Experimental Analysis of Vibrating Guitar Strings

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Introduction

Vibrating strings and their standing waves have been a common place of study in the physics world for many years. High speed pictures have been taken of sections of them to analyze but never the entire length of the string. For our project, we decided to take on the challenge of filming the entire length of a guitar string. Then we used a theoretical model of the string to see how well the standing waves produced on the real string compared to what was predicted by the theoretical model.

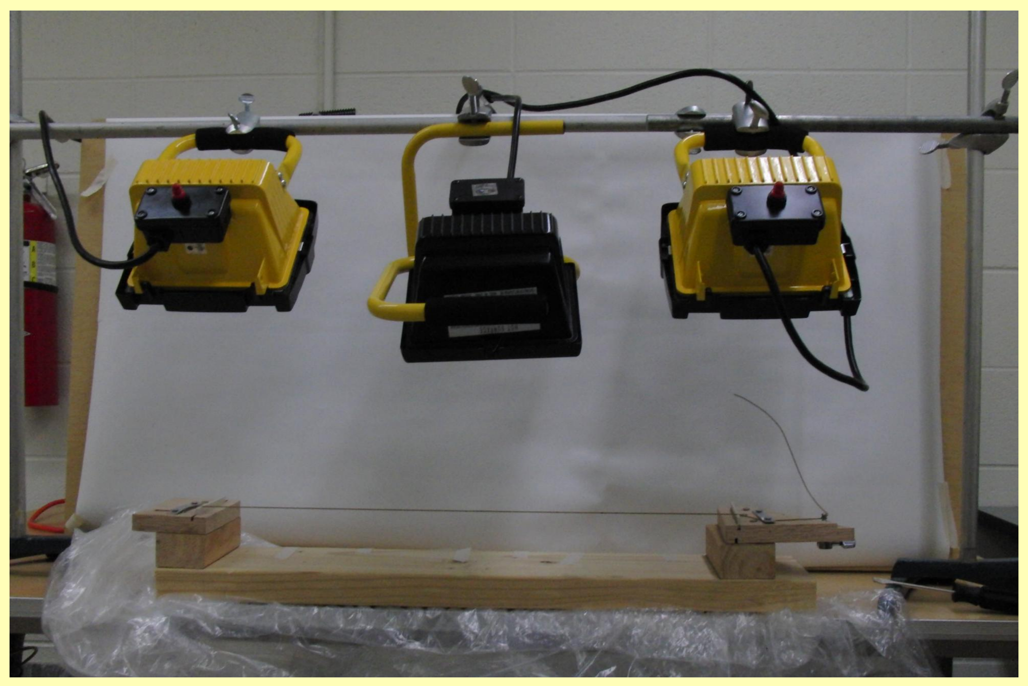
Methods

Experimental

In our experiments, we used a nickel wound guitar string tuned to E2. We mounted the string on a homemade "guitar" to enable us to pluck the string at varying amplitudes (Fig. 1). The pictures were taken with a Casio EX-F1 operating at 1200 frames per second.

Fig. 1

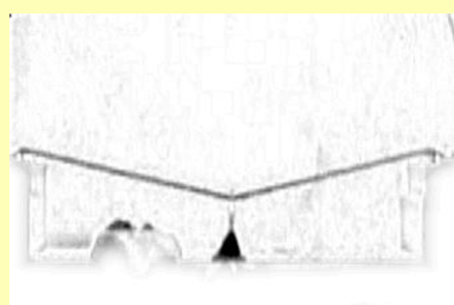
Fig. 2



We set up a stage for our pictures (Fig. 2) with our "guitar" in front of a white background to produce as much contrast as possible. We mounted three halogen construction lamps above it to provide enough light to see a good image with the extremely high shutter speed.

Fig. 3

Fig. 4



We loaded the image onto a computer program called ImageJ (Fig. 3) and used it to subtract the background and increase the contrast of the image (Fig. 4). Then we used ImageJ to select the string and convert the selection to xy pairs in a text file, which were then adjusted in Excel in order to be compared to the theoretical model.

Theoretical

The theoretical model we used was a Fourier series describing the standing waves on the string. The following equation was used for our wave function to describe both the forward and backward propagating wave forms, where *n* is the mode, *t* is the time, and *freq* is the frequency of the string:

$$y_n(x,t) = \sin(n\cdot\pi(x-2\cdot freq\cdot t)) + \sin(n\cdot\pi(x+2\cdot freq\cdot t))$$

The amplitude contribution of each mode was described by the following equation, where *n* is the mode and *F* is the fraction of the string at which it was plucked:

$$A_n = \frac{1}{n^2}\frac{\sin(2\pi\cdot F)}{\sin(2n\pi\cdot F)}$$

We used Maple to calculate the standing wave pattern for a given *t* to give us the *y* values for a string at points ranging from *x*=0 to *x*=1:

$$y(x,t) = \left[\sum_{n=1}(A_n\cdot y_n(x,t))\right]\cdot e^{C\cdot freq\cdot t}$$

The exponential component at the end was to model the damping factor the string had where *freq* and *t* were frequency and time and *C* was a constant chosen to match the data.

Results

In our analysis, we found that strings plucked with small amplitudes fit the model better than those with large amplitudes. Figures 5 and 6 both display the model (blue) and the data (red) at approximately the same time for two different cases when the string was plucked in the center, but Figure 5 was plucked with a much greater amplitude than 6.

Fig. 5

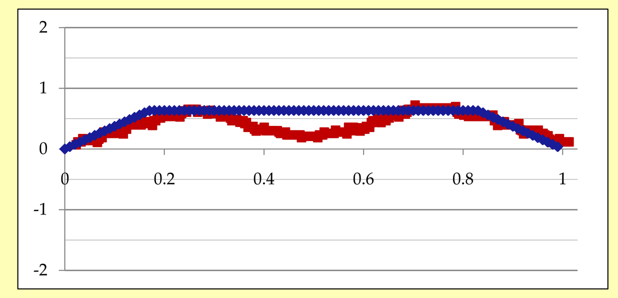
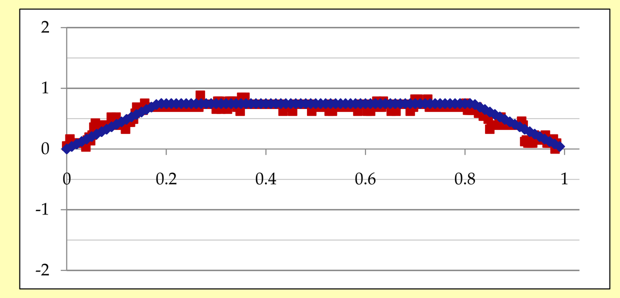


Fig. 6



Our analysis also showed that the string was unable to have the sharp kinks that the theoretical model predicted. Figure 7 displays a case when the string was plucked off-center and Figure 8 shows one that was plucked at the center. The black circles highlight spots where the string had extreme deviation from the sharp angle, which then caused the string to bow out in other spots.

Fig. 7

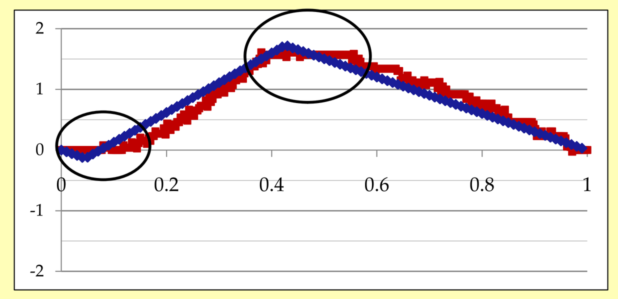
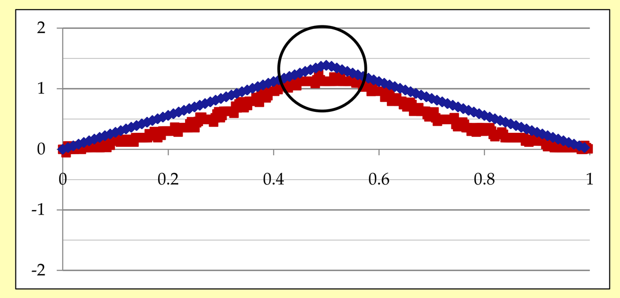
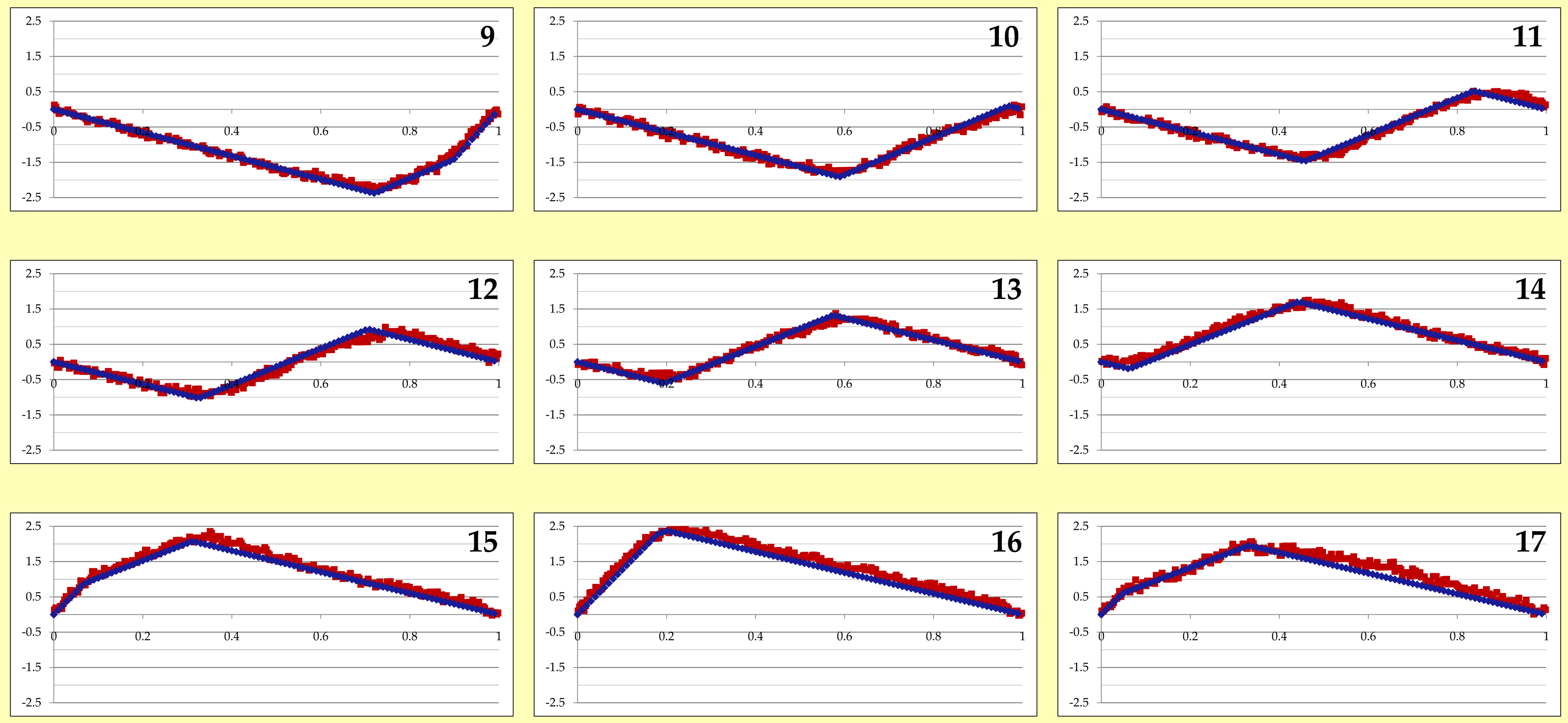


Fig. 8



Figures 9-17



Figures 9 through 17 depict the propagation of standing waves when the string is plucked at 1/5 of the length of the string away from the right side with low amplitude. Note the shape of the wave and how well the string matches what the theoretical model predicts.

Conclusions

Through our study, we found that, in general, a plucked string follows the shape defined by a simple Fourier series. Additional terms in the equation would need to be added to the simple model we used to get more agreement with the data. For example, a term describing the rigidity of the string could be used to predict what would happen at the sharp angles in our model. When plucked with a large amplitude, we believe that the string's fundamental frequency actually changed with time, decreasing as the amplitude decayed, so there would need to be a part of the equation that described a time varying frequency.

Future work will look at how different string compositions affect the vibrating pattern and how a struck string, like in a piano, vibrates compared to a plucked string.

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