University of Wisconsin-Synchrotron Radiation Center Technical Note	File Number: SRC-191	Page: 1 of 13
Subject: Dynamic aperture of Aladdin.	**Name:** R. A. Bosch	
	Date: revised January 6, 2000	

ABSTRACT

The 1000-turn dynamic aperture of Aladdin has been computed by tracking electron trajectories from the middle of Long Straight Section 1 (LSS1) using the Methodical Accelerator Design (MAD) code. The ideal Aladdin lattice was studied, as well as two imperfect lattices with quadrupole excitation errors. One imperfect lattice is the "Mar. 28, 1997 fit" in which quadrupole errors were determined by a fit of the measured steering response matrix. The other imperfect lattice incorporates quadrupole errors determined by magnetic field measurements before assembly of the storage ring. For these imperfect lattices, the horizontal dynamic aperture is ~20% smaller than that of the ideal lattice while the vertical dynamic aperture is ~35% smaller.

The physical aperture was computed by tracking with the sextupoles off, while including apertures at the quadrupole locations, septum location, and in the long straight sections. The imperfect lattices have a horizontal physical aperture equal to that of the ideal lattice, while their vertical physical aperture is ~20% smaller.

For all three lattices, the dynamic aperture exceeds the physical aperture for momentum errors $\Delta p/p$ obeying $-0.03 \leq \Delta p/p \leq 0.05$. With 1% emittance coupling or full coupling, the horizontal physical aperture exceeds $15\sigma_x$ while the vertical physical aperture exceeds $15\sigma_y$, where (σ_x, σ_y) is the rms beam size in the middle of LSS1.

The dynamic aperture consists of those transverse positions where an electron which initially travels parallel to the design orbit is on a stable trajectory. Here, we compute the dynamic aperture of Aladdin at the center of Long Straight Section 1 (LSS1), tracking for 1000 turns with three tracking methods provided by the Methodical Accelerator Design (MAD) code [1]. The RF-cavity voltage is zero, so that synchrotron oscillations are not included in the tracking. For Aladdin, 1000 turns equals ~2.5 synchrotron oscillation periods and ~1/50th of the radiation damping time.

Because the tracking is performed with sextupoles set for zero chromaticity, all three methods give similar results; this is not the case when the sextupoles are off. The dynamic aperture is much smaller with sextupoles on, indicating that the dynamic aperture with sextupoles on is limited by the nonlinear sextupole fields.

We first studied the ideal Aladdin lattice with tunes of $(\nu_x,\nu_y) = (7.139, 7.229)$; this lattice does not include dipole magnet fringe fields. The tracking methods provided by MAD (TRANSPORT, LIE3, and LIE4) give similar results for 1000-turn tracking starting at the center of LSS1. Results are shown in Fig. 1. Disregarding the abnormally-large horizontal aperture obtained when the vertical position is identically zero, the dynamic aperture for zero energy error obeys: max(x) = 0.033 m, min(x) = -0.025 m, max(y) = -min(y) = 0.03 m. These values are shown in Table I. For an energy error $\Delta p/p$ with $\Delta p/p \geq 10\%$ or $\Delta p/p \leq -5\%$, the dynamic aperture is shrunk by more than a factor of 2 in both directions. For $\Delta p/p = \pm 25\%$, electrons may still be tracked for 1000 turns, but such results are not trustworthy since MAD tracking is not accurate unless $|\Delta p/p| \ll 1$ [2].

In comparison, previous 10,000-turn tracking with the MARYLIE [3] code yielded a dynamic aperture for zero energy error with max(x) = 0.024m, min(x) = -0.020 m, max(y) = -min(y) = 0.018 m in the center of LSS1 [4]. These MARYLIE results neglect the abnormally-large horizontal aperture obtained when the vertical position is identically zero. The tunes used for MARYLIE tracking, $(\nu_x,\nu_y) = (7.138, 7.227)$, were nearly the same as those used with MAD. When using the TRANSPORT method of the MAD code, the 10,000-turn dynamic aperture for zero energy error obeys max(x) = 0.027m, min(x) = -0.022 m, max(y) = -min(y) = 0.017 m in the center of LSS1. Thus, the MAD tracking results agree with those from MARYLIE within ~10%.

The horizontal dynamic aperture computed for 10,000-turn tracking is ~15% smaller than that computed for 1000-turn tracking while the vertical dynamic aperture is ~45% smaller. It is expected that the dynamic aperture decreases with the number of turns tracked, asymptotically approaching an ∞–turn value. A study of the LHC proton accelerator suggests that the 1000-turn and 10,000 turn dynamic apertures exceed the ∞–turn dynamic aperture by ~25–33% in that case [5]. For electrons in Aladdin, transverse oscillations are damped by radiation damping over ~50,000 turns, so that survival for 1000–10,000 turns may ensure indefinite survival. This suggests that tracking for 1000 or 10,000 turns provides a reasonable estimate for the long-term dynamic aperture of Aladdin.

The physical aperture (also shown in Fig. 1) was determined by tracking electrons with the sextupoles turned off, while including a circular physical aperture at the quadrupole locations (radius = 0.032 m), a rectangular aperture in the long straight sections (0.06 m wide x 0.019 m high) and a horizontal aperture at the septum (0.0414 m wide). A rectangular physical aperture is found, indicating that the vertical physical aperture results from the vacuum chamber height in the long straight sections, while the horizontal physical aperture is determined by the vacuum chamber width in the long straight sections and/or the septum. The physical aperture in the center of LSS1 has max(x) = -min(x) = 0.015 m, max(y) = -min(y) = 0.0085 m.

For zero energy error, the physical aperture is smaller than the 1000-turn and 10,000-turn dynamic apertures. In Table I, the limiting aperture for zero energy error (the physical aperture) is compared with the computed beam dimensions in the middle of LSS1.

Next, we studied imperfect lattices with quadrupole excitation errors. One of these lattices, the "Mar. 28, 1997 fit", has quadrupole errors that were computed to reproduce the measured steering response matrix by the LOCO code [6,7]. The other imperfect lattice incorporates quadrupole errors determined by magnetic field measurements before assembly of the storage ring. Both imperfect lattices model dipole magnet fringe fields with a "field integral" of 0.45 [1] and have tunes of (ν_x,ν_y) = (7.139, 7.229). Results are shown in Table I and Figs. 2 and 3. The horizontal dynamic apertures shown in Table I are estimated while disregarding the abnormally-large horizontal aperture computed when the vertical position is identically zero.

For these imperfect lattices, the horizontal dynamic aperture is ~20% smaller than that of the ideal lattice while the vertical dynamic aperture is ~35% smaller. The horizontal physical aperture approximately equals that of the ideal lattice while the vertical physical aperture is ~20% smaller.

For all three lattices, the dynamic aperture exceeds the physical aperture for momentum errors $\Delta p/p$ in the range $-0.03 \le \Delta p/p \le 0.05$. With 1% emittance coupling or full coupling, the horizontal physical aperture exceeds $15\sigma_x$ while the vertical physical aperture exceeds $15\sigma_y$, where (σ_x, σ_y) is the rms beam size in the middle of LSS1.

---

[1] H. Grote and F. C. Iselin, "The MAD Program (Methodical Accelerator Design) Version 8.1 User's Reference Manual", CERN/SL/90-13 (1990), CERN, CH-1211 Geneva 23, Switzerland.

[2] F. C. Iselin, private communication (1998).

[3] A. J. Dragt, R. D. Ryne, L. M. Healy, F. Neri, D. R. Douglas and E. Forest, "MARYLIE 3.0, A Program for Charged Particle Beam Transport Based on Lie Algebraic Methods", University of Maryland, 1985.

[4] W. S. Trzeciak and D. C. Morin, "Aladdin II", Proc. 1991 IEEE PAC, 2643 (1992); D. C. Morin, Jr. "Dynamic aperture calculations", SRC Technical Note SRC-100, Synchrotron Radiation Center (1991).

[5] M. Böge and F. Schmidt, "Estimates for long-term stability for the LHC", Proc. 1997 PAC, 1439 (1997).

[6] J. Safranek, "Experimental determination of linear optics including quadrupole rotations", Proc. 1995 IEEE PAC, 2817 (1996); "Experimental determination of storage ring optics using orbit response measurements", BNL Note 63382, Brookhaven National Laboratory.

[7] R. A. Bosch, "Calibration of the SRC quadrupoles, BPM's, and steering magnets", SRC Technical Note SRC-183, Synchrotron Radiation Center (1997).

		Ideal Aladdin lattice	Mar. 28, 1997 fit lattice	Lattice with measured quad errors
Dynamic	x_{max} (mm)	33	28	25
aperture	x_{min} (mm)	-25	-22	-20
(for zero energy error)	y_{max} (mm)	30	20	18
Physical	x_{max} (mm)	15	15	14.5
aperture	x_{min} (mm)	-15	-15	-14.5
	y_{max} (mm)	8.5	6.8	7.3
Horizontal emittance	(πnm-rad)	126.5	126.6	121.4
Touschek lifetime	(minutes)	338	366	351
Beam size	σ_x (mm)	0.958	1.027	0.960
(1% coupling)	σ_y (mm)	0.064	0.058	0.067
(Physical aperture)/ (beam size)	x_{max}/σ_x	16	15	15
@ 1% coupling	y_{max}/σ_y	133	117	109
@ full coupling	y_{max}/σ_y	19	17	15

Table I: Dynamic aperture, physical aperture, and beam size in the center of LSS1 for 800 MeV ring energy. Also shown are the natural emittance, Touschek lifetime (for scattering out of an 80-kV RF bucket with a 200-mA current and 1% emittance coupling), and the ratio of physical aperture to beam size.

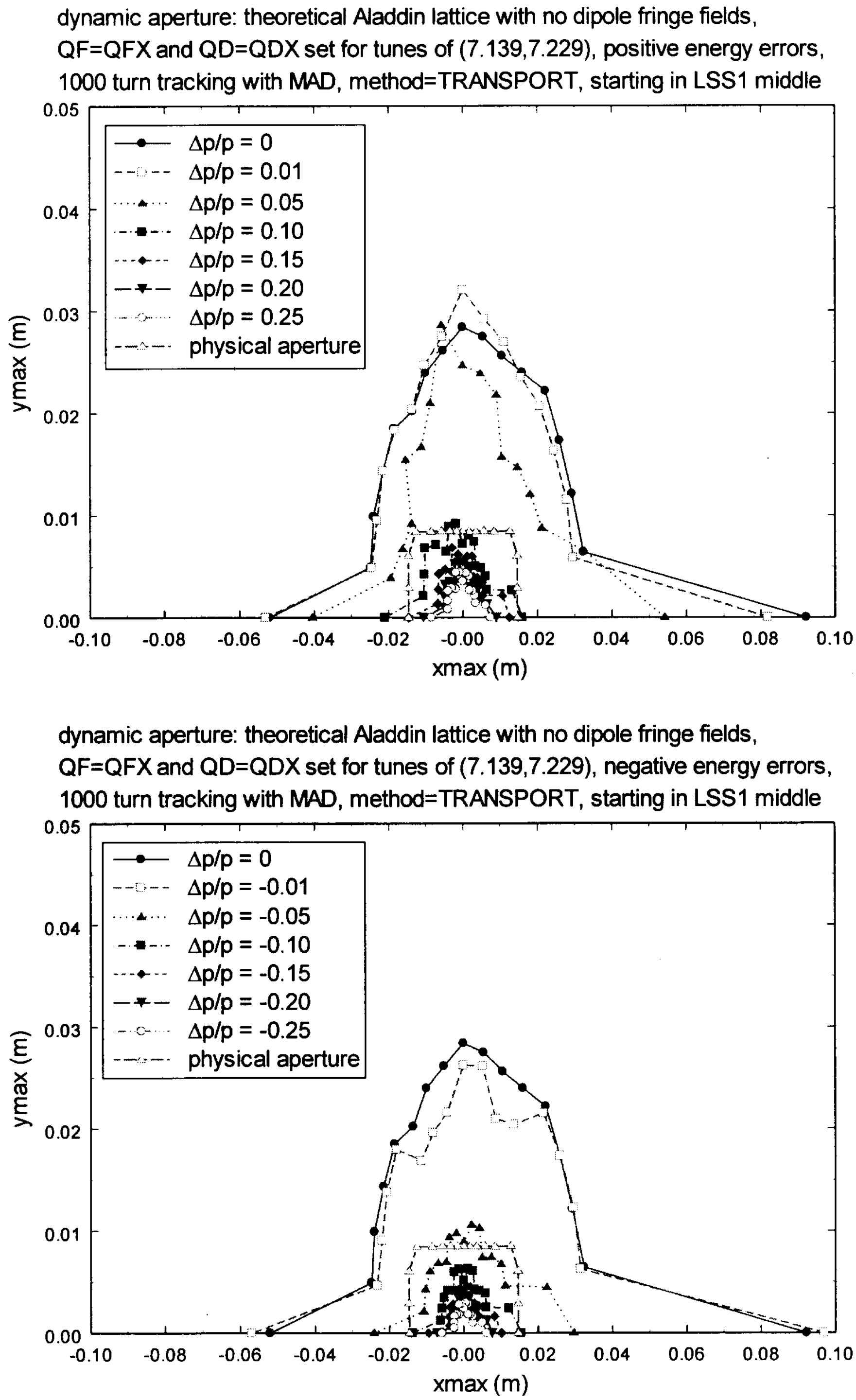
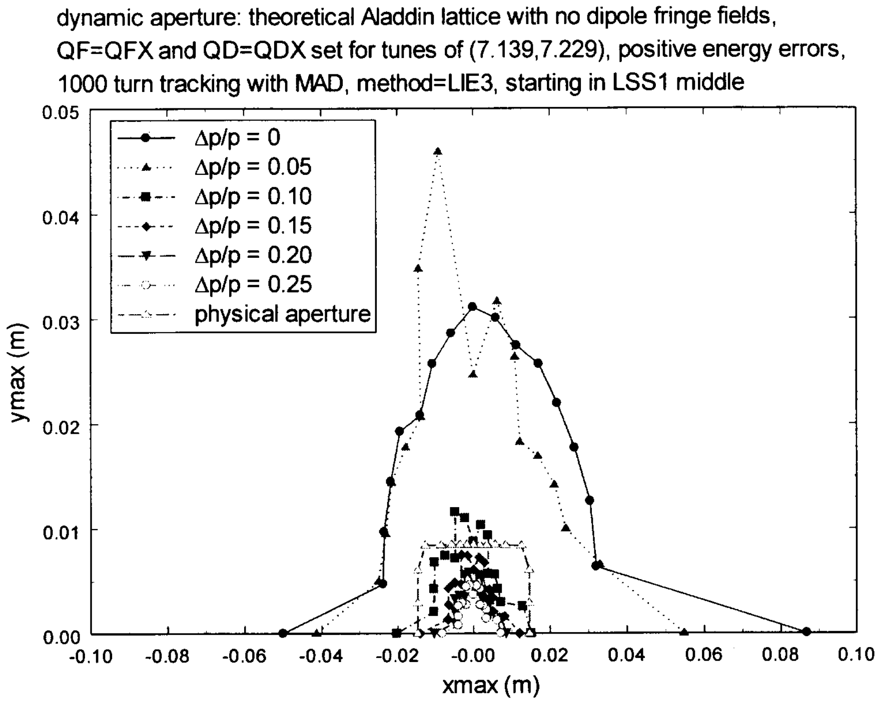


Figure 1(a). 1000-turn dynamic aperture computed by the TRANSPORT method for the ideal Aladdin lattice.



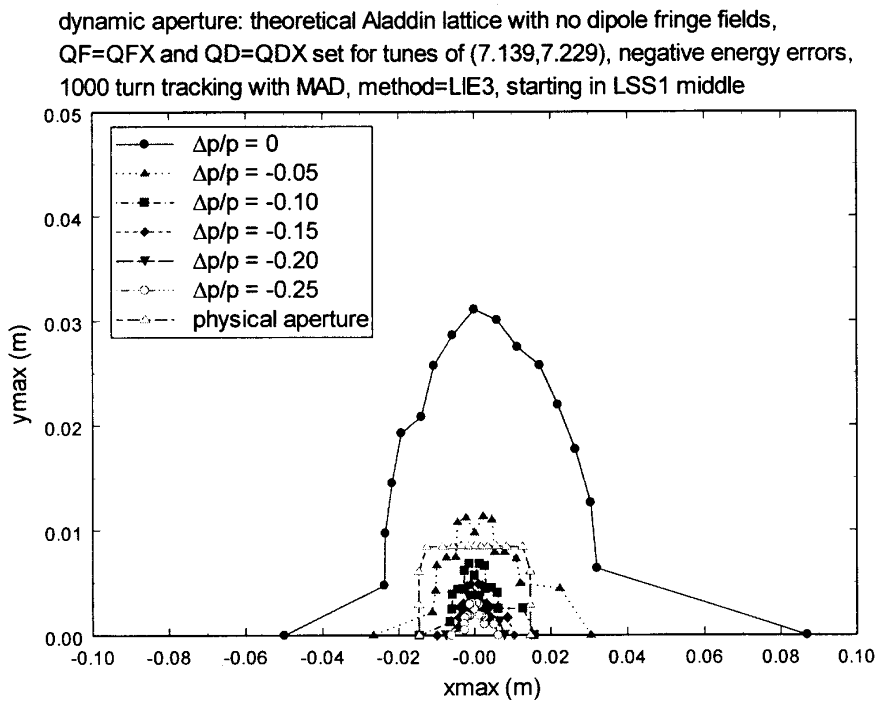
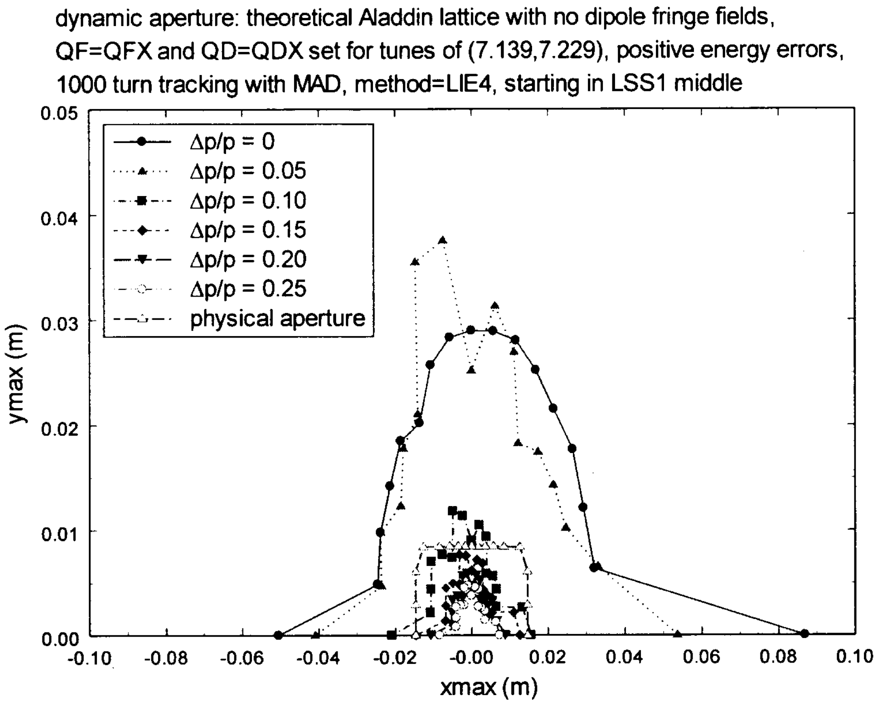


Figure 1(b). 1000-turn dynamic aperture computed by the LIE3 method for the ideal Aladdin lattice.



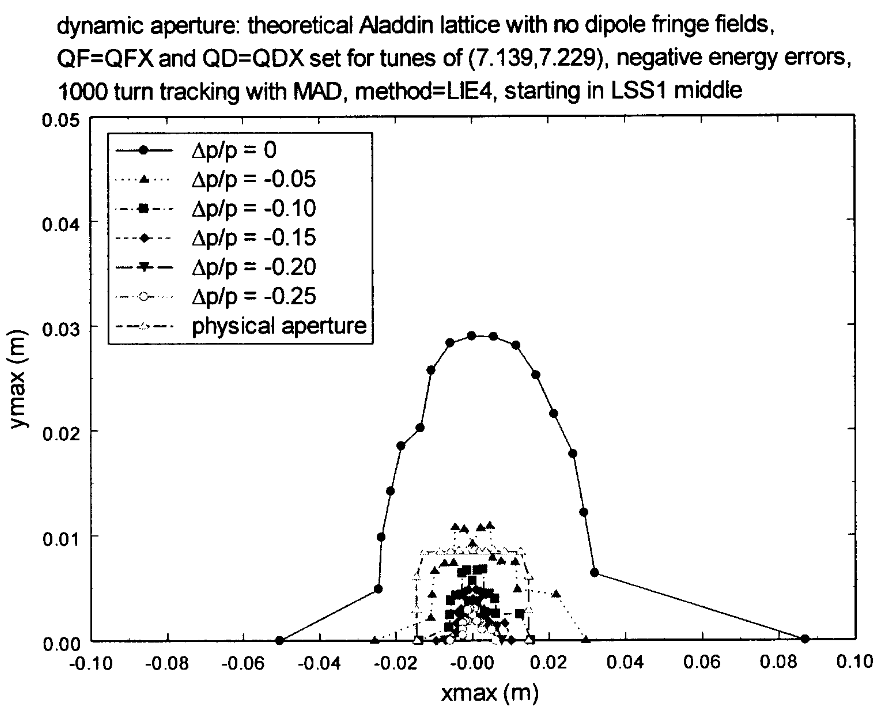


Figure 1(c). 1000-turn dynamic aperture computed by the LIE4 method for the ideal Aladdin lattice.

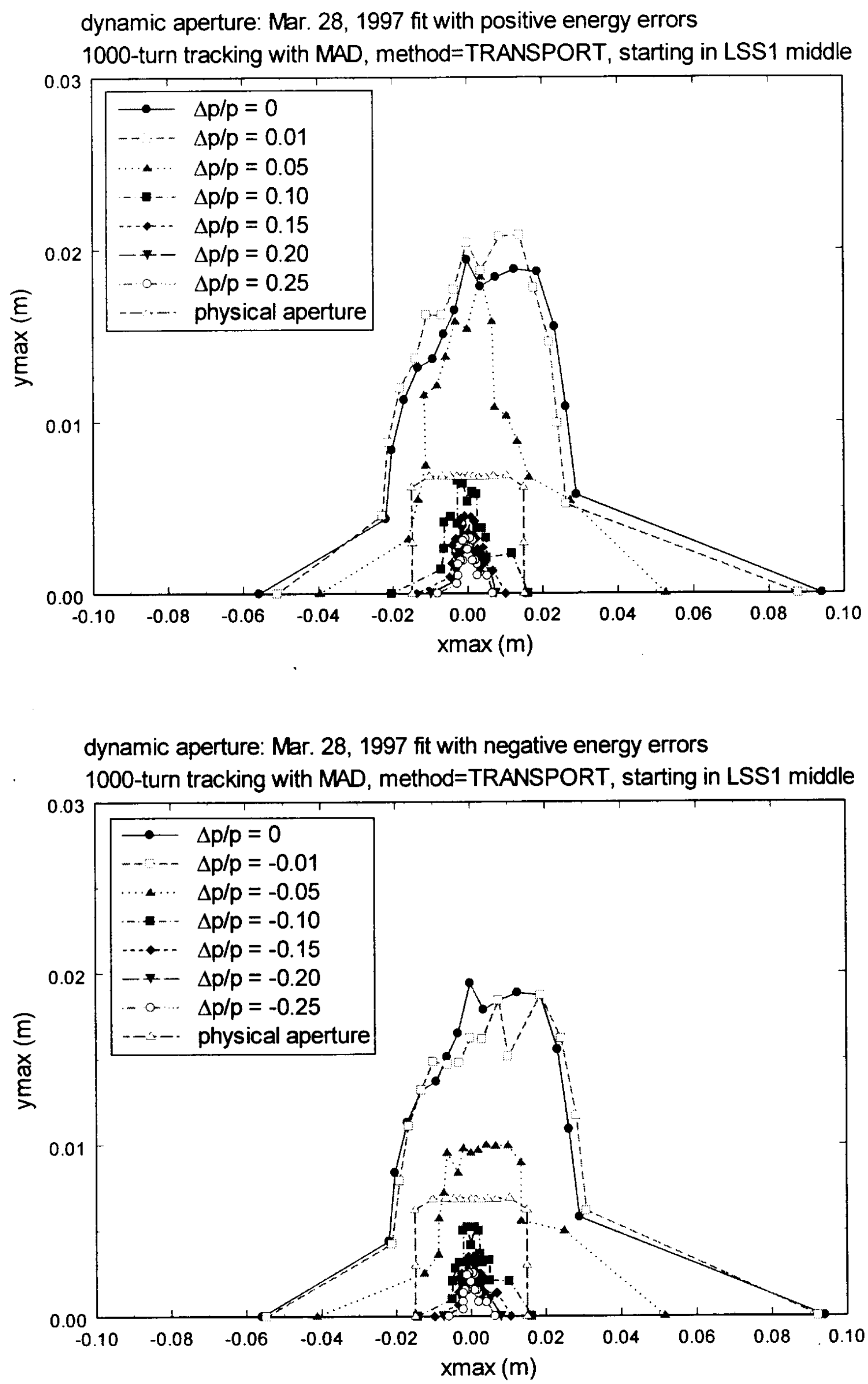
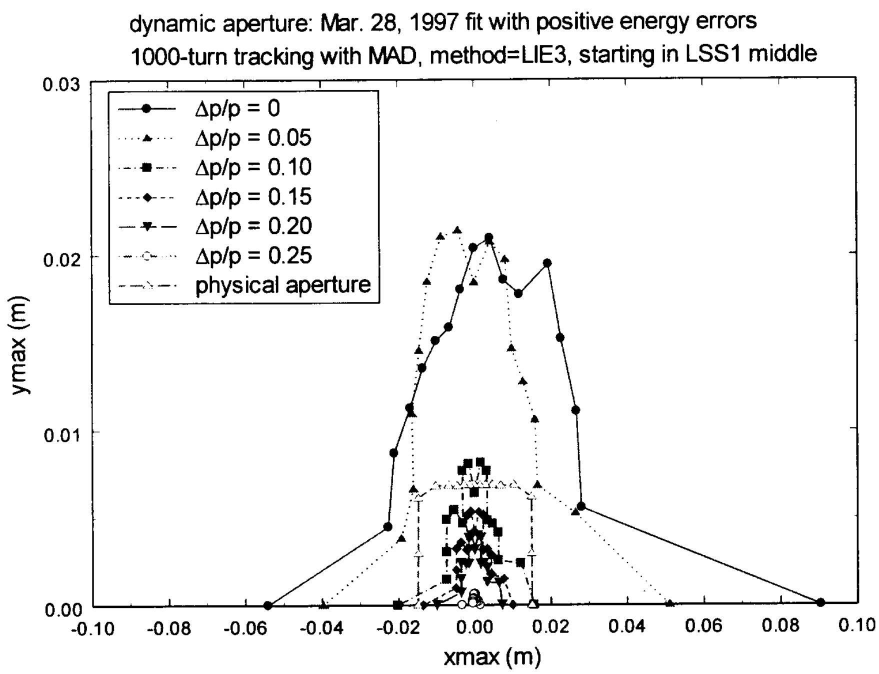


Figure 2(a). 1000-turn dynamic aperture computed by the TRANSPORT method for the Mar. 28, 1997 fit lattice.



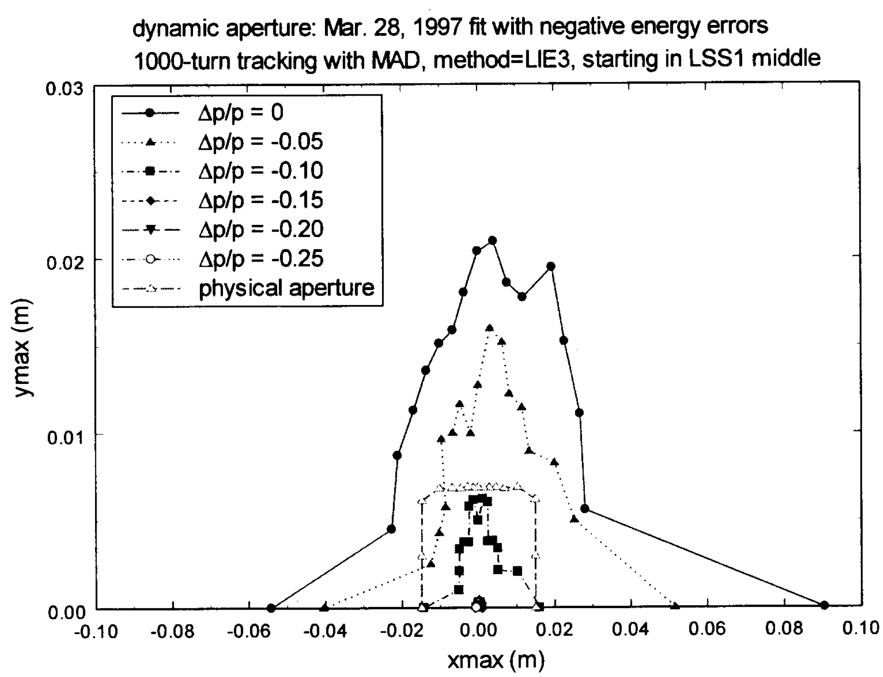
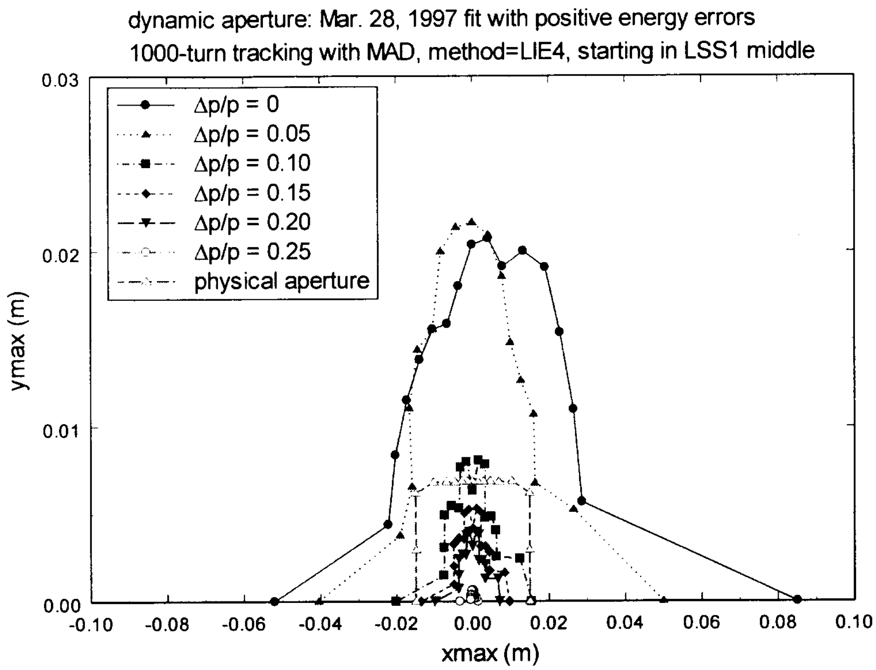


Figure 2(b). 1000-turn dynamic aperture computed by the LIE3 method for the Mar. 28, 1997 fit lattice.



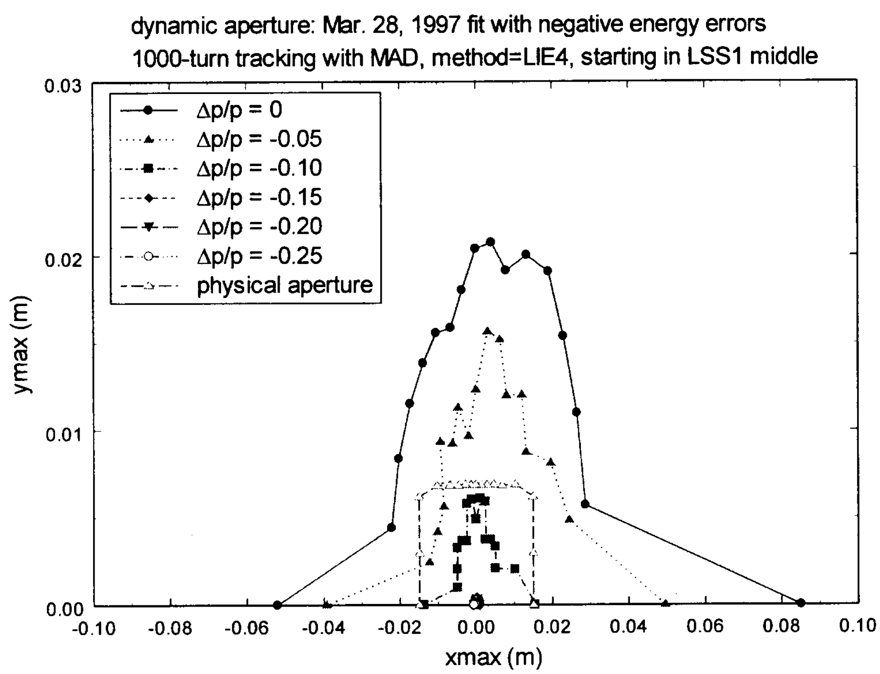
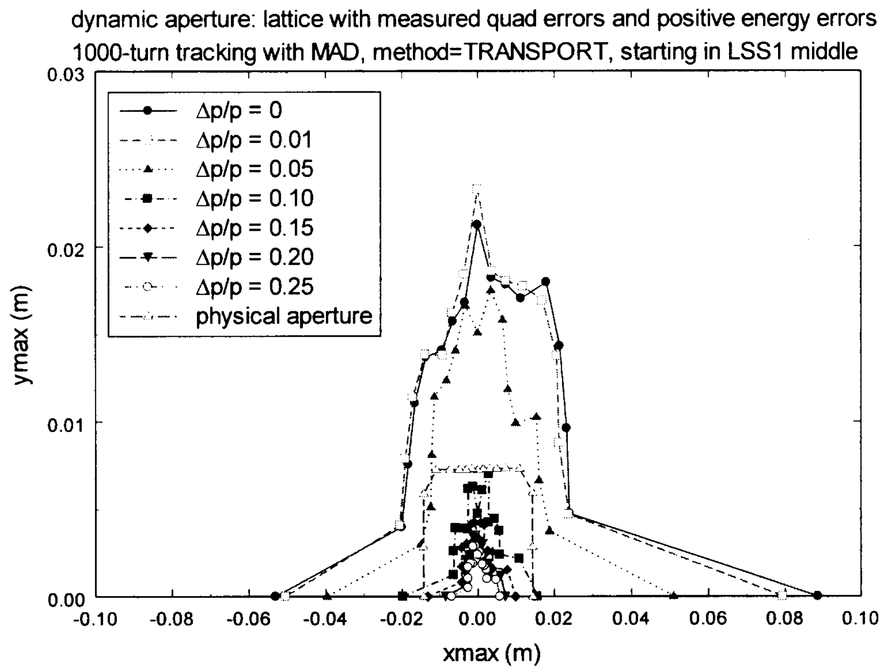


Figure 2(c). 1000-turn dynamic aperture computed by the LIE4 method for the Mar. 28, 1997 fit lattice.



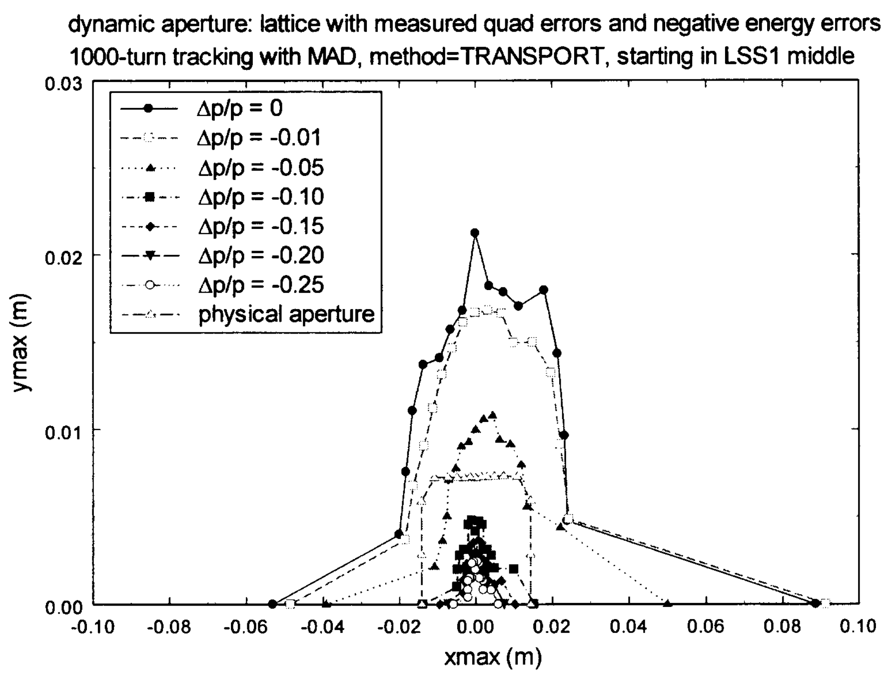


Figure 3(a). 1000-turn dynamic aperture computed by the TRANSPORT method for a lattice with quadrupole errors determined from magnetic field measurements.

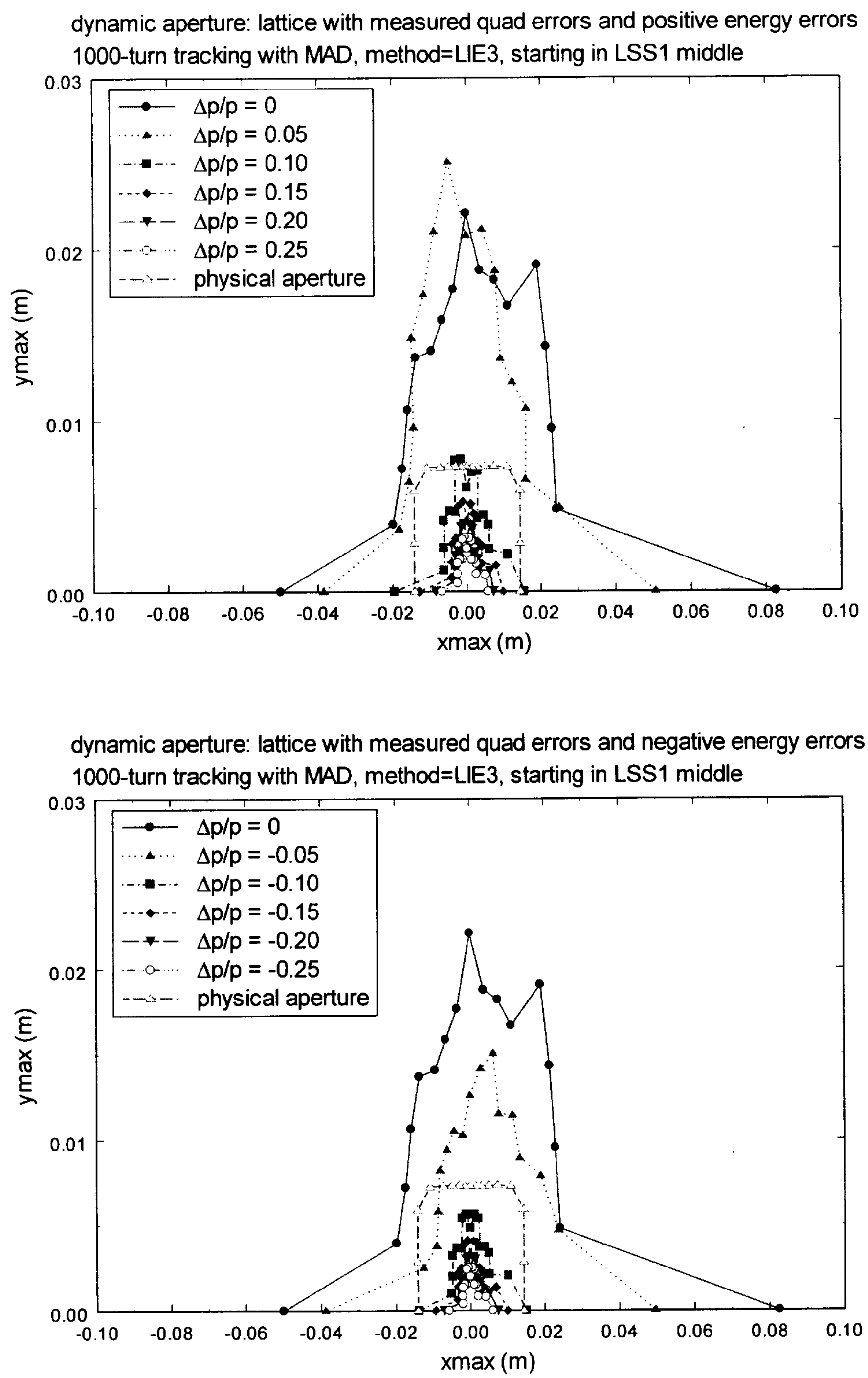
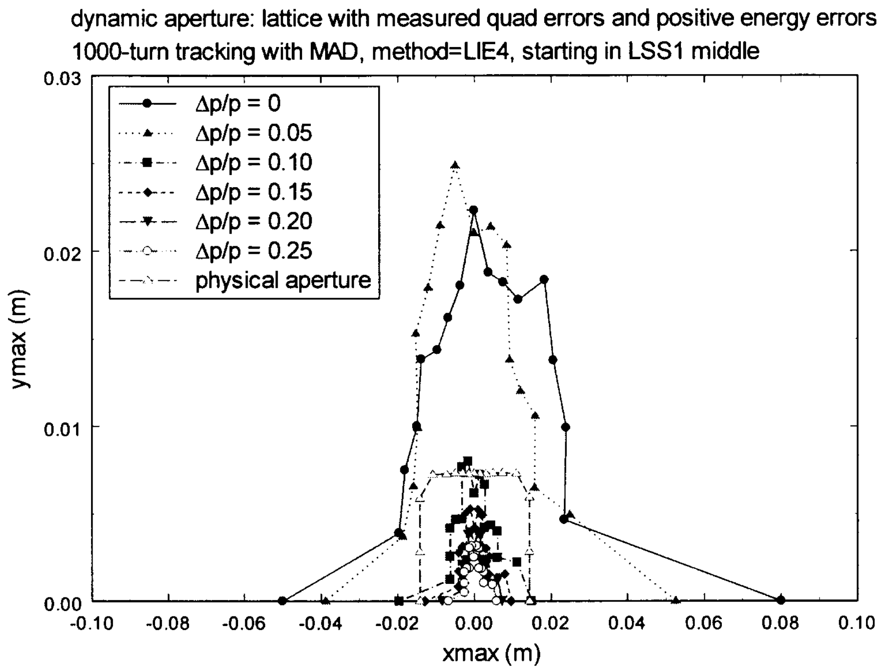


Figure 3(b). 1000-turn dynamic aperture computed by the LIE3 method for a lattice with quadrupole errors determined from magnetic field measurements.



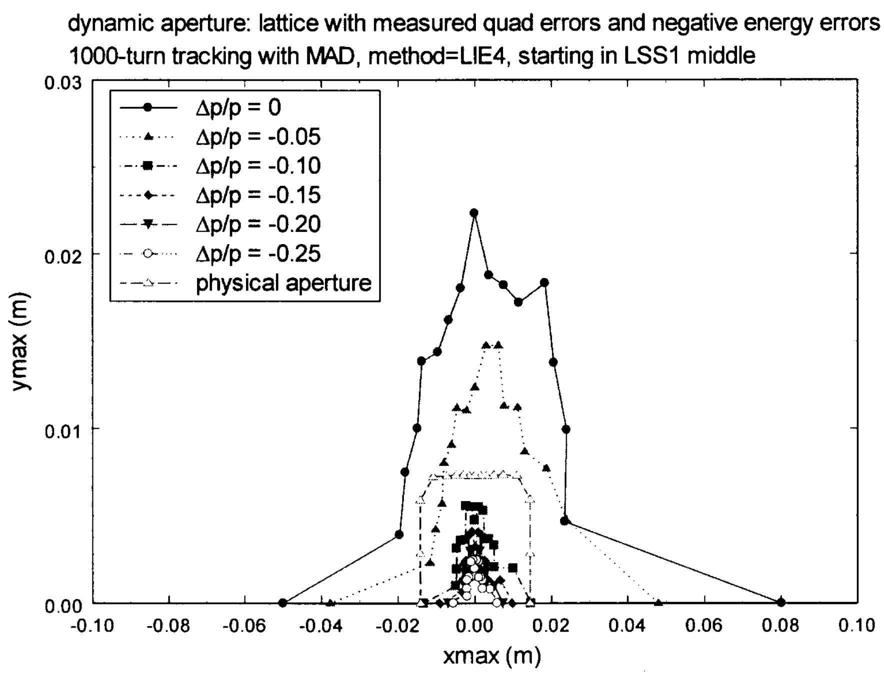


Figure 3(c). 1000-turn dynamic aperture computed by the LIE4 method for a lattice with quadrupole errors determined from magnetic field measurements.