Markov Chains and Student Academic Progress II

Jessica Porath ❖ Department of Mathematics ❖ University of Wisconsin-Eau Claire
Faculty Mentor: Dr. Don Reynolds

Abstract

With data obtained from student enrollment records, absorbing Markov chains are used to model the academic progress of students attending the University of Wisconsin-Eau Claire over a specific period of time. Useful statistics, such as the length of time a student enrolling in UWEC as a freshman can expect to spend before graduating, are derived from the Markov model. With this information, the University may more accurately measure the academic progress of its students, and thus better reflect on its own institutional effectiveness.

Background

An **absorbing Markov chain** consists of:

- A finite number, r, of **states** of existence from which there are periodic transitions
  - Four **non-absorbing states**:
    - Freshman Class (fewer than 30 credits)
    - Sophomore Class (30 to 59 credits)
    - Junior Class (60 to 89 credits)
    - Senior Class (90 or more credits)
  - At least one **absorbing state** where, once reached, it is impossible to leave:
    - Graduation

  Transition from each non-absorbing state to some absorbing state is possible and the probability that the process will eventually reach an absorbing state is 1.
- A **transition matrix** P= $[p_{ij}] \in M(r,r)$
  - The entry p_{ij} is the probability of moving from the i^{th} state to the j^{th} state.
  - If the states are ordered so the absorbing states are listed first then the transition matrix will be of the following form:

    $$P = \begin{bmatrix} I & R \\ 0 & Q \end{bmatrix}$$
  - Then the matrix $P^n = \begin{bmatrix} I & R + RQ + \cdots + RQ^{n-1} \\ 0 & Q^n \end{bmatrix}$

    gives the probabilities of moving between states after n steps.
  - Each entry of sub-matrix Q gives the frequency of a visit to that position after n steps. In order to calculate the total number of visits to a specific state until absorption, we need to sum the same entry for every possible power of Q. Thus, if the system begins in the j^{th} nonabsorbing state, the expected number of visits to the i^{th} nonabsorbing state is the ij-entry of $I + Q + Q^2 + Q^3 + \ldots$
  - This series of matrices will converge to $(I-Q)^{-1}$ (called the **fundamental matrix**) provided that each entry in Q is less than 1.

Our Data and Calculations

- Data files were obtained from the UWEC Registrar's Office that identified the gender, ethnicity, entrance year, and the class rank for every student enrolled from 1997 through 2005.
- The following assumptions were then made:
  - Only full-time students were considered. A student in the same class (except for the Senior class) for three years or longer was assumed not to be full-time.
  - Students who transferred or dropped out permanently were not considered. If a student transferred/ studied abroad for a semester, the student is assumed to be in their previous class rank.
  - Senior class students who failed to return were assumed to have graduated.
- **Transition Matrix**
  - The entries are the weighted averages of the probabilities of moving from one state to another from 1997 to 2005:

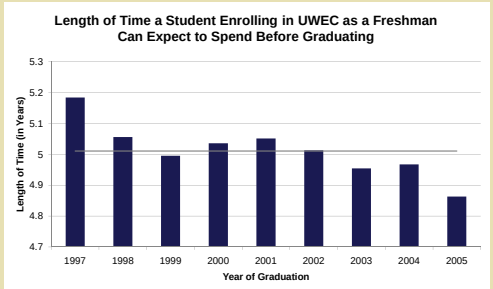
    P =

	Gr.	Fr.	So.	Jr.	Sr.
Gr.	1	0	0	0	0.6665
Fr.	0	0.1926	0	0	0
So.	0	0.5694	0.1343	0	0
Jr.	0	0.0384	0.6708	0.1047	0
Sr.	0	0	0.0621	0.7832	0.3334
- Then the **fundamental matrix** is:

  $(I-Q)^{-1} =$

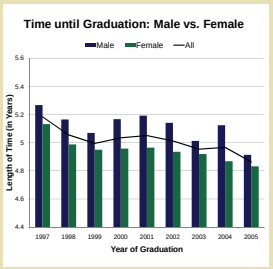
  $$\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.1926 & 0 & 0 & 0 \\ 0.5694 & 0.1343 & 0 & 0 \\ 0.0384 & 0.6708 & 0.1047 & 0 \\ 0 & 0.0621 & 0.7832 & 0.3334 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 1.2385 & 0 & 0 & 0 \\ .8146 & 1.1551 & 0 & 0 \\ .6635 & .8655 & 1.1169 & 0 \\ .8554 & 1.1245 & 1.3123 & 1.5001 \end{bmatrix}$$
- The sum of the entries in the diagonal of the fundamental matrix will give us the average length of time a student enrolling in UWEC as a freshman can expect to spend before graduating.
- 1.2385 + 1.1551 + 1.1169 + 1.5001 = 5.0106 years

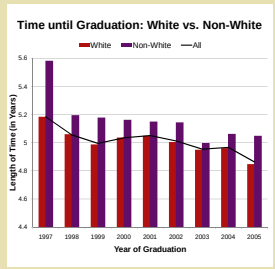
Results and Conclusions



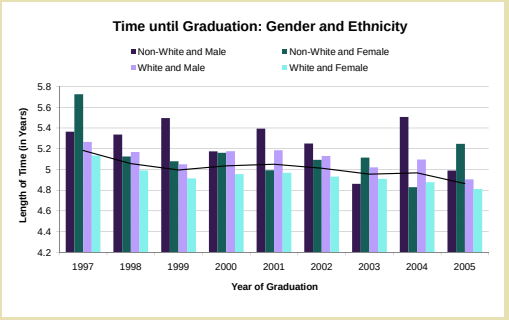
- Based on the mean of the data for graduates between 1997 and 2005, a student enrolling in UWEC as a freshman can expect to spend an average of 5.0106 years before graduating.

Results and Conclusions, cont.





- From 1997 through 2005, the UWEC full time student population was:
  - 38.9% male and 61.1% female
  - 93.5% white and 6.5% non-white (black, Native American, Asian, Hispanic)



- In every year from 1997 through 2005:
  - The length of time spent before graduation was shorter for females than for males
  - The length of time spent before graduation was shorter for those students of white ethnicity than for those of non-white ethnicity
  - For all groups the length of time until graduation is decreasing

Sources

- Maki, Daniel P., and Thompson Maynard. Mathematical Models and Applications. New Jersey: Prentice-Hall, 1973.
- Messer, Robert. Linear Algebra: The Gateway to Mathematics. New York: Harper Collins College, 1993.
- Williams, Gareth. Linear Algebra with Applications. Massachusetts: Jones and Bartlett Publishers, 2005.

We would like to thank the Office of Research and Sponsored Programs at the University of Wisconsin-Eau Claire for supporting our research.