

Center for Quality and Productivity Improvement
UNIVERSITY OF WISCONSIN
610 Walnut Street
Madison, Wisconsin 53705
(608) 263-2520
(608) 263-1425 FAX

Report No. 157

**How to Reduce Costs
Using a Tolerance Analysis Formula
Tailored to Your Organization**

Spencer Graves

April 1997

The Center for Quality and Productivity Improvement cares about your reactions to our reports. Please direct comments (general or specific) to: Report Editor, Center for Quality and Productivity Improvement, 610 Walnut Street, Madison, WI 53705; (608) 263-2520. All comments will be forwarded to the author(s).

How to Reduce Costs Using a Tolerance Analysis Formula Tailored to Your Organization

Spencer Graves¹

Center for Quality and Productivity
Improvement

*University of Wisconsin
Madison, Wisconsin*

ABSTRACT

In volume production, a few pennies per unit can add up quickly. In such cases, it may be worth considering whether cheaper components might be used without jeopardizing quality. One element in many tolerance analyses is the formula used to relate product tolerances to component tolerances. This article (a) discusses deficiencies with traditional tolerancing, (b) outlines a simple procedure for converting process capability information into an improved tolerancing formula tailored to a specific class of products, and (c) describes how this analysis can contribute to substantive improvements in profits by helping to identify improvement opportunities in production.

KEYWORDS: *tolerance analysis and allocation/synthesis; reducing manufacturing costs*

¹ Dr. Graves is a research scientist at the Center for Quality and Productivity Improvement. He is a member of ASQC.
Copyright © 1997 Spencer Graves

How to Reduce Costs Using a Tolerance Analysis Formula Tailored to Your Organization

Spencer Graves

In volume production, a few pennies per unit can add up quickly. In such cases, it may be worth considering whether cheaper components might be used without jeopardizing quality. One element in many tolerance analyses is the formula used to relate product tolerances to component tolerances. This article (a) discusses deficiencies with traditional tolerancing, (b) outlines a simple procedure for converting process capability information into an improved tolerancing formula tailored to a specific class of products, and (c) describes how this analysis can contribute to substantive improvements in profits by helping to identify improvement opportunities in production.

INTRODUCTION

In volume production of, say, a thousand units per day, savings of a penny per unit is roughly \$2,500 per year. Many Fortune 500 companies have single products produced in such volumes. In such cases, a better understanding of tolerancing may help reduce scrap and rework for products with production problems and may contribute to cost reductions for products with excess process capability.

This article describes a simple modification to traditional statistical tolerancing that can make it more useful for any manufacturing organization. The discussion begins by reviewing "worst case" and "statistical" tolerancing. This is followed by a description of "inflated statistical tolerancing," which practitioners in industry have recommended as preferable to either of the two traditional alternatives.

In fact, if a product characteristic could theoretically be toleranced using traditional statistical tolerancing, then any tolerance for that characteristic could be described as "inflated statistical tolerancing" for some inflation factor. The applicable inflation factor can be obtained by dividing the current tolerance by the relevant theoretical statistical tolerance. We show below that this "estimated inflation factor" is the product of the process capability and a factor that is essentially independent of the method used to determine the current tolerance. (In this article, C_{pk} is used for process capability; Kotz and Johnson (1993). The procedure can also be used with C_p or C_{pm} ; the algebra for that is slightly different from what is presented here but is derived in essentially the same way.)

The general use of inflated statistical tolerancing proposed in this article can be introduced slowly into many manufacturing organizations at negligible cost and with negligible disruption to the way people currently work. Initially, someone needs to

review relevant data bases to determine which parts of the required data are already available and how any additional data collection should be added. A year or two after the necessary enhanced data collection is adopted, the organization should be ready to start analyzing the data in ways that can lead to improvements in tolerancing practices.

A more aggressive implementation effort would include special projects to review tolerancing on a few current products that have excessive problems or waste money on manufacturing precision that makes negligible contribution to customer satisfaction ("excess process capability"). The advantage of a more aggressive implementation is that a study of tolerancing might help reduce costs with current products, whether it be customer complaints, warranty, scrap, rework, or money spent on unnecessary precision. Also, a tolerancing study of certain current products might help reduce the time required to transfer similar new products from engineering to production. The current article outlines a four-step process for this more aggressive implementation.

WORST CASE TOLERANCING

It is typical in manufacturing to require that a product characteristic, X , lie between upper and lower specification limits, U and L , respectively. In this article, the word "tolerance" refers to the width of this interval, $T = U - L$. (One-sided tolerance limits, where one but not both of U and L is specified, are not discussed here.) We also assume symmetric tolerancing with target $\theta = (U + L)/2$, midway between U and L .

In many cases, a product characteristic, X_a , can be approximated as a linear function of component characteristics, X_i 's, as

$$X_a \cong \theta_a + \sum b_i(X_i - \theta_i), \quad (1)$$

where θ_i is the nominal or target value for X_i , the b_i 's are coefficients that make expression (1) a reasonable approximation to the relationship between a product characteristic and component characteristics, and Σ denotes the sum over $i = 1, \dots, n$ (n = the number of component characteristics assumed to impact appreciably the product characteristic of interest). For example, if X_a is a continuously differentiable function of X_1, \dots, X_n , then a first-order Taylor series approximation has the form (1). Expression (1) is called the "variation transmission formula".

Unfortunately, reality could deviate from (1), either because we omitted an important X_i or because we used an incorrect value for one of the b_i 's. We model these sources of discrepancy by adding an error term, ϵ , to (1):

$$X_a = \theta_a + \sum b_i(X_i - \theta_i) + \epsilon. \quad (1')$$

Tolerancing problems can originate in the difference between (1) and (1'): Publications on tolerancing, with rare exceptions, assume that the user knows all the factors, X_i , that contribute to the variability in the assembly characteristic of interest, X_a . For a standard one-dimensional mechanical assembly, the difference between (1) and (1') may be negligible. However, in general, the assumption of perfect knowledge is not realistic. At its most basic, philosophical level, the real world is infinite in complexity and "unknown and unknowable" (Deming (1986)). Tolerances may be assigned using theory, but that theory must ultimately be grounded in experimentation (Bisgaard (1997)). Problems can be expected whenever the unmodeled variability, ε , is not negligible but is nevertheless neglected.

Ignoring these considerations, it can be shown that if (1) holds and if $L_i \leq X_i \leq U_i$, $i = 1, \dots, n$, then $L_a \leq X_a \leq U_a$, where $(L_a, U_a) = \theta_a \pm T_a^{(2)}/2$,

$$T_a^{(2)} = \sum |b_i| T_i, \quad (2)$$

and $T_a^{(j)}$ is the relationship between component and assembly tolerances described by expression (j) in this article. Expression (2) is called "worst case" tolerancing and typically requires the T_i 's to be tighter than necessary to achieve a desired T_a , although the opposite problem is possible: If the error ε in (1') is appreciable, even "worst case" tolerancing, (2), can lead to production problems, though such problems seem to be relatively rare. A more common complaint is that manufacturers pay more than necessary for components, especially with long tolerance chains (large n). We now discuss the traditional approach to reducing manufacturing costs by considering random variability in production.

TRADITIONAL STATISTICAL TOLERANCING

If in (1) the component characteristics, X_i , are uncorrelated random variables with standard deviations σ_i , then (1) implies that the standard deviation of the assembly is given by (Shewhart (1931, ch. 17))

$$\sigma_a = \sqrt{b_1^2 \sigma_1^2 + \dots + b_n^2 \sigma_n^2} \quad (3)$$

or more generally using (1') rather than (1),

$$\sigma_a = \sqrt{b_1^2 \sigma_1^2 + \dots + b_n^2 \sigma_n^2 + \sigma_\varepsilon^2} \quad (3')$$

It has been common at least since Rüdénberg (1929), Brandenberger (1946, pp. 72-77), Grant (1946, p. 326), and Bruyevich (1946) to assume that the width of the

tolerance interval is $T = 2Z\sigma$, where Z is typically 3. If we assume that $Z_a = Z_1 = \dots = Z_n$, and substitute $T_i/(2Z)$ for σ_i in (3), we get

$$T_a^{(4)} = \sqrt{\sum b_i^2 T_i^2} \quad (4)$$

Expression (4) is commonly called “statistical tolerancing”. Unfortunately, while worst case tolerancing, (2), has typically pushed manufacturers to spend more for materials than necessary, statistical tolerancing, (4), has often surprised users with unexpected high rates of problems in assembly. This happens because (4) is quite non-robust to departures from assumptions. This point is made by Graves and Bisgaard (1997), who discuss “five ways statistical tolerancing can fail”:

(a) Deficiencies in the variation transmission model, (1), as mentioned above.

(b) A desire for more process capability in the assembly than in the components (i.e., $Z_a > Z_i$). For example, if the inputs all follow uniform distributions between L_i and U_i , this makes $\sigma_i = T_i/(2\sqrt{3})$ so $Z_i = \sqrt{3}$, and $C_{pk,i} = 0.58$, where

$$C_{pk} = \frac{\min\{U - \mu, \mu - L\}}{3\sigma}, \quad (5)$$

and μ is the process average (Kotz and Johnson (1993)). If the thicknesses of two flat washers both follow a uniform distribution between L and U and specification limits on the height of the assembly are established by statistical tolerancing, then 8.6 percent of assemblies will be outside specifications.

(c) Biases in the distributions of component characteristics. If statistical tolerancing is applied to the height of a stack of four flat washers where the distribution of the thicknesses of individual washers was biased, the assembly could have $C_{pk} = -1$ even though $C_{pk} = +1$ for all components!

(d) Correlations among component characteristics. If certain characteristics of numerous transistors in an integrated circuit all have the same positive correlation with others and $b_i = 1$, $i = 1, \dots, n$, then $\sigma_a^2 = n\sigma^2[1 + (n-1)\rho]$, where $\sigma^2 =$ variance of an individual characteristic, and $\rho =$ correlation between two such characteristics. If $n\rho$ is large, this makes $\sigma_a \cong n\sigma\sqrt{\rho}$; this can be much larger than the independent case where $\sigma_a = \sigma\sqrt{n}$. In such cases, worst case tolerancing, (2), is more appropriate than traditional statistical tolerancing, (4). However, it would be better still to model the correlation directly.

(e) Non-normality in the distributions of component characteristics. Even when this occurs, it is unlikely to cause a problem *except* in the rare cases that non-normal distributions are so pathological that the Central Limit Theorem does not apply.

Any of these problems can be modeled directly (Graves (1994) or Chase and Greenwood (1988)). However, *if their effect is fairly consistent* across a class of products or quality characteristics, it is often reasonable (and cheaper) to account for them by adding an “inflation factor” to (4), as we now explain.

INFLATED STATISTICAL TOLERANCING

Gilson (1951) and Bender (1962) studied tolerancing applications in their respective organizations to determine if worst case, (2), or statistical tolerancing, (4), or something else gave the best answer. They concluded that

$$T_a^{(6)} = f \sqrt{\sum b_i^2 T_i^2} \tag{6}$$

with $f = 1.5$ or 1.6 worked well for them. We call this “inflated statistical tolerancing,” with f being the “inflation factor”. One purpose of this article is to describe how you can evaluate whether this formula seems appropriate for your organization or for a particular class of problems, and if so, what would be an appropriate inflation factor, f .

Different situations can justify the use of different values for f ; see Table 1. For example, consider a product that uses several copies of the same component. Any temporary bias in the process for producing that component would likely be the same in all units used at the same time. For many product designs, this could suggest a need for a larger inflation factor, f , than otherwise.

Alternatively, suppose the theory and the metrology is better developed for one important quality characteristic than for another. In this case, σ_ϵ^2 might be a larger proportion of (3') for the second type of quality characteristic. This would support the use of a larger inflation factor, f , where the theory is not as well worked out.

As yet another alternative, it may be desired to use a larger inflation factor, f , for low volume products than for high volume, with the anticipation that the higher production volume could support the investment of more engineering time to understand and control sources of variability.

Similarly, a concurrent engineering program may get a product to market faster by using a fairly large inflation factor, f , for initial production. The product introduction plan could call for experiments in production to verify or refute the variation transmission formula used, (1) or (1'), to identify sources of variability not previously considered, and to revise tolerancing to achieve adequate process capability at a lower cost. This work could be done *after* product introduction, so an early introduction could (a) contribute to the competitive position of the organization in the marketplace, (b) reduce the cash

f small	f large
<ul style="list-style-type: none"> • High volume product • Mature production • Part of an aggressive technology-development program 	<ul style="list-style-type: none"> • Low volume product • Initial production • Theory or metrology not well worked out • Multiple copies of the same component

Table 1. Alternative Justifications for Different Inflation Factors

required before the product starts generating revenue, and (c) allow the organization to invest in tightening tolerances only for products that are truly successful in the marketplace.

In many high tech applications, the opposite is done: Tolerances are selected to challenge current technology, resulting in low initial yields. However, yields rise fairly rapidly because the product introduction plan includes technical resources to fix problems. This kind of management policy could justify the use of a smaller inflation factor, f .

In sum, many different issues can influence the choice of a tolerance inflation factor, f , for a particular organization and class of products. The next two sections describe how process capability data can be used to help select a value or values for this inflation factor, f .

TOLERANCING AND PROCESS CAPABILITY

A central element of the analysis described below is a comparison of process capability with f computed from (6), as

$$f^{(7)} = \frac{T_a}{\sqrt{\sum b_i^2 T_i^2}} = \frac{T_a}{T_a^{(4)}}, \quad (7)$$

where T_a is the assembly tolerance currently used, and $T_a^{(4)}$ is the width of a statistical tolerance interval based on the variation transmission model and component tolerances actually used. If tolerances were set using statistical tolerancing, (4), $f^{(7)} = 1$; if worst case, (2), were used, $f^{(7)} \cong \sqrt{n}$ (provided the $|b_i|$'s are roughly equal). Since $f^{(7)}$ is a result of a computation, we call it "the *estimated* inflation factor".

In this section, we will decompose the estimated inflation factor, $f^{(7)}$, into the product of $C_{pk,a}$, the (observed) process capability for the assembly, and a term representing the contributions to tolerancing of the problems discussed following (4). We shall see that $f^{(7)}$ depends on the actual tolerance, T_a , only through $C_{pk,a}$, and the remaining factor is independent of the procedure (such as (2), (4) or (6)) used to select T_a .

To understand this decomposition, we need the process centering coefficient (Liggett (1993, pp. 60-62)):

$$C_c = 2|\mu - \theta|/T.$$

By way of comparison, Motorola's "six sigma" program (Liggett (1993, pp. 60-62), Harry (1992)) assumes that $T = 12\sigma$ and the process average is at most 1.5σ from the target; this makes $C_c \leq 2(1.5)/12 = 0.25$.

We substitute C_c into the definition of C_{pk} , (5), recalling that $\theta = (U + L)/2$ so $U - \mu = (U - \theta) - (\mu - \theta) = \frac{1}{2}T(1 \pm C_c)$, and solve for σ to get

$$\sigma = \frac{T(1 - C_c)}{6C_{pk}}$$

We add subscripts and substitute into (3') to get

$$\frac{T_a(1 - C_{c,a})}{6C_{pk,a}} = \sqrt{\sum b_i^2 \left[\frac{T_i(1 - C_{c,i})}{6C_{pk,i}} \right]^2} + \sigma_\epsilon^2,$$

which implies

$$T_a^{(8)} = \frac{C_{pk,a}}{(1 - C_{c,a})} \sqrt{\sum b_i^2 \left[\frac{T_i(1 - C_{c,i})}{C_{pk,i}} \right]^2} + 36\sigma_\epsilon^2. \quad (8)$$

If we now let

$$f^+ = \frac{\sqrt{\sum b_i^2 \left[\frac{T_i(1 - C_{c,i})}{C_{pk,i}} \right]^2} + 36\sigma_\epsilon^2}{(1 - C_{c,a}) \sqrt{\sum b_i^2 T_i^2}}, \quad (9)$$

then (8) can be rewritten as

$$T_a^{(10)} = C_{pk,a} f^+ \sqrt{\sum b_i^2 T_i^2} = C_{pk,a} f^+ T_a^{(4)}, \quad (10)$$

which is just (6) with

$$f = C_{pk,a} f^+. \quad (11)$$

Thus, (11) decomposes the inflation factor, f , into the product of the process capability for the assembly, $C_{pk,a}$, and the "adjusted inflation factor," f^+ . Note that f^+ is a dimensionless quantity. It is a useful concept if it can be found to be roughly constant over a class of products or product characteristics; we explore this possibility and its implications in the remainder of this article.

To check the sensibility of this decomposition, recall that the assumptions for statistical tolerancing, (4), imply that $C_{c,i} = C_{c,a} = 0$, $C_{pk,i} = 1$, and $\sigma_\epsilon = 0$. We substitute these values for $C_{c,i}$, $C_{pk,i}$, and σ_ϵ into (9) to get $f^+ = 1$. By (11), this would make $f = C_{pk,a}$. This helps explain the decomposition.

In reality, σ_ε may rarely be negligible, and $C_{pk,i}$ may be less than 1 for a critical component characteristic. Moreover, if $C_{c,i}$'s are non-zero, they often accumulate through (1) to make $C_{c,a}$ much larger than any of the individual $C_{c,i}$'s. These effects can make f^+ substantially greater than 1. To see the implications of this, we note that inflated statistical tolerancing, (6), with $f = 1$ reduces to traditional statistical tolerancing, (4). In that case, by (11), $C_{pk,a} = 1/f^+$. Thus, if statistical tolerancing were used in situations where (9) gave $f^+ > 1$, it would result in poor process capability, $C_{pk,a} < 1$. While no surveys of tolerancing practice are known to the present author (apart from attendees at a workshop (Greenbaum et al. (1988, p. 117))), there are indications that statistical tolerancing may not be widely used. This apparently limited usage may be due in part to the deficiencies described here.

With data available on process capability of assemblies, $C_{pk,a}$'s, we can solve (11) for the adjusted inflation factor, f^+ , as

$$f^{+(12)} = \frac{f^{(7)}}{C_{pk,a}} = \frac{T_a}{C_{pk,a} \sqrt{\sum b_i^2 T_i^2}} = \frac{T_a}{C_{pk,a} T_a^{(4)}} \quad (12)$$

This expression captures the key idea of this article: $f^{+(12)}$ summarizes the effect of all the deficiencies in statistical tolerancing, and is largely independent of how the assembly was actually toleranced; the effect of the method used to establish the tolerances is captured in the factor $C_{pk,a}$ in (11) and (12). ($f^{+(12)}$ is precisely independent of how the assembly was toleranced only if the assembly characteristic is unbiased, $C_{c,a} = 0$. In that case, doubling T_a would also double $C_{pk,a}$, and the change would cancel in (12).) Regardless of how the tolerances were set, if the assumptions required for statistical tolerancing are off by the same constant percentage in different products, the "estimated adjusted inflation factor", $f^{+(12)}$, will be (approximately) constant across products.

The estimated adjusted inflation factor, $f^{+(12)}$, is the value for f whose use in tolerance analysis, determining T_a from the T_i 's based on (6), would have produced a C_{pk} of 1. Its use in (6) for tolerance allocation (determining the T_i 's to achieve a given T_a) may or may not produce a C_{pk} of 1, depending on what happens to the other terms in (8) as we change some of the T_i 's.

The next section describes what data to collect and how to analyze it using (12) to select a value for the target inflation factor, f , to use in inflated statistical tolerancing, (6), in the future. The analysis also includes two different Pareto analyses that can help organizations get better use of their engineers and their production trouble-shooting efforts.

A POST-INTRODUCTION TOLERANCE ANALYSIS

This analysis takes place in four steps:

- Step 1. Prioritize improvement opportunities based on potential for cost reductions.
- Step 2. Compare current tolerances with statistical tolerances.
- Step 3. Use the results of Step 2 to develop an action plan for the opportunities identified in Step 1.
- Step 4. Convert the overall analysis into a value of f to use in inflated statistical tolerancing, (6), in the future.

The next four subsections discuss an example of the application of this four-step process. While the data here are hypothetical, data of this nature are routinely collected in many manufacturing organizations and could be used as described here to good advantage.

Step 1. Prioritize Improvement Opportunities

The first part of this analysis is illustrated in Table 2. This includes two different Pareto analyses. First, column [6] illustrates the computation of the anticipated scrap / rework for next year by product characteristic: (forecasted scrap & rework, [6]) = (forecasted sales, [4]) \times (current scrap & rework rate, [5]). By this measure, Characteristic 2 of Product B should be the subject of some cost reduction effort; meanwhile, Product C has a higher problem *rate* but such low sales that it may not justify any improvement effort unless a fix for the problem could be developed and implemented at negligible cost.

The second Pareto is an analysis of excess process capability, given in Column [8] of Table 2: This is a rough estimate of the potential or "anticipated" savings from relaxing tolerances on certain components. Some who preach "continuous improvement" may object to the concept of "excess process capability": Any deviation from the nominal carries a cost to society, and organizations should strive to reduce that variability, not increase it! While I agree with the sentiment, there are many situations where an increase in uniformity makes a negligible contribution to customer satisfaction. For example, few people look at the inside of the case of their home television set; money spent on an attractive interior finish will not likely be rewarded by an increase in sales and may therefore reduce the ability of that manufacturer to survive in a market economy. The present analysis attempts to find cases where costs can be reduced with negligible impact on customers.

For the present example, these potential savings were computed as 1% of sales times the excess of C_{pk} over $C_{pk,0} = 2$. This threshold $C_{pk,0}$ might be lowered with good process stability or raised if the need for quality were high. Similarly, the exact percentage of sales to use with this C_{pk} excess could be adjusted. An organization whose sales are based primarily on engineering innovation should use a relatively small percentage here because the cost of components is a relatively small portion of the selling price.

Product			Forecast				Tolerance		Remarks
[1] Product	[2] Product characteristic	[3] ^(a) Why was this product selected for this study?	[4] Sales forecast for next year (K\$)	[5] Scrap and rework rate, last 3 months (%)	[6] Forecasted scrap/rework for the next year (K\$)	[7] C_{pk} (last 3 months)	[8] Potential savings (K\$) from looser tolerances = 1% of sales times $(C_{pk} - C_{pk,0})$; $C_{pk,0} = 2$	[9] Actual Tolerance T_a	
A		2	9,000	0.1	9	1.31		0.002	
B	1	1	230	4	9	0.81		5	
	2	1	230	19	44	0.50		10	<-Substantial problems forecasted for next year. We should try to fix this.
	3	1	230			1.33		0.1	<-Highest problem rate, lowest C_{pk} , but low forecasted sales; we ignore this.
C		2	9	40	4	0.33		100	
D	1	3	1,400	0.2	3	1.31		10	
	2	3	1,400	0.1	1	1.52		0.001	
E		3	290			4.64	8	60	<-Excess C_{pk} , but low sales; not worth trying to save money here.
F		3	10			1.85		5	<-High sales and high C_{pk} ; substantial savings from looser tolerances?
G		2	30,000			4.21	663	8	

(a) 1 = scrap, rework, customer complaints, and warranty problems
 2 = high forecasted sales.
 3 = representative of future products. (The introduction of products currently under development might be accelerated by insights gained from studying similar current products.)

Table 2. Prioritizing Opportunities to Improve Tolerancing

Conversely, manufacturers that pay a larger percent of sales for components might be able to find greater savings in high volume products with large C_{pk} 's. After several projects of this nature, an organization could develop a better formula for estimating anticipated savings.

In the example of Table 2, this evaluation of excess process capability estimated that \$663,000 might be saved next year by relaxing tolerances on Product G. Product E has greater potential per-unit savings because its C_{pk} is higher. However, Product E's lower forecasted sales translated into an estimated savings of only \$8,000.

The decision of whether to review the tolerances on a particular product depend heavily on how much money might be saved from that review. The savings estimated by this procedure are very imprecise. Fortunately, the money involved is often of such magnitude that project assignment decisions would be similar if the numbers changed by a factor of 2 or 3. For example, whether the savings next year from improving product G were \$1,200,000 or \$300,000, it could still justify some effort to try to improve it. Conversely, a focus on Product E may pay for itself only if a change can be developed and tested fairly cheaply or if the actual savings are, say, four times the \$8,000 per year forecasted here.

Another caveat is in order here: We assume that the apparent excess process capability identified in Table 2 for product G is on a critical product characteristic in the sense that if a tolerance were relaxed on a certain component, it would not likely create problems with another product characteristic. Many products carry multiple specifications, only a few of which are difficult to achieve. We need to focus our search for excess process capability on critical product characteristics, and then verify that proposed changes do not jeopardize quality with other product characteristics that currently are easily achieved.

In sum, the analysis of Table 2 suggests (a) we study Product B to reduce scrap and rework in production, and (b) we consider relaxing tolerances on Product G if we can save money doing so. In the next subsection, we discuss how information on tolerances and process capability can assist in analyzing these two cases.

Table 2 includes other information that deserves mention here: the reason each product was selected for this study, column [3]. This would only be needed for initial applications of this methodology, and then only if the organization did not want to wait a year or two before the data would be available for the analysis described here. A more aggressive implementation effort should focus on <1> current problems, <2> high volume products that provide opportunities for substantial cost savings, and <3> products whose study might speed current new product development projects.

Table 2 ends with current tolerances in Column [9]. These numbers are required for the computation of C_{pk} , and should therefore be available whenever C_{pk} is available. The next task in this four-step analysis process is to collect the one other piece of required information: the tolerance that would be required by statistical tolerancing, $T_a^{(4)}$.

Step 2. Compare Current Tolerances with Statistical Tolerances

The computations for both worst case, $T_a^{(2)}$, and statistical tolerancing, $T_a^{(4)}$, are illustrated in Table 3 for the product characteristics discussed in Table 2. ($T_a^{(2)}$ is included only for comparison; it is not used in the analysis.) If either worst case or statistical tolerancing was used, the information needed for this computation could be readily available. If not, some further effort might be required to get it.

The analysis is completed by (a) adding the applicable statistical tolerances, $T_a^{(4)}$, from Table 3 to Table 2 and (b) using the information now available to compute the estimated inflation factor, $f^{(7)}$, and the estimated adjusted inflation factor, $f^{+(12)}$; see Table 4.

Step 3. Action Plans for Products B and G

For Characteristic 2 of Product B, $f^{(7)}$ is 1.4. A value for $f^{(7)}$ this large should produce an acceptable process capability, much better than the C_{pk} of 0.50 observed in this case. There are two possible explanations for this deficiency:

- (a) An important component of Product B has a very poor process capability; it may have an excessive standard deviation or not be adequately centered (e.g., $C_c > 0.3$ with $C_{pk} < 1.2$.)
- (b) There is a substantive error in the variation transmission formula, (1).

Diagnosing this problem may involve a review of statistical control charts and inspection records, tests of components awaiting assembly, an "is / is-not" analysis (Kepner and Trago (1976)), and / or a designed experiment (Bisgaard (1997)). The experiment should include sufficient runs to estimate the standard deviation σ_ε of the unmodeled variability, ε , in (1'), and the analysis should compare predicted variability with data on process capability for components and assembly.

Product G has the opposite "problem": Its C_{pk} of 4.21 suggests that the organization may currently be spending money for unnecessary precision in components. In this hypothetical example, we have supposed that a check of the records revealed that the coefficient b_2 in Table 3 should have been 0.3 rather than 3. The last two rows of Table 3 contain alternative analyses designed to correct this error. The first simply recomputes statistical tolerancing $T_a^{(4)} = 1.76$ using this corrected value for b_2 . The second relaxes the tolerance for component 2 from $T_2 = 2$ to $T_2 = 12$. This change increases the estimated statistical tolerance, $T_a^{(4)}$, to 3.96.

We can predict that after this change, C_{pk} for the assembly should fall approximately in proportion to the increase in σ_a , by the definition of C_{pk} , (5). But $\sigma_a = T_a^{(4)} / (2Z_a)$. Since changing T_2 from 2 to 12 would likely increase $T_a^{(4)}$ from 1.76 to 3.96, it would also increase σ_a from $1.76 / (2Z_a)$ to $3.96 / (2Z_a)$. This in turn implies that

Product characteristic	Predicted Tolerance per "Worst Case," (2)	Stat. Tol., (4)	Component ^(b)											
			1	2	3	4	5	6	7					
	b_1	T_1	b_2	T_2	b_3	T_3	b_4	T_4	b_5	T_5	b_6	T_6	b_7	T_7
A	0.0024	0.00117	1	0.0005	0.2	0.0005	1	0.0007	0.001	0.5	0.001	0.6		
B	6	3.74	1	1	1	3	1	2						
2	13	7.14	1	1	3	1	2	3	2	1	1	1		
3	0.12	0.0636	1	0.05	3	0.005	2	0.01	3	0.01	1	0.005		
C	178	100	1	50	50	1	70	1	0.1	80				
D	11.15	5.92	2	2	1	3	0.5	0.3	1	1	1	3		
2	0.00099	0.000598	1	0.0003	1	0.0001	1	0.0005	0.3	0.0003				
E	58.4	28.7	1	4	1.1	20	0.4	5	0.3	8	5	1	2	4
F	5	2.55	0.5	1	0.5	2	1	1	1	0.5	1	2	6	2.5
G	8.46	6.22	0.4	1.5	3 ^(c)	2	0.6	0.6	0.5	3				
Alternative Analyses for Product G:														
Alternative 1. Correcting the Error Transmission Model														
	1.76	0.4	1.5			2	0.6	0.6	0.5	3				
					0.3 ^(d)									
(d) corrected weight, b_2														
Alternative 2. Loosening a Tolerance after Correcting the Error Transmission Model														
	3.96	0.4	1.5	0.3	12 ^(e)	0.6	0.6	0.6	0.5	3				
(e) loosened tolerance, T_2														

Table 3. Computation of Predicted Assembly Tolerance from Component Tolerances

(b) Different tolerance chains have different lengths.

[1]	[2]	[3] ^(a)	[7]	[9]	[10]	[11]	[12]		
FROM TABLE 1					ANALYSIS				
Product	Product characteristic	Why was this product selected for this study?	C_{pk} (last 3 months)	Actual Tolerance T_a	Tolerance Predicted from Statistical Tolerancing (Table 2) $T_a^{(4)}$	Estimated Tolerance Inflation Factor $f^{(7)} = \text{actual} / \text{stat. tol.}$	Estimated Adjusted Tolerance Inflation Factor, $f^{+(12)}$		
A		2	1.51	0.002	0.00117	1.71	1.14		
B	1	1	0.81	5	3.74	1.34	1.65		
	2	1	0.50	10	7.14	1.40	2.80		
	3	1	1.33	0.1	0.0636	1.57	1.18		
C		2	0.33	100	100	1.00	3.04		
D	1	3	1.31	10	5.92	1.69	1.29		
	2	3	1.52	0.001	0.000598	1.67	1.10		
E		3	4.64	60	28.7	2.09	0.45		
F		3	1.85	5	2.55	1.96	1.06		
G		2	4.21	8	6.22	1.29	0.31		
Alternative Analyses for Product G:									
Alternative 1. After Correcting the Variation Transmission Model					4.21	8	1.76(f)	4.54	1.08
Alternative 2. Loosening a Tolerance after Correcting the Error Transmission Model					1.87(h)	8	3.96(g)	2.02	1.08

<-Might the variation transmission model omit important sources of variability?

<-Might one or more weights in the variation transmission model, (1), be too large?

(f) corrected $T_a^{(4)}$; see Table 2.

(g) revised $T_a^{(4)}$; see Table 2.

(h) predicted C_{pk} based on the increase in $T_a^{(4)}$ discussed in Table 2.

- (a) 1 = problems
- 2 = forecasted sales
- 3 = representative of future products

Table 4. Hypothetical Example of Tolerances and Process Capability

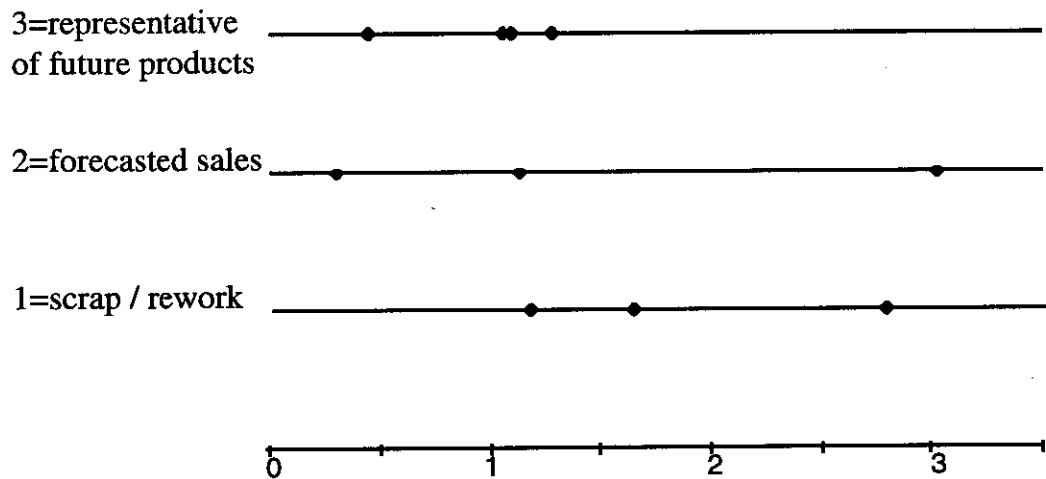
C_{pk} would fall to approximately $4.21(1.76)/3.96 = 1.87$. (See the last two rows of column [10] in Table 4.) Personnel responsible for setting specifications can estimate the cost savings anticipated from loosening T_2 from 2 to 12. If this number is anywhere close to the estimated \$663,000 given in Table 2, the change will most likely be implemented – unless there is some other characteristic not considered in the present analysis that could be jeopardized by this change.

Step 4. Select f to Use in Inflated Statistical Tolerancing, (6), in the Future.

Having reviewed the existing data for improvement opportunities, we now turn to selecting a value for the inflation factor, f , in (6) to use in the future. As noted with (12), if the assumptions for standard statistical tolerancing, (4), are consistently biased, the estimated adjusted inflation factor, $f^{+(12)}$, will be (approximately) a constant. To check this, Figure 1 presents a dot diagram of $f^{+(12)}$ for each of the three reasons for including products in this study. Unfortunately, the values for $f^{+(12)}$ in this hypothetical example show substantial variability. A study of these cases might lead to improved engineering practices that could reduce this variability, thereby making it easier to set tolerances so future new products are more manufacturable.

With or without such a study, we recall that production problems produce a high value of $f^{+(12)}$; this corresponds to the value of the inflation factor f whose use might have produced a C_{pk} of 1, thereby eliminating those problems. Conversely, an excessive

reason for including product in this study



Adjusted Tolerance Inflation Factor, $f^{+(12)}$

Figure 1. Distribution of Adjusted Tolerance Inflation Factor

C_{pk} suggests that the tolerancing model overestimated the variability in the assembly. Similar plots of $f^{+(12)}$ vs. other aspects of the product or characteristic (e.g., the length n of the tolerance chain) might help us better understand the issues involved in tolerancing and might lead to improved tolerancing, possibly using different inflation factors, f , for different types of products and / or quality characteristics.

After a target adjusted inflation factor, f^+ , is selected (or a system for selecting a value is devised), we multiply that number by a target C_{pk} per (11) to get a target inflation factor, f ; this is used with inflated statistical tolerancing, (6), in the future. We generally set targets for f^+ and C_{pk} jointly. For example, if we choose a target $f^+ = 2$ at roughly the 80th percentile of the distribution in Figure 1 (ignoring the facts that (a) this is not a random sample, and (b) the design process is not stable), we may use a target C_{pk} of only 1; this produces a target $f = C_{pk}f^+ = 2$ (per (11)). Since this f^+ was at roughly the 80th percentile, the actual $f^{+(12)}$ computed from subsequent production should be less than this for roughly 80 percent of products; those cases would then have $C_{pk} > 1$ (because f is fixed at 2).

On the other hand, if we choose a target f^+ closer to the median of 1.16, we may want a target C_{pk} of, for example, 1.5. This would make the target $f = 1.16(1.5) = 1.72$. If actual production conditions made $f^{+(12)} = 2(1.16)$, we would still have $C_{pk} = 1.5/2$ (per (11)); this would still allow us to ship products to customers while an effort was mounted to understand the issues and fix the problems for this and similar future products.

Organizations following Motorola's "six sigma" program may want to choose a target C_{pk} of at least 1.5. They would then probably want to use a target f^+ exceeding the median in Figure 1 to ensure that they get $C_{pk} \geq 1.5$ in production for most products.

By applying this kind of logic, exploring alternative scenarios, a user will be able to derive a tolerance inflation factor, f , that should help engineers set tolerances in the future that reduce simultaneously (a) the cost of components and (b) other costs of production and use (scrap, rework, customer complaints, warranty).

CONCLUSIONS

This article has outlined a method for analyzing process capability data to help (a) prioritize cost reduction efforts and (b) improve tolerancing practices in an organization based on the use of a tolerance inflation factor introduced in expression (6). It was assumed that C_{pk} data was available on products but not on components, and process centering information was not readily available. If more process capability information were available, the methodology described above could be revised to use this other information to produce better tolerances.

ACKNOWLEDGMENTS

This research was supported by grant number DMI 9500140 from the National Science Foundation. This article benefited from suggestions by Søren Bisgaard and by the Reports Committee at the Center for Quality and Productivity Improvement at UW-Madison

REFERENCES

- Bender, A. (1962). "Benderizing Tolerances". *Graphic Science*, Dec. 1962, pp. 17-21.
- Bisgaard, S. (1989). "The Quality Detective: A Case Study". *Philosophical Transactions of the Royal Society*, A 327, pp. 499-511.
- Bisgaard, S. (1997). "Designing Experiments for Tolerancing Assembled Products". *Technometrics*, 39, no. 2, pp. 142-152.
- Brandenberger, H. (1946). *Toleranzen, Passung und Konstruktion*. Schweizer Druck- und Verlagshaus, Zürich, Switzerland.
- Bruyevich, N. G. (1946). *Tochnost Mekhanizmov*. (in Russian) Mashgiz, cited from Sutherland and Roth (1975).
- Chase, K. W., and Greenwood, W. H. (1988). "Design Issues in Mechanical Tolerance Analysis". *Manufacturing Review* 1(1), pp. 50-59.
- Deming, W. E. (1986). *Out of the Crisis*. MIT Center for Advanced Engineering Studies, Cambridge, MA.
- Gilson, J. (1951). *A New Approach to Engineering Tolerances*. The Machinery Publishing Co., London, UK.
- Grant, E. L. (1946). *Statistical Quality Control*. McGraw-Hill, New York, NY.
- Graves, S. B. (1994). "Tolerance Analysis Considering Manufacturing Variability and the Cost of Deviating from the Nominal". Report 117, Center for Quality and Productivity, University of Wisconsin-Madison.
- Graves, S. B. and Bisgaard, S. (1997). "Five Ways Statistical Tolerancing Can Fail and What to Do About Them". Report 159, Center for Quality and Productivity, University of Wisconsin-Madison.

- Greenbaum, J. R., Besser, L., Biehl, B., Pollard, B. D., and Osann, R. (1988) *Analysis and Design of Electronic Circuits Using PCs*. Van Nostrand Reinhold, New York, NY.
- Harry, M. J. (1992). "The Vision of Six Sigma Quality: Producibility Measurement and Analysis". in Tipnis (1992b). pp. 41-64.
- Kepner, C. H., and Tregoe, B. B. (1976). *The Rational Manager*, 2nd ed. Kepner-Tregoe, Princeton, NJ.
- Kotz, S., and Johnson, N. L. (1993). *Process Capability Indices*. Chapman and Hall, New York, NY.
- Liggett, J. V. (1993). *Dimensional Variation Management Handbook*. Prentice Hall, Englewood Cliffs, NJ.
- Rüdenberg, R. (1929). "Die Beurteilung elektrischer Maschinen und Apparate durch Toleranzen auf Grund statistischer Methoden". *Zeitschrift für angewandte Mathematik und Mechanik* 9 (August), pp. 318-334; reprinted in Plaut, H. C. (1930), *Fabrikationskontrolle auf Grund statistischer Methoden*. VDI-Verlag, Berlin, Germany, pp. 40-56.
- Shewhart, W. A. (1931). *Economic Control of Quality of Manufactured Product*. Van Nostrand, New York, NY. (reprinted 1980, American Society for Quality Control, Milwaukee, WI).
- Sutherland, G. H., and Roth, R. (1975). "Mechanism Design: Accounting for Manufacturing Tolerances and Costs in Function Generation Problems". *Transactions of the ASME: Journal of Engineering for Industry*, February, pp. 283-286.