 \leq b_3, 0 \leq \mathfrak{d}^0 \leq b_4, \\ 0 \leq \mathfrak{d}^1 \leq b_5, 0 \leq d \leq b_6, 0 \leq \lambda^0 \leq b_7, 0 \leq \lambda^1 \leq b_8, 0 \leq \lambda^2 \leq b_9, 0 \leq \mu^0 \leq b_{10}, 0 \leq \mu^1 \leq b_{11}\},$$

where $b = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}) \geq 0$ and $Q^0 \leq b_1, Q^1 \leq b_2, Q^2 \leq b_3, \mathfrak{d}^0 \leq b_4, \mathfrak{d}^1 \leq b_5, d \leq b_6, \lambda^0 \leq b_7, \lambda^1 \leq b_8, \lambda^2 \leq b_9, \mu^0 \leq b_{10}, \mu^1 \leq b_{11}$ means that $x_{jm}^{in} \leq b_1, x_s^{jm} \leq b_2, x_{tk}^{js} \leq b_3, \delta_g^{in} \leq b_4, \delta_g^{jm} \leq b_5, d_{jk} \leq b_6, \lambda_i^0 \leq b_7, \lambda_{jm}^1 \leq b_8, \lambda^2 \leq b_9, \mu_{ing}^0 \leq b_{10}, \mu_{jmg}^1 \leq b_{11}$ for all i, j, n, m, t, k . Then \mathcal{K}_b is a bounded, closed, convex subset of $R_+^{\mathcal{N}}$. Tangentially, the existence of b is sensible from an economic perspective. Thus, the VI expression

$$\langle F(X^b), X - X^b \rangle \geq 0, \quad \forall X^b \in \mathcal{K}_b \quad (4.38)$$

admits at least one solution. Following [Kinderlehrer and Stampacchia \(1980\)](#), and [Nagurney et al. \(2002\)](#), it is straightforward to establish:

Lemma 3

The VI (4.37) admits a solution if and only if there exists a $b > 0$ such that the VI (4.38) admits a solution in \mathcal{K}_b , with

$$Q^0 < b_1, Q^1 < b_2, Q^2 < b_3, \mathfrak{d}^0 < b_4, \mathfrak{d}^1 < b_5, d < b_6, \\ \lambda^0 < b_7, \lambda^1 < b_8, \lambda^2 < b_9, \mu^0 < b_{10}, \mu^1 < b_{11}. \quad (4.39)$$

Theorem 15 (Existence)

The VI problem (4.33) admits at least one solution in \mathcal{K}_b .

Proof: By virtue of theorem 3.1 in [Harker and Pang \(1990b\)](#), one can easily verify that \mathcal{K}_b is non-empty, compact, and convex, and that the mapping F representing (4.33) is continuous. Therefore, there exists a solution to the problem (4.33) in \mathcal{K}_b . \square

Next, I provide a set of sufficient conditions for the uniqueness properties. In general, uniqueness is often associated with the monotonicity of the F that enters the VI problem. Here, I begin by redefining a set of well-known monotonicities (definition 2.3.1 by [Facchinei and Pang \(2003\)](#)).

I will also use the following established lemma (theorem 1.6 in [Nagurney \(1999\)](#)).

Lemma 4

Suppose that $F(X)$ is strictly monotone on \mathcal{K} , then the solution to the VI(F, K) problem is unique, if one exists.

To further characterize the nature of the network, I need to adopt an additional set of notations. Following [Melo \(2018\)](#), I define the game Jacobian of $F(X)$ as an \mathcal{N} -by- \mathcal{N} matrix

$$J(X) := \nabla_X F(X), \quad \forall X \in R_+^{\mathcal{N}}. \quad (4.40)$$

Decomposing the Jacobian in terms of its diagonal and off-diagonal element yields

$$J(X) := D(X) + N(X), \quad (4.41)$$

where $D(X)$ is a diagonal matrix, and $N(X)$ is an off-diagonal matrix, whose elements are described as [\(2.18\)](#) and [\(2.19\)](#), respectively.

Finally, I denote the lowest eigenvalue of a square matrix as $\lambda_{min}(\cdot)$. Clearly, both $D(X)$ and $\bar{N}(X)$ have real-numbered eigenvalues because they are symmetric. Now I present the main results for the uniqueness.

Theorem 16 (Sufficient Conditions for Uniqueness)

Assuming the condition for [Theorem 15](#) is satisfied, the solution X^ to VI [\(4.33\)](#) is unique, if*

- (i) F is strictly monotone on \mathcal{K} , or*
- (ii) F is strongly monotone on \mathcal{K} , or*
- (iii) u is strictly concave, and the condition $|\lambda_{min}(\bar{N}(X))| < \lambda_{min}(D(X))$ holds $\forall X \in \mathcal{K}$, where $u(X) = -\int F(X)$.*

Proof:

- (i) The proof is immediate with [Lemma 4](#) and [Theorem 15](#).
- (ii) See the proof of theorem 1.8 in [Nagurney \(1999\)](#).
- (iii) Because all cost, incentive, and demand functions in equation [\(4.33\)](#) are continuously differentiable, all of their partial derivatives are well defined. By virtue of [Proposition 3](#), $F(X)$ is

strictly monotone on \mathcal{X} . Thus, according to Theorem 13, (iii) is proved. \square

Remark

A few observations can be made. First, it is well-known that the monotonicity conditions in Definition 4 are ranked in the ascending order of “strength” (Facchinei and Pang, 2003), i.e., (c) implies (b), and (b) implies (a). In Theorem 16, the uniqueness condition (i) and (iii), in essence, are established under strict monotonicity, whereas (ii) is under strong monotonicity. Hence, one can easily infer a relation of “(ii) \Rightarrow (i)”, as well as “(ii) \Rightarrow (iii)” in Theorem 16.

Second, to characterize the uniqueness of variational equilibrium (4.33), one could also establish similar sufficient conditions by the semidefiniteness of $J(X)$ as it pertains to the analogous monotonicity features (cf. Parise and Ozdaglar (2019); Melo (2018)).

4.5 Algorithm

To solve a VI problem in standard form, I propose an algorithm with theoretical measures of the result. The algorithm is extragradient method first proposed by Korpelevich (1976), which is later promoted as the modified projection method in Nagurney (1999) by setting the step length to 1. The solution is guaranteed to converge as long as the function F that enters the standard form is monotone and Lipschitz continuous. The realization of the algorithm for the cross-sector multi-product scarce resource supply chain network model is given as follows.

The modified projection method:

Step 0. Initialization

Set $X^0 = (Q^{00}, Q^{10}, Q^{20}, \bar{d}^{00}, \bar{d}^{10}, \bar{d}^{20}, \lambda^{00}, \lambda^{10}, \lambda^{20}, \mu^{00}, \mu^{10}) \in \mathcal{X}$. Set $\tau = 1$ and select φ such that $0 < \varphi \leq 1/L$, where L is the Lipschitz constant for function F .

Step 1. Construction and computation

Compute $\bar{X}^{\tau-1} = (\bar{Q}^{0\tau}, \bar{Q}^{1\tau}, \bar{Q}^{2\tau}, \bar{d}^{0\tau}, \bar{d}^{1\tau}, \bar{d}^{2\tau}, \bar{\lambda}^{0\tau}, \bar{\lambda}^{1\tau}, \bar{\lambda}^{2\tau}, \bar{\mu}^{0\tau}, \bar{\mu}^{1\tau}) \in \mathcal{X}$ by solving the VI sub-problem

$$\langle \bar{X}^{\tau-1} + \varphi F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^{\tau-1} \rangle \geq 0, \quad \forall X \in \mathcal{X}. \tag{4.42}$$

Step 2. Adaptation

Compute $X^\tau = (Q^{0\tau}, Q^{1\tau}, Q^{2\tau}, \mathfrak{d}^{0\tau}, \mathfrak{d}^{1\tau}, d^\tau, \lambda^{0\tau}, \lambda^{1\tau}, \lambda^{2\tau}, \mu^{0\tau}, \mu^{1\tau}) \in \mathcal{X}$ by solving the VI sub-problem

$$\langle X^\tau + \varphi F(\bar{X}^{\tau-1}) - X^{\tau-1}, X - X^\tau \rangle > 0, \quad \forall X \in \mathcal{X}. \quad (4.43)$$

Step 3. Convergence verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, for $\epsilon > 0$, a pre-specified tolerance, then, stop; otherwise, set $\tau =: \tau + 1$ and go to step 1.

I now further discuss the modified projection method. The projection map of X , $P_{\mathcal{X}}$, on the feasible set \mathcal{X} is given by:

$$P_{\mathcal{X}}(X) = \operatorname{argmin}_{z \in \mathcal{X}} \|X - z\|, \quad (4.44)$$

where, $\|\cdot\|$ is the Euclidean norm (i.e., for an arbitrary vector ν , $\|\nu\| = \langle \nu, \nu \rangle$). The solution to the variational inequality sub-problem (4.42), $\bar{X}^{\tau-1}$, generated by the modified projection method is the projection of $X^{\tau-1} - \varphi F(X^{\tau-1})$ on the closed convex set \mathcal{X} . Hence, I have

$$\bar{X}^{\tau-1} = P_{\mathcal{X}}[X^{\tau-1} - \varphi F(X^{\tau-1})]. \quad (4.45)$$

The solution to the variational inequality sub-problem (4.43), in turn, corresponds to the following

$$X^\tau = P_{\mathcal{X}}[X^{\tau-1} - \varphi F(\bar{X}^{\tau-1})]. \quad (4.46)$$

Next, I state the expanded form of $F(X)$ for the current model.

Step 0. Initialization

Set $(Q^{00}, Q^{10}, Q^{20}, \mathfrak{d}^{00}, \mathfrak{d}^{10}, d^0, \lambda^{00}, \lambda^{10}, \lambda^{20}, \mu^{00}, \mu^{10}) \in \mathcal{X}$. Set $\tau =: 1$ and select ψ such that $0 < \psi \leq 1/L$, where L is the Lipschitz constant for function F .

Step 1. Construction and computation

Compute $(\bar{Q}^{0\tau}, \bar{Q}^{1\tau}, \bar{Q}^{2\tau}, \bar{\mathfrak{d}}^{0\tau}, \bar{\mathfrak{d}}^{1\tau}, \bar{d}^\tau, \bar{\lambda}^{0\tau}, \bar{\lambda}^{1\tau}, \bar{\lambda}^{2\tau}, \bar{\mu}^{0\tau}, \bar{\mu}^{1\tau}) \in \mathcal{X}$ by solving the variational in-

equality sub-problem

$$\begin{aligned}
& \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\bar{x}_{jm}^{in(\tau-1)} + \varphi \left(\frac{\partial f^{in}(x_{jm}^{in(\tau-1)})}{\partial x_{jm}^{in}} + \frac{\partial f^{jm}(x_{jm}^{\tau-1})}{\partial x_{jm}^{in}} + \frac{\partial c_{jm}^{in}(x_{jm}^{in(\tau-1)})}{\partial x_{jm}^{in}} \right. \right. \\
& \quad \left. \left. - \frac{\partial \alpha_0(x_{jm}^{in(\tau-1)})}{\partial x_{jm}^{in}} + \lambda_{in}^{0(\tau-1)} + \psi_{jm}^{in} \lambda_{jm}^{1(\tau-1)} + \sum_{g=1}^G \mu_{ing}^{0(\tau-1)} \right) - x_{jm}^{in(\tau-1)} \right] \times (x_{jm}^{in} - \bar{x}_{jm}^{in(\tau-1)}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[\bar{\delta}_g^{in(\tau-1)} + \varphi \left(- \frac{\partial \alpha_g(\delta_g^{in(\tau-1)})}{\partial \delta_g^{in}} \right) - \delta_g^{in(\tau-1)} \right] \times (\delta_g^{in} - \bar{\delta}_g^{in(\tau-1)}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{s=1}^{S_j} \left[\bar{x}_s^{jm(\tau-1)} + \psi \left(\frac{\partial c_s^{jm}(x_s^{jm(\tau-1)})}{\partial x_s^{jm}} - \frac{\partial \beta_0(x_s^{jm(\tau-1)})}{\partial x_s^{jm}} \right. \right. \\
& \quad \left. \left. - \lambda_{jm}^{1(\tau-1)} - \lambda_{js}^{2(\tau-1)} + \sum_{g=1}^G \mu_{jmg}^{1(\tau-1)} \right) - x_s^{jm(\tau-1)} \right] \times (x_s^{jm} - \bar{x}_s^{jm(\tau-1)}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[\bar{\delta}_g^{jm(\tau-1)} + \varphi \left(- \frac{\partial \beta_g(\delta_g^{jm(\tau-1)})}{\partial \delta_g^{jm}} \right) - \delta_g^{jm(\tau-1)} \right] \times (\delta_g^{jm} - \bar{\delta}_g^{jm(\tau-1)}) \\
& \quad + \sum_{j=I}^I \sum_{s=1}^{S_j} \sum_{t=1}^{T_j} \sum_{k=1}^K \left[\bar{x}_{tk}^{js(\tau-1)} + \varphi \left(\frac{\partial f^{js}(x_{tk}^{js(\tau-1)})}{\partial x_{tk}^t} + \frac{\partial c_{tk}^{js}(x_{tk}^{js(\tau-1)})}{\partial x_{tk}^t} \right. \right. \\
& \quad \left. \left. + \tilde{c}_{tk}^{js}(x_{tk}^{js(\tau-1)}) + \lambda_{js}^{2(\tau-1)} \right) - x_{tk}^{js(\tau-1)} \right] \times (x_{tk}^{js} - \bar{x}_{tk}^{js(\tau-1)}) \\
& \quad + \sum_{j=1}^I \sum_{k=1}^K \left[\bar{d}^{\tau-1} + \varphi \left(- \rho_{3k}^j(d^{\tau-1}) \right) - d^{\tau-1} \right] \times (d - \bar{d}^{\tau-1}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \left[\bar{\lambda}_{in}^{0(\tau-1)} + \varphi \left(U_{in} - \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in(\tau-1)} \right) - \lambda_{in}^{0(\tau-1)} \right] \times (\lambda_{in}^0 - \bar{\lambda}_{in}^{0(\tau-1)})
\end{aligned} \tag{4.47}$$

$$\begin{aligned}
& + \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\bar{\lambda}_{jm}^{1(\tau-1)} + \varphi \left(\sum_{s=1}^{S_j} x_s^{jm(\tau-1)} - \sum_{i=1}^I \sum_{n=1}^{N_i} x_{jm}^{in(\tau-1)} \cdot \psi_{jm}^{in} \right) - \lambda_{jm}^{1(\tau-1)} \right] \times (\lambda_{jm}^1 - \bar{\lambda}_{jm}^{1(\tau-1)}) \\
& \quad + \sum_{j=I}^I \sum_{s=1}^{S_j} \left[\bar{\lambda}_{js}^{2(\tau-1)} + \varphi \left(\sum_{m=1}^{M_j} x_s^{jm(\tau-1)} - \sum_{k=1}^K \sum_{t=1}^{T_j} x_{tk}^{js(\tau-1)} \right) - \lambda_{js}^{2(\tau-1)} \right] \times (\lambda_{js}^2 - \bar{\lambda}_{js}^{2(\tau-1)}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[\bar{\mu}_{ing}^{0(\tau-1)} + \varphi \left(A_g^{in} - \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in(\tau-1)} + \delta_g^{in(\tau-1)} \right) - \mu_{ing}^{0(\tau-1)} \right] \times (\mu_{ing}^0 - \bar{\mu}_{ing}^{0(\tau-1)}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[\bar{\mu}_{jmg}^{1(\tau-1)} + \varphi \left(B_g^{jm} - \sum_{s=1}^{S_j} x_s^{jm(\tau-1)} + \delta_g^{jm(\tau-1)} \right) - \mu_{jmg}^{1(\tau-1)} \right] \times (\mu_{jmg}^1 - \bar{\mu}_{jmg}^{1(\tau-1)}) \geq 0,
\end{aligned}$$

$$\forall (Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1) \in \mathcal{H}.$$

Step 2. Adaptation

Compute $(Q^{0\tau}, Q^{1\tau}, Q^{2\tau}, \mathfrak{d}^{0\tau}, \mathfrak{d}^{1\tau}, d^\tau, \lambda^{0\tau}, \lambda^{1\tau}, \lambda^{2\tau}, \mu^{0\tau}, \mu^{1\tau}) \in \mathcal{K}$ by solving the variational inequality sub-problem

$$\begin{aligned}
& \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} \left[x_{jm}^{in\tau} + \varphi \left(\frac{\partial f^{in}(\bar{x}^{in(\tau-1)})}{\partial x_{jm}^{in}} + \frac{\partial f^{jm}(\bar{x}_{jm}^{\tau-1})}{\partial x_{jm}^{in}} + \frac{\partial c_{jm}^{in}(\bar{x}_{jm}^{in(\tau-1)})}{\partial x_{jm}^{in}} \right. \right. \\
& \quad \left. \left. - \frac{\partial \alpha_0(\bar{x}_{jm}^{in(\tau-1)})}{\partial x_{jm}^{in}} + \bar{\lambda}_{in}^{0(\tau-1)} + \psi_{jm}^{in} \bar{\lambda}_{jm}^{1(\tau-1)} + \sum_{g=1}^G \bar{\mu}_{ing}^{0(\tau-1)} \right) - x_{jm}^{in(\tau-1)} \right] \times (x_{jm}^{in} - x_{jm}^{in\tau}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[\delta_g^{in\tau} + \varphi \left(-\frac{\partial \alpha_g(\bar{\delta}_g^{in(\tau-1)})}{\partial \delta_g^{in}} \right) - \delta_g^{in(\tau-1)} \right] \times (\delta_g^{in} - \delta_g^{in\tau}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{s=1}^{S_j} \left[x_s^{jm\tau} + \varphi \left(\frac{\partial c_s^{jm}(\bar{x}_s^{jm(\tau-1)})}{\partial x_s^{jm}} - \frac{\partial \beta_0(\bar{x}_s^{jm(\tau-1)})}{\partial x_s^{jm}} \right. \right. \\
& \quad \left. \left. - \bar{\lambda}_{jm}^{1(\tau-1)} - \bar{\lambda}_{js}^{2(\tau-1)} + \sum_{g=1}^G \bar{\mu}_{jmg}^{1(\tau-1)} \right) - x_s^{jm(\tau-1)} \right] \times (x_s^{jm} - x_s^{jm\tau}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[\delta_g^{jm\tau} + \varphi \left(-\frac{\partial \beta_g(\bar{\delta}_g^{jm(\tau-1)})}{\partial \delta_g^{jm}} \right) - \delta_g^{jm(\tau-1)} \right] \times (\delta_g^{jm} - \delta_g^{jm\tau}) \\
& \quad + \sum_{j=I}^I \sum_{s=1}^{S_j} \sum_{t=1}^{T_j} \sum_{k=1}^K \left[x_{tk}^{js\tau} + \varphi \left(\frac{\partial f^{js}(\bar{x}^{js(\tau-1)})}{\partial x_{sk}^t} + \frac{\partial c_{tk}^{js}(\bar{x}_{tk}^{js(\tau-1)})}{\partial x_{sk}^t} \right. \right. \\
& \quad \left. \left. + \tilde{c}_{tk}^{js}(\bar{x}_{tk}^{js(\tau-1)}) + \bar{\lambda}_{js}^{2(\tau-1)} \right) - x_{tk}^{js(\tau-1)} \right] \times (x_{tk}^{js} - x_{tk}^{js\tau}) \\
& \quad + \sum_{j=1}^I \sum_{k=1}^K \left[d^\tau + \varphi \left(-\rho_{3k}^j(\bar{d}^{\tau-1}) \right) - d^{\tau-1} \right] \times (d - d^\tau) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \left[\lambda_{in}^{0\tau} + \varphi \left(U_{in} - \sum_{j=1}^I \sum_{m=1}^{M_j} \bar{x}_{jm}^{in(\tau-1)} \right) - \lambda_{in}^{0(\tau-1)} \right] \times (\lambda_{in}^0 - \lambda_{in}^{0\tau}) \quad (4.48) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\lambda_{jm}^{1\tau} + \varphi \left(\sum_{s=1}^{S_j} \bar{x}_s^{jm(\tau-1)} - \sum_{i=1}^I \sum_{n=1}^{N_i} \bar{x}_{jm}^{in(\tau-1)} \cdot \psi_{jm}^{in} \right) - \lambda_{jm}^{1(\tau-1)} \right] \times (\lambda_{jm}^1 - \lambda_{jm}^{1\tau}) \\
& \quad + \sum_{j=I}^I \sum_{s=1}^{S_j} \left[\lambda_{js}^{2\tau} + \varphi \left(\sum_{m=1}^{M_j} \bar{x}_s^{jm(\tau-1)} - \sum_{k=1}^K \sum_{t=1}^{T_j} \bar{x}_{tk}^{js(\tau-1)} \right) - \lambda_{js}^{2(\tau-1)} \right] \times (\lambda_{js}^2 - \lambda_{js}^{2\tau}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[\mu_{ing}^{0\tau} + \varphi \left(A_g^{in} - \sum_{j=1}^I \sum_{m=1}^{M_j} \bar{x}_{jm}^{in(\tau-1)} + \bar{\delta}_g^{in(\tau-1)} \right) - \mu_{ing}^{0(\tau-1)} \right] \times (\mu_{ing}^0 - \mu_{ing}^{0\tau}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[\mu_{jmg}^{1\tau} + \varphi \left(B_g^{jm} - \sum_{s=1}^{S_j} \bar{x}_s^{jm(\tau-1)} + \bar{\delta}_g^{jm(\tau-1)} \right) - \mu_{jmg}^{1(\tau-1)} \right] \times (\mu_{jmg}^1 - \mu_{jmg}^{1\tau}) \geq 0, \\
& \quad \forall (Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1) \in \mathcal{K}.
\end{aligned}$$

Step 3. Convergence verification

If $|x_{jm}^{in\tau} - x_{jm}^{in(\tau-1)}| \leq \epsilon$, $|\delta_g^{in\tau} - \delta_g^{in(\tau-1)}| \leq \epsilon$, $|x_s^{jm\tau} - x_s^{jm(\tau-1)}| \leq \epsilon$, $|\delta_g^{jm\tau} - \delta_g^{jm(\tau-1)}| \leq \epsilon$, $|x_{tk}^{js\tau} - x_{tk}^{js(\tau-1)}| \leq \epsilon$, $|d^\tau - d^{\tau-1}| \leq \epsilon$, $|\lambda_{in}^{0\tau} - \lambda_{in}^{0(\tau-1)}| \leq \epsilon$, $|\lambda_{jm}^{1\tau} - \lambda_{jm}^{1(\tau-1)}| \leq \epsilon$, $|\lambda_{js}^{2\tau} - \lambda_{js}^{2(\tau-1)}| \leq \epsilon$, $|\mu_{ing}^{0\tau} - \mu_{ing}^{0(\tau-1)}| \leq \epsilon$, $|\lambda_{jm}^{1\tau} - \lambda_{jm}^{1(\tau-1)}| \leq \epsilon$, $|\mu_{jmg}^{1\tau} - \mu_{jmg}^{1(\tau-1)}| \leq \epsilon$, for all $i = 1, \dots, I$, $j = 1, \dots, I$, $n = 1, \dots, N_i$, $m = 1, \dots, M_j$, $t = 1, \dots, T_j$, $k = 1, \dots, K$, $g = 1, \dots, G$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to step 1.

Closed form expressions

Because the feasible set is that of the non-negative orthant, the projections (4.45) and (4.46) decompose across the coordinates of the feasible set. I present the closed form expressions for the solution of problems (4.47). Analogous ones for (4.48) can be determined accordingly. The closed form expressions for the path flows at iteration τ are as follows.

$$\begin{aligned} \bar{x}_{jm}^{in(\tau-1)} &= \max \left\{ 0, \varphi \left(- \frac{\partial f^{in}(x^{in(\tau-1)})}{\partial x_{jm}^{in}} - \frac{\partial f^{jm}(x_{jm}^{\tau-1})}{\partial x_{jm}^{in}} - \frac{\partial c_{jm}^{in}(x_{jm}^{in(\tau-1)})}{\partial x_{jm}^{in}} \right. \right. \\ &\quad \left. \left. + \frac{\partial \alpha_0(x_{jm}^{in(\tau-1)})}{\partial x_{jm}^{in}} - \lambda_{in}^{0(\tau-1)} - \psi_{jm}^{in} \lambda_{jm}^{1(\tau-1)} - \sum_{g=1}^G \mu_{ing}^{0(\tau-1)} \right) + x_{jm}^{in(\tau-1)} \right\} \\ \bar{x}_s^{jm(\tau-1)} &= \max \left\{ 0, \varphi \left(- \frac{\partial c_s^{jm}(x_s^{jm(\tau-1)})}{\partial x_s^{jm}} + \frac{\partial \beta_0(x_s^{jm(\tau-1)})}{\partial x_s^{jm}} \right. \right. \\ &\quad \left. \left. + \lambda_{jm}^{1(\tau-1)} + \lambda_{js}^{2(\tau-1)} - \sum_{g=1}^G \mu_{jmg}^{1(\tau-1)} \right) + x_s^{jm(\tau-1)} \right\} \\ \bar{x}_{tk}^{js(\tau-1)} &= \max \left\{ 0, \varphi \left(- \frac{\partial f^{js}(x^{js(\tau-1)})}{\partial x_{sk}^t} - \frac{\partial c_{tk}^{js}(x_{tk}^{js(\tau-1)})}{\partial x_{sk}^t} - \hat{c}_{tk}^{js}(x_{tk}^{js(\tau-1)}) - \lambda_{js}^{2(\tau-1)} \right) + x_{tk}^{js(\tau-1)} \right\} \\ \bar{d}^{\tau-1} &= \max \left\{ 0, \varphi \rho_{3k}^j(d^{\tau-1}) + d^{\tau-1} \right\} \\ \bar{\delta}_g^{in(\tau-1)} &= \max \left\{ 0, \varphi \frac{\partial \alpha_g(\delta_g^{in(\tau-1)})}{\partial \delta_g^{in}} + \delta_g^{in(\tau-1)} \right\} \\ \bar{\delta}_g^{jm(\tau-1)} &= \max \left\{ 0, \varphi \frac{\partial \beta_g(\delta_g^{jm(\tau-1)})}{\partial \delta_g^{jm}} + \delta_g^{jm(\tau-1)} \right\} \\ \bar{\lambda}_{in}^{0(\tau-1)} &= \max \left\{ 0, \varphi \left(- U_{in} + \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in(\tau-1)} \right) + \lambda_{in}^{0(\tau-1)} \right\} \\ \bar{\lambda}_{jm}^{1(\tau-1)} &= \max \left\{ 0, \varphi \left(- \sum_{s=1}^{S_j} x_s^{jm(\tau-1)} + \sum_{i=1}^I \sum_{n=1}^{N_i} x_{jm}^{in(\tau-1)} \cdot \psi_{jm}^{in} \right) + \lambda_{jm}^{1(\tau-1)} \right\} \end{aligned}$$

$$\begin{aligned}\bar{\lambda}_{js}^{2(\tau-1)} &= \max\left\{0, \varphi\left(-\sum_{m=1}^{M_j} x_s^{jm(\tau-1)} + \sum_{k=1}^K \sum_{t=1}^{T_j} x_{tk}^{js(\tau-1)}\right) + \lambda_{js}^{2(\tau-1)}\right\} \\ \bar{\mu}_{ing}^{0(\tau-1)} &= \max\left\{0, \varphi\left(-A_g^{in} + \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in(\tau-1)} - \delta_g^{in(\tau-1)}\right) + \mu_{ing}^{0(\tau-1)}\right\} \\ \bar{\mu}_{jmg}^{1(\tau-1)} &= \max\left\{0, \varphi\left(-B_g^{jm} + \sum_{s=1}^{S_j} x_s^{jm(\tau-1)} - \delta_g^{jm(\tau-1)}\right) + \mu_{jmg}^{1(\tau-1)}\right\}\end{aligned}$$

The algorithm converges to the solution of $VI(F, \mathcal{X})$ under the following conditions.

Theorem 17 (Convergence)

Assume that $F(X)$ is monotone, as is in expression (2.12), and that $F(X)$ is also Lipschitz continuous, then the modified projection method converges to a solution of VI (4.37).

Proof: see Theorem 2.5 in [Nagurney \(1999\)](#).

4.6 Small Scale Examples

In this section, I construct several numerical cases to illustrate this model’s utility. Example 1 is an application to the medical PPE glove supply. Example 2 broadens the application to an interconnected abstract resource-trio supply chain. The aforementioned algorithm is implemented in MATLAB installed on an ordinary ASUS VivoBook F510 personal laptop computer with an Intel Core i5 CPU 2.5 GHz and RAM 8.00 GB. I exhibit and discuss the highlights of each example.

Example 1.1: A medical gloves supply chain network (benchmark)

The COVID-19 pandemic of 2020 has reportedly caused a demand surge for medical gloves ([Finkenstadt et al., 2020](#)). In a single 100-day wave during early 2020, the estimated need for medical gloves was 3.939 billion ([Toner, 2020](#)), followed by a subsequent official guideline on conservation and optimizing usage of gloves during medical practice in the U.S. ([CDC, 2020](#)). It has become clear that the scarcity of medical gloves calls for a boost in the supply chain to the extent of better coordination and stimulus effort. Commonly used medical glove materials include latex,

made from natural rubber, and nitrile, made from petroleum-based materials (Anedda et al., 2020; Henneberry, 2020).

This example illustrates a resource-duo supply chain network with natural rubber and petroleum as the resources, and medical gloves as their end-products. Specifically, the network contains 2 owners, 2 producers, and 2 retailers, as in Figure 4.12. The corresponding end-products, latex and nitrile gloves, are shipped to in 2 demand markets, medical and residential facilities, via 1 available transportation mode. I use this example as the benchmark case, in which the supply chain network has unlimited resources and is imposed no fiscal policies. I will continue to use the same topology

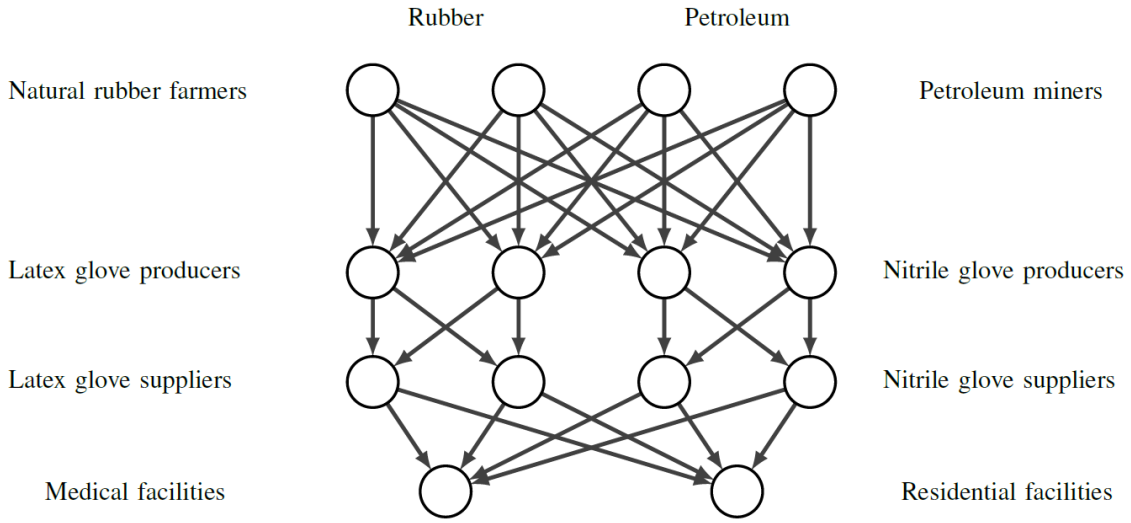


Figure 4.3: A resource-duo supply chain network for medical gloves

on example 1.2-1.5, with variations of setting. I examine the output quantities of firms, market prices, and welfare estimates.

The cost functions are constructed for all $i = 1, \dots, I$ and $j = 1, \dots, J$, with $I = J = N_i = M_j = K_j = 2$ and $T_j = 1$, as the following.

$$f^{1n}(x^{1n}) = 2.5 \left(\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{1n} \right)^2 + \left(\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{1n} \right) \left(\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{2n} \right) + 2 \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{1n},$$

$$f^{2n}(x^{2n}) = 0.5 \left(\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{2n} \right)^2 + \left(\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{1n} \right) \left(\sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{2n} \right) + 2 \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{2n},$$

$$c_s^{jm}(x_s^{jm}) = 0.5(x_s^{jm})^2, \quad f^{js}(x^{js}) = 0.1 \left(\sum_{m=1}^{M_j} x_s^{jm} \right)^2, \quad \hat{c}_{tk}^{js}(x_{tk}^{js}) = 0.01x_{tk}^{js},$$

$$c_{11}^{11} = 0.5(x_{11}^{11})^2 + 3.5x_{11}^{11}, \quad c_{12}^{11} = 0.5(x_{12}^{11})^2 + 3.5x_{12}^{11}, \quad c_{11}^{12} = 0.5(x_{11}^{12})^2 + 2x_{11}^{12}, \quad c_{12}^{12} = 0.5(x_{12}^{12})^2 + 2x_{12}^{12},$$

$$c_{11}^{21} = 0.4(x_{11}^{21})^2 + 3.5x_{11}^{21}, \quad c_{12}^{21} = 0.4(x_{12}^{21})^2 + 3.5x_{12}^{21}, \quad c_{11}^{22} = 0.4(x_{11}^{22})^2 + 2x_{11}^{22}, \quad c_{12}^{22} = 0.4(x_{12}^{22})^2 + 2x_{12}^{22}.$$

All other costs are set to zero. The price-demand functions at the markets are:

$$p_{3k}^j(d_{jk}) = -d_{jk} + 300, \quad \forall j, k.$$

In addition, the conversion rates of production by resource producers are $\psi_{jm}^{in} = 0.9$. The weights of resource commodities at the markets are $w_{11} = 0.5, w_{12} = 0.5$. The parameters concerning the resource capacities and policy instruments, U_i, A_g^i, B_g^i , are all set to a sufficiently large number, given their absence.

I initialize the algorithm by setting all the flow quantity to be 1, the step-size φ to be 0.01 (unless noted otherwise), the convergence tolerance ϵ to be 10^{-4} . The computation process takes approximately 2.0 seconds. I display the solution in Table 4.2. Among the equilibrium solution, the zero values of δ_g^{in} and δ_g^{jm} simply reaffirm the incentives to be flat-rate, whereas the zero values of all Lagrange multipliers suggest that the corresponding constraints are inactive.

Table 4.2: The equilibrium solution of example 1.1

Variable	Result	Variable	Result	Variable	Result	Variable	Result
x_{11}^{11*}	8.88	x_1^{11*}	13.79	δ_1^{11*}	0.00	λ_1^0*	0.00
x_{12}^{11*}	8.88	x_2^{11*}	18.19	δ_1^{12*}	0.00	λ_2^0*	0.00
x_{21}^{11*}	9.50	x_1^{12*}	13.79	δ_1^{21*}	0.00	λ_{11}^1*	247.28
x_{22}^{11*}	9.50	x_2^{12*}	18.19	δ_1^{22*}	0.00	λ_{12}^1*	247.28
x_{11}^{12*}	8.88	x_1^{21*}	14.53	δ_1^{11*}	0.00	λ_{21}^1*	247.29
x_{12}^{12*}	8.88	x_2^{21*}	19.65	δ_1^{12*}	0.00	λ_{22}^1*	247.29
x_{21}^{12*}	9.50	x_1^{22*}	14.53	δ_1^{21*}	0.00	λ_{11}^2*	266.59
x_{22}^{12*}	9.50	x_2^{22*}	19.65	δ_1^{22*}	0.00	λ_{12}^2*	263.64
x_{11}^{21*}	8.88	x_{11}^{11*}	13.79	μ_{111}^0*	0.00	λ_{21}^2*	267.64
x_{12}^{21*}	8.88	x_{12}^{11*}	13.79	μ_{121}^0*	0.00	λ_{22}^2*	264.97
x_{21}^{21*}	9.50	x_{11}^{12*}	18.18	μ_{211}^0*	0.00	d_{11}^*	15.98
x_{22}^{21*}	9.50	x_{12}^{12*}	18.18	μ_{221}^0*	0.00	d_{12}^*	15.98

x_{11}^{22*}	8.88	x_{11}^{21*}	14.53	μ_{111}^{1*}	0.00	d_{21}^*	17.10
x_{12}^{22*}	8.88	x_{12}^{21*}	14.53	μ_{121}^{1*}	0.00	d_{22}^*	17.10
x_{21}^{22*}	9.50	x_{11}^{22*}	19.67	μ_{211}^{1*}	0.00		
x_{22}^{22*}	9.50	x_{12}^{22*}	19.67	μ_{221}^{1*}	0.00		

Example 1.2: The commons¹⁰ with a resource capacity limit

The natural rubber shipments from the commons of farming and harvesting, which are concentrated in Southeast Asia, can be severely affected by external shocks, such as natural disasters, geopolitical shifts, regulations, and pandemics (Chou, 2020; Lee, 2020). Therefore, it bears merit to investigate the resilience of these supply chains. I perform a sensitivity study on the rubber capacity limit, by inheriting all settings from example 1.1, with the additional imposition of resource capacity on natural rubber. Acknowledging U_1 's level when such a limit can be reached, I vary U_1 between 10 and 100. As the natural rubber's capacity increases, as expected, each glove producer,

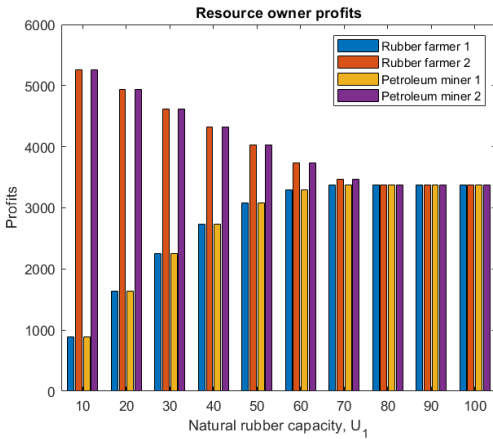


Figure 4.4: Owner profits under resource capacity

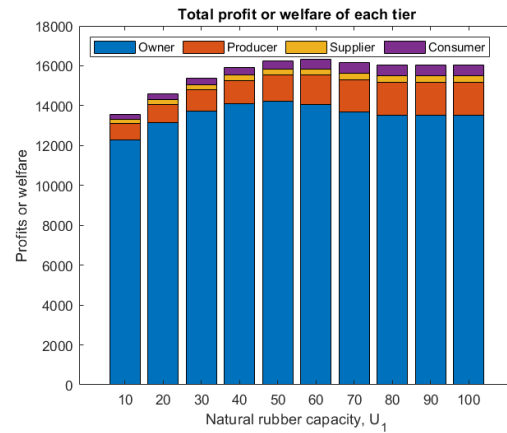


Figure 4.5: Total profits and welfare under resource capacity

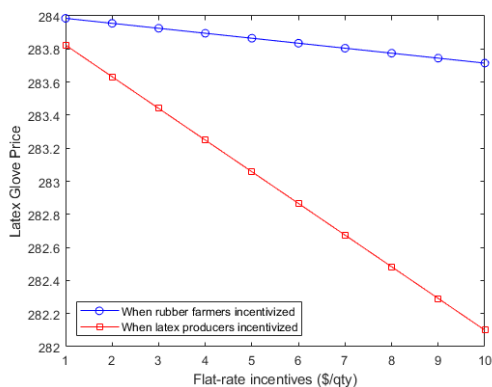
supplier's profit, as well as the consumer surplus, increases. The petroleum miners, however, suffer from the abundance of natural rubber, as Figure 4.4 shows. The total profits or surplus of each supply chain tier are displayed in Figure 4.5. It is worth noting that the overall height of each stacked bar indicates social welfare of the entire network. Interestingly, it is found that social welfare peaks at a U_1 's level of around 60, likely owing to a similar trend of the owners' total profit

¹⁰A commons is where the natural resources are owned and shared collectively (Ostrom, 1990).

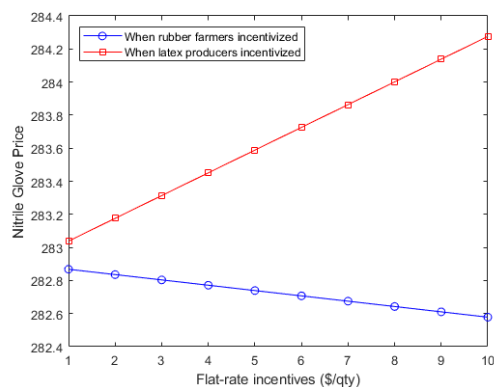
that peaks at the similar level of U_1 .

Example 1.3: Who should get the incentive, owners or producers?

In this example, I compare the scenarios in which either the natural rubber farmers or the latex glove producers receive a fairly small flat-rate incentive on their production quantity. As such, I inherit all settings from example 1.1, with the additional imposition of a fiscal policy in the form of quantity incentives. Specifically, I separately incentivize the natural rubber farmers and the latex glove producers with a flat-rate of α_0^1 and β_0^1 , respectively.



(a) Latex glove market price



(b) Nitrile glove market price

Figure 4.6: Glove market prices under flat-rate incentives

I elect to examine only the glove prices at the markets, as the rest of the equilibrium results can be examined invariably. In Figure 4.6, one observes that with the flat-rate incentives administered to the latex glove supply chain, the corresponding latex glove prices at the market are reduced, whereas the prices of the substitute product, nitrile glove, change in opposite trends.

From the standpoint of the supply chain participants, the farmers, producers, and consumers each as a tier, benefit mildly, because of the relatively small amount of incentives. See Figure 4.7. The suppliers, however, enjoy a discernible gain in total profit when the incentives are given to the rubber farmers.

From the standpoint of the incentive administer, it bears meaning to examine such a fiscal policy's social benefit and efficiency. I use the Benefit-Cost ratio, i.e., the dollar amount of social-welfare gain for every \$1 incentive administered to the system, to represent the efficiency. In Figure

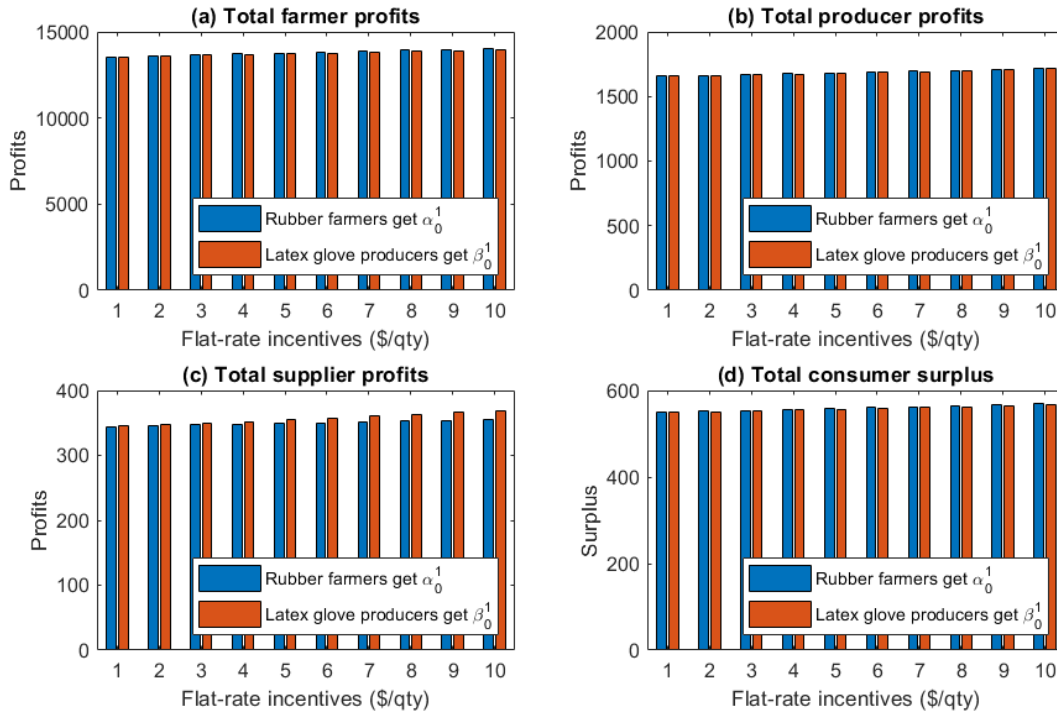


Figure 4.7: Total profits and welfare under flat-rate incentives

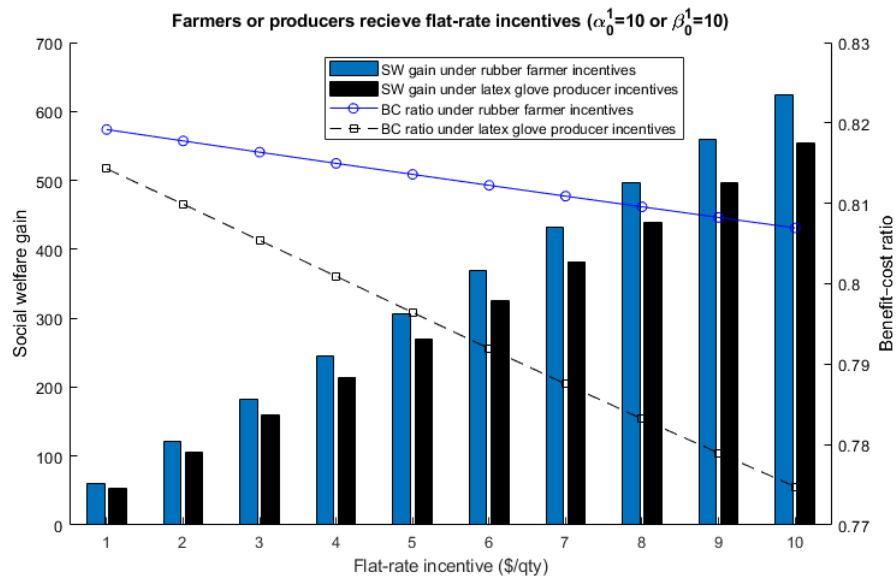


Figure 4.8: Social-welfare gains and policy efficiencies under flat-rate incentives

4.8, I show the social-welfare gains, as well as the efficiency of the incentive. It can be seen that the efficiency of the incentive given to the resource producer suffers more severely.

Example 1.4: Regressive vs. flat-rate incentives

In this example, I examine how a regressive incentive policy differs from a flat-rate one, as well as how the incentive bracket affects the system performance. Once again, I inherit all settings from example 1.1, plus the additional imposition of regressive incentive policy, with $\alpha_0^1 = 11$ and $\alpha_1^1 = -2.2$ on both natural rubber farmers, to be selected to provide a sensible comparison against a flat-rate, $\alpha_0^1 = 10$, incentive policy. The only incentive bracket, A_1^1 , will be left varying as a sensitivity study parameter.

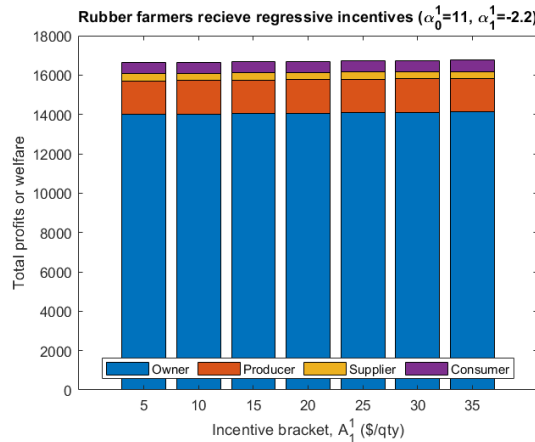


Figure 4.9: Total profits and welfare under regressive incentives

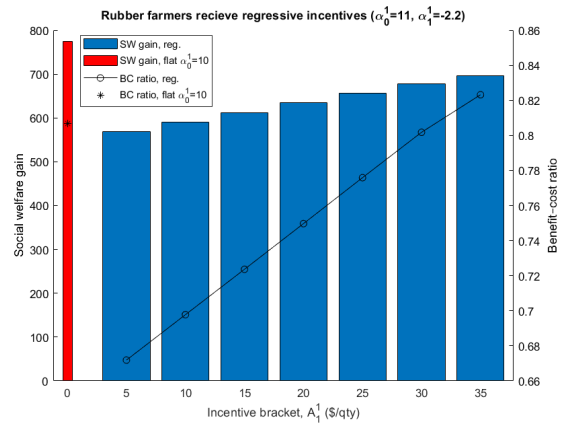


Figure 4.10: Social-welfare gains and benefit-cost ratio under regressive incentives

The range of A_1^1 displayed in Figure 4.9 and 4.10 are selected to ensure each farmer’s total output quantity exceeds such a bracket value for it to take effect. In Figure 4.9, one can observe that the change of the regressive incentive bracket does not influence each supply chain tier’s profit and the consumer welfare significantly. Such a result is consistent with the findings with respect to the effect of the “tax brackets” on firm’s profits in a study by Yu et al. (2018). In Figure 4.10, a mildly increasing gain of social welfare can be gleaned as the bracket increases, though falling short of the comparable flat-rate policy. The dollar amount of social benefit, embodied by the Benefit-Cost ratio, on the other hand, trends up with the bracket level.

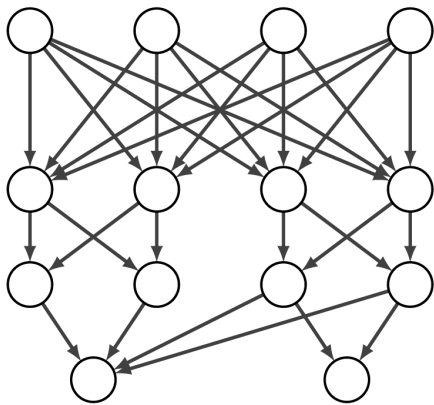
Example 1.5: Critical resource shortage relief

In this example, I use a flat-rate incentive to relieve a latex glove shortage caused by a demand surge at the medical facilities. First, I construct a distressed supply chain in which the natural rubber emerges to be a critical shortage in its supplies at the medical facilities. In doing so, I

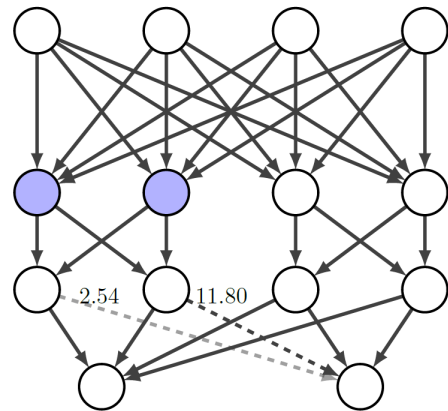
inherit all settings from example 1.1 and set the price-demand function of the latex gloves at medical facilities to be

$$p_{31}^1(d_{11}) = -d_{11} + 420.$$

The algorithm returns the result including the shipments from two of the suppliers to a market, x_{12}^{11} and x_{12}^{12} being 0. To illustrate, Figure 4.11(a) displays the topology of this disrupted supply chain, in which the residential facilities are completely cut from the supply of latex gloves. To aid such



(a) A latex glove shortage occurs at residential



(b) Shortage relieved by producer incentives

Figure 4.11: The relief of a critical resource shortage by production stimulus

circumstances, I impose a flat-rate incentive on both latex glove producers, with $\beta_0^1 = 50$, and re-run the model. Immediately, the previously disrupted supply can be restored. I use Figure 4.11(b) to display the recovered supply chain status. Similar results of shortage relief can be achieved by imposing a flat-rate incentive on both rubber farmers.

Example 2: A Mixed Fiscal Policy in An Abstract Scarce Resource Supply Chain

In many developed economies, governments tend to be concerned with levels of income inequality, and are, therefore, interested in making redistribution of societal wealth a substantial objective for economic development. Thus, it is meaningful to evaluate the utility of a mixed fiscal policy in redistributing social welfare. In this example, I construct an abstract resource-trio network to illustrate a mixed fiscal policy with a combination of incentives and taxes. In the network topology

shown in Figure 4.12, the red nodes indicate the firms that are being taxed whereas the blue ones incentivized.

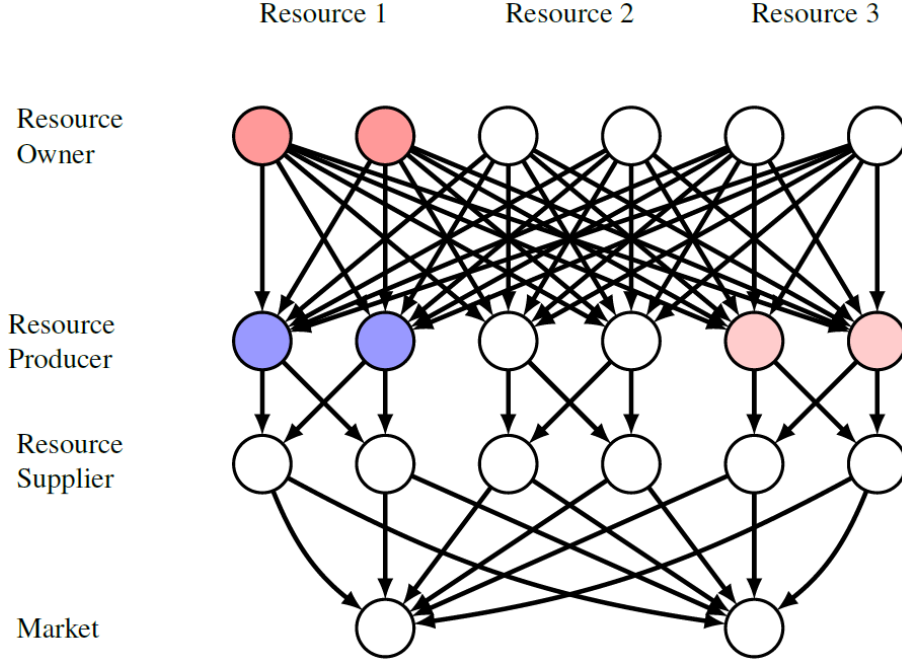


Figure 4.12: A resource-trio supply chain network with a mixed fiscal policy

The cost functions are constructed, for all $i = 1, \dots, I$ and $j = 1, \dots, J$, with $I = J = 3$, $N_i = M_j = K_j = 2$ and $T_j = 1$, as the following.

$$\begin{aligned}
 f^{11} &= 2.5q_{11}^2 + q_{11}(q_{21} + q_{31}) + 2q_{11}, & f^{12} &= 2.5q_{12}^2 + q_{12}(q_{22} + q_{32}) + 2q_{12}, \\
 f^{13} &= 2.5q_{13}^2 + q_{13}(q_{23} + q_{33}) + 2q_{13}, & f^{21} &= 0.5q_{21}^2 + q_{21}(q_{11} + q_{31}) + 2q_{21}, \\
 f^{22} &= 0.5q_{22}^2 + q_{22}(q_{12} + q_{32}) + 2q_{22}, & f^{23} &= 0.5q_{23}^2 + q_{23}(q_{13} + q_{33}) + 2q_{23}, \\
 f^{31} &= 0.5q_{31}^2 + q_{31}(q_{11} + q_{21}) + 2q_{31}, & f^{32} &= 0.5q_{32}^2 + q_{32}(q_{12} + q_{22}) + 2q_{32}, \\
 f^{33} &= 0.5q_{33}^2 + q_{33}(q_{13} + q_{23}) + 2q_{33}, & \text{where, } q_{in} &= \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in}; \\
 c_1^{11} &= 0.5(x_1^{11})^2, & c_2^{11} &= 0.25(x_2^{11})^2, & c_1^{12} &= 0.5(x_1^{12})^2, & c_2^{12} &= 0.25(x_2^{12})^2, \\
 c_1^{21} &= 0.5(x_1^{21})^2, & c_2^{21} &= 0.25(x_2^{21})^2, & c_1^{22} &= 0.5(x_1^{22})^2, & c_2^{22} &= 0.25(x_2^{22})^2, \\
 c_1^{31} &= 0.5(x_1^{31})^2, & c_2^{31} &= 0.25(x_2^{31})^2, & c_1^{32} &= 0.5(x_1^{32})^2, & c_2^{32} &= 0.25(x_2^{32})^2; \\
 f^{11} &= 0.1(x_1^{11} + x_1^{12})^2, & f^{12} &= 0.1(x_2^{11} + x_2^{12})^2,
 \end{aligned}$$

$$\begin{aligned}
f^{21} &= 0.1(x_1^{21} + x_1^{22})^2, & f^{22} &= 0.1(x_2^{21} + x_2^{22})^2, \\
f^{31} &= 0.1(x_1^{31} + x_1^{32})^2, & f^{32} &= 0.1(x_2^{31} + x_2^{32})^2; \\
c_{11}^{11} &= 0.5(x_{11}^{11})^2 + 3.5x_{11}^{11}, & c_{12}^{11} &= 0.5(x_{12}^{11})^2 + 3.5x_{12}^{11}, \\
c_{11}^{12} &= 0.5(x_{11}^{12})^2 + 2x_{11}^{12}, & c_{12}^{12} &= 0.5(x_{12}^{12})^2 + 2x_{12}^{12}, \\
c_{11}^{21} &= 0.4(x_{11}^{21})^2 + 3.5x_{11}^{21}, & c_{12}^{21} &= 0.4(x_{12}^{21})^2 + 3.5x_{12}^{21}, \\
c_{11}^{22} &= 0.4(x_{11}^{22})^2 + 2x_{11}^{22}, & c_{12}^{22} &= 0.4(x_{12}^{22})^2 + 2x_{12}^{22}, \\
c_{11}^{31} &= 0.45(x_{11}^{31})^2 + 3.5x_{11}^{31}, & c_{12}^{31} &= 0.45(x_{12}^{31})^2 + 3.5x_{12}^{31}, \\
c_{11}^{32} &= 0.45(x_{11}^{32})^2 + 2x_{11}^{32}, & c_{12}^{32} &= 0.45(x_{12}^{32})^2 + 2x_{12}^{32}, \\
\hat{c}_{tk}^{js}(x_{tk}^{js}) &= 0.1x_{tk}^{js}.
\end{aligned}$$

All other costs are set to zero. The price-demand functions at the markets are

$$p_{3k}^j(d_{jk}) = -d_{jk} + 300, \quad \forall j, k.$$

Similar to example 1-5, I set the production conversion rates $\psi_{jm}^{in} = 0.9$, the market resource commodity weights $w_{11} = 0.5, w_{12} = 0.5$, the parameters concerning the resource capacities, U_i , sufficiently large. In contrast to previous examples, here I set the step-size φ to be 10^{-5} , the convergence tolerance ϵ to be 6×10^{-4} for this example.

To examine the efficacy of this mixed fiscal policy, I first establish the equilibrium of the benchmark scenario, i.e., the setting without such policy. As such, I present the results in table 4.3. With the ex-ante knowledge that the resource owners captures most of the supply chain profit, I then impose such policy in which the producers of resource 1 are given a flat-rate incentive of $\beta_0^1 = 12$, whereas the owners of resource 1 and producers of resource 3 are charged a flat-rate tax of $\alpha_0^1 = -10$ and $\beta_0^1 = -2$, respectively.

The projection method takes approximately 120 seconds for this problem of total 99 variables to converge to the preset tolerance. I include all equilibrium results in the supplemental file while displaying only the profit-related outcome in Table 4.3 again. It is worth pointing out that the “net incentive” is the total taxes collected net the total incentive disbursed by the government.

Table 4.3: Profit and welfare results of example 2

Profit		Benchmark	with policy	Welfare		Benchmark	with policy
Owner	π_{11}	2573.50	2359.58	Consumer	CS_{11}	104.26	141.64
Owner	π_{12}	2573.50	2700.43	Consumer	CS_{12}	120.11	97.84
Owner	π_{21}	2573.50	2700.43	Consumer	CS_{21}	107.24	77.30
Owner	π_{22}	2573.50	2359.58	Consumer	CS_{22}	104.26	141.64
Owner	π_{31}	2573.50	2700.43	Consumer	CS_{31}	120.11	97.84
Owner	π_{32}	2573.50	2700.43	Consumer	CS_{32}	107.24	77.30
Producer	π_{11}	248.25	340.20	Tot. owner	π_{total}	15441.00	15520.90
Producer	π_{12}	433.20	345.63	Tot. producer	π_{total}	1977.61	1963.28
Producer	π_{21}	307.36	295.82	Tot. supplier	π_{total}	1825.59	667.13
Producer	π_{22}	248.25	340.20	Tot. consumer	CS_{total}	663.21	633.57
Producer	π_{31}	433.20	345.63	Soc. welfare	SW	19907.42	18784.87
Producer	π_{32}	307.36	295.82	Net Incentive		0.00	99.15
Supplier	π_{11}	323.04	175.60	Soc. welfare gain	ΔSW	0.00	-1122.55
Supplier	π_{12}	302.42	60.78	Benefit-to-cost	BC	-	-11.32
Supplier	π_{21}	255.43	95.10				
Supplier	π_{22}	319.75	182.51				
Supplier	π_{31}	291.83	58.54				
Supplier	π_{32}	333.12	94.60				

In Figure 4.13, one can observe that under the mixed fiscal policy, the directly affected firms, i.e., the top two tiers of the network, do not consistently respond to the intended policy elements, as there are both increases and decreases of profit on both the incentivized and taxed firms. On the bottom two tiers of the network, the supplier profits drop unanimously, whereas the distribution of consumer surplus across the markets becomes more uneven.

Overall, with a positive net incentive, \$99.15, administered to the network, one can observe, however, that social welfare gains a negative amount, owing to the significant decrease in supplier profits. See Table 4.3 and Figure 4.14.

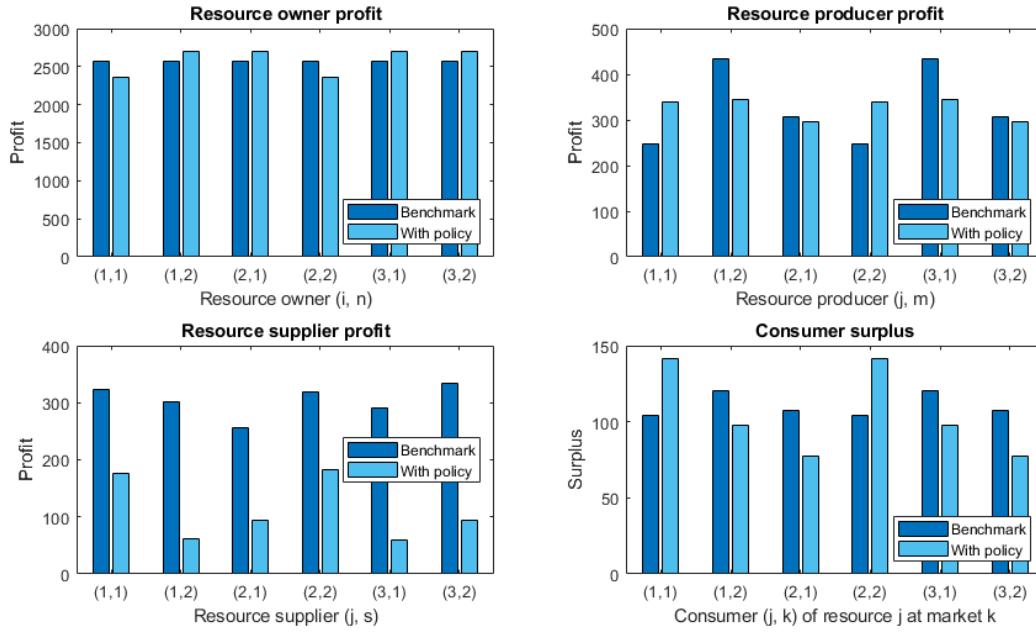


Figure 4.13: Profits and surplus under a mixed fiscal policy



Figure 4.14: Total profits and welfare under a mixed fiscal policy

4.7 Managerial Implications

Government plays a key role in maintaining a balance between the firm's profitability, consumer's well-being, supply chain's overall health, and society's sustainable growth. Yet in the face of widespread resource shortages that stifle the economic growth in supply chains, balancing the tradeoffs between the goals has been enormously challenging for governments. The proposed cross-

sector multi-product scarce resource supply chain network model can serve as a support system for both the supply chain practitioners and governments to make operational and strategic planning decisions. With a particular focus on the practice that can be adopted by the government to help address the insurgent scarcity and achieve overall sustainability, I herein instill the following managerial implications.

1. From the viewpoint of designing a stimulative fiscal policy, the legislator will likely face a caveat between the choice of a flat-rate policy and a regressive one. My analysis shows that the flat-rate incentive generally performs more effectively than its regressive counterpart when administered to the resource owners in stimulating supply chain welfare. This result is in general agreement with a dual study by [Yu et al. \(2018\)](#), in which they pointed out the strength of flat-rate tax when imposed to discourage adverse environmental activities. This result is also supported by the advantages of approximately linear tax in maintaining welfare ([Mirrlees, 1971](#)). Furthermore, as a case in point, it is also found that if the legislator's focus of policy design shifts to the efficiency of it, i.e., the social-welfare gain on every dollar of fiscal investment, then a carefully designed two-bracket regressive incentive can be more advantageous in policy efficiency than its flat-rate counterpart. That is, with a large enough "bracket", the legislator may be able to devise a regressive incentive scheme that can achieve a higher Benefit-Cost ratio, and almost as high of a net welfare gain as the flat-rate scheme. Though delicate, the value of such a bracket can be practically narrowed down by determining the largest effective bracket, followed by determining the largest achievable net welfare gain.
2. For the choice of incentive recipients in a supply chain, i.e., the resource owners or the resource producers, the decision-makers should anticipate a tradeoff. When the incentives are administered to the owners, both the social welfare gain and the policy efficiency will remain at a relatively high level. When the incentives are administered to the producers, the supplier's profit will enjoy the most gains, whereas the market price of the affected products will be reduced significantly. In practice, the market price of a scarce resource not only influences the consumer's behaviors, but also serves as a key indicator for the broader economy. Under a supply chain disruption, I suggest that the governments first administer an appropriate amount of incentives to the producers so that the commodity prices can be reduced at the

market level in short term. I note that, at the same time, any commodities experiencing shortage at the market level may also be relieved by the same incentives. Once the cause for the disruption has subsided and the supply chain's performance measures stabilized, governments can then reelect the appropriate supply chain tier(s) as the new recipients of the next round of incentives, depending on the economic objectives and legislative priorities.

3. "Income inequality" is believed to hinder economic growth. Using fiscal policy to reduce income gaps has become the goal of many advanced economies (Coady and Gupta, 2012). In an observation, the large profit difference across the supply chain tiers entices a growth-minded government to pursue a redistributive strategy for social surplus. It has been found, however, that the redistribution of welfare among resource owners and producers via a mixed fiscal policy may result in a net loss of welfare. While I acknowledge that such a finding resembles the rhetoric of the opposition to the dominant social policy notion that the resources generated by economic growth should be redistributed to fund social programs (Midgley, 1999), it is pertinent to note that the current analytical takeaway is derived from a microeconomic framework with standard assumptions. Nonetheless, I note further that the welfare loss caused by my experimental mixed fiscal policy has been previously associated with the elasticity of demand from the classic oligopoly theories (Worcester, 1975). In particular, it is also widely acknowledged in literature that taxation can be ineffective in reducing wealth inequalities as opposed to what conventional wisdom would have anticipated (Mirrlees, 1971). As is demonstrated before, the proposed fiscal policy can effectively relieve resource shortages and stimulate growth in supply chains. But for the governments or practitioners who oversee the supply chain grand strategies in a post-crisis stage, I caution that the well-intended fiscal interventionist may derail the overall economic sustainability of society. I advise that the use of a mixed fiscal policy in supply chains should follow the principle of configuring a mild or less redistributive policy, as is recognized in Coady and Gupta (2012).
4. Finally, it is noted an interesting link between resource capacity and welfare within a competitive supply chain. In related literature, Chen and Chou (2008) imposed firm-wise resource capacity in their supply chain and found that capacity limit restricts welfare. In both Nagurney et al. (2019b) and Hu and Qiang (2013), though the shared capacity limit was modeled,

neither study associated such a limit with its impact on social welfare. The current study, however, proffers further results. I find that the capacity limit of a given type of scarce (more often, natural) resource does not strictly curtail social welfare. Rather, an appropriate level of the limit can even benefit the social outcome. In practice, the ownership and the right to use a natural resource as a shared public good is often governed by its respective common laws or local policies¹¹. Thus, if a natural resource of critical importance belongs to a government’s jurisdiction, it is advisable for the government to legislate and impose a mild restriction on the usage of such natural resources. The restriction, if properly selected, will not only preserve the quantity of the shared resource but also create a “sweet spot” that induces higher social welfare.

Admittedly, the above managerial implications are based on stylized numerical experiments and thus, are limited to the extent of the characteristics of the network, e.g., the competitive nature of the nodes, the substitutability of the flows, the mix of policies, etc. Moreover, because of the number of features incorporated in this model, it is probable that in the presence of multiple features, their interactions could result in more profound managerial implications than what I have uncovered. Nonetheless, practitioners and decision-makers should carefully verify and validate the premises of the model before expanding the aforementioned insights.

4.8 Chapter Summary

In this chapter, I construct the first general decentralized cross-sector scarce resource SCNE model with a unifying supply-side fiscal policy. A rigorous VI formulation for the governing equilibrium conditions of the network model is provided. Such a substitute network provides a versatile tool for the evaluation of profit, welfare, policy instruments, cost structure, transportation, conservation, competition, and interdependence of resources throughout the supply chains. The generality of this model also allows for a variety of extensions, i.e., dynamics, stochastic features, multi-criteria decision-making, disequilibrium behaviors, etc, to be furnished.

Second, the model also constitute a general Nash equilibrium problem. I formulate the GNE

¹¹Public goods has been a widely discussed topic in studies of economics and law. See Réaume (1988), Holcombe (1997) and the reference therein.

of the network model in VI. There are only a few GNEP studies that incorporate fiscal policy in the SCNE literature. The utility of this model is not limited to the scarce resource supply chains, but also eligible to any resource commodity that pertains to the aforementioned characteristics of scarce resources.

Third, from a technical aspect, I introduce a recently uncovered approach to characterize the network equilibrium by adopting a novel set of theoretical properties, including λ_{min} . To the best of my knowledge in the supply chain network literature, such a means of characterization for the uniqueness property of network equilibrium has yet to appear heretofore.

Lastly, I furnish the model with numerical studies and extract managerial insights that provide governments, resource owners, and firms useful advice in expansion, cost restructuring, resource conservation, competition/collaboration strategy, shortage handling, and post-crisis stimulation. In particular, I provide guidance on supply-side policy design and administration in relieving and stimulating the PPE shortage caused by the COVID-19 global pandemic. The findings enrich the political discussion on public resource legislation, income inequality, and sustainable development.

Chapter 5

MODELING RARE EARTH MATERIAL RECYCLING, CHIP SHORTAGE, AND POLICY UNCERTAINTIES

5.1 Introduction

In Chapter 1, the argument is made that the criticality and vulnerability of rare earth element (REE) is not only a cause, but also an effect of policy uncertainties. It is imperative to develop a shared understanding of the risks associated with REE supply-side policies within the context of competition and conservation, by constructing a framework designed for this purpose. Recent literature on REEs has emphasized the importance of recycling, as highlighted by [Omodara et al. \(2019\)](#), [Li et al. \(2020\)](#), and [Schmid \(2019\)](#). For instance, [Li et al. \(2020\)](#) concluded from a simulation model that material recycling, efficiency, production expansion, and technical innovation are promising approaches for mitigating REE supply shortages in the long term. [Schmid \(2019\)](#) referred to Japan's strategies as a reference for the US to adopt recycling as a more promising approach to mitigate supply risks. [Mancheri et al. \(2019\)](#) called for a detailed quantitative analysis of Chinese REE policies to gain insights into global REE flows and resilience parameters. [Omodara](#)

et al. (2019) further suggested that intensive research be conducted to develop efficient and effective recycling and substitution schemes.

To address these issues, this chapter presents a novel framework, the robust closed-loop supply chain network equilibrium (RCLSCNE) model, which characterizes the production, consumption, and recycling activities of REE resources, along with their associated utilities. Robust optimization techniques are employed to account for the uncertainties in supply induced by policies and resource capacities, given the lack of public data in these critical areas of the commodity (Mancheri et al., 2019). The proposed model can also be formulated as a generalized Nash equilibrium problem, taking into consideration of the scarce resource’s pooling effect discussed in Chapter 4. Moreover, variational inequality theories are adopted to establish the network equilibrium and theoretical properties of the game-theoretic model. Finally, a pathway to solving this problem with an algorithmic procedure is outlined. This chapter is derived from the paper by Hu and Li (2023)¹².

5.1.1 Research Gap and Relevance to Literature

The research in closed-loop supply chain (CLSC) and robust optimization (RO) applications has yielded significant progress in the past decade, as evidenced by several notable studies (Bertsimas and Thiele, 2004; Ben-Tal et al., 2005; Pishvaei et al., 2011). However, there are still notable gaps that need to be addressed. First, despite the development of tractable RO techniques in the 2000s, the application of RO in CLSC remained limited until the 2010s (Guan et al., 2020). Therefore, there is a need for further exploration of data-driven modeling at the intersection of CLSC and RO, with a focus on sustainable development and multi-objective models that consider economic, legislative, fiscal, and social factors (González-Sánchez et al., 2020).

Second, existing RO models primarily focus on uncertainties related to supplier costs or demand quantities (§1.2.6), with limited attention given to uncertainties arising from supply-side policies or resource capacities. Hence, there is a gap in the literature for addressing these types of uncertainties within the context of supply chain network equilibrium (SCNE) studies.

Third, in terms of addressing the criticality of REEs and the resulting supply chain crises, the broader supply chain modeling literature has lacked sufficient coverage. While previous studies have focused on cost minimization at the firm’s production level using optimization techniques

¹²I thank Dr. Peng Li, who co-authored this paper, and in turn, contributed to this chapter.

Table 5.1: A review of supply chain network models since 2010

Article	Closed-loop	Game theory	Robust optm.	Stochastic optm.	Policy
Allevi et al. (2018)	•	•			•
Tao et al. (2015)	•	•			•
Wan and Hong (2019)	•	•			•
Wu et al. (2019)	•	•			•
Duan et al. (2020)	•	•			
Qiang et al. (2013)	•	•		•	
Mohammed et al. (2018)	•		•		•
Hasani et al. (2012)	•		•		
Hatefi and Jolai (2014)	•		•		
Pishvaei et al. (2011)	•		•		
Ma et al. (2016)	•		•		
Soleimani et al. (2021)	•		•		
Talaee et al. (2016)	•		•		
Nayeri et al. (2020)	•		•		
Yavari and Geraeli (2019)	•		•		
Heidari-Fathian and Pasandideh (2018)	•		•		
Ramezani et al. (2013)	•		•		
Shi et al. (2011)	•				
Amin and Zhang (2013)	•			•	
Zeballos et al. (2013)	•			•	
Hirano and Narushima (2019)		•	•		
Yu et al. (2016)		•			•
Hu and Li (2022)		•			•
Nagurney et al. (2021)		•		•	
This chapter	•	•	•		•

([Knutson et al., 2015, 2017](#)), few analytical models have integrated competition, risk, and scarcity considerations.

Finally, a comprehensive review of the most relevant papers since 2010 is provided in [Table 5.1](#). To the best of my knowledge, this study is the first to integrate robust optimization techniques into a closed-loop supply chain equilibrium network model.

The remainder of this chapter is organized as follows. [Section 5.2](#) presents the RCLSCNE model. [Section 5.3](#) shows the solution concept of the model. [Section 5.4](#) reformulates the equilibrium problem and proposes an algorithmic procedure to solve it. Lastly, [section 5.5](#) serves as an epilogue with highlights and extensions of this study.

5.2 The Robust Closed-Loop Supply Chain Network Model

I propose a robust closed-loop supply chain network in which REE raw material suppliers, chip producers, and demand markets participate a forward supply flow, whereas the demand markets, recovery centers, and chip producers participate in the reverse logistics by collecting recyclable EoL products from demand markets. Through the markets and producers, the forward and reverse chains join and form a closed-loop. See Figure 5.1. I now outline the behaviors of each tier of the network and state the model assumptions.

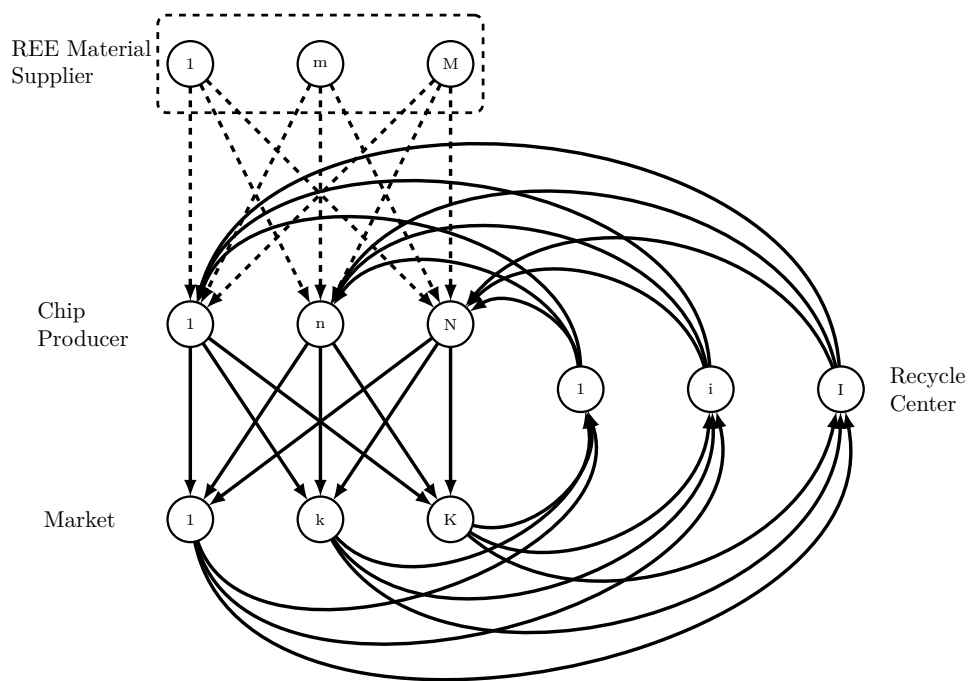


Figure 5.1: The closed-loop chip supply chain network model

REE Material Suppliers. Each REE material supplier (denoted as $m \in \{1, \dots, M\}$) extracts, refines, and sells one type of virgin REE material to the chip producers. All suppliers excavate and extract from a shared REE mineral source with a capacity. At the same time, each of them is subject to an influence by some policies (more in §5.2.1).

Chip Producers. Each chip producer (denoted as $n \in \{1, \dots, N\}$) uses either virgin materials from the suppliers or recycled materials from the recycle centers to produce semiconductor chips. Then they sell the semiconductor chips to consumers located in different demand markets.

Recycle Centers. The recycle centers (denoted as $i \in \{1, \dots, I\}$) collect the recyclable EoL

products from the demand markets, transport them back to their own facilities, where they sort and process them accordingly before the recycled materials are sold to the chip producers. It is assumed that the recycle centers do not recycle products from sources other than the demand markets.

Markets. Each market (denoted as $k \in \{1, \dots, K\}$) determines the procurement and return quantity based on the conditions detailed in §5.2.4.

It is assumed that all firms of the same tier supply a homogeneous product, i.e., the output of a given firm is indistinguishable from others of the same tier. I also assume that firms or markets in this model act independently of all others in both the same and other tiers. All firms compete in a non-cooperative fashion by determining their optimal production quantity and shipments, given the simultaneous best response of all competitors. The governing equilibrium concept that encompasses such behavior is a generalized Nash equilibrium (more in §5.3). All the decision variables are determined before any of the uncertain parameters are realized. All of the notations regarding the model are listed in §2.6.

5.2.1 REE Raw Material Suppliers' Optimal Behavior

Constructing a risk-immune optimization problem

A typical REE material supplier m strategically maximizes its profit π_m by determining its REE raw material shipment quantity x_{mn} to some chip producers of the next tier. A column vector, Q^1 , is used to group all x_{mn} . Each supplier incurs an operating cost $f_m = f_m(x_m)$, where $x_m \equiv (x_{m1}, \dots, x_{mn}, \dots, x_{mN}) \in \mathbb{R}^N$ is a column vector, and a transaction cost $c_{mn} = c_{mn}(x_{mn})$, $\forall m, n$. It is assumed that both functions are continuously differentiable. Each supplier is also subject to a direct impact on profit caused by a policy adversity: $\Phi_m(x_{mn}, w_m) = w_m^{\mathbf{T}} x_m$,¹³ where $w_m \equiv (w_{m1}, \dots, w_{mn}, \dots, w_{mN}) \in \mathbb{R}^N$, is a column vector. It is assumed that the strength parameter of policy adversity, w_{mn} , is a random number whose distribution information is unknown. It is assumed that the the vector of strength parameter resides within a certain set enclosed by an ellipsoid. I hence define an ellipsoidal uncertainty set for w_m as the following:

$$\mathcal{U}_m^{ell} \equiv \left\{ w_m \in \mathbb{R}^{S_m} : w_{m,s} = \bar{w}_m + H_{ms} u_s, \text{ where } \|u_s\| \leq 1, \quad \forall s = 1, \dots, S_m \right\}, \quad \forall m. \quad (5.1)$$

¹³Throughout this chapter, I use the boldface superscript \mathbf{T} to denote the transpose operator, as $\cdot^{\mathbf{T}}$, in distinguishing from the index T .

Table 5.2: Data structure of supplier’s policy uncertainty parameters

Supplier	Nominal Policy-Strength (\mathbb{R}^N)	Data Point	Policy-Strength Based on Data (\mathbb{R}^N)	Policy Uncertainty-Scale Matrix ($\mathbb{R}^{N \times N}$)
1	\bar{w}_1	1	w_{11}	H_{11}
		\vdots	\vdots	\vdots
		s	w_{1s}	H_{1s}
		\vdots	\vdots	\vdots
\vdots	\vdots	S_1	w_{1S_1}	H_{1S_1}
m	\bar{w}_m	1	w_{m1}	H_{m1}
		\vdots	\vdots	\vdots
		s	w_{ms}	H_{ms}
		\vdots	\vdots	\vdots
\vdots	\vdots	S_m	w_{mS_m}	H_{mS_m}
M	\bar{w}_M	1	w_{M1}	H_{M1}
		\vdots	\vdots	\vdots
		s	w_{Ms}	H_{Ms}
		\vdots	\vdots	\vdots
		S_M	w_{MS_M}	H_{MS_M}

In the uncertainty set (5.1), the column vector $w_{m,s} \equiv (w_{m1,s}, \dots, w_{mn,s}, \dots, w_{mN,s})$ is a data point of w_m , indexed by $s \in \{1, \dots, S_m\}, \forall m$, with a nominal value vector, \bar{w}_m . The notation $\|\cdot\|$ represents the Euclidean norm, with which, I define $u_s \in \mathbb{R}^N$ to be a unit Euclidean ball. The matrix $H_{ms} \in \mathbb{R}^{N \times N}, \forall m, s$, assumed to be symmetric positive-definite¹⁴, contains historical correlation information between the policy strengths. With this definition of \mathcal{U}_m^{ell} , it is expected that the conservatism of the risk-immune outcome is to be reduced (see Ben-Tal et al. (2005) for extended discussions on “conservatism”). A data-structure depiction of the aforementioned parameters is given in Table 5.2.

All REE material suppliers share a common pool of resources (e.g., a mine) with an uncertain capacity Z and unknown distributional information. It is assumed that each uncertain capacity resides within a certain interval given by historical data. Following Ben-Tal et al. (2005) and Pishvae et al. (2011), I define Z ’s interval before its realization, as the following box uncertainty

¹⁴More specifically, Hirano and Narushima (2019) delineates that H_{ms} is often chosen to be $H_{ms} = g\Omega_{ms}^{-1}$, where g is a non-negative scalar denoting the uncertainty scale of the policy adversity, and Ω_{ms} is the variance-covariance matrix of historical data with respect to x_m . Thus, H_{ms} is guaranteed to be positive-definite, and \mathcal{U}_m^{ell} an ellipsoid.

set:

$$\mathcal{U}^{box} \equiv \left\{ Z \in \mathbb{R}^T : |Z_t - \bar{Z}_t| \leq \rho G_t, \quad \forall t = 1, \dots, T \right\}. \quad (5.2)$$

where, Z_t is the indexed value of a realized Z , \bar{Z}_t is the nominal value of Z_t , ρ is the uncertain level (with $\rho > 0$), and G_t is the uncertainty scale. Clearly, \mathcal{U}^{box} is a T -dimensional polytope. To ensure the random variable Z to be non-negative in practice, I select $G_t = \bar{Z}_t$ and restrict $0 < \rho \leq 1$. To this end, a supplier's profit under the uncertain policy adversity is:

$$\tilde{\pi}_m^1 = \sum_{n=1}^N p_{mn} x_{mn} - f_m(x_m) - \sum_{n=1}^N c_{mn}(x_{mn}) - \Phi_m(x_{mn}, w_m), \quad \forall m = 1, \dots, M. \quad (5.3)$$

Furthermore, I define the REE material supplier's disutility $\tilde{\psi}_m$ to be the negation of its profit, i.e., $\tilde{\psi}_m = -\tilde{\pi}_m^1$. Hence, a supplier's disutility minimization problem can be expressed in the following robust optimization problem:

$$(P) \quad \begin{aligned} & \min_{x_{mn} \geq 0} \quad \tilde{\psi}_m \\ & \text{s.t.} \quad \sum_{m=1}^M \sum_{n=1}^N x_{mn} \leq Z, \quad Z \in \mathcal{U}^{box}. \end{aligned} \quad (5.4)$$

The robust approach I adopt here is appropriate for the given modeling purpose, in that [Mancheri et al. \(2019\)](#) pointed out the difficulty to quantify the public policy issues and their impact on REE supply chain resilience, particularly due to the lack of data. In essence, problem (P) is no different from a profit-maximization problem. Moreover, because of the aforementioned criticality of rare earth supplies, one takes interest in the worst-case scenario under the uncertain policy effect. For this reason, it is appropriate to represent each supplier's strategic decisions as a robust counterpart (RC) problem of (P):

$$(RC) \quad \min_{x_m \in \mathcal{S}} \max_{\substack{w_m \in \mathcal{U}_m^{ell} \\ Z \in \mathcal{U}^{box}}} \tilde{\psi}_m \quad \forall m = 1, \dots, M. \quad (5.5)$$

where, $\mathcal{S} \equiv \{x_{mn} \in \mathbb{R}_+^{MN} : (5.4) \text{ holds}\}$ is the feasible set of x_{mn} . Note that both \mathcal{U}_m^{ell} and \mathcal{U}^{box} are convex sets. The problem (RC) seeks the x_{mn} that minimizes the worst disutility and, hence,

is regarded as a risk-immune optimization problem (Ben-Tal and Nemirovski, 1998, 2000).

Reformulating as tractable robust counterpart (TRC)

Noticing that both Z and w_m are affinely uncertain parameters, one will be able to reformulate the problem (RC) as a tractable forms. Further assuming independence of one another, the uncertain parameters Z and w_m can be handled separately within the inner “max” problem of (RC), as I proceed. First, re-express the uncertain parameter w_m and its set \mathcal{U}_m^{ell} . Denote $\psi_m \equiv \max_{w_m \in \mathcal{U}_m^{ell}} \tilde{\psi}_m$, then

$$\psi_m = - \sum_{n=1}^N p_{mn} x_{mn} + f_m(x_m) + \sum_{n=1}^N c_{mn}(x_{mn}) + \max_{s=1, \dots, S_m} \left\{ w_{m,s}^{\mathbf{T}} x_m, \|u_s\| \leq 1 \right\} \quad (5.6)$$

$$= - \sum_{n=1}^N p_{mn} x_{mn} + f_m(x_m) + \sum_{n=1}^N c_{mn}(x_{mn}) + \bar{w}_m^{\mathbf{T}} x_m + \max_{s=1, \dots, S_m} \left\{ u_s^{\mathbf{T}} H_{ms}^{\mathbf{T}} x_m, \|u_s\| \leq 1 \right\} \quad (5.7)$$

$$= - \sum_{n=1}^N p_{mn} x_{mn} + f_m(x_m) + \sum_{n=1}^N c_{mn}(x_{mn}) + \bar{w}_m^{\mathbf{T}} x_m + \max_{s=1, \dots, S_m} \|H_{ms}^{\mathbf{T}} x_m\|. \quad (5.8)$$

From (5.7) to (5.8), the derivation is achieved by virtue of Cauchy-Schwartz inequality, i.e., $u_s^{\mathbf{T}} H_{ms}^{\mathbf{T}} x_m \leq \|u_s^{\mathbf{T}}\| \cdot \|H_{ms}^{\mathbf{T}} x_m\|$. The equality is preserved due to the “max” operator. Hence, an affine robust optimization problem ψ_m is cast as a second-order cone problem (SCOP), a sub-class of convex optimization problem. Further more, it is worth noting that the last term of (5.8) is also a maximum of norms. Because the norm term is non-differentiable and therefore is undesirable in an objective function, I introduce auxiliary variables, v_{ms} and \bar{v}_m , $\forall m = 1, \dots, M$ and $s = 1, \dots, S_m$, such that $\|H_{ms}^{\mathbf{T}} x_m\| \leq v_{ms}$, and $v_{ms} \leq \bar{v}_m$, $\forall m, s$. (For more details of such formulation technique, see Lobo et al. (1998).) Thus, the tractable form of ψ_m , $\forall m$ is :

$$\psi_m = - \sum_{n=1}^N p_{mn} x_{mn} + f_m(x_m) + \sum_{n=1}^N c_{mn}(x_{mn}) + \bar{w}_m^{\mathbf{T}} x_m + \bar{v}_m, \quad (5.9)$$

where, $\|H_{ms}^{\mathbf{T}} x_m\| \leq v_{ms}$, $\forall m, s$, serves as a conic constraint. The column vectors, v , are used to group all v_{ms} and \bar{v} to group all \bar{v}_m .

Next, re-express the uncertain parameter Z and its set \mathcal{U}^{box} . By finding the extreme points of \mathcal{U}^{box} , as described by Ben-Tal et al. (2005) and Pishvae et al. (2011), the constraint (5.4) that is

associated with the uncertainty set (5.2) can be rewritten as the following tractable form:

$$\sum_{m=1}^M \sum_{n=1}^N x_{mn} \leq \bar{Z}_t - \rho \bar{Z}_t, \quad \forall t = 1, \dots, T. \quad (5.10)$$

Combining (5.9) and (5.10), it is finally possible to reformulate the problem (*RC*) as a tractable robust counterpart problem:

$$(TRC) \quad \min_{\substack{x_{mn}, v_{ms}, \\ \bar{v}_m \geq 0}} \psi_m = - \sum_{n=1}^N p_{mn} x_{mn} + f_m(x_m) + \sum_{n=1}^N c_{mn}(x_{mn}) + \bar{w}_m^{\mathbf{T}} x_m + \bar{v}_m$$

$$\text{s.t.} \quad \sum_{m=1}^M \sum_{n=1}^N x_{mn} \leq \bar{Z}_t - \rho \bar{Z}_t, \quad \forall t \quad (\lambda_t^0) \quad (5.11)$$

$$v_{ms} \leq \bar{v}_m, \quad \forall s \quad (\lambda_{ms}^1) \quad (5.12)$$

$$\|H_{ms}^{\mathbf{T}} x_m\| \leq v_{ms}, \quad \forall s. \quad (5.13)$$

In problem (*TRC*), λ_t^0 is the Lagrange multiplier associated with constraint (5.11) and λ_{ms}^1 is the Lagrange multiplier associated with constraint (5.12). I group all λ_t^0 and λ_{ms}^1 into column vectors λ^0 and λ^1 , respectively. Because all constraints (5.11), (5.12), and (5.13) are convex constraints, the entire problem (*TRC*) is a convex optimization problem. To this end, I have completed reformulating the REE material supplier's affinely uncertain problem as a deterministic problem.

Reformulating as VIP

Because the REE material suppliers compete in a non-cooperative fashion, each supplier determines the optimal output, given the simultaneously optimal ones of all competitors. Hence, the optimality condition for the convex optimization problem, (*TRC*), can be expressed in VIs: determine the solution $(Q^{1*}, \lambda^{0*}, v^*)$ satisfying:

$$\begin{aligned} \sum_{m=1}^M \sum_{n=1}^N \left[-p_{mn}^* + \frac{\partial f_m(x_m^*)}{\partial x_{mn}} + \frac{\partial c_{mn}(x_{mn}^*)}{\partial x_{mn}} + \bar{w}_{mn} + \sum_{t=1}^T \lambda_t^{0*} \right] \times (x_{mn} - x_{mn}^*) \\ + \sum_{t=1}^T \left[\bar{Z}_t - \rho \bar{Z}_t - \sum_{m=1}^M \sum_{n=1}^N x_{mn}^* \right] \times (\lambda_t^0 - \lambda_t^{0*}) \\ + \sum_{m=1}^M \sum_{s=1}^{S_m} \left[\bar{v}_m - v_{ms} \right] \times (\lambda_{ms}^1 - \lambda_{ms}^{1*}) \end{aligned} \quad (5.14)$$

$$\begin{aligned}
& + \sum_{m=1}^M \left[1 - \sum_{s=1}^{S_m} \lambda_{ms}^{1*} \right] \times (\bar{v}_m - \bar{v}_m^*) \\
& + \sum_{m=1}^M \sum_{s=1}^{S_m} \lambda_{ms}^{1*} \times (v_{ms} - v_{ms}^*) \geq 0 \\
& \forall (Q^1, \lambda^0, \lambda^1, \bar{v}, v) \in \mathcal{K}^1,
\end{aligned}$$

where $\mathcal{K}^1 \equiv \{(Q^1, \lambda^0, \lambda^1, \bar{v}, v) \mid Q^1 \geq 0, \lambda^0 \geq 0, \lambda^1 \geq 0, \bar{v} \geq 0, \|H_{ms}^T x_m\| \leq v_{ms}, \forall m = 1, \dots, M, s = 1, \dots, S_m\}$.

5.2.2 Chip Producers' Optimal Behavior

A chip producer manufactures a semiconductor end-product that can be consumed by markets. There are two sources a producer can acquire rare earth raw materials from: the virgin materials from the REE material suppliers, and the recycled materials from the recycle centers. Both types of materials can be converted to the end-product at quantity rates of β_r and β_u , respectively. A typical producer n strategically maximizes its profit π_n^2 by producing semiconductor chips of quantity x_{nk} and shipping them to some demand markets of the next tier. I use a column vector Q^2 to group all x_{nk} .

Each chip producer incurs an operating cost $f_n = f_n(x^n, x_n), \forall n$, where $x^n \equiv (x_{1n}, \dots, x_{mn}, \dots, x_{Mn}) \in \mathbb{R}^M$, $x_n \equiv (x_{1n}, \dots, x_{in}, \dots, x_{In}) \in \mathbb{R}^I$ are both column vectors, and a transaction cost $c_n = c_n(\sum_{k=1}^K x_{nk}), \forall n$. I assume both cost functions to be continuously differentiable. Thus, the profit maximizing problem of the producer n is

$$\begin{aligned}
\max_{x_{nk} \geq 0} \quad & \pi_n^2 = \sum_{k=1}^K p_{nk} x_{nk} - \sum_{m=1}^M p_{mn} x_{mn} - \sum_{i=1}^I p_{in} x_{in} - f_n(x^n, x_n) - c_n\left(\sum_{k=1}^K x_{nk}\right) \\
\text{s.t.} \quad & \beta_r \sum_{m=1}^M x_{mn} + \beta_u \sum_{i=1}^I x_{in} \geq \sum_{k=1}^K x_{nk}. \tag{5.15}
\end{aligned}$$

The constraint (5.15) is each producer's flow conservation, taking account of the rare earth material supply from both the virgin materials and the recycled EoL products. λ_n^2 is the Lagrange multiplier associated with constraint (5.15). I group all λ_n^2 into a column vector λ^2 . Because the producers are assumed to compete with one another in a non-cooperative fashion by determining the optimal output production quantities, similar to condition (5.14), one could derive the optimal

behaviors of all producers in a VI form: determine $(Q^{1*}, Q^{2*}, Q^{3*}, \lambda^{2*}) \in \mathcal{X}^2$, satisfying:

$$\begin{aligned}
& \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial f_n(x^{n*}, x_n^*)}{\partial x_{mn}} + p_{mn}^* - \beta_r \lambda_n^{2*} \right] \times (x_{mn} - x_{mn}^*) \\
& + \sum_{i=1}^I \sum_{n=1}^N \left[\frac{\partial f_n(x^{n*}, x_n^*)}{\partial x_{in}} + p_{in}^* - \beta_u \lambda_n^{2*} \right] \times (x_{in} - x_{in}^*) \\
& + \sum_{n=1}^N \sum_{k=1}^K \left[\frac{\partial c_n(\sum_{k=1}^K x_{nk}^*)}{\partial x_{nk}} - p_{nk}^* + \lambda_n^{2*} \right] \times (x_{nk} - x_{nk}^*) \\
& + \sum_{n=1}^N \left[\beta_r \sum_{m=1}^M x_{mn}^* + \beta_u \sum_{i=1}^I x_{in}^* - \sum_{k=1}^K x_{nk}^* \right] \times (\lambda_n^2 - \lambda_n^{2*}) \geq 0 \tag{5.16} \\
& \forall (Q^1, Q^2, Q^3, \lambda^2) \in \mathcal{X}^2 \subset \mathbb{R}_+^{N(M+I+K+1)}.
\end{aligned}$$

5.2.3 Recycle Centers' Optimal behavior

In a rare earth supply chain, the role of a typical recycle center is to acquire recycled EoL products from the consumers, extract or recondition the valuable materials containing certain REEs, and resell the recycled REE materials to the chip producers. In this model, a recycle center i strategically maximizes its profit π_n^3 by determining its shipment quantity x_{in} to some demand markets of the next tier. I use a column vector Q^3 to group all x_{in} . At the same time, recycle center i also offers a buy-back price p_k to the market k , and hence receives a quantity of the recycled EoL product shipments, denoted by x_{ki} , from the same market. The mechanism of the buy-back price and reverse flows will be discussed in detail in the next subsection.

I assume each recycle center incurs an operating cost $f_i = f_i(x_i)$, $\forall i$, where $x_i \equiv (x_{1i}, \dots, x_{ki}, \dots, x_{Ki}) \in \mathbb{R}^K$ is a column vector, and a transaction cost $c_{ki} = c_{ki}(x_{ki})$, $\forall k, i$. Both of the functions are assumed to be continuously differentiable. Immediately, the profit maximizing problem of the recycle center i can be expressed as:

$$\begin{aligned}
\max_{x_{in} \geq 0} \quad & \pi_i^3 = \sum_{n=1}^N p_{in} x_{in} - \sum_{k=1}^K p_{ki} x_{ki} - f_i(x_i) - \sum_{k=1}^K c_{ki}(x_{ki}) \\
\text{s.t.} \quad & \sum_{n=1}^N x_{in} \leq \sum_{k=1}^K x_{ki}, \quad \forall i. \tag{5.17}
\end{aligned}$$

The constraint (5.17) is each recycle center's flow conservation. λ_i^3 is the Lagrange multiplier associated with constraint (5.17). I group all λ_i^3 in a column vector λ^3 . Because the recycle centers

are assumed to compete with one another in a non-cooperative fashion by determining the optimal output production quantities, similar to the treatment before, one could derive the optimal behavior of all recycle centers in a VI form: determine $(Q^{3*}, Q^{4*}, \lambda^{3*}) \in \mathcal{X}^3$, satisfying:

$$\begin{aligned}
& \sum_{i=1}^I \sum_{k=1}^K \left[p_{ki}^* + \frac{\partial f_i(x_i^*)}{\partial x_{ki}} + \frac{\partial c_{ki}(x_{ki}^*)}{\partial x_{ki}} - \lambda_i^{3*} \right] \times (x_{ki} - x_{ki}^*) \\
& + \sum_{i=1}^I \sum_{n=1}^N \left[\lambda_i^{3*} - p_{in}^* \right] \times (x_{in} - x_{in}^*) + \sum_{i=1}^I \left[\sum_{k=1}^K x_{ki}^* - \sum_{n=1}^N x_{in}^* \right] \times (\lambda_i^3 - \lambda_i^{3*}) \geq 0 \quad (5.18) \\
& \forall (Q^3, Q^4, \lambda^3) \in \mathcal{X}^3 \subset \mathbb{R}_+^{I(N+K+1)}.
\end{aligned}$$

5.2.4 Demand Markets' Optimal Behavior

A demand market is a collection of representative semiconductor chip consumers and therefore serves as the key hub between the forward and reverse parts of the supply chain. In this model, each market makes two decisions: the quantity of the semiconductor chips, x_{nk} , to purchase from the chip producers, and the quantity of the EoL products, x_{ki} , to return to the recycle centers. All of x_{ki} is grouped by a column vector Q^4 .

At the market level, I further denote the demand price of the product at market k to be p_k , all of which are grouped in a column vector p . Each demand price corresponds with a demand quantity d_k in the form of a price-demand function.

$$d_k = d_k(p_k), \quad \forall k. \quad (5.19)$$

In the forward logistics chain, it is assumed that the transaction cost $\hat{c}_{nk} = \hat{c}_{nk}(x_{nk})$, $\forall n, k$, is continuously differentiable. The consumers take the price charged by the producers, p_{nk}^* , plus the unit transaction cost incurred for obtaining the product. The equilibrium conditions for consumers at demand market k are represented by a well-known spatial price equilibrium (SPE) (for more details of such concept and formulation technique, see [Samuelson \(1952\)](#); [Takayama and Judge \(1964\)](#); [Dafermos and Nagurney \(1984\)](#), inter alia.) model as the following:

$$p_{nk}^* + \hat{c}_{nk}(x_{nk}^*) \begin{cases} = p_k^* & \text{if } x_{nk}^* > 0 \\ \geq p_k^* & \text{if } x_{nk}^* = 0 \end{cases} \quad \forall n, k; \quad (5.20)$$

and

$$d_k(p^*) \begin{cases} = \sum_{n=1}^N x_{nk}^* & \text{if } p_k \geq 0 \\ \leq \sum_{n=1}^N x_{nk}^* & \text{if } p_k = 0 \end{cases} \quad \forall k. \quad (5.21)$$

The intuition of equation (5.20) is that the consumption of products at the markets k will remain positive if the producers' purchase price of such a resource plus all associated cost is equal to the price consumers are willing to pay; however, if the producers' purchase price plus the cost turns out to be higher than what consumers are willing to pay, then there will be no shipments between the producer and the market. The condition (5.21), on the other hand, suggests that as long as the price consumers are willing to pay for the products at a demand market is positive, the quantity consumed by the consumers at the demand market is precisely equal to the demand.

In the reverse logistics chain, the consumer's behaviors can be represented by a similar set of equilibrium conditions. I define the consumer aversion as $\alpha_k(Q^4)$, a monotonically increasing function. The consumer determines the quantity of EoL to recycle by weighing in between its aversion against the buy-back payment it may receive. Hence, the reverse flow equilibrium condition is given by (5.22) with a flow conservation constraint (5.23). λ_k^4 is the Lagrange multiplier associated with constraint (5.23). Same as before, All λ_k^4 are grouped into a column vector λ^4 .

$$\alpha_k(Q^{4*}) \begin{cases} = p_{ki}^* & \text{if } x_{ki}^* > 0 \\ \geq p_{ki}^* & \text{if } x_{ki}^* = 0 \end{cases} \quad \forall k, i. \quad (5.22)$$

$$\text{s.t.} \quad \sum_{i=1}^I x_{ki} \leq \sum_{n=1}^N x_{nk}, \quad \forall k. \quad (\lambda_k^4) \quad (5.23)$$

Sensibly, the consumer aversion depends on the amount of used products that are returned to the recycle centers, i.e., the more products to be collected in the reverse chain, the higher the buy-back price a recycle center has to offer. From the standpoint of recycle centers, a higher buy-back price would persuade the consumer's recycling activities. As such, for any given buy-back price p_{ki}^* , the model segments the consumers at demands markets into two groups: the ones that will be persuaded to return the recyclable products and the ones that will not (see also [Hammond and Beullens \(2007\)](#); [Yang et al. \(2009\)](#)).

Combining the consumer behavior conditions in both forward and reverse supply chains, the

spatial price equilibrium in a VI form can be written as: determine $(Q^{2*}, Q^{4*}, p^*, \lambda^{4*}) \in \mathcal{X}^4$ satisfying:

$$\begin{aligned}
& \sum_{n=1}^N \sum_{k=1}^K \left[p_{nk}^* + \hat{c}_{nk}(x_{nk}^*) - p_k^* - \lambda_k^{4*} \right] \times (x_{nk} - x_{nk}^*) \\
& + \sum_{k=1}^K \left[\alpha_k(Q^{4*}) - p_{ki}^* + \lambda_k^{4*} \right] \times (x_{ki} - x_{ki}^*) \\
& + \sum_{k=1}^K \left[\sum_{n=1}^N x_{nk}^* - d_k(p^*) \right] \times (p_k - p_k^*) \\
& + \sum_{k=1}^K \left[\sum_{n=1}^N x_{nk}^* - \sum_{i=1}^I x_{ki}^* \right] \times (\lambda_k^4 - \lambda_k^{4*}) \geq 0 \\
& \forall (Q^2, Q^4, p, \lambda^4) \in \mathcal{X}^4 \subset \mathbb{R}_+^{K(N+I+2)}.
\end{aligned} \tag{5.24}$$

It is worth noting that one could easily infer from the second summand of condition (5.24) that the optimal buy-back price the recycle center i should offer to the market k equals the sum of the disutility and the shadow price of the total product flow difference at the same market.

5.3 Equilibrium of the Robust Closed-Loop Supply Chain Network

Definition 9 (The generalized Nash equilibrium of the closed-loop supply chain network)

A product flow pattern of each REE material supplier, chip producer, and recycle center constitutes a generalized Nash equilibrium if it satisfies the sum of the equilibrium and optimality conditions (5.14), (5.16), (5.18), and (5.24).

This definition implies that the strategic outcome of each REE material supplier, chip producer, recycle center, as well as market, is determined not only by the strategy of their own but also of others in the same tier. In addition, because the set \mathcal{S} pools all the REE material supplier's decisions and thus results in a crossover, this problem is not of a Cournot-Nash equilibrium concept (Cournot, 1838; Nash, 1950, 1951), but a generalized Nash equilibrium one (Debreu, 1952; Facchinei and Kanzow, 2007). Furthermore, according to Kulkarni and Shanbhag (2012), a generalized Nash

equilibrium can be formulated as the solution to a finite-dimensional variational inequality problem.

Theorem 18 (Variational inequality formulation)

The generalized Nash equilibrium according to Definition 1 is equivalent to the solution to the following VIP:

Determine $(Q^{1*}, Q^{2*}, Q^{3*}, Q^{4*}, p^*, \lambda^{1*}, \lambda^{2*}, \lambda^{3*}, \lambda^{4*}, v^*) \in \mathcal{H}$, satisfying

$$\begin{aligned}
& \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial f_m(x_m^*)}{\partial x_{mn}} + \frac{\partial c_{mn}(x_{mn}^*)}{\partial x_{mn}} + \frac{\partial f_n(x^{n*}, x_n^*)}{\partial x_{mn}} + \bar{w}_{mn} + \sum_{t=1}^T \lambda_t^{0*} - \beta^r \lambda_n^{2*} \right] \times (x_{mn} - x_{mn}^*) \\
& + \sum_{n=1}^N \sum_{k=1}^K \left[\frac{\partial c_n(\sum_{k=1}^K x_{nk}^*)}{\partial x_{nk}} + \hat{c}_{nk}(x_{nk}^*) - p_k^* + \lambda_n^{2*} - \lambda_k^{4*} \right] \times (x_{nk} - x_{nk}^*) \\
& + \sum_{i=1}^I \sum_{k=1}^K \left[\frac{\partial f_i(x_i^*)}{\partial x_{ki}} + \frac{\partial c_{ki}(x_{ki}^*)}{\partial x_{ki}} + \alpha_k(Q^{4*}) - \lambda_i^{3*} + \lambda_k^{4*} \right] \times (x_{ki} - x_{ki}^*) \\
& + \sum_{i=1}^I \sum_{n=1}^N \left[\frac{\partial f_n(x^{n*}, x_n^*)}{\partial x_{in}} - \beta^u \lambda_n^{2*} + \lambda_i^{3*} \right] \times (x_{in} - x_{in}^*) \\
& + \sum_{t=1}^T \left[\bar{Z}_t - \rho \bar{Z}_t - \sum_{m=1}^M \sum_{n=1}^N x_{mn}^* \right] \times (\lambda_t^0 - \lambda_t^{0*}) + \sum_{m=1}^M \sum_{s=1}^{S_m} \left[\bar{v}_m - v_{ms} \right] \times (\lambda_{ms}^1 - \lambda_{ms}^{1*}) \\
& + \sum_{n=1}^N \left[\beta^r \sum_{m=1}^M x_{mn}^* + \beta^u \sum_{i=1}^I x_{in}^* - \sum_{k=1}^K x_{nk}^* \right] \times (\lambda_n^{2*} - \lambda_n^{2*}) + \sum_{i=1}^I \left[\sum_{k=1}^K x_{ki}^* - \sum_{n=1}^N x_{in}^* \right] \times (\lambda_i^{3*} - \lambda_i^{3*}) \\
& + \sum_{k=1}^K \left[\sum_{n=1}^N x_{nk}^* - \sum_{i=1}^I x_{ki}^* \right] \times (\lambda_k^{4*} - \lambda_k^{4*}) + \sum_{k=1}^K \left[\sum_{n=1}^N x_{nk}^* - d_k(p^*) \right] \times (p_k - p_k^*) \\
& + \sum_{m=1}^M \left[1 - \sum_{s=1}^{S_m} \lambda_{ms}^{1*} \right] \times (\bar{v}_m - \bar{v}_m^*) + \sum_{m=1}^M \sum_{s=1}^{S_m} \lambda_{ms}^{1*} \times (v_{ms} - v_{ms}^*) \geq 0 \\
& \forall (Q^1, Q^2, Q^3, Q^4, p, \lambda^0, \lambda^1, \lambda^2, \lambda^3, \lambda^4, \bar{v}, v) \in \mathcal{H},
\end{aligned} \tag{5.25}$$

where, $\mathcal{H} \equiv \mathcal{H}^1 \times \mathcal{H}^2 \times \mathcal{H}^3 \times \mathcal{H}^4 = \{(Q^1, Q^2, Q^3, Q^4, p, \lambda^0, \lambda^1, \lambda^2, \lambda^3, \lambda^4, \bar{v}, v) \in \mathbb{R}_+^{\mathcal{N}} : \|H_{ms}^T x_m\| \leq v_{ms}, \forall m = 1, \dots, M; s = 1, \dots, S_m\}$ and $\mathcal{N} = MN + NK + KI + IN + M + N + 2K + I + T$.

Proof: See Appendix C.

Retrieving the price variables

It should be noted that variable p_{mn}^* , p_{nk}^* , p_{ki}^* , and p_{in}^* do not appear within the formulation of Theorem 1, as they are endogenous to the model and can be retrieved once the solution is obtained.

To retrieve p_{nk}^* , for all n, k , recall equilibrium condition (5.20). Since p_k^* is readily available from (5.25), if $x_{nk}^* > 0$ for some n, k , then p_{nk}^* can be obtained by the equality

$$p_{nk}^* = p_k^* - \hat{c}_{nk}(x_{nk}^*). \quad (5.26)$$

Invariably, if $x_{mn}^* > 0$ for some m, n , then from the first summand in (5.16), one may immediately obtain

$$p_{mn}^* = \beta^r \lambda_n^{2*} - \frac{\partial f_n(x^{n*}, x_n^*)}{\partial x_{mn}}. \quad (5.27)$$

If $x_{in}^* > 0$ for some i, n , then from the second summand of (5.18), one can retrieve

$$p_{in}^* = \lambda_i^{3*}. \quad (5.28)$$

And, if $x_{ki}^* > 0$ for some k, i , then from the first summand of (5.18), one can retrieve

$$p_{ki}^* = -\frac{\partial f_i(x_i^*)}{\partial x_{ki}} - \frac{\partial c_{ki}(x_{ki}^*)}{\partial x_{ki}} + \lambda_i^{3*}. \quad (5.29)$$

5.4 Solution Approach

Because the VIP (5.25) is known as a second-order cone constrained variational inequality problem, solving it remains a challenge, as it contains non-differentiable functions. However, a procedure curated by [Hirano and Narushima \(2019\)](#) allows for an alternative solution to this problem via a reformulation to a mixed nonlinear second-order cone complementarity problem, as the latter problem can be solved by a host of smoothing and regularization algorithms (see also [Sun et al. \(2012\)](#)).

In general, a $(1 + \zeta)$ -dimensional second-order cone $\mathcal{K}^{1+\zeta}$ can be defined as (see [Lobo et al. \(1998\)](#), [Sun et al. \(2012\)](#), inter alia)

$$\mathcal{K}^{1+\zeta} \equiv \{y = (y_1, y_2) \mid y_1 \geq \|y_2\|, y_1 \in \mathbb{R}, y_2 \in \mathbb{R}^\zeta\}. \quad (5.30)$$

Following [Hirano and Narushima \(2019\)](#), first re-express the set \mathcal{K} in condition (5.25) as:

$$\begin{aligned} \mathcal{K} \equiv & \{(Q^{1*}, Q^{2*}, Q^{3*}, Q^{4*}, p^*, \lambda^{0*}, \lambda^{1*}, \lambda^{2*}, \lambda^{3*}, \lambda^{4*}, \bar{v}^*, v^*) | \\ & (Q^{1*}, Q^{2*}, Q^{3*}, Q^{4*}, p^*, \lambda^{0*}, \lambda^{1*}, \lambda^{2*}, \lambda^{3*}, \lambda^{4*}, \bar{v}^*) \in \mathbb{R}_+^\sigma; \\ & (v^*, H_{ms}^{\mathbf{T}} x_m^*) \in \mathcal{K}^{1+S_m}, \forall m = 1, \dots, M, s = 1, \dots, S_m\} \end{aligned}$$

where $\sigma \equiv MN + NK + KI + IN + N + 2K + I + T$ and \mathcal{K}^{1+S_m} is a $(1 + S_m)$ -dimensional second-order cone. Clearly, \mathcal{K}^1 is given by \mathbb{R}_+ . Then define \mathcal{K} and $G_1(x)$ as:

$$\mathcal{K} \equiv \mathbb{R}_+^\sigma \times \prod_{m=1}^M \mathcal{K}^{1+S_m}. \quad (5.31)$$

$$G_1(x) \equiv \begin{pmatrix} Q^1 \\ Q^2 \\ Q^3 \\ Q^4 \\ \lambda^0 \\ \lambda^1 \\ \lambda^2 \\ \lambda^3 \\ \lambda^4 \\ v_{11} \\ H_{11}^{\mathbf{T}}x_1 \\ \vdots \\ v_{1S_1} \\ H_{1S_1}^{\mathbf{T}}x_1 \\ v_{21} \\ H_{21}^{\mathbf{T}}x_2 \\ \vdots \\ v_{MS_M} \\ H_{MS_M}^{\mathbf{T}}x_M \end{pmatrix}. \quad (5.32)$$

Next, rewrite the set \mathcal{K} in condition (5.25) as $\mathcal{K} = \{x | G_1(x) \in \mathcal{K}\}$. Thus, the VIP (5.25) can be reformulated as the following MNSOCCP (see Sun et al. (2012) inter alia):

$$\begin{aligned} \text{Find } & (x, y) \in \mathbb{R}^{\mathcal{N}} \times \mathbb{R}^{\tau} \\ \text{such that } & F(x) - \nabla G_1(x)y = 0, \\ & y \in \mathcal{K}, G_1(x) \in \mathcal{K}, y^{\mathbf{T}}G_1(x) = 0, \end{aligned}$$

or, equivalently, by substituting $z = G_1(x)$ and $G_2(x, y) = F(x) - \nabla G_1(x)y$,

$$\begin{aligned}
 & \text{Find } (x, y, z) \in \mathbb{R}^{\mathcal{N}} \times \mathbb{R}^\tau \times \mathbb{R}^\tau \\
 & \text{such that } z = G_1(x), \\
 & \quad G_2(x) = 0, \\
 & \quad y \in \mathcal{K}, z \in \mathcal{K}, y^{\mathbf{T}}z = 0,
 \end{aligned} \tag{5.33}$$

where $\tau = \sigma + M + \sum_{m=1}^M S_m$. Finally, I propose the smoothing and regularization method by [Hayashi et al. \(2005\)](#) to solve MNSOCCP (5.33).

5.5 Chapter Summary

In this chapter, I develop the first closed-loop supply chain network equilibrium model for the REE supply, production, and recycling. A robust approach is adopted to model the supply-side policy impact on the rest of the supply chain by taking into account the criticality of such minerals and their lack of data. The model is formulated as a generalized Nash equilibrium problem using finite-dimensional variational inequality theories. Finally, I propose an algorithmic procedure with which the problem can be solved. The RCLSCNE model embodies the scarcity of REEs with policy and capacity uncertainties and a means of conservation through recycling.

Chapter 6

CONCLUSIONS AND FUTURE RESEARCH

6.1 Conclusions

The sustainability and growth of individual firms, societies, and humanity depend on the munificence of scarce resources. However, in the face of intensifying societal changes, shocks, crises, and interconnectedness, the competition for scarce resources has become more intense. Coping with complex crises such as climate change and the COVID-19 pandemic requires lifestyle changes, which lead to an explosive increase in demand for products and resources such as semiconductor components. Fiscal policy remains the primary governmental instrument to relieve the shortage of supplies and stimulate economies. In the context of monopolized REE supplies, chip shortages, and curtailed end-product sales, understanding competition, policy risk, and recycling activity in global supply chains becomes crucial.

The principal objective of this dissertation is to examine and better understand the critical issues and challenges in the supply chains of scarce resources. Globalization has resulted in worldwide supply chain networks, which have been beneficial to multinational corporations seeking low-cost supplies. However, lean productions and Just-in-Time inventories, proven cost-cutting measures, have stretched global supply chains to the breaking point during times of stress. This has made firms vulnerable to disruptions and restrictive policies. The dissertation's major contributions

encompass modeling, theories, applications, and managerial insights.

The dissertation focuses on several key matters: the limited pool of natural resources, the impact and uncertainties of supply-side policies, and operations management in supply chain settings. All these matters are addressed through the lens of multi-agent mathematical optimization. Throughout my work, the devised models are game-theoretic with non-cooperative competitions in nature. In Chapter 3, I tackle a popular operations management problem by modeling the inventories that are carried strategically by the competing retailers in a supply chain. The purpose of this study is to understand the strategic interactions, the role of inventories, and the effect of operational parameters, i.e., contracts and costs, under the competition within a supply chain. In Chapter 4, I construct a cross-sector multi-product general scarce resource supply chain network with resource common pools and a unified fiscal policy scheme. I examine the cause and effect of scarcity under the context of resource capacities, disruptive demand surge, government interventions, and social outcomes. In Chapter 5, I zoom in on the resource-focused perspective on REE, a critical raw material whose supply is known to control the arteries of nearly all of today's technological devices. I dedicate the research to uncovering the effect of adverse supply-side policies and the role of recycling precious materials.

On a technical landscape, in Chapter 2, I provide a technical overview of the methodologies employed in the subsequent chapters, and I contribute to the theoretical development of the network-game literature by introducing a new theorem. In all Chapters 3, 4, and 5, I seek to establish the equilibrium decisions across the non-cooperative agents. Chapter 3 presents a simple two-period dynamic Cournot-type game with affine inventory costs and normalized (to zero) operating and transaction costs. The closed-form solution is obtained using the backward reasoning approach. In Chapter 4, I present a cross-sector multi-product supply chain network model that is more complex in both topology and strategic interplay due to its generality, relaxed cost assumptions, inclusion of resource capacity, and policy instruments. The solution procedure is to reformulate the convex-optimization representation of each profit-seeking firm's payoff into a VIP, which is eventually solved via the modified projection algorithm. Chapter 5 addresses a pressing real-world issue by inheriting the same type of network interplay among the decision agents as Chapter 4. I propose a robust closed-loop supply chain network equilibrium model that regresses from the multi-product model in the previous chapter, but advances the network complexity by further incorporating a reverse

chain in the network structure and stochastic components on the supply-side parameters. While the additional closed-loop topology does not alter the solution viability of the variational approach, the distribution-unknown uncertainties associated with the supply nodes require special treatment. To do so, I adopt the robust optimization approach to handle these uncertainties, reformulate the problem into a SOCP, and then further reformulate it into a VIP as part of the whole network. The global problem eventually takes the form of a SOCCVI problem. I further transform it to an MNSOCCP, in order to be solved by the smoothing regularization Newton method.

I wish fervently to use the models covered in this dissertation to inform meaningful dialogues that can turn into valuable insights for supply chain practitioners and legislators. For instance, operations managers can learn from Chapter 3 about how to select inventory contracts when facing a rising stocking cost or new competitive entrants. At the same time, when it comes to mitigating shortages amid an existential crisis, they will have the foresight on the ramifications of cost restructuring, expansionary funds, and resource preservation from Chapter 4. Government legislators can also benefit from this dissertation by learning about the design and effect of supply-side fiscal policies. With this knowledge, they may reconsider their policy balance act, taking into account potential risks and adverse effects, as well as their impact on economic growth and income equality.

6.2 Future Research

The topic of strategic inventory, as discussed in Chapter 3 of this dissertation, has proven to be enduring since its introduction by [Anand et al. \(2008\)](#). Recent studies, such as those by ([Xin et al., 2022](#); [Chen et al., 2022](#); [Wang et al., 2022](#); [Li et al., 2022](#)), continue to emerge, indicating renewed interest in the subject. This renewed interest can be attributed to the significant shortages of goods during the supply chain disruptions caused by the pandemic, where the JIT inventory-handling approach failed to cope with demand surges. These latest research efforts have expanded the scope of strategic inventory to include topics such as chain-to-chain competition, purchasing strategies, pricing decisions, incentive mechanisms, contractual outcomes, and information transparency, inter alia. Further refinement and exploration of these areas offer potential avenues for future research. Additionally, this dissertation has examined specific contracts in the context of strategic inventory, but other contracts, such as revenue sharing, cost sharing, and two-part tariff, could also be inte-

grated into supply chain models to study the roles of manufacturers and retailers (Xin et al., 2022). Another interesting direction for future research would be to consider demand uncertainty in the presence of strategic inventories (Li et al., 2022; Xin et al., 2022).

The scarce resource supply chain network model presented in Chapter 4 aims to investigate the impact of legislation on conservation, competition, and crisis relief by incorporating the economic properties of scarce resources. The generality of this network model provides a versatile framework that can be expanded with additional features. Future research could explore alternative network structures (e.g., reverse flow), uncertain elements (e.g., information asymmetry), asynchronous decision sequences, and dynamic games as methodological enhancements. In terms of applications, peripheral issues related to resource scarcity, such as environmental concerns (Chen et al., 2020), quality measures (Besik et al., 2023), and corporate social responsibilities (Lee and Tang, 2018), could be potential future research topics. Chapter 5 included in this dissertation is indeed an extension of the model presented in Chapter 4. Considering global issues in the post-pandemic era of the 2020s, it is anticipated that further extension of this model could provide insights into the stimulation and relief efforts related to medical supply distribution, humanitarian efforts, economic recoveries, migrations, and technology adoption.

The extension of the RCLSCNE model from Chapter 5 can be pursued in several directions. First, from a network topology perspective, future research could consider alterations to the network flows by incorporating dual chains (as discussed in Fan et al. (2022), Liu et al. (2022), inter alia) or additional supply chain entities. Second, as an SCNE model, the RCLSCNE can be extended in various ways. This may involve incorporating demand uncertainties, production capacities, transportation costs, congestion, or other policy-related elements. Third, considering that trade policy uncertainty is recognized as a significant source of economic uncertainty (Novy and Taylor, 2020; Handley and Limão, 2022), studying supply chain models with uncertain trade elements could provide valuable insights for policy discussions and managerial implications. Fourth, exploring alternative representations of uncertainties, such as stochastic and distributionally robust approaches, could be worth considering if data permits. Lastly, developing time-dependent dynamic models could yield intriguing insights with practical implications.

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Acronyms

AVI	affine variational inequality
CLSC	closed-loop supply chain
CLSCNE	closed-loop supply chain network equilibrium
CP	complementarity problem
CS	consumer surplus
EoL	end-of-life
EV	electric vehicle
GNE	general Nash equilibrium
GNEP	general Nash equilibrium problem
JIT	Just-in-Time
KKT	Karush–Kuhn–Tucker
LCP	linear complementarity problem
MiCP	mixed complementarity problem
MLCP	mixed linear complementarity problem
MNSOCCP	mixed nonlinear second-order cone complementarity problem
NCP	nonlinear complementarity problem
NE	Nash equilibrium
NSOCP	nonlinear second-order cone program
PPE	personal protective equipment

RCLSCNE	robust closed-loop supply chain network equilibrium
REE	rare earth element
REPM	rare earth permanent magnet
RO	robust optimization
SCM	supply chain management
SCNE	supply chain network equilibrium
SI	strategic inventory
SOCCVI	second-order cone constrained variational inequality
SOCP	second-order cone program
SPNE	subgame-perfect Nash equilibrium
SW	social welfare
TRC	tractable robust counterpart
VI	variational inequality
VIP	variational inequality problem
WEF	water-energy-food

Appendix A

FULL RESULTS FROM CHAPTER 3

Table [A.1](#), [A.2](#), [A.3](#), and [A.4](#) exhibit four comparisons of scenarios discussed in this chapter.

Table A.1: No inventory (i.e., JIT) vs. inventory carried in a retailer duopoly

		No Inventory		Inventory
Wholesale Price	w_1	$a/2$	<	$3/124(21a - 4h)$
	w_2	$a/2$	>	$1/124(55a + 84h)$
	w_{avg}	$a/2$	>	$(125a^2 - 24ah + 144h^2)/(4(65a - 36h))$
Retail Price	p_1	$2a/3$	<	$1/186(125a - 12h)$
	p_2	$2a/3$	>	$1/62(39a + 28h)$
	p_{avg}	$2a/3$	>	$(7849a^2 - 1632ah - 3600h^2)/(186(65a - 36h))$
Consumer Purchase Qty	Q_1	$a/3b$	>	$(61a + 12h)/186b$
	Q_2	$a/3b$	<	$(23a - 28h)/62b$
	Q_{total}	$2a/3b$	<	$(65a - 36h)/93b$
Inventory	I	0	<	$(7(a - 12h))/186b$
Retailer Sales (ea.)	q_1	$a/6b$	>	$(61a + 12h)/372b$
	$q_2 + I$	$a/6b$	<	$(23a - 28h)/124b$
	q_{total}	$a/3b$	<	$(65a - 36h)/186b$
Retailer Profit (ea.)	πR_1	$a^2/36b$	>	$(1075a^2 + 28512ah + 56592h^2)/138384b$
	πR_2	$a^2/36b$	<	$(2357a^2 - 11928ah - 11760h^2)/46128b$
	πR_{total}	$a^2/18b$	<	$(4073a^2 - 3636ah + 10656h^2)/69192b$
Manufacturer Profit	πM_1	$a^2/6b$	<	$(3(25a - 52h)(21a - 4h))/7688b$
	πM_2	$a^2/6b$	>	$(55a + 84h)^2/23064b$
	πM_{total}	$a^2/3b$	<	$(125a^2 - 24ah + 144h^2)/372b$
SC Profit	π_1	$(2a^2)/9b$	>	$(7625a^2 - 1836ah + 31104h^2)/34596b$
	π_2	$(2a^2)/9b$	<	$(897a^2 - 448ah - 784h^2)/3844b$
	π_{total}	$(4a^2)/9b$	<	$(7849a^2 - 2934ah + 12024h^2)/17298b$
Consumer Surplus	CS_1	$a^2/18b$	>	$(61a + 12h)^2/69192b$
	CS_2	$a^2/18b$	<	$(23a - 28h)^2/7688b$
	CS_{total}	$a^2/9b$	<	$(4241a^2 - 5064ah + 3600h^2)/34596b$
Social Welfare	SW_1	$(5a^2)/18b$	>	$(18971a^2 - 2208ah + 62352h^2)/69192b$
	SW_2	$(5a^2)/18b$	<	$(2323a^2 - 2184ah - 784h^2)/7688b$
	SW_{total}	$(5a^2)/9b$	<	$(19939a^2 - 10932ah + 27648h^2)/34596b$

Table A.2: Retailer's monopoly vs. duopoly under a dynamic contract when the holding cost is low ($h < \frac{a}{12}$)

		Monopoly (Anand et al., 2008)		Duopoly	
Wholesale Price	w_1	$1/17(9a - 2h)$	>	$3/124(21a - 4h)$	
	w_2	$2/17(3a + 5h)$	<	$1/124(55a + 84h)$	
	w_{avg}	$\frac{9a^2 - 4ah + 8h^2}{19a - 8h}$	<	$\frac{125a^2 - 24ah + 144h^2}{4(65a - 36h)}$	
Retail Price	p_1	$1/17(13a - h)$	>	$1/186(125a - 12h)$	
	p_2	$1/34(23a + 10h)$	>	$1/62(39a + 28h)$	
	p_{avg}	$\frac{461a^2 - 84ah - 104h^2}{34(19a - 8h)}$	>	$\frac{7849a^2 - 1632ah - 3600h^2}{186(65a - 36h)}$	
Consumer Purchase Qty	Q_1	$\frac{4a+h}{17b}$	<	$(61a + 12h)/186b$	
	Q_2	$(11a - 10h)/34b$	<	$(23a - 28h)/62b$	
	Q_{total}	$(19a - 8h)/34b$	<	$(65a - 36h)/93b$	
Inventory	I	$(5(a - 4h))/34b$	>	$(7(a - 12h))/186b$	
Retailer Sales (ea.)	q_1	$(13a - 18h)/34b$	>	$(61a + 12h)/372b$	
	$q_2 + I$	$(3a + 5h)/17b$	> (mostly ¹⁵)	$(23a - 28h)/124b$	
	q_{total}	$(19a - 8h)/34b$	>	$(65a - 36h)/186b$	
Retailer Profit (ea.)	πR_1	$(-13a^2 + 121ah + 302h^2)/578b$	<	$\frac{1075a^2 + 28512ah + 56592h^2}{138384b}$	
	πR_2	$\frac{181a^2 - 360ah - 300h^2}{1156b}$	>	$\frac{2357a^2 - 11928ah - 11760h^2}{46128b}$	
	πR_{total}	$\frac{155a^2 - 118ah + 304h^2}{1156b}$	>	$\frac{4073a^2 - 3636ah + 10656h^2}{69192b}$	
Manufacturer Profit	πM_1	$\frac{(13a - 18h)(9a - 2h)}{578b}$	< (mostly)	$\frac{3(25a - 52h)(21a - 4h)}{7688b}$	
	πM_2	$(2(3a + 5h)^2)/289b$	<	$(55a + 84h)^2/23064b$	
	πM_{total}	$(9a^2 - 4ah + 8h^2)/34b$	<	$\frac{125a^2 - 24ah + 144h^2}{372b}$	
SC Profit	π_1	$\frac{104a^2 - 67ah + 338h^2}{578b}$	<	$\frac{7625a^2 - 1836ah + 31104h^2}{34596b}$	
	π_2	$\frac{(11a - 10h)(23a + 10h)}{1156b}$	<	$\frac{897a^2 - 448ah - 784h^2}{3844b}$	
	π_{total}	$\frac{461a^2 - 254ah + 576h^2}{1156b}$	<	$\frac{7849a^2 - 2934ah + 12024h^2}{17298b}$	
Consumer Surplus	CS_1	$(4a + h)^2/578b$	<	$(61a + 12h)^2/69192b$	
	CS_2	$(11a - 10h)^2/2312b$	<	$(23a - 28h)^2/7688b$	
	CS_{total}	$\frac{185a^2 - 188ah + 104h^2}{2312b}$	<	$\frac{4241a^2 - 5064ah + 3600h^2}{34596b}$	
Social Welfare	SW_1	$\frac{120a^2 - 59ah + 339h^2}{578b}$	<	$\frac{18971a^2 - 2208ah + 62352h^2}{69192b}$	
	SW_2	$\frac{(11a - 10h)(57a + 10h)}{2312b}$	<	$\frac{2323a^2 - 2184ah - 784h^2}{7688b}$	
	SW_{total}	$\frac{1107a^2 - 696ah + 1256h^2}{2312b}$	<	$\frac{19939a^2 - 10932ah + 27648h^2}{34596b}$	

Table A.3: Retailer's monopoly vs. duopoly under a dynamic contract when the holding cost is high ($\frac{a}{12} \leq h < \frac{a}{4}$)

		Monopoly (Anand et al., 2008)		Duopoly
Wholesale Price	w_1	$1/17(9a - 2h)$	$>$	$a/2$
	w_2	$2/17(3a + 5h)$	$<$	$a/2$
	w_{avg}	$(9a^2 - 4ah + 8h^2)/(19a - 8h)$	$<$	$a/2$
Retail Price	p_1	$1/17(13a - h)$	$>$	$2a/3$
	p_2	$1/34(23a + 10h)$	$>$	$2a/3$
	p_{avg}	$\frac{461a^2 - 84ah - 104h^2}{34(19a - 8h)}$	$>$	$2a/3$
Consumer Purchase Qty	Q_1	$(4a + h)/17b$	$<$	$a/3b$
	Q_2	$(11a - 10h)/34b$	$<$	$a/3b$
	Q_{total}	$(19a - 8h)/34b$	$<$	$2a/3b$
Inventory	I	$(5(a - 4h))/34b$	$>$	0
Retailer Sales (ea.)	q_1	$(13a - 18h)/34b$	$>$	$a/6b$
	$q_2 + I$	$(3a + 5h)/17b$	$>$	$a/6b$
	q_{total}	$(19a - 8h)/34b$	$>$	$a/3b$
Retailer Profit (ea.)	πR_1	$(-13a^2 + 121ah + 302h^2)/578b$	$<$ (if $h < 0.14969$); $>$ if ($h > 0.14969$)	$a^2/36b$
	πR_2	$(181a^2 - 360ah - 300h^2)/1156b$	$>$	$a^2/36b$
	πR_{total}	$(155a^2 - 118ah + 304h^2)/1156b$	$>$	$a^2/18b$
Manufacturer Profit	πM_1	$((13a - 18h)(9a - 2h))/578b$	$>$	$a^2/6b$
	πM_2	$(2(3a + 5h)^2)/289b$	$<$	$a^2/6b$
	πM_{total}	$(9a^2 - 4ah + 8h^2)/34b$	$<$	$a^2/3b$
SC Profit	π_1	$(104a^2 - 67ah + 338h^2)/578b$	$<$	$(2a^2)/9b$
	π_2	$((11a - 10h)(23a + 10h))/1156b$	$<$	$(2a^2)/9b$
	π_{total}	$(461a^2 - 254ah + 576h^2)/1156b$	$<$	$(4a^2)/9b$
Consumer Surplus	CS_1	$(4a + h)^2/578b$	$<$	$a^2/18b$
	CS_2	$(11a - 10h)^2/2312b$	$<$	$a^2/18b$
	CS_{total}	$(185a^2 - 188ah + 104h^2)/2312b$	$<$	$a^2/9b$
Social Welfare	SW_1	$(120a^2 - 59ah + 339h^2)/578b$	$<$	$(5a^2)/18b$
	SW_2	$((11a - 10h)(57a + 10h))/2312b$	$<$	$(5a^2)/18b$
	SW_{total}	$(1107a^2 - 696ah + 1256h^2)/2312b$	$<$	$(5a^2)/9b$

Table A.4: Retailer's monopoly vs. duopoly under a commitment contract

		Monopoly (Anand et al., 2008)		Duopoly
Wholesale Price	w_1	$a/2$	=	$a/2$
	w_2	$a/2$	=	$a/2$
	w_{avg}	$a/2$	=	$a/2$
Retail Price	p_1	$3a/4$	>	$2a/3$
	p_2	$3a/4$	>	$2a/3$
	p_{avg}	$3a/4$	>	$2a/3$
Consumer Purchase Qty	Q_1	$a/4b$	<	$a/3b$
	Q_2	$a/4b$	<	$a/3b$
	Q_{total}	$a/2b$	<	$2a/3b$
Inventory	I	0	=	0
Retailer (ea.) Sales	q_1	$a/4b$	>	$a/6b$
	$q_2 + I$	$a/4b$	>	$a/6b$
	q_{total}	$a/2b$	>	$a/3b$
Retailer (ea.) Profit	πR_1	$a^2/16b$	>	$a^2/36b$
	πR_2	$a^2/16b$	>	$a^2/36b$
	πR_{total}	$a^2/8b$	>	$a^2/18b$
Manufacturer Profit	πM_1	$a^2/8b$	<	$a^2/6b$
	πM_2	$a^2/8b$	<	$a^2/6b$
	πM_{total}	$a^2/4b$	<	$a^2/3b$
SC Profit	π_1	$(3a^2)/16b$	<	$(2a^2)/9b$
	π_2	$(3a^2)/16b$	<	$(2a^2)/9b$
	π_{total}	$(3a^2)/8b$	<	$(4a^2)/9b$
Consumer Surplus	CS_1	$a^2/32b$	<	$a^2/18b$
	CS_2	$a^2/32b$	<	$a^2/18b$
	CS_{total}	$a^2/16b$	<	$a^2/9b$
Social Welfare	SW_1	$(7a^2)/32b$	<	$(5a^2)/18b$
	SW_2	$(7a^2)/32b$	<	$(5a^2)/18b$
	SW_{total}	$(7a^2)/16b$	<	$(5a^2)/9b$

Appendix B

PROOF FROM CHAPTER 4

B.1 Proof of Theorem 14

First, I prove that an equilibrium according to Definition 1 coincides with the solution of VI (4.33). The summation of (4.11), (4.20), (4.25), and (4.30), after algebraic simplifications, yields (4.33).

Next, I prove the converse, that is, a solution to the VI (4.33) satisfies the sum of conditions (4.11), (4.20), (4.25), and (4.30), and thereby, is a cross-sector multi-product scarce resource SCNE pattern, in accordance with Definition 1.

In (4.33), I begin by adding the term $\sum_{j=1}^I \sum_{m=1}^{M_j} (p_{0jm}^{in*} - p_{0jm}^{in*})$ to the first summand expression over i and n , $\sum_{s=1}^{S_j} (p_{1s}^{jm*} - p_{1s}^{jm*})$ to the third summand expression over j and m , and lastly, $p_{2tk}^{js*} - p_{2tk}^{js*}$ to the fifth summand expression over i , s , t , and k . Since these terms are all equal to zero, (4.33) holds true. Hence, I obtain the following inequality:

$$\begin{aligned}
& \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\frac{\partial f^{in}(x^{in*})}{\partial x_{jm}^{in}} + \frac{\partial f^{jm}(x_{jm}^*)}{\partial x_{jm}^{in}} + \frac{\partial c_{jm}^{in}(x_{jm}^{in*})}{\partial x_{jm}^{in}} - \frac{\partial \alpha_0^i(x_{jm}^{in*})}{\partial x_{jm}^{in}} \right. \\
& \quad \left. + \lambda_i^{0*} - \psi_{jm}^{in} \lambda_{jm}^{1*} + \sum_{g=1}^G \mu_{ing}^{0*} + (p_{0jm}^{in*} - p_{0jm}^{in*}) \right] \times (x_{jm}^{in} - x_{jm}^{in*}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[-\frac{\partial \alpha_g^i(\delta_g^{in*})}{\partial \delta_g^{in}} - \mu_{ing}^{0*} \right] \times (\delta_g^{in} - \delta_g^{in*}) \\
& + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{s=1}^{S_j} \left[\frac{\partial f^{js}(x^{js*})}{\partial x_s^{jm}} + \frac{\partial c_s^{jm}(x_s^{jm*})}{\partial x_s^{jm}} - \frac{\partial \beta_0^j(x_s^{jm*})}{\partial x_s^{jm}} \right. \\
& \quad \left. + \lambda_{jm}^{1*} - \lambda_{js}^{2*} + \sum_{g=1}^G \mu_{jmg}^{1*} + (p_{1s}^{jm*} - p_{1s}^{jm*}) \right] \times (x_s^{jm} - x_s^{jm*}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[-\frac{\partial \beta_g^j(\delta_g^{jm*})}{\partial \delta_g^{jm}} - \mu_{jmg}^{1*} \right] \times (\delta_g^{jm} - \delta_g^{jm*})
\end{aligned}$$

$$\begin{aligned}
& + \sum_{j=I}^I \sum_{s=1}^{S_j} \sum_{t=1}^{T_j} \sum_{k=1}^K \left[\frac{\partial c_{tk}^{js}(x_{tk}^{js*})}{\partial x_{tk}^{js}} + \hat{c}_{tk}^{js}(x_{tk}^{js*}) + \lambda_{js}^{2*} + (p_{2tk}^{js*} - p_{2tk}^{js*}) \right] \times (x_{tk}^{js} - x_{tk}^{js*}) \\
& \quad - \sum_{j=1}^I \sum_{k=1}^K p_{3k}^j(d^*) \times (d_{jk} - d_{jk}^*) \tag{B.1} \\
& \quad + \sum_{i=1}^I \left[U_i - \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in*} \right] \times (\lambda_i^0 - \lambda_i^{0*}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\sum_{i=1}^I \sum_{n=1}^{N_i} x_{jm}^{in*} \cdot \psi_{jm}^{in} - \sum_{s=1}^{S_j} x_s^{jm*} \right] \times (\lambda_{jm}^1 - \lambda_{jm}^{1*}) \\
& \quad + \sum_{j=I}^I \sum_{s=1}^{S_j} \left[\sum_{m=1}^{M_j} x_s^{jm*} - \sum_{k=1}^K \sum_{t=1}^{T_j} x_{tk}^{js*} \right] \times (\lambda_{js}^2 - \lambda_{js}^{2*}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[A_g^i - \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in*} + \delta_g^{in*} \right] \times (\mu_{ing}^0 - \mu_{ing}^{0*}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[B_g^j - \sum_{s=1}^{S_j} x_s^{jm*} + \delta_g^{jm*} \right] \times (\mu_{jmg}^1 - \mu_{jmg}^{1*}) \geq 0, \\
& \quad \forall (Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1) \in \mathcal{K}.
\end{aligned}$$

Rearranging (B.1) yields:

$$\begin{aligned}
& \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\frac{\partial f^{in}(x_{jm}^{in*})}{\partial x_{jm}^{in}} + \frac{\partial c_{jm}^{in}(x_{jm}^{in*})}{\partial x_{jm}^{in}} - p_{0jm}^{in*} - \frac{\partial \alpha_0^i(x_{jm}^{in*})}{\partial x_{jm}^{in}} \right. \\
& \quad \left. + \lambda_i^{0*} + \sum_{g=1}^G \mu_{ing}^{0*} \right] \times (x_{jm}^{in} - x_{jm}^{in*}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[-\frac{\partial \alpha_g^i(\delta_g^{in*})}{\partial \delta_g^{in}} - \mu_{ing}^{0*} \right] \times (\delta_g^{in} - \delta_g^{in*}) \\
& \quad + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\frac{\partial f^{jm}(x_{jm}^{in*})}{\partial x_{jm}^{in}} + p_{0jm}^{in*} - \psi_{jm}^{in} \lambda_{jm}^{1*} \right] \times (x_{jm}^{in} - x_{jm}^{in*}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{s=1}^{S_j} \left[\frac{\partial c_s^{jm}(x_s^{jm*})}{\partial x_s^{jm}} - p_{1s}^{jm*} - \frac{\partial \beta_0^j(x_s^{jm*})}{\partial x_s^{jm}} + \lambda_{jm}^{1*} + \sum_{g=1}^G \mu_{jmg}^{1*} \right] \times (x_s^{jm} - x_s^{jm*}) \\
& \quad + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[-\frac{\partial \beta_g^j(\delta_g^{jm*})}{\partial \delta_g^{jm}} - \mu_{jmg}^{1*} \right] \times (\delta_g^{jm} - \delta_g^{jm*}) \\
& \quad + \sum_{j=I}^I \sum_{s=1}^{S_j} \sum_{t=1}^{T_j} \sum_{k=1}^K \left[\frac{\partial c_{tk}^{js}(x_{tk}^{js*})}{\partial x_{tk}^{js}} - p_{2tk}^{js*} + \lambda_{js}^{2*} \right] \times (x_{tk}^{js} - x_{tk}^{js*})
\end{aligned}$$

$$\begin{aligned}
& + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{s=1}^{S_j} \left[\frac{\partial f^{js}(x^{js*})}{\partial x_s^{jm}} + p_{1s}^{jm*} - \lambda_{js}^{2*} \right] \times (x_s^{jm} - x_s^{jm*}) \\
& + \sum_{j=1}^I \sum_{s=1}^{S_j} \sum_{t=1}^{T_j} \sum_{k=1}^K [p_{2tk}^{js*} + \hat{c}_{tk}^{js}(x_{tk}^{js*})] \times (x_{tk}^{js} - x_{tk}^{js*}) \\
& - \sum_{j=1}^I \sum_{k=1}^K p_{3k}^j(d^*) \times (d_{jk} - d_{jk}^*) \tag{B.2} \\
& + \sum_{i=1}^I \left[U_i - \sum_{n=1}^{N_i} \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in*} \right] \times (\lambda_i^0 - \lambda_i^{0*}) \\
& + \sum_{j=1}^I \sum_{m=1}^{M_j} \left[\sum_{i=1}^I \sum_{n=1}^{N_i} x_{jm}^{in*} \cdot \psi_{jm}^{in} - \sum_{s=1}^{S_j} x_s^{jm*} \right] \times (\lambda_{jm}^1 - \lambda_{jm}^{1*}) \\
& + \sum_{j=1}^I \sum_{s=1}^{S_j} \left[\sum_{m=1}^{M_j} x_s^{jm*} - \sum_{k=1}^K \sum_{t=1}^{T_j} x_{tk}^{js*} \right] \times (\lambda_{js}^2 - \lambda_{js}^{2*}) \\
& + \sum_{i=1}^I \sum_{n=1}^{N_i} \sum_{g=1}^G \left[A_g^i - \sum_{j=1}^I \sum_{m=1}^{M_j} x_{jm}^{in*} + \delta_g^{in*} \right] \times (\mu_{ing}^0 - \mu_{ing}^{0*}) \\
& + \sum_{j=1}^I \sum_{m=1}^{M_j} \sum_{g=1}^G \left[B_g^j - \sum_{s=1}^{S_j} x_s^{jm*} + \delta_g^{jm*} \right] \times (\mu_{jmg}^1 - \mu_{jmg}^{1*}) \geq 0, \\
& \forall (Q^0, Q^1, Q^2, \mathfrak{d}^0, \mathfrak{d}^1, d, \lambda^0, \lambda^1, \lambda^2, \mu^0, \mu^1) \in \mathcal{H}.
\end{aligned}$$

Clearly, (B.2) is the sum of the optimality condition (4.11), (4.20), (4.25), and (4.30), and thereby, is, according to Definition 8, a cross-sector multi-product scarce resource SCNE pattern. \square

Remarks:

Although the proof of Theorem 14 contains both the proof of the condition and its converse, it shall not be confused with a proof of necessity and sufficiency. This is because Definition 8, upon which Theorem 14 relies, defines the sum of conditions (4.11), (4.20), (4.25), and (4.30) as the equilibrium condition, making the proof of the Theorem trivial with only a need for verifying what is being stated. The proof of the converse, included in the proof, simply serves as a cross-check.

Mathematically, Definition 8 is also intuitive. An equilibrium that supposedly satisfies each of the conditions (4.11), (4.20), (4.25), and (4.30) individually will certainly satisfy the sum of them.

This style of constructing and representing a network equilibrium was inaugurated by Nagurney et al. (2002), a highly cited work, and has been adopted by many SCNE works, including the papers cited in the last two paragraphs of §1.2.2.

Appendix C

PROOF FROM CHAPTER 5

C.1 Proof of Theorem 18

First, I prove that an equilibrium according to Definition 1 coincides with the solution of VIP (5.25). The summation of (5.14), (5.16), (5.18), and (5.24), after algebraic simplifications, yields (5.25).

Next, I prove the converse, that is, a solution to the VIP (5.25) satisfies the sum of conditions (5.14), (5.16), (5.18), and (5.24), and thereby, is a robust closed-loop supply chain network equilibrium pattern, in accordance with Definition 1.

In (5.25), I begin by adding the term $(p_{mn} - p_{mn}^*)$ to the first summand expression over m and n , $(p_{nk} - p_{nk}^*)$ to the second summand expression over n and k , $(p_{ki} - p_{ki}^*)$ to the third summand over k and i , and lastly, $(p_{in} - p_{in}^*)$ to the fourth summand expression over i and n . Since these terms are all equal to zero, (5.25) holds true. Hence, I obtain the following inequality:

$$\begin{aligned}
& \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial f_m(x_m^*)}{\partial x_{mn}} + \frac{\partial c_{mn}(x_{mn}^*)}{\partial x_{mn}} + \frac{\partial f_n(x^{n*}, x_n^*)}{\partial x_{mn}} + \bar{w}_{mn} \right. \\
& \quad \left. + \sum_{t=1}^T \lambda_t^{0*} - \beta^r \lambda_n^{2*} + (p_{mn} - p_{mn}^*) \right] \times (x_{mn} - x_{mn}^*) \\
& + \sum_{n=1}^N \sum_{k=1}^K \left[\frac{\partial c_n(\sum_{k=1}^K x_{nk}^*)}{\partial x_{nk}} + \hat{c}_{nk}(x_{nk}^*) - p_k^* + \lambda_n^{2*} - \lambda_k^{4*} + (p_{nk} - p_{nk}^*) \right] \times (x_{nk} - x_{nk}^*) \\
& + \sum_{i=1}^I \sum_{k=1}^K \left[\frac{\partial f_i(x_i^*)}{\partial x_{ki}} + \frac{\partial c_{ki}(x_{ki}^*)}{\partial x_{ki}} + \alpha_k(Q^{4*}) - \lambda_i^{3*} + \lambda_k^{4*} + (p_{ki} - p_{ki}^*) \right] \times (x_{ki} - x_{ki}^*) \quad (C.1) \\
& + \sum_{i=1}^I \sum_{n=1}^N \left[\frac{\partial f_n(x^{n*}, x_n^*)}{\partial x_{in}} - \beta^u \lambda_n^{2*} + \lambda_i^{3*} + (p_{in} - p_{in}^*) \right] \times (x_{in} - x_{in}^*) \\
& + \sum_{t=1}^T \left[\bar{Z}_t - \rho \bar{Z}_t - \sum_{m=1}^M \sum_{n=1}^N x_{mn}^* \right] \times (\lambda_t^0 - \lambda_t^{0*}) \\
& + \sum_{m=1}^M \sum_{s=1}^{S_m} \left[\bar{v}_m - v_{ms} \right] \times (\lambda_{ms}^1 - \lambda_{ms}^{1*})
\end{aligned}$$

$$\begin{aligned}
& + \sum_{n=1}^N [\beta^r \sum_{m=1}^M x_{mn}^* + \beta^u \sum_{i=1}^I x_{in}^* - \sum_{k=1}^K x_{nk}^*] \times (\lambda_n^2 - \lambda_n^{2*}) \\
& + \sum_{i=1}^I \left[\sum_{k=1}^K x_{ki}^* - \sum_{n=1}^N x_{in}^* \right] \times (\lambda_i^3 - \lambda_i^{3*}) + \sum_{k=1}^K \left[\sum_{n=1}^N x_{nk}^* - \sum_{i=1}^I x_{ki}^* \right] \times (\lambda_k^4 - \lambda_k^{4*}) \\
& + \sum_{k=1}^K \left[\sum_{n=1}^N x_{nk}^* - d_k(p^*) \right] \times (p_k - p_k^*) \\
& + \sum_{m=1}^M \left[1 - \sum_{s=1}^{S_m} \lambda_{ms}^{1*} \right] \times (\bar{v}_m - \bar{v}_m^*) + \sum_{m=1}^M \sum_{s=1}^{S_m} \lambda_{ms}^{1*} \times (v_{ms} - v_{ms}^*) \geq 0 \\
& \forall (Q^1, Q^2, Q^3, Q^4, p, \lambda^0, \lambda^1, \lambda^2, \lambda^3, \lambda^4, \bar{v}, v) \in \mathcal{H},
\end{aligned}$$

where, $\mathcal{H} \equiv \mathcal{H}^1 \times \mathcal{H}^2 \times \mathcal{H}^3 \times \mathcal{H}^4 = \{(Q^1, Q^2, Q^3, Q^4, p, \lambda^0, \lambda^1, \lambda^2, \lambda^3, \lambda^4, \bar{v}, v) \in \mathbb{R}_+^{\mathcal{N}} : \|H_{ms}^{\mathbf{T}} x_m\| \leq v_{ms}, \forall m = 1, \dots, M; s = 1, \dots, S_m\}$

Rearranging (C.1) yields:

$$\begin{aligned}
& \sum_{m=1}^M \sum_{n=1}^N \left[-p_{mn}^* + \frac{\partial f_m(x_m^*)}{\partial x_{mn}} + \frac{\partial c_{mn}(x_{mn}^*)}{\partial x_{mn}} + \bar{w}_{mn} + \sum_{t=1}^T \lambda_t^{0*} \right] \times (x_{mn} - x_{mn}^*) \\
& + \sum_{t=1}^T \left[\bar{Z}_t - \rho \bar{Z}_t - \sum_{m=1}^M \sum_{n=1}^N x_{mn}^* \right] \times (\lambda_t^0 - \lambda_t^{0*}) \\
& + \sum_{m=1}^M \sum_{s=1}^{S_m} \left[\bar{v}_m - v_{ms} \right] \times (\lambda_{ms}^1 - \lambda_{ms}^{1*}) \\
& + \sum_{m=1}^M \left[1 - \sum_{s=1}^{S_m} \lambda_{ms}^{1*} \right] \times (\bar{v}_m - \bar{v}_m^*) \\
& + \sum_{m=1}^M \sum_{s=1}^{S_m} \lambda_{ms}^{1*} \times (v_{ms} - v_{ms}^*) \\
& + \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial f_n(x_n^*, x_n^*)}{\partial x_{mn}} + p_{mn}^* - \beta^r \lambda_n^{2*} \right] \times (x_{mn} - x_{mn}^*) \\
& + \sum_{i=1}^I \sum_{n=1}^N \left[\frac{\partial f_n(x_n^*, x_n^*)}{\partial x_{in}} + p_{in}^* - \beta^u \lambda_n^{2*} \right] \times (x_{in} - x_{in}^*) \\
& + \sum_{n=1}^N \sum_{k=1}^K \left[\frac{\partial c_n(\sum_{k=1}^K x_{nk}^*)}{\partial x_{nk}} - p_{nk}^* + \lambda_n^{2*} \right] \times (x_{nk} - x_{nk}^*) \\
& + \sum_{n=1}^N [\beta^r \sum_{m=1}^M x_{mn}^* + \beta^u \sum_{i=1}^I x_{in}^* - \sum_{k=1}^K x_{nk}^*] \times (\lambda_n^2 - \lambda_n^{2*}) \\
& + \sum_{i=1}^I \sum_{k=1}^K \left[p_{ki}^* + \frac{\partial f_i(x_i^*)}{\partial x_{ki}} + \frac{\partial c_{ki}(x_{ki}^*)}{\partial x_{ki}} - \lambda_i^{3*} \right] \times (x_{ki} - x_{ki}^*) \\
& + \sum_{i=1}^I \sum_{n=1}^N \left[\lambda_i^{3*} - p_{in}^* \right] \times (x_{in} - x_{in}^*)
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
& + \sum_{i=1}^I \left[\sum_{k=1}^K x_{ki}^* - \sum_{n=1}^N x_{in}^* \right] \times (\lambda_i^3 - \lambda_i^{3*}) \\
& + \sum_{n=1}^N \sum_{k=1}^K \left[p_{nk}^* + \hat{c}_{nk}(x_{nk}^*) - p_k^* - \lambda_k^{4*} \right] \times (x_{nk} - x_{nk}^*) \\
& + \sum_{k=1}^K \left[\alpha_k(Q^{4*}) - p_{ki}^* + \lambda_k^{4*} \right] \times (x_{ki} - x_{ki}^*) \\
& + \sum_{k=1}^K \left[\sum_{n=1}^N x_{nk}^* - d_k(p^*) \right] \times (p_k - p_k^*) \\
& + \sum_{k=1}^K \left[\sum_{n=1}^N x_{nk}^* - \sum_{i=1}^I x_{ki}^* \right] \times (\lambda_k^4 - \lambda_k^{4*}) \geq 0 \\
& \forall (Q^1, Q^2, Q^3, Q^4, p, \lambda^0, \lambda^1, \lambda^2, \lambda^3, \lambda^4, \bar{v}, v) \in \mathcal{H},
\end{aligned}$$

where, $\mathcal{H} \equiv \mathcal{H}^1 \times \mathcal{H}^2 \times \mathcal{H}^3 \times \mathcal{H}^4 = \{(Q^1, Q^2, Q^3, Q^4, p, \lambda^0, \lambda^1, \lambda^2, \lambda^3, \lambda^4, \bar{v}, v) \in \mathbb{R}_+^{\mathcal{N}} : \|H_{ms}^{\mathbf{T}} x_m\| \leq v_{ms}, \forall m = 1, \dots, M; s = 1, \dots, S_m\}$

Clearly, (5.25) is the sum of the optimality condition (5.14), (5.16), (5.18), and (5.24), and thereby, is, according to Definition 9, a robust closed-loop supply chain network equilibrium pattern.

□