Measuring Δ' from electron temperature fluctuations in the Tokamak Fusion Test Reactor

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(Received 29 July 1997; accepted 24 October 1997)

A method is developed for determining directly from experimental data the classical tearing mode stability parameter Δ'. Specifically, an analytical fit function is derived for the electron temperature fluctuations ($\tilde{T}_e$) in the vicinity of a magnetic island. Values of Δ' determined from the fit function parameters for $m/n=2/1$, 3/2 and 4/3 modes (m and n are poloidal and toroidal mode numbers) are obtained using the high resolution $\tilde{T}_e$ profile data from major radius shift (''jog'') experiments on the Tokamak Fusion Test Reactor (TFTR) [D. Meade *et al.*, *Proceedings of the International Conference on Plasma Physics and Controlled Nuclear Fusion.* Washington, District of Columbia, 1990 (International Atomic Energy Agency, Vienna, 1991), Vol. I, pp. 9–24]. It is found that the $n\geqslant 2$ modes have $\Delta'<0$. © *1998 American Institute of Physics.* [S1070-664X(98)00602-8]

I. INTRODUCTION

In order to find a successful magnetic confinement system for fusion, it is very important to know if an equilibrium configuration is stable. There are two kinds of magnetohydrodynamic (MHD) instabilities: Ideal modes and resistive modes. The tearing mode[1] is the fundamental paradigm resistive instability driven by the current gradient that causes resistive magnetic reconnection. It can change the topology of the magnetic surfaces by forming a magnetic island about the rational surface, and thereby degrade plasma confinement.[2] In the classic paper by Furth, Killeen and Rosenbluth,[1] a parameter Δ', which can be determined from the equilibrium current profile, was shown to represent the stability criterion. The parameter Δ' represents the free energy available to the mode from the current gradient; a positive Δ' implies tearing instability. To date no satisfactory method exists for determining this fundamental tearing mode stability parameter directly from experimental data.

Recently, it has been estimated[3] that for the $m/n=3/2$ and 4/3 tearing modes on the Tokamak Fusion Test Reactor (TFTR),[4] Δ' is negative and the mode is instead driven by the pressure gradient according to the neoclassical tearing mode theory.[5] In these calculations of Δ', the radial derivative of the current profile, dJ/dr, is required. Usually, the current profile is obtained from the derivative of the experimental q-profile, either from a direct measurement via a motional Stark effect (MSE) diagnostic or from an interpretive code (like TRANSP or SNAP).[6] Such methods have practical difficulties since eventually the second derivative of the experimentally measured q profile is required. Therefore, a more direct, reliable and independent method for measuring Δ' is needed.

In this paper, a method is proposed for measuring Δ' using the electron temperature fluctuation profile in the vicinity of a magnetic island obtained from an electron cyclotron emission (ECE) diagnostic. By definition,[1] Δ' is the difference between the slope of the perturbed radial magnetic field on the two sides of the resonant rational surface and is related to the shape of the magnetic surfaces on the two sides of the magnetic island. Since in a high temperature plasma the heat conductivity along a magnetic field line is extremely high, the electron temperature T_e is well approximated by a (helical) magnetic flux surface function. Therefore, a detailed T_e profile in the vicinity of a magnetic island should provide information about the shape of magnetic surfaces. In the experiment, a stationary ECE diagnostic samples a fluctuating local T_e from different magnetic flux surfaces as a magnetic island that is frozen into the plasma rotates past it. The $\tilde{T}_e(r)$ profile can then be used to calculate Δ'.

In order to obtain an accurate Δ', the ECE measurements must have a high spatial resolution. However, the typical distance between two ECE channels in TFTR is about 5 to 6 cm, which is often the same order as the island width. Thus, it is difficult to use ordinary ECE data to find Δ'. On TFTR, a special experiment called a jog experiment[7] is conducted to achieve the necessary high resolution for the ECE measurement. In the jog experiment the plasma is radially compressed or expanded rapidly ($\Delta x/\Delta t\geqslant 30$ cm/60 ms) during an otherwise steady-state operation phase. Since the jog is nearly adiabatic, the $\tilde{T}_e(t)$ data from each channel obtained during the jog can be mapped into $\tilde{T}_e(r)$ before the jog. With the jog and a high data collection rate (500 kHz), the obtained $\tilde{T}_e$ profile can have a much higher spatial resolution. We assume that the plasma properties remain unchanged during the jog since the plasma as a whole is moved rapidly.

In the rest of the paper, an analytical fit function is derived which relates this $\widetilde{T_e}$ profile to Δ' in Sec. II. In Sec. III

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this formula will then be applied to the ECE data from the jog experiment on TFTR to determine Δ' for the tearing modes observed there. A short discussion is given in Sec. IV.

II. FIT FUNCTION

We use cylindrical coordinates (r,θ,z) and only consider a single mode. The coupling between different poloidal modes due to the toroidal geometry is neglected here since it is small (on the order of inverse aspect ratio) and is usually suppressed by the differential rotation at the different rational surfaces. The equilibrium magnetic field can be written as:

$$\mathbf{B}=B_z\hat{\mathbf{z}}+B_\theta(r)\hat{\boldsymbol{\theta}}. \tag{1}$$

For a thin magnetic island, i.e., $w/r_s \ll 1$, where w is the island width and r_s the radius of the rational surface, the equilibrium field near the rational surface can be expanded as:

$$\mathbf{B}=B_z\hat{\mathbf{z}}+B_\theta(r_s)\hat{\boldsymbol{\theta}}+\nabla\psi_0\times\hat{\mathbf{z}},$$

$$\psi_0\equiv-\left[\tfrac{1}{2}B_\theta'(r_s)x^2+\tfrac{1}{6}B_\theta''(r_s)x^3\right], \tag{2}$$

$$x\equiv r-r_s.$$

It is necessary to retain the x^3 term in ψ_0 because, as we will show later, it is of the same order as the Δ' effect.

The island can be modeled by superimposing a perturbed field $\tilde{\mathbf{B}}$ on the equilibrium field:

$$\tilde{\mathbf{B}}=\nabla\psi_1\times\hat{\mathbf{z}},$$

$$\psi_1\equiv-\frac{r}{m}B_r\left(1+\frac{x}{\alpha_\pm}\right)\cos\vartheta, \tag{3}$$

$$\vartheta\equiv m\theta-nZ/R_0.$$

Here, the subscripts "+" and "−" are for $x>0$ and $x<0$, respectively. Therefore, Δ' can be parameterized by the difference between the reciprocals of α_+ and α_-:

$$\Delta'\equiv[(d\psi_1/dr)_+-(d\psi_1/dr)_-]/\psi_1(0)=1/\alpha_+-1/\alpha_-. \tag{4}$$

In principle, there is another term, proportional to $x \ln x$, in the expression of ψ_1.[8] However, this term will not contribute to Δ' and is also not found to be significant in fitting the experimental data. Therefore, we neglect this term.

The equation for a given magnetic field line, as illustrated in Fig. 1a in the $x-\vartheta$ plane, is $\psi_0+\psi_1=\text{const}$, and becomes

$$\frac{1}{2}x^2(1+\epsilon_1 x)+\left(\frac{w}{4}\right)^2(1+\epsilon_2 x)\cos\vartheta=y, \tag{5}$$

where $\epsilon_1\equiv B_\theta''(r_s)/3B_\theta'(r_s)$, $\epsilon_2\equiv(1/\alpha_\pm+1/r_s)$, $(w/4)^2\equiv r_sB_r/mB_\theta'$. Here, y is a constant which labels different magnetic field lines. The value of y on the separatrix is $y=(w/4)^2$. Compared to the conventional magnetic island equation, the new terms $\epsilon_1 x$ and $\epsilon_2 x$ describe both the effect of the cylindrical geometry and Δ'. This equation illustrates that these effects are of higher order in w/r_s than the other terms. Therefore, what defines Δ' is the fine details of the island shape—the small difference between the two sides of the magnetic island.

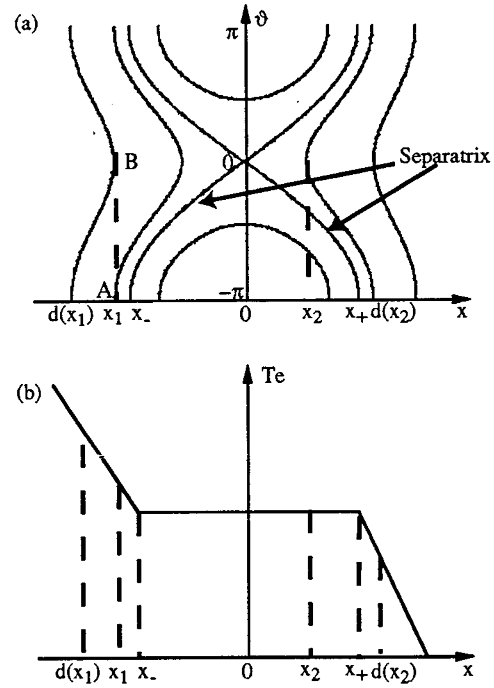

FIG. 1. (a) Contour plot of lines of constant y (constant ψ) for Eq. (5) in x,ϑ plane. Dashed lines represent trajectories scanned by two ECE channels located at x_1 and x_2. The amplitude of the electron temperature fluctuations recorded by the channel at x_1 is proportional to the difference of T_e between A and B. Since T_e is constant along the contour, it is also equal to the difference of T_e between $d(x_1)$ and x_1. (b) The electron temperature profile across the island O-point ($\vartheta=\pi$). The T_e is assumed to be flat within the separatrix (from x_- to x_+).

Considering the large parallel electron heat conductivity, we assume that T_e is a magnetic flux function outside the island and constant inside the island as shown in Fig. 1b. The equilibrium temperature gradient on the two sides of the island is denoted by $(dT/dx)_\pm$. From Fig. 1a it can be seen that the amplitude of the temperature fluctuation, $\tilde{T}_e(x_1)$, as seen by an ECE channel located at a radial position x_1 outside the separatrix, is proportional to the temperature difference between points A and B, which can be written as $(dT/dx)_-[d(x_1)-x_1]$. For a point x_2 inside the separatrix, we have $\tilde{T}_e(x_2)=(dT/dx)_+[d(x_2)-x_+]$. Here $d(x)$ is the radial coordinate at the O-point (where $\vartheta=-\pi$) of a field line whose radial coordinate at the X-point (where $\vartheta=0$) is x. In general, we have

$$\tilde{T}_e(x)=\begin{cases}|dT/dx|_\pm|d(x)-x_\pm| & \text{for } |x|\le|x_\pm|,\\ |dT/dx|_\pm|d(x)-x| & \text{for } |x|>|x_\pm|.\end{cases} \tag{6}$$

Here, $x_\pm$ are the radial coordinates of the separatrix at the O-point, i.e., they are the half widths of the island on the two sides of the rational surface.

Both $x_\pm$ and $d(x)$ can be calculated from Eq. (5). To the lowest order in w/r_s where the Δ' effect can be considered, the results are

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$$x_{\pm}=\pm\frac{w}{2}-\left(\frac{w}{4}\right)^2(2\epsilon_1-\epsilon_2), \tag{7}$$

$$\begin{aligned}d(x)=&\pm\sqrt{(w/2)^2+x^2}\\&\pm[\epsilon_1 x^3/2+(w/4)^2\epsilon_2 x]/\sqrt{(w/2)^2+x^2}\\&-\epsilon_1[(x^2/2)^2+2(w/4)^2]+\epsilon_2(w/4)^2.\end{aligned} \tag{8}$$

Now, to determine $(dT/dx)_{\pm}$ we first write

$$(dT/dx)_{\pm}=(dT/dy)|_{y=(w/4)^2}\times(\partial y/\partial x)|_{\vartheta=-\pi,x=x_{\pm}}. \tag{9}$$

From Eq. (5), we obtain

$$(\partial y/\partial x)|_{\vartheta=-\pi,x=x_{\pm}}=\pm w/2+\epsilon_1(w/2)^2. \tag{10}$$

To find dT/dy, we assume that no sources or sinks of heat exist within the narrow region near the rational surface and that the heat flux across the field line is continuous.[9] Therefore, we obtain

$$\int \nabla T\cdot \mathbf{dS}=\text{const}$$

or

$$\frac{dT}{dy}\cdot\int|\nabla y|dl=\text{const}. \tag{11}$$

Both $|\nabla y|$ and dl (the differential line length along a field line) can be found from Eq. (5):

$$|\nabla y|=\left|x+\frac{3}{2}\epsilon_1 x^2+\left(\frac{w}{4}\right)^2\epsilon_2\cos\vartheta\right|, \tag{12}$$

$$dl=(r_s+x)d\vartheta. \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (11) and expressing x in terms of (y,ϑ) from Eq. (5), the integral in Eq. (11) can be performed over ϑ (from $-\pi$ to π). Then, setting $y=(w/4)^2$ and using Eqs. (9) and (10), $(dT/dx)_{\pm}$ can be determined to within a constant:

$$\left(\frac{dT}{dx}\right)_{\pm}=A\left\{1\pm\frac{w}{2}\left[\left(1-\frac{\pi}{4}\right)\epsilon_1-\frac{\pi}{4r_s}\right]\right\}. \tag{14}$$

Finally, an analytical formula for the electron temperature fluctuation profile $\widetilde{T_e}$ is obtained from Eqs. (6–8) and (14):

$$\widetilde{T_e}(x)=\left|A\left\{1\pm\frac{w}{2}\left[\left(1-\frac{\pi}{4}\right)\epsilon_1-\frac{\pi}{4r_s}\right]\right\}\right|\times\left|\sqrt{\left(\frac{w}{2}\right)^2+x^2}-\frac{w}{2}+\frac{\epsilon_1 x^3/2+(w/4)^2\epsilon_2 x}{\sqrt{(w/2)^2+x^2}}\mp\epsilon_1\frac{x^2}{2}\right|, \tag{15}$$

when $|x|\leqslant|x_{\pm}|$, and

$$\widetilde{T_e}(x)=\left|A\left\{1\pm\frac{w}{2}\left[\left(1-\frac{\pi}{4}\right)\epsilon_1-\frac{\pi}{4r_s}\right]\right\}\right|\times\left|\sqrt{\left(\frac{w}{2}\right)^2+x^2}-\frac{w}{2}+\frac{\epsilon_1 x^3/2+(w/4)^2\epsilon_2 x}{\sqrt{(w/2)^2+x^2}}\mp\epsilon_1\left[\frac{x^2}{2}+2\left(\frac{w}{4}\right)^2\right]\pm\left[\epsilon_2\left(\frac{w}{4}\right)^2-x\right]\right|, \tag{16}$$

when $|x|>|x_{\pm}|$. The five parameters, $A,w,\epsilon_1,1/\alpha_-$ and $1/\alpha_+$ (in ϵ_2), are to be found by fitting the formula to the experimental data. The value of Δ' is then given by Eq. (4).

III. Δ' FROM ECE DATA

Figure 2 shows a plot of a typical $\widetilde{T_e}$ profile from the jog experiment and its fitting by Eqs. (15) and (16). The experimental profile is taken from one ECE channel which swept the island during the jog. The typical average relative error of the fitting (defined as the ratio of average deviation to maximum amplitude) is about 10%.

Table I shows a comparison between the results from the jog ECE data and the Δ's calculated from the q-profile obtained from the interpretive codes. The two schemes basically agree with each other. This result also supports the idea that the 3/2 and 4/3 modes on TFTR are not the classical current-driven tearing modes since $\Delta'<0$ for both modes.

An estimate of the appropriate error bars for these results is difficult. However, to illustrate the degree of reliability of the fitting results in Table I, an example is given in Table II in which three ECE channels (Ch. 12–14) have scanned one entire island. The fluctuations at a certain radial position are calculated from the raw data for two different durations—every 240 points (0.48 ms) or every 360 points (0.72 ms). (The results in Table I were obtained using the 240 point duration). With the data collection rate of 500 kHz and the mode rotation frequency on the order of 10 kHz, the 360 point duration corresponds to surveying the island about 7 times during the calculation of the fluctuations. The plasma moves radially in this duration only about 0.3 cm due to the jog; thus the radial resolution is not compromised. Six possible $r_s\Delta'$s for the same mode are listed in Table II. The systematic uncertainty of the fitting results is about 25%. For the Δ's from the usual calculations using q-profile, it is well known that the results are very sensitive to the first and second derivatives of the $q(r)$ near the rational surface. To obtain the interpretive code q-profile results in Table I we have used a standard spline interpolation scheme to increase the spatial resolution. Therefore, we effectively assumed a smooth q profile. The fact that the q-profile results are in good agreement with the results from ECE data justifies the use of the smoothed q profile.

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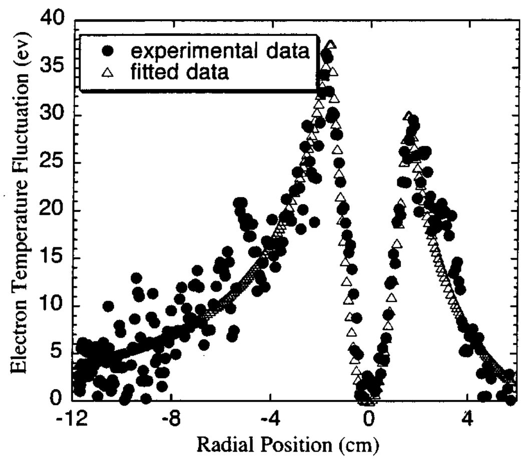

FIG. 2. Comparison of experimental data and fitted data [using Eqs. (15) and (16)] for the electron temperature fluctuation profile for a 3/2 mode (shot 82443, channel 12, 240-point smoothing). The fitting parameters are: $A=57.5$, $w=1.66$, $1/\alpha_+=-0.0763$, $1/\alpha_-=0.107$, $\epsilon_1=0.235$. Therefore, $r_s\Delta'=-3.7$.

IV. DISCUSSION

The underlying assumptions used in the derivation of the analytical expression Eqs. (15) and (16) are: 1) the electron temperature is constant on a magnetic field line or flux surface; and 2) the heat flux from diffusion is constant across the flux surface. Thus, this method can be applied to any quantity that satisfies these two assumptions—such as density or pressure fluctuations. As a verification of these assumptions, we apply Eqs. (15) and (16) to the pressure fluctuation data generated by a full three-dimensional initial value resistive/neoclassical MHD code.[10] The initial equilibrium is set up to be classically stable (with $r_s\Delta'=-4.4$ at the $q=2/1$ surface) but neoclassically unstable for the 2/1 mode. Then, the pressure is allowed to equilibrate on the perturbed magnetic flux surfaces. The final pressure fluctuations at a certain minor radius are analyzed in a similar way as the ECE electron temperature fluctuations. Shown in Table III are the values of $r_s\Delta'$ obtained by applying Eqs. (15) and (16) to the simulated pressure fluctuation data for different ratios of parallel to perpendicular heat conductivity.

TABLE I. Values of $r_s\Delta'$ obtained from two different methods. The ''fitting'' values are obtained using the formulas developed in this paper. The last column lists the Δ's calculated from the q-profile. Except for shot 86138, the data are from a single ECE channel. The data for shot 86138 are from two adjacent channels. ''SN'' means using the q-profile from the SNAP code, while ''TR'' means from TRANSP. Usually the ones using the TRANSP q-profiles are more reliable.

Shot	Mode	$r_s\Delta'$ (fitting)	$r_s\Delta'$ (q-profile)
73413	2/1	−1.3	−2.3 (SN)
82443	3/2	−3.7	−2.7 (TR)
82444	4/3	−5.9	−5.8 (TR)
82445	3/2	−4.3	−4.0 (TR)
84665	2/1	0.52	1.4 (TR)
86138	2/1	4.2	10.2 (SN)
91621	3/2	−7.0	−5.2 (SN)

TABLE II. Values of $r_s\Delta'$ for one island from 3 different ECE channels (Ch. 12–14). Here, ''240'' and ''360'' denote the number of points used in local fluctuation calculation from the raw data. The average $r_s\Delta'$ is −3.9.

	Ch.12		Ch.13		Ch.14	
No. of points	240	360	240	360	240	360
$r_s\Delta'$	−3.7	−4.2	−4.1	−4.9	−3.5	−3.1

As the parallel heat conductivity becomes higher and the pressure becomes more of a surface quantity, the method developed here can obtain ever more accurate measurements of Δ'.

Eqs. (15) and (16) are derived under the assumption of a flat temperature profile inside the island (Fig. 1b). This assumption is based on the theoretical consideration[11] and has experimental support.[2] However, a similar fit function for a non-flat profile can be easily obtained using the method developed here, requiring only a modification of the $|x|\leq|x_\pm|$ part in Eq. (6) (and Eq. (15) correspondingly).

The intrinsic band-width of a single ECE channel limits its radial resolution to about 2 cm (full width). However, the jog technique and the assumed linear radial temperature profile near the rational surface (Fig. 1b) allow the fluctuation profile to be measured with a radial resolution less than 2 cm (see the data in Fig. 2).

Finally, we should point out that the ECE data used here are usually taken when the magnetic island width is greater than the resistive layer width—the so-called Rutherford regime.[12] We implicitly assume that for a classically stable but neoclassically unstable mode, Δ' in the presence of an island with finite width remains about the same as its value when no island is present.[3,5]

To summarize, we have presented a new, direct method of measuring Δ', the fundamental tearing mode stability parameter, from electron temperature fluctuation data. Unlike previous methods, the technique does not suffer from the difficulty of relying on the derivatives of the q-profile. This method gives values of Δ' for the 2/1, 3/2, 4/3 modes in TFTR that agree with those obtained by the previous method. Furthermore, it provides an important independent measurement of the previous inference that 3/2 and 4/3 modes on TFTR have negative Δ' values and therefore are stable to the classical tearing mode. Thus, the observed 3/2 and 4/3 magnetic islands in TFTR[2] must have been induced by unstable neoclassical tearing modes,[5] as deduced in Ref. 3.

TABLE III. Values of $r_s\Delta'$ obtained by applying Eqs. (15) and (16) to the computer-simulation-generated pressure fluctuation data for different $\chi_\parallel/\chi_\perp$, the ratio of parallel to perpendicular heat conductivity. On TFTR, $\chi_\parallel/\chi_\perp\approx10^9$. The theoretical input value of $r_s\Delta'$ is −4.4.

$\chi_\parallel/\chi_\perp$	10^5	10^6	10^7	10^8
$r_s\Delta'$	−1.87	−2.37	−2.79	−3.04

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ACKNOWLEDGMENT

This work was supported by United States Department of Energy under grant DE-FG02-92ER54139 (UW) and Contract No. DE-AC02-76-CHO-3073 (TFTR/PPPL).

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