Achieving a Closed Orbit Around Neptune Through Aerobraking

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Introduction

Changing the orbit of a spacecraft requires large changes in energy and angular momentum. If a spacecraft approaches a target planet with too much angular momentum, it will be routed around the planet and not captured into orbit.

To be captured, the spacecraft must shed excess energy. In most cases, the spacecraft will slow itself down with rockets, but this can be a very costly process as it requires substaintial amounts of fuel.

An alternative method called 'aerobraking' can be used to slow the spacecraft down by letting it pass through the upper atmosphere of a planet to burn off excess energy.

We examine the parameters needed to break a spacecraft around Neptune using the process of aerobraking.

Theory

A common maneuver in orbital mechanics, the Hohmann transfer, involves applying a force to slow down a spacecraft at its periapse to change its orbit. We assumed an initial velocity of 30 km/s relative to Neptune.

At this velocity, the spacecraft's orbit is hyperbolic. The goal is to enter an elliptical orbit of eccentricity e around the planet. To achieve this, the force must be applied at the periapse of the orbit, which will remain stationary with each subsequent pass. The change in velocity at this point is related to how much force is applied by the atmosphere. The desired change in velocity is given by

$$\Delta v = v_{incoming} - v_{capture} = \sqrt{{v_\infty}^2 + \frac{2\mu}{r_p}} - \sqrt{\frac{\mu\,(1+e)}{r_p}}$$

The quantity μ is the reduced mass of the system, e is the eccentricity of the resultant orbit, and r_p is the distance of the periapse from the center of the planet.

This relationship assumes that the only change in velocity occurs at the periapse of the orbit. While this is not precisely the case, it is a necessary estimation for our model.

The relationship between drag force and velocity is a differential equation that has the solution

$$v = v_0 e^{-\frac{1}{2}\frac{C_d A \rho}{m}}$$

A is the cross sectional area of the spacecraft, ρ is the density of the atmosphere, and C_d is the coefficient of drag.

Atmospheric Profile

Information about Neptune's atmosphere is necessary to determine the drag force on the spacecraft. Like Earth, Neptune's atmosphere can be divided into several regions. In our model, the spacecraft remains only in the uppermost regions, the exosphere and thermosphere, in which the composition of the atmosphere can be considered constant.

Element	Symbol	Relative Amount
Hydrogen	H_2	(80.0 ± 3.2)%
Helium	He	(19 .0 ± 3.2)%
Methane	CH_4	(1.5 ± 0.5)%

Table 1: Chemical composition of Neptune's upper atmosphere

Table 1 contains information about the composition of Neptune's atmosphere. This information was used to determine the average molecular mass of the air that the spacecraft would fly through. Additional data regarding pressure and temperature values at various depths were used to construct a relationship between density and depth.

Methods

Analytically calculating the trajectory of a spacecraft through an atmosphere is difficult because the velocity decreases over the length of the trajectory. Instead, we made the approximation that the velocity changes only at the periapse of the orbit. Such an approximation made it possible to consider the orbit change a Hohmann transfer.

The resulting trajectory can therefore be directly related to the depth of atmospheric penetration by the spacecraft. Combining the penetration depth with our atmospheric profile of Neptune, we can determine the average pressure, temperature, drag force, and the change in velocity at the periapse.

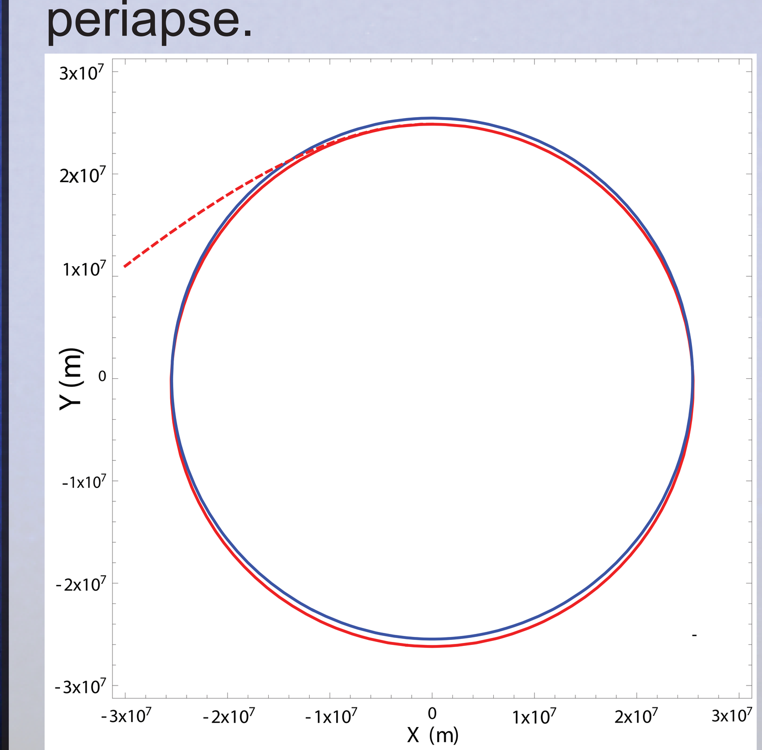

Figure 1: Example trajectory. The dashed line is the incoming hyperbolic trajectory. The solid red line is the resultant elliptic orbit.

We could then calculate the eccentricity of the resultant trajectory using the equations described in the theory section. We chose a variety of penetration depths and computed their corresponding eccentricities until we found a range that produced fairly circular orbits.

Results

The range of acceptable penetration depths is a window of approximately 25 km. If the spacecraft passes too deep into the atmosphere, too much energy will be lost causing it to spiral into the planet. This resultant orbit is indicated by a negative eccentricity.

Within the 25 km window, small variations in initial conditions result in large variations in the resulting orbit. After the first pass through the atmosphere, the spacecraft may have a highly elliptical orbit. In such a case, rockets are still required to adjust the trajectory to that of a circle. However, this would require much less fuel than a purely rocket based orbit change.

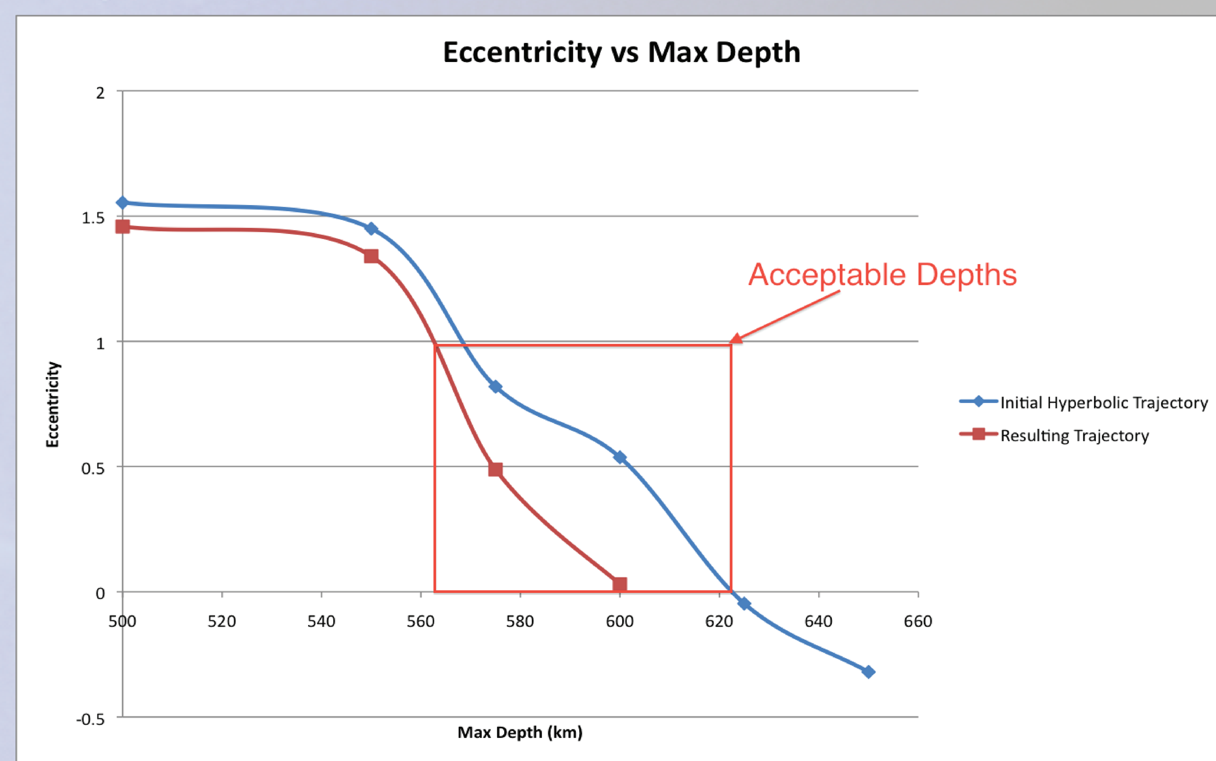

Figure 2: Eccentricity of inital and final orbits for various periapse depths. Negative values denote unstable orbits.

Future Work

We are currently refining our model to account for changes in velocity that occur along the path through the atmosphere, rather than at one particular point.

To achieve this, we will approach the problem numerically rather than analytically. We are using MATLAB to program the new model.

We hope that this new model will allow us to calculate trajectories that are more accurate by eliminating our greatest source of error.

Acknowledgments

This research was started as our entry in the 2010 University Physics Competition:
http://www.uphysicsc.com/