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REGIME-SWITCHING ADVANTAGE IN STATISTICAL ARBITRAGE STRATEGIES CONDITIONED ON TIME SERIES MOMENTUM AND VOLATILITY IN LEVERAGED EXCHANGE TRADED FUNDS: THEORY AND EVIDENCE

presented by Nisheeth Saini

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REGIME-SWITCHING ADVANTAGE IN STATISTICAL ARBITRAGE STRATEGIES
CONDITIONED ON TIME SERIES MOMENTUM AND VOLATILITY IN LEVERAGED
EXCHANGE TRADED FUNDS: THEORY AND EVIDENCE

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By
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DEDICATION

To Vajra, my Thunderbolt,

his Mom

and

his Grandparents

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ABSTRACT

The phenomena of volatility decay (also known as time decay) and path dependence in leveraged exchange traded funds (ETF) markets have been documented in the literature. This dissertation examined whether it is possible to exploit these market conditions for leveraged ETF (LETF) trading using statistical arbitrage (StatArb) strategies. The study proposed a regime switching model tailored for LETF markets to predict volatility and time-series momentum in the behavior of the underlying indexes of the LETFs. The study then used this model to test short pair trading strategies on a varied set of commodity LETFs to see if theoretical intuitions informed by these analyses were empirically supported by data. The study also introduced the concept of lag relative expected volatility (LREV) based on inductive learning in a binary classification framework to model upward shocks in expected volatility on any given trading day.

The results of this study showed that an active short pair trading strategy in commodity LETFs, conditioned on momentum and volatility, outperforms an unconditioned and passive sell-and-hold StatArb trading strategy on a risk-adjusted basis. This outperformance was,

however, found to be present in Sortino ratios only. The study did not find any evidence of

outperformance for the active trading strategy in either Sharpe ratios or absolute returns. The

results also provided further evidence that LETFs tracking equity indexes are poor candidates for

active StatArb trading strategies due to low volatility. Further, the results also indicated that any

incremental deterioration in the efficiency of LETF products in rapidly fluctuating markets

appears to be mostly attributable to systemic jumps in the implied volatility and less due to any

incremental inefficiency in their daily rebalancing process. This finding may be of interest to the

regulators.

Lastly, the study also provided evidence from the LETF markets for an inverse

relationship between volatility and momentum, as established in some recent studies.

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Evidence

Leveraged Exchange Traded Funds (LETFs hereon) as an asset class are relatively new.

The first LETF was launched by Proshares in 2006 (Cummans, 2013). Being among the newer

product innovations in the financial market, academic research about LETFs is still in its early

stages. The history of exchange traded stocks and bonds goes back more than 400 years to 1602

AD when an earlier avatar of Euronext Amsterdam was established by Dutch East India

Company (Beattie, 2018). Stock price datasets go back almost 150 years (Shiller, n.d.), while

LETF datasets go back only 12 years, with commodity LETF datasets being available for an

even shorter time period. Nonetheless, the LETF market has grown rapidly since their inception

in 2006. According to a Reuters report, as of June 2014, the market for LETF and related

products was already worth $60 billion (McLaughlin and Ablan, 2014).

Motivation

The microstructure of LETFs has begun to attract academic curiosity. Cheng and

Madhavan (2009), Jarrow (2010), Avellaneda and Zhang (2010) and other scholars cited later in

this study have produced insightful tracts on LETFs. However, a lot of territory pertaining to

LETF behavior is still unchartered in both academic and practitioner spaces. As the name

suggests, these products have leverage embedded in them and considerable anxiety exists about

them among regulatory bodies as to their suitability for both institutional and retail investors. All

these attributes make LETF market a promising avenue to gain new insights about market

behavior, especially about derivative financial innovations such as exchange traded notes (ETNs), which is what LETFs essentially are.

Two of the well-known properties about LETF's pricing behavior are: (a) path dependence and (b) volatility decay (also called time decay). Although path dependence and volatility decay have been known for some years in mathematical finance literature, a study examining them in the context of return predictability and arbitrage literature has yet to emerge. Further, their implications for industry practitioners also remains unexplored.

The present study sought to address this gap in knowledge. It also built on the existing studies in order to empirically validate their microstructure implications for both the practice and development of econometric theory. Market efficiency and arbitrage theoretical literature have continued to evolve. Most of the existing studies in this area have dealt with common equities, commodities, and the foreign exchange (forex) and derivatives markets. LETF markets can and often do combine the properties of all the foregoing. A study such as the present one enriches this field with unique insights from the perspective of synthetic LETFs markets, which may depart in behavior from pure equities, commodities, and the forex and derivatives markets in more than one way. Hence, the ability of study like this to contribute to market efficiency and arbitrage literature in a novel and original way is not without promise.

Problem Statement

Volatility decay and path dependent behavior of LETFs has been studied by scholars such as Avellaneda and Zhang (2010), who showed that LETF returns are conditioned by the market state of the reference index they track such that the reference in index is trending, these LETFs outperform their target daily market return. Conversely, when the reference index is volatile, the

LETFs underperform their target daily market return. Guo and Leung (2014) further showed

volatility decay can benefit a trading strategy based on shorting both long and short commodity

LETFs even though this trading strategy is subject to significant risk in trending market states of

the reference indexes. The present dissertation extended this earlier work further into investment

theory and examined its implications for the statistical arbitrage (StatArb) framework given in

Avellaneda and Zhang (2010). More specifically, the present study exploited volatility decay

and path dependence behavior of LETFs to develop active algorithmic StatArb trading strategies

and study their absolute and risk-adjusted performance. It capitalized on the findings of the

existing studies on LETFs and explored the possibility of extracting StatArb by modifying sell-

and-hold LETF trading strategies by adding trading signals conditioned on volatility and time-

series momentum. The intuition for these StatArb strategies came from the 'long momentum

and short variance' property of the LETFs, as discussed later in this dissertation. In summary,

the return of the LETF has a known property of path dependence which indicates that it loses or

gains value based on behavior of the reference index it tracks. If it could be predicted whether

the reference index will be exhibiting high variance or momentum or another econometric

property over a given timeframe, a trading strategy could then possibly be developed which

could beat the passive sell-and-hold LETF trading strategies.

Research Questions

Hogan, Jarrow, Teo , and Warachka (2004) defined StatArb as a time series concept

analogous to limited arbitrage opportunity as discussed by Ross (1976). Avellaneda and Zhang

(2010) said that the term StatArb can be used for a variety of trading strategies which share

number of common attributes; these common attributes are: (a) trading signals are generated

systematically in a rules-based mode instead of being generated by fundamental analysis, (b)

trading portfolio are market and beta neutral, and (c) generation of excess returns is based on a statistical process. The study of potential arbitrage opportunities in a relatively new asset class such as LETFs has profound implications for the evolving body of literature on market efficiency, return predictability, and arbitrage pricing theory. More specifically, this study sought to address following questions:

1. Is there return predictability in the LETF market?
2. Can the known LETF properties of volatility decay and path dependence be successfully exploited in trading to give superior results?
3. Can active LETF short pairs trading strategies outperform passive sell-and-hold trading strategies?
4. Does active trading in commodity LETFs lead to better returns due to higher volatility?

To this author's knowledge, this specialized study is the first its kind for this relatively new financial market innovation, and it is only a matter of time before this or a similar study emerges in published research.

Hypotheses

The essence of the research questions is best operationalized using the following hypotheses tested by the present study:

H_{1a}: An active short pair trading strategy conditioned on volatility and momentum in commodity LETFs outperforms a passive unconditioned short pair trading strategy on an absolute return basis.

H_{1b}: An active short pair trading strategy conditioned on volatility and momentum in commodity LETFs outperforms a passive unconditioned short pair trading strategy on a risk-adjusted basis.

H_2: An active short pair trading strategy conditioned on volatility and momentum in LETFs outperforms a passive unconditioned short pair trading strategy only for commodity LETFs and not for non-commodity LETFs as the latter tend to have less volatility.

H_3: Underperformance of LETFs in reference to their stated leverage multiples in a manner that yields any trading advantage is not just restricted to continuously high volatility market states of the underlying indexes, but it also exhibits in low volatility market states when there are abrupt upward shocks in expected volatility.

Contribution

In a general sense, the contributions of this study can be summarized as follows. First, it enriches the literature about LETF research, which is still in its adolescence. Second, the present study provides empirical evidence to enrich literature on return predictability, with special reference to exchange traded derivative products such as LETFs.

More elaborately, this is the first study that has examined the return predictability of commodity LETFs. Secondly, this study provides evidence on the inverse relationship between volatility and momentum in LETF markets. This finding also enriches the ongoing research exploring the relationship between volatility and momentum. Thirdly, this study shows how an active pair trading strategy conditioned on momentum and volatility performs in comparison to a passive unconditioned trading strategy in the LETF market. This is a contribution both to theoretical and practitioner literature. Fourthly, this study provides empirical evidence as to how

commodity LETFs behave differently from non-commodity LETFs and under which conditions. Fifthly, this study develops a new statistical framework called theoretical compound annual growth (TCAGR) based on the cost of capital and discounted cashflow approaches for the performance measurement of StatArb portfolios. This new performance measurement framework can be generalized to all kinds of StatArb strategies, not just the ones based on LETFs. This framework solves a key problem for both academics and practitioners by making available a theoretically sound way to not only measure the performance of StatArb strategies against other StatArb portfolios but also against all kinds of benchmark portfolios based on traditional investment strategies. Lastly, the study introduces a new framework called the lag relative expected volatility (LREV) statistical framework to model the shocks in expected volatility in a regime agnostic way.

Leveraged Exchange Traded Funds and Sample Periods

The study focused on both commodity and non-commodity LETFs. Commodity LETFs cover reference indexes based on crude oil, natural gas, gold, gold mining, and silver. Non-commodity LETFs cover major equity indexes, namely the Standard & Poor's 500 Index (S&P 500), Dow Jones Industrial Average (DJIA), and Russell 2000. Holding periods for the trading strategies ranged from five to nine years.

Annual performance metrics of these trading strategies were also considered to understand time-varying elements in return distributions.

Findings

Following are the key findings of this study:

No evidence was found that an active short pair trading strategy conditioned on volatility and momentum in commodity LETFs outperforms a passive unconditioned short pair trading strategy on absolute return basis.

There was significant evidence that an active short pair trading strategy conditioned on volatility and momentum in commodity LETFs outperformed a passive unconditioned short pair trading strategy on risk-adjusted basis. However, this outperformance only manifested in Sortino ratios and was asymmetrically distributed.

An active short pair trading strategy conditioned on volatility and momentum in LETFs outperformed a passive unconditioned short pair trading strategy only for commodity LETFs and not for non-commodity LETFs. This was because the non-commodity LETFs track indexes which do not generally have the required amount of volatility. LETFs appeared to significantly underperform their stated leverage multiple in response to their reference index return only in continuously high volatility market states but not in low volatility market states, even when there were abrupt upward shocks in expected volatility.

Although LETFs may empirically experience marginally higher rebalancing costs on upward shocks in expected volatility in low-volatility regimes, in this study these costs did not appear to be high enough to offer any trading advantage or arbitrage opportunity. Another implication of this finding is that any incremental deterioration in the efficiency of LETF products in restive markets appears mostly attributable to systemic jumps in the implied volatility and less to any incremental inefficiency in the daily rebalancing process. This observation may be of interest to regulators.

Apart from adding new insights for the practitioner community and regulators, these findings inform the emergent academic research on LETFs. They also enrich empirical finance studies focused on time series momentum and volatility.

These results also provide the evidence for an inverse relationship between momentum and volatility from LETF markets. Outside the field of traditional finance, these findings also have implications for methods research in computational finance, a specialized field which combines investment finance and operations research.

Literature Review

Time Decay and Path-Dependence of Leveraged ETFs

Avellaneda and Zhang (2010) showed the unsuitability of leveraged ETFs for buy-and-hold investors; they showed that if the price of the underlying reference index, which is tracked by LETF, is bound in a narrow range and is volatile over the holding period, the investor in the LETF always underperforms its underlying reference index. Avellaneda et al. and Guo and Leung (2014) further demonstrated that this behavior is observed due to the time decay (also known as volatility decay) property of LETFs, which is applicable to both long and short LETFs and , if the underlying reference index trends in either direction in a significant way, the investor in LETF always outperforms the underlying reference index. Avellaneda et al. implied that this behavior of the LETF, which makes its performance reliant on the direction taken by the underlying reference index, is what renders its path-dependence property. Accordingly, LETFs have also been described as long momentum and short variance (Guo and Leung , 2014).

A ProShares (2018) investor advisory illustrated that LETF tracking errors result primarily due to daily rebalancing and compounding effects. Other expenses like management fees play a minor role in causing large tracking errors for LETFs over time. Table 1, excerpted from the ProShare website, demonstrates the compounding effect with the help of a hypothetical example.

Table 1

Illustration of compounding effect in LETF returns

Return Type	Benchmark Daily Return	2x Fund Daily Return	$100 Investment Daily Value	Underlying Market State
Day 1 Return	+5.00%	+10.00%	$110.00	Upward Trend
Day 2 Return	+5.00%	+10.00%	$121.00	Upward Trend
Compounded 2-Day Return	+10.25%	+21.00%	$121.00	Upward Trend
Day 1 Return	-5.00%	-10.00%	$90.00	Downward Trend
Day 2 Return	-5.00%	-10.00%	$81.00	Downward Trend
Compounded 2-Day Return	-9.75%	-19.00%	$81.00	Downward Trend
Day 1 Return	+5.00%	-10.00%	$90.00	Volatile
Day 2 Return	-5.00%	+10.00%	$99.00	Volatile
Compounded 2-Day Return	-0.25%	-1.00%	$99.00	Volatile

Note. Adapted from "Effects of daily rebalancing and compounding on geared investing." by ProShares, 2018. Retrieved from http://www.proshares.com/media/documents/compounding.pdf

Daily rebalancing entails maintaining LETF's stated exposure to the underlying reference index. This sort of rebalancing, based on buy high and sell low, comes with substantial transaction costs, which are passed on to the LETFs. Tang and Xu (2013) and Cheng and Madhavan (2009) stated that LETFs use complex derivatives, mostly total return swaps, to provide the stated exposure to the underlying reference index. In periods of higher volatility, this daily rebalancing is thus expensive, which exposes the fund's NAV to greater negative compounding effects. Higher rebalancing costs in volatile periods coupled with negative compounding effects create an amplified volatility decay. In trending periods for the underlying reference index, a positive compounding effect, along with diminished daily rebalancing costs due to low volatility, is primarily responsible for the LETF's superior performance.

Given the complicated behavior of these securities, all ETF sponsoring companies now include in their prospectuses descriptive warnings against holding LETFs for periods longer than very short time horizons (no more than 2-3 weeks). The volatility decay property of the LETFs has made the securities attractive to short sellers. This is based on their knowledge of the volatility decay phenomenon. Barring the periods when the underlying reference index moves up or down in major way, the default behavior of the LETFs is to lose value. This gives short sellers an intuitive sense to short sell both long and short LETFs simultaneously in a seemingly market neutral way and close the trade when the pair has lost enough value. Before going any further, it needs to be commented that market neutrality of this passive sell-and-hold strategy is at best naïve. Even though the long and short positions appear to offset each other, there is obvious tail risk associated with this trading strategy due to the possibility of the underlying reference index experiencing trending regimes during the holding period. Another factor which makes true market neutrality difficult is the fact that the inverse, or the short LETF, is not the

exact mirror image of the long LETF over longer periods (Cheng and Madhavan 2009). The timely availability of the securities for shorting and borrowing cost considerations cannot be ignored either. That said, this passive sell-and-hold trading strategy, requiring minimal rebalancing, is still not without its promises. Xinxin and Stanley (2017) analyzed investment strategies involving LETF pairs using simulated daily returns for a period of 48 years and found superior performance of these strategies on a risk-adjusted basis in reference to a static strategy of holding a long position in S&P 500. Shorting the short LETF which offers three times the inverse return of underlying index (3x LETF) and the long 3x LETF in a 2:1 proportion, while staying long in treasuries, Xinxin et al. discovered the average annual Sharpe ratio for this sell-and-hold strategy to be four times superior than for the S&P 500. Xinxin et al. further found that this strategy outperformed S&P 500 in 43 out of 48 years. Guo and Leung (2014) discussed a similar trading strategy in reference to commodity LETFs, while noting the associated tail risk. Studies such as Kent , Jagannathan, and Kim (2012) and Daniel and Moskowitz (2016) have documented the tail risk in traditional momentum literature for trading strategies that seek momentum. However, it needs to be commented that the tail risk in the context of this study had exactly the opposite implication in terms of the trading strategy development. As opposed to traditional momentum seeking strategies, the strategies primarily using LETF pairs shorting need to avoid momentum.

Leveraged Exchange Traded Funds Return Predictability vis-à-vis Return Predictability in Exchange Traded Funds and Commodities

Return predictability in LETFs has yet to appear as an express theme in the available literature, but studies such as Cheng and Madhavan (2009), Avellaneda and Zhang (2010), Tang and Xu (2013), Jarrow (2012) and Guo and Leung (2014) have focused on modeling of LETF

returns and the findings have strongly implied return predictability in LETFs. The securities like LETFs can also be studied as a special class of ETFs. The idea of return predictability in ETF literature is not new. Guedj and McCann (2011) showed that the returns of unleveraged commodity ETFs are predictable. Unlike equity ETFs, commodity ETFs do not hold actual or even representative positions in the constituents of the reference indexes they are tracking. They instead use commodity futures to give the stated exposure to the investor in the commodity ETFs. By demonstrating the predictability in the term structure of the futures used in daily rebalancing, Guedj and McCann (2011) showed predictability in commodity ETF returns as well. Similarly, Fulkerson, Jordan, and Riley (2014) (2014) showed predictability in bond ETF returns by showing persistence of alternating ETF premiums and discounts. Intuitively, the return predictability framework in ETFs can easily be generalized to LETFs as this ETF class by design promises a known leverage multiple regarding the underlying reference indexes comprising of equity and bond indexes, commodity futures, and so forth. Additionally, the predictability of returns of the asset classes from which LETF reference indexes are derived has been extensively documented in studies such as Ferson and Harvey (1991), Campbell and Thompson (2008), and Cochrane (2008). Just by theoretical deduction alone it should not be difficult to assert that if a given asset class exhibits return predictability, then returns of a derivative product class based on the asset class must have similar properties. The present dissertation provides further empirical evidence of this by incorporating path-dependence and volatility decay properties of LETFs into trading strategies to obtain predictable returns. It is noteworthy that Cochrane (1999) and other scholars before, such as Stevenson and Bear (1970), Cargill and Rausser (1975), and Leuthold (1972), had already demonstrated that commodity futures prices do not exhibit random walks and have predictable return patterns that can possibly be exploited.

Market Efficiency

Sharpe (1964) pointed out the basic risk-reward equilibrium existing in financial markets when capital asset pricing model (CAPM) was introduced. Sharpe's (1964) CAPM was essentially an elaboration of the modern portfolio theory (MPT) originally proposed by Markowitz (1952) in the seminal paper on portfolio selection. Lintner (1965) showed that under the CAPM market equilibrium, rewards are directly proportional to the risk, illustrating how covariance between the security and market portfolio, rather than the security's own volatility, is the formative influence over the return beyond the risk-free rate. Jensen (1968) defined alpha as the abnormal rate of return on a security or portfolio in excess of what would be predicted by an equilibrium model like the CAPM. The idea of efficient market hypothesis (EMH) expressed in terms of MPT implies that the alpha of any asset or portfolio on a consistent basis will be zero or an insignificant value close to it, otherwise, anomalous returns are implied. Fama (1970) forcefully argued in favor of EMH, which states that the price and accounting value of an asset rapidly adjust to each other due to symmetric and efficient assimilation of the information by the market participants. In short, EMH says that price is equal to value (Lee, 2001). In response to the theoretical and empirical attacks on MPT and EMH, Scott and Horvath (1980) investigated moments of higher order than the variance, which is the standard proxy for risk in academic literature; their findings implied that mere outperformance of a purely variance-based measure, like a market portfolio's Sharpe ratio, by a trading strategy does not necessarily imply violation of EMH as the market participant's preference for skewness and kurtosis also play their relative part in determination of a risk–reward tradeoff. Shiller (2003) regarded semi-strong form of EMH to be the general efficient markets model., Lee (2001) also said that although the conceptual impossibility of instantaneous price adjustments is generally well understood, much

of the capital market literature has treated this adjustment process as trivial, and this has in turn

biased the way much of the academic research has been conducted and reported.

Studies such as Jegadeesh (1993) and Chan, Jegadeesh,and Lakonishok (1996)

demonstrated existence of anomalous returns in momentum strategies. Shiller (1981, 2003)

argued that the anomaly of excess volatility challenges EMH in a much more troubling way than

other anomalies such as the January effect and end-of-the week effect. Shiller (1981, 2003)

attributed anomaly of excess volatility to behavioral factors operating in the market. Speaking of

market studies partly as a function of investor sentiment, Barberis, Huang, and Santos (1999)

used prospect theory to show that the investors not only derive utility from consumption levels

but also from fluctuation in the financial wealth. They further showed that the investors are much

more sensitive to reduction in wealth than to increases.. Daniel, Hirshleifer, and Subrahmanyam

(1998) proposed a theory of securities market underreaction and overreaction based on

psychological biases, such as investor overconfidence and biased self-attribution. Daniel et al.

also showed in asset returns negative long-lag autocorrelations, positive short-lag

autocorrelations, excess volatility, and return predictability under certain conditions . Hong and

Stein (1999) studied news-watchers and momentum traders as two groups of boundedly rational

market participants and showed similar evidence of price underreaction and overreaction based

on the less than instantaneous rate of information diffusion which appears to be a contradiction

of EMH. Fama (1998) argued overreaction and underreaction are randomly countervailing each

other and that the statistical methodology used in the tests are a factor that biases the study of

long-term anomalies. Fama (1991) also indicated a joint hypothesis problem whereby any test of

market efficiency also requires a concomitant equilibrium model to fairly price an asset. The

implication of the argument is that market efficiency is not testable directly. In the case of an

anomalous return in the market, it can never be established that it exists due to falsification of market efficiency or simply due to misspecification of the equilibrium model.

As already noted in motivation section, most of the existing studies in market efficiency and return predictability area mostly have dealt with common equities, commodities, and forex and derivatives markets. Markets trading LETFs can and often do combine the properties of all the above asset classes. The present study sought to address this gap with unique insights from the perspective of synthetic LETFs markets, which differ from common equities, commodities, and forex and derivatives markets in several ways.

Methods: Trading Strategies

In this section, the methodological approach for hypothesis testing is outlined. First, a three-state regime switching framework was developed based on time series momentum and volatility using existing theory. The three modeled states are momentum regime, variant regime, and inert regime. These states model momentum, volatility, and momentum reversal with low volatility, respectively, in behavior of the reference index. Next, the details of the benchmark trading strategy, unconditionally short pair trading strategy (USPTS), are described. Different variations of this trading strategy have already been discussed in recent literature. Thereafter, two variants of the volatility short momentum neutral (VSMN) trading strategy, namely VSMN1 and VSMN2, are developed.

The first variant, VSMN1, was based on the explicit regime switching framework discussed previously. This strategy, consisting of a long and short LETF pair, remains in cash if momentum is detected and when volatility is otherwise low. It goes short when volatility is high. The second variant, VSMN2, relied the idea of implicit momentum. It not only acted as a robustness check for VSMN1, but it also helped test low volatility non-commodity LETFs, which do not respond well to the high volatility threshold set in VSMN1. A new inductive learning based framework to model volatility shocks even in low volatility market regimes was introduced and used in this trading strategy.

After this, a robustness check was developed to empirically test the effectiveness of the momentum detection oscillator using the three-state regime switching framework. Further, another robustness check was developed to ensure that the parameter values used in the regime switching framework were robust to selection bias.

The selection of industry sectors from which LETFs were drawn are discussed next. Finally, in order to select volatility-based trading signals using a methodologically rigorous approach, various commonly used volatility prediction models are discussed.

A Regime-Switching Framework for Leveraged Exchange Traded Funds

The test of two of the hypotheses required a regime sensitive trading strategy. The details of the regime-switching model for LETFs are laid out in the next two sections. As noted in the foregoing sections, studies such as Avellaneda and Zhang (2010) have documented the volatility or time decay and path dependence properties of LETF returns. The regime-switching framework used in this study was operationalized using these properties and a combination thereof. Accordingly, three regimes for the behavior of the reference index were classified as follows: momentum regime, variant regime, and inert regime. As the names suggest, when the reference index was trending either upward or downward, it was classified as momentum regime. When it was in a period when it was range bound and volatile at the same time, it was classified as variant regime. When the reference index was range bound with low expected variance, it was classified as inert regime. Path dependence and volatility properties of LETFs were inherent in all three regimes.

Modeling Trading Strategies Based on Leveraged Exchange Traded Funds Regime-Switching

The basic intuition behind the trading strategy used in this dissertation is described next. It is based on the development of a mechanism to predict the state of the reference index and then using this prediction to model trading signals to open and close short positions in respective legs of the LETF pair. By doing this, at least in theory, the risks or losses associated with holding short position in the LETF pair while the reference index is in unfavorable regime could

be greatly minimized, if not eliminated. It also reduces unnecessary borrowing costs for the shorted securities, which are roughly 6% annually based on information obtained from Interactive Brokers by this author via a telephonic survey in 2016.

To capture the volatility- and momentum-related excess return, two trading strategies were examined as already mentioned. Accordingly, the three possible regimes of the underlying reference indexes, discussed earlier, were operationalized in concrete econometric terms. Moskowitz, Ooi, and Pedersen (2012) demonstrated time series momentum in equity index, currency, commodity, and bond futures using a time series momentum factor (TSMOM) indicator which was based on the autoregression coefficient between return of month t and the signage of return of holding period $t - h$ (where h is a previous month such that $t > h$).

This study used similar but more refined trading rules based on moving averages to identify the hypothesized market regimes in reference index returns. Although these trading rules were original to this study and were conditioned by unique properties of LETF market, the approach they are based on does have support in prior literature. The use of moving averages for the development of relative strength indicators in momentum literature goes back over 50 years to Levy (1967). Marshall and Visaltanachoti (2017) showed how time series momentum-based trading rules modeled using moving averages and TSMOM are closely related and showed a correlation more than 0.8 between the returns generated by the two indicators.

Econometric Operationalization of Leveraged Exchange Traded Funds Market Regime-Switching

The regime detection strategy was broken down as follows. First, the rolling moving averages of daily returns of the reference index over 6-month, 3-month, 6-week, and 2-week

time horizons were calculated. A momentum regime was inferred from these rolling moving averages as per following rules:

- If the 6-month return rolling moving average (MA) is greater than the 3-month return rolling MA and the 6-week return rolling MA is greater than the 2-week return rolling MA, then momentum regime equals positive momentum.

- If the 6-month return rolling MA is greater than the 3-month return rolling MA and the 6-week return rolling MA is less than the 2-week return rolling MA, then momentum regime equals positive momentum reversal.

- If the 6-month return rolling MA is less than the 3-month return rolling MA and the 6-week return rolling MA is less than the 2-week return rolling MA, then momentum regime equals negative momentum.

- If the 6-month return rolling MA is less than the 3-month return rolling MA and the 6-week return rolling MA is greater than the 2-week return rolling MA, then momentum regime equals negative momentum reversal.

Second, high expected volatility was explored. What constitutes high or low volatility can be subjective. In order to remove this subjectivity from the definition, reasonable assumptions were made. First, the S&P 500 index was taken as a proxy for the broader market and annualized volatility was calculated using daily closing values for a period of 15 years between 1993 and 2008. All the data samples used in this study fell outside this period, which ruled out any risk of look-ahead bias. The annualized volatility of the S&P 500 index was computed as 18.76%. This was incremented by 1% and then rounded up to nearest hundred to arrive at a figure of 20%. The purpose for this increment was to create a noise filter for the trading signal. This figure gives a margin of error of approximately 7% of the average expected

volatility (18.76% x 1.07 $\approx$ 20%). Accordingly, whenever expected volatility for any given trading day was over 20%, a high volatility regime was inferred.

Third, in order to model abrupt upward shocks in expected volatility (see H_3), the concept of LREV was introduced. For the purpose of this study, LREV was defined as the expected volatility on any given day in relation to the median of the distribution of expected volatility values of past 20 trading days. In other words, on any given trading day t, a high LREV was inferred if the expected volatility on that day exceeded the median of the distribution comprising of expected volatility values of trading days t through t - 21. High and low volatility was inferred concretely using a binary or binomial classification approach of machine learning as outlined:

- If LREV exceeded the 50% quantile (i.e., median) of the distribution of volatilities for day t - 1 through day $t – 21$, it was deemed high.

- If LREV was less than the 50% quantile of the distribution of volatilities for day t - 1 through day $t – 21$, it was deemed low.

Note that using the 30% or 40% quantile may have been enough to filter out relatively low volatility periods and concomitant momentum regimes. It was set at the 50% quantile to give some margin of error to filter out any noise in the trading signal.

Finally, variant and inert market regimes were inferred as follows. If the momentum regime equaled positive or negative momentum reversal and market volatility was high, then the regime was variant. If the momentum regime equaled positive or negative momentum reversal and volatility was low, then the regime was inert.

In short, the market regimes and their rules can be summarized as follows. Momentum regime indicates both moving averages in long and short time horizon moving average pairs have the same sign. Variant regime indicates both moving averages in long and short time horizon moving average pairs have opposite signs and volatility is high. Inert regime indicates both moving averages in long and short time horizon moving average pairs have opposite signs and volatility is low.

Trading Strategies

As already indicated, two volatility conditioned trading strategies developed in this study to test the hypothesis were tested against the benchmark portfolio formed by using a passive sell-and-hold trading strategy which was neither conditioned on volatility or momentum. The specifics of these trading strategies are discussed next.

Unconditionally short pair trading strategy. This is the benchmark strategy. It shorted the LETF pair from day one without any regard to the regime in which the underlying reference index existed. The portfolio formed by this strategy was used as yardstick to measure the performance of the other two strategies. As discussed in the literature review section, per the published literature, this passive sell-and-hold strategy delivers superior returns during high volatility periods but underperforms the market when the underlying reference index is trending (Guo & Leung, 2014; Xinxin,& Stanley, 2017).

Volatility Short and Momentum Neutral Pair Trading Strategy: Variant 1

This strategy (VSMN1) capitalized on the regimes defined in the earlier section. Trading rules were as follows:

1. Select a long and short LETF pair.

2. Calculate the daily volatility of the reference index which the LETF pair tracks.

3. Calculate the rolling volatility of the reference index using 10 lagging daily returns from the trade day. If this volatility was higher than 20%, short the pair. This was the variant regime defined previously. Conversely, buy the pair back if this rolling 10-day volatility drops below 20%.

4. If the underlying reference index was in momentum regime, stay in cash.

5. If the underlying reference index was in inert regime, stay in cash.

In summary regime switching and trading actions for VSMN1 can also summarized as follows. The momentum regime involves either positive or negative momentum; traders should close the short position because a disadvantageous trending regime is expected. The variant regime involves either a positive or negative momentum reversal and volatility is high; traders should open the short position because LETFs are expected to underperform their indexes due to higher rebalancing costs. The inert regime involves either a positive or negative momentum reversal and volatility is low; traders should close the short position because low volatility does not create enough rebalancing costs for the LETFs and minor profits cannot offset much higher security borrowing costs.

Volatility Short and Momentum Neutral Pair Trading Strategy: Variant 2

The second variant, VSMN2, is a simpler yet effective version of the hypothesis test to check the robustness of the results. This trading strategy was implemented using the following steps:

1. Select the long and short LETF pair.

2. Calculate the daily volatility of the reference index which the LETF pair tracks.

3. If the daily volatility of the reference index exceeded its 50% quantile (or median) over the last 20 days, then short the LETF using 4:00 p.m. (eastern standard time) prices. Conversely, if the reference index volatility was below the 50% quantile, then the pair should be bought back.

Although this strategy did not reference momentum explicitly, it capitalized on the idea of implicit momentum. Wang and Xu (2015) demonstrated an inverse relationship between momentum and volatility. When volatility was low per this trading rule, the only other possible market regime was either inert regime or momentum regime, both of which this strategy sought to avoid. A valid criticism of the VSMN2 trading strategy could be that using a trading signal based on relative expected volatility would cause it to miss remaining shorts on many profitable variant regime days when the long-term expected volatility was still high ($> 20\%$) but the daily rolling LREV was still below the median.

Conversely, in a low long-term volatility market state, the strategy could still end up being short on the days it needed to remain in cash. This could again deteriorate the performance of the LETF pair shorting strategy. At the first look, the probability of this outcome should appear to impact only shorting of the most efficiently rebalanced LETFs which adjust to even minor volatility shocks with low rebalancing costs. This may not apply to commodity LETFs, which are known to show higher tracking errors. However, this still needed to be tested empirically and this investigation was completed through the testing of H_3.

Robustness Check for Momentum Detector used in Variant 1 of Volatility Short Momentum Neutral Trading Strategy

To check whether the moving average-based momentum detection worked, a robustness check was applied. To accomplish this, the VSMN1 strategy was tested by removing the

momentum-based signal and retaining only the volatility signal. This modification of VSMN1 is hereafter referred to as the momentum detection robustness check (MDRC). The robustness check logic here was that if the momentum detector worked as theorized, a strategy that used it must deliver superior returns as compared to the one that used a volatility-based signal only.

Robustness Check for Variant 1 of Volatility Short Momentum Neutral Trading Strategy

In order to make sure that the success or failure of the moving average-based momentum detection strategy was not dependent on specific values of the moving average parameters, this strategy was rerun after varying the lag parameters to check the robustness of results. This approach minimized any selection bias in the choice of moving average parameters.

Accordingly, as indicated earlier, the base VSMN1 strategy had the following long-term and short-term moving average parameters: (a) long-term MA 1 was six months, (b) long-term MA 2 was three months, (c) short-term MA 1 was six weeks, and (d) short-term MA 2 was two weeks. This version of the VSMN1 strategy is hereafter referred to as VSMN1-6362.

The second version of this strategy meant for the robustness check had the following MA lag parameters: (a) long-term MA 1 was five months, (b) long-term MA 2 was two months, (c) short-term MA 1 was four weeks, and (d) short-term MA 2 was two weeks. This robustness check version of VSMN1 strategy is hereafter referred to as VSMN1-5242.

Selection of Leveraged Exchange Traded Funds Industry Sectors

The study was primarily based on LETFs drawn from commodity sectors. Within the commodity sector, mining-based LETFs were also used in study. Following is the list of industry sectors involved in this study: (a) crude oil, (b) natural gas, (c) energy, (d) gold (COMEX), (e) gold (mining), and (f) silver (COMEX).

Additionally, LETFs based on popular equity indexes such as the S&P 500, Rusell 2000 and DJIA were used to investigate whether the trading strategies deployed on low-volatility indexes behave the same way.

The choice of commodity-based LETFs was based largely on the need to have an adequate level of volatility in the reference index prices, which is generally not present in equity-based indexes. Since volatility decay was an important motivator of this study, reference indexes known to be more volatile were naturally more suitable choices. More details regarding LETF and indexes used in this study can be found in data section.

Selection of Trading Signal

Having an optimal trading signal was an important factor for the accuracy of the results of the hypothesis testing. To select the model which best predicted the volatility, reference indexes' daily end of day price data was loaded onto the following volatility forecasting models: seasonal random walk model, implied volatility model, simple moving average model, general autoregressive conditional heteroscedasticity model , popularly known as just GARCH, exponentially weighted moving average model,, and autoregressive integrated moving average model , popularly known as just ARIMA. Volatility forecasting methods are discussed in detail in the next section.

Methods: Volatility Forecasting Models

Volatility Forecasting Models

Following is a brief overview of predictive models used to forecast the volatility of the underlying reference indexes. Since different return series may have different volatility patterns variation in the microstructure of each reference index, study used an inductive approach to learn empirically from the data to discover which volatility model provided best forecast for a given reference index. Loss minimization functions were used to measure the forecasting errors. The volatility models given below are the ones used most commonly across the industry. They were selected to give a wide array of volatility predictive techniques to discover the model which best fitted a given return series.

Seasonal Random Walk -SRW

This model, abbreviated as RW, is based on volatility forecast which is assumed to be previous season's historical volatility plus the mean of the seasonal difference. It also assumes clustering in volatility, but it is agnostic to any assumptions about asymmetric shifts in volatility. This model can be expressed follows:

$$\hat{\sigma}_t = \sigma_{t-m} + \mu \qquad\qquad \text{----------(1)}$$

Where, $\hat{\sigma}_t$ is the volatility forecast for trading day t, σ_{t-m} is the historical volatility for trading day t-m and μ is the mean seasonal difference.

For the purpose of this study, m was assumed to be 1, which means that the model predicted volatility for any given trading day based on volatility of last trading day. μ was assumed to be 0 as well.

Implied Volatility- IV

This model, abbreviated as IV, gives volatility forecast based on expectation of market participants. It is volatility forecast based on well-known Black-Scholes model. In this study , the implied volatility was derived from Chicago Board Options Exchange (Cboe) volatility indexes for each underlying reference index. Cboe uses its proprietary VIX methodology to build these volatility indexes which uses an intricate calculation methodology and is given on Cboe's official website (Cboe, 2018) . It is calculated by using the mid-point of underlying reference index in real-time based on bid/ask quotes on the options contracts written on it. VIX indexes are designed to be a real-time market estimate of the expected volatility of a given index for a period of 30 days (Cboe, 2018).

Refer to Table 2 for the list of CBOE volatility indexes used to derive this volatility forecast.

Simple Moving Average-SMA

This model, abbreviated as SMA, gives volatility forecast based on rolling arithmetic mean of historical volatility for a given number of trading days. This model is expressed as follows:

$$\hat{\sigma}_t = \frac{1}{k} \sum_{i=1}^{k} \sigma_{t\text{-}i} \qquad\qquad \text{--------(2)}$$

Where, $\hat{\sigma}_t$ is the volatility forecast for trading day t, k is number of lags falling between trading days t and i.

Exponentially Weighted Moving Average-EWMA

This model, abbreviated as EWMA, gives volatility forecast using weights that exponentially decrease as we progress back in time. This model is expressed as follows:

$$\hat{\sigma}_t = [(1-\lambda) \sum_{i=1}^{k} \lambda^{i-1} u^2_{t-i+} \lambda^t \sigma^2_{t-k}]^{1/2}$$ ---------(3)

Where, $\hat{\sigma}_t$ is the volatility forecast for trading day t, k is number of lags falling between trading days t and i, λ represents the rate at which weights, α_i, for u_i decline as we progress back in time and u_{t-i} represents percentage change in most recent daily volatility.

Python's *pandas.ewma* function was used estimate this model using its *span* decay parameter. The value of *span* parameter was set as 10 in model estimation.

EWMA model does not incorporate parameter of mean reversion.

Autoregressive Integrated Moving Average- ARIMA (0,1,0)

This model, abbreviated as ARIMA (0,1,0), gives volatility forecast based on the first order differencing of daily volatility. Autoregressive and moving average parameters were modeled to be of degree 0 which assumes the underlying ARIMA process to be comprising a random walk.

$$\hat{\sigma}_t - \sigma_{t-1} = \Delta_t$$ ----------(4)

Alternately, this equation can also be written as follows:

$$\hat{\sigma}_t = \Delta_t + \sigma_{t-1}$$ ----------(5)

Where, $\hat{\sigma}_t$ is the volatility forecast for trading day t, σ_{t-1} is the daily realized volatility of the previous trading day and Δ_t is daily predicted change in realized volatility derived from the estimated parameter value for first degree differencing. Δ_t represents long-term drift and is updated daily using a regression model.

Python's *statsmodels.tsa.arima_model.ARIMA* and *ARIMA.predict* functions were used to estimate the ARIMA regression model and the predicted value for Δ_t.

Generalized Autoregressive Conditional Heteroscedasticity- GARCH (p,q)

This model, abbreviated as GARCH (p,q) , gives volatility forecast based on long-run average variance rate , V_L, in addition to variables used in EWMA model. ARIMA (p,d,q) was

This model is expressed as follows:

$$r_t = \sigma_t k_t \qquad \qquad \text{----------(6)}$$

Where , $\{ r_t \}$ is a time series of daily returns, $\{ k_t \}$ is discrete white noise, with zero mean and unit variance and the forecast of σ_t is denoted by the following model:

$$\hat{\sigma}_t = [\alpha_0 + \sum_{i=1}^{q} \alpha_i \, r^2_{t-I} + \sum_{j=1}^{p} \beta_j \, \sigma^2_{t-j}]^{1/2} \qquad \qquad \text{----------(7)}$$

Where, α_i and β_j are the estimated parameters of the model.

Model implementation approach is given as follows.

1. Calculate the log return series using the closing price of the underlying reference index.

2. Fit ARIMA (p,d,q) model on the return series and estimate parameters p, d and q.

3. Plot the correlogram of the residuals to check for the realization of discrete white noise.

4. If white noise is evidenced (which indicates good fit for the ARIMA model) , square the residuals and plot correlogram again to check for the conditional heteroskedastic behavior.

5. Subject to the evidence of serial correlation in squared residuals, fit GARCH (p,q) model and estimate its parameters.

6. In the final step, square the residuals from GARCH (p,q) model and check for serial correlation. If discrete white noise is obtained use the estimated GARCH model for volatility prediction.

Note that the above standard methodology to empirically use GARCH model and is widely used in the both industry and academia.

Python's *arch.univariate.arch_model* function was used estimate this model.

Loss Functions for Model Selection

The following loss functions were used to calculate the forecasting errors:

1. Mean Absolute Error (MAE)

2. Mean Squared Error (MSE)

3. Root Mean Squared Error (RMSE)

The following formula was used to calculate the forecasting errors.

$$Forecasting\ Error = (MAE+MSE+RMSE)/3 \qquad \text{---------(8)}$$

The model which gave the minimum forecasting error was used to predict volatility to be used as trading signal from respective trading strategies for any given reference index.

Python's *statsmodels.tools.eval_measures* module was used to calculate the values for the loss functions.

Calculation of Realized Volatility used in Volatility Forecasting Models

Realized volatility estimate for the reference index was calculated over a rolling 10-day (or two-week) lagging daily return count at the end of trading day t using the following equations:

$$\sigma_t^d = \sqrt{\frac{1}{k-1}\sum_{t=1}^{k}(r_t - \bar{r})^2} \qquad \text{---------(9)}$$

Where, σ_t^d is the estimated daily standard deviation or volatility of the underlying index on

trading day t, k is the number of trading days over which volatility was estimated, r_t is the return

for trading day t and $\bar{r}$ is mean return over trading days k.

Daily return r_t was calculated using the following equation:

$$r_t = ln\left(\frac{p_t}{p_{t-1}}\right) \qquad\qquad \text{----------(10)}$$

Where, p_t is the closing price for trading day t and p_{t-1} is the price at the end of trading day t-1.

Estimated annualized volatility was calculated using the following equation:

$$\sigma_t^a = \sigma_t^d\sqrt{252} \qquad\qquad \text{----------(11)}$$

Where, σ_t^a is annualized realized volatility at the end of trading day t, σ_t^d is the estimated daily

volatility of the underlying index on trading day.

Python's *numpy* module was used estimate this model.

Calculation of Forecasting Errors for Model Selection

The realized volatility series calculated using the above equations were then were lagged

11 days such that the 11th day realized volatility could be compared with the forecast made 11

days prior. The volatility forecasting model giving the lowest forecasting error based on the

average of all three forecasting error statistics (see equation 8) was then used to generate a

volatility-based trading signal.

Out-of-Sample Data for Calculation of Forecasting Errors

The forecast models giving the lowest forecasting errors were selected using out-of-sample data. The trading strategy execution was done on in-sample data without varying the parameters of the most successful forecasting model derived from out-of-sample data.

For example, if for any LETF pair the sample testing period was August 16 , 2011 through August 15, 2016, volatility model selection for back testing the trading strategy was based on sample data prior to August 16 , 2011. Volatility model used for generating the trading signal was not varied for the in-sample period August 16 , 2011 through August 15, 2016.

Minimum out-of-sample period for the calculation of forecasting errors was kept at one year and maximum period was kept at three years.

Methods: Transaction Costs

Commission Cost Model

Following Interactive Brokers, commissions were calculated per trade based on $0.005 per share (Interactive Brokers, n.d.). The portfolio was rebalanced to ensure complete long and short offset each time a trade was executed for the active trading strategy.

Slippage Model

Slippage refers to the difference between the intended price for order execution and the price realized upon actual order execution. Slippage is thus a function of execution quality. Keim and Madhavan (1998) showed how order-placement-strategy and trading difficulty influence equity trading costs in different ways. They also showed that factors affecting trading difficulty include order size, trading duration, order type, relative illiquidity, market momentum, market volatility , trader skill, etc. Slippage can often also work favorably for the trader through price improvement whereby a better price than what was intended could be realized due to market factors such as momentum and volatility. Trading costs prediction is a vast topic in its own right

Greer, Brorsen, and Liu (1992) showed slippage costs to be around 0.14% of the value of futures contract. Although Keim and Madhavan (1998) did not model slippage costs separately , they showed estimated transaction costs to be around 9 basis points or 0.09% for equity funds of the size of $1 billion using a single-day trading order execution strategy.

In this study conservative estimates were made to model the slippage costs even though in certain market scenarios on an average positive and negative slippage often offset each other with a negligible residual either way. This study modeled slippage to be 0.30% of the order

value. This means that for opening the short position order the price fill was discounted at 0.30% and for closing the short position a like premium was applied to the order value.

The reason for this conservative assumption about slippage costs , the study does not model any systematic order execution algorithm like Volume Weighted Average Price (VWAP) which could always ensure the highest execution quality.

Measurements

Performance Metrics

For testing H_{1a}, which focused on the outperformance of trading strategies VSMN1 and VSMN2 over the benchmark strategies on the basis of absolute returns, compound annual growth rate (CAGR) was computed. For testing H_{1b}, which did the same but on a risk adjusted basis, the Sharpe ratio and Sortino ratio were computed.

Sharpe Ratio

The following formula was used to compute the Sharpe ratio:

$$Sharpe\ Ratio = (R_p - R_f) / ò_p \qquad \text{----------(12)}$$

Where R_p is the daily return of the portfolio, R_f is the risk free rate, and $ò_p$ is the daily standard deviation of portfolio.

Sortino Ratio

The following formula was used to compute the Sortino ratio:

$$Sortino\ Ratio = (R_p - R_f) / ò_{nrp} \qquad \text{----------(13)}$$

Where R_p is the daily return of the portfolio, R_f is the risk-free rate, and $ò_{nrp}$ is the daily standard deviation of negative returns of the portfolio.

Assumptions about the Minimal Acceptable Return for the Sortino Ratio

For most studies, a minimal acceptable return (MAR) equal to the expected market return would be more appropriate. In the present study, however, expected market return did not add as much value as MAR because the study's objective was to measure the performance of the active

trading strategy against a passive sell-and-hold based benchmark and not against the market

portfolio-based benchmark. For this reason, MAR was assumed to be 0% for both the active

LETF trading strategy and the passive sell-and-hold benchmark.

Relevance and Efficacy of the Sortino Ratio as a Performance Measure

Sortino and Vandermeer (1991) provided a downward variance-based risk framework

and demonstrated its utility for goal driven investment plans. Sharpe (1994) offered a now very

popular Sharpe ratio performance measure with an underlying assumption of symmetrical return

distributions. Further, Sharpe's (1994) framework assumed that investors or traders treat both

upward and downward variances in returns the same way. Sortino and Price (1994) argued for a

performance measure that did not penalize upward variance in returns which are above MAR and

also relaxed the normal distribution assumption for the return series which is implicit in Sharpe's

performance measure framework. The Sortino ratio as a performance measure was created as

part of this discourse.

Pair trading strategies based on LETFs tend to show skewed return distributions in most

cases. Chaudhary and Johnson (2008) showed that the Sortino ratio achieves higher power when

return distributions are asymmetric and its relative bias is somewhat lower than the Sharpe ratio

as a performance measure. From a practitioner viewpoint, the Sortino ratio as a performance

measure appears to make more practical sense as traders tend not to treat upward variance in

returns in the same way as downward variance. Jarrow, Teo, Tse, and Warachka (2012) offered

a modified revised test of statistical arbitrage motivated by the same idea. For these reasons, the

Sortino ratio was included as a performance measure in this study.

Assumptions about the Risk-Free Rate

For the purpose of this study, the risk-free rate was assumed to be 0%. Since the objective of this study was to compare the relative performance of active trading strategies against a benchmark strategy and the same risk-free rate was assumed in each calculation, the assumptions about the risk-free rate do not make any material difference to comparative performance measures.

Derivation of the Theoretical Compound Annual Growth Rate

The following is the standard formula for CAGR:

$$CAGR = (Ending\ Value\ /\ Beginning\ Value)^{(1\ /\ Number\ of\ Years)} - 1 \qquad \text{----------(14)}$$

This study used a theoretical approach using the cost of capital concept and the discounted cash flow method to define the beginning value of the portfolio. It was defined as the opportunity cost of collateral posted to borrow shorted securities over the entire holding period discounted to its present value.

The reason behind this approach was that even though statistical arbitrage strategies are self-financing in the sense that the portfolio is formed without the purchase of any securities, there are intangible borrowing costs involved beyond the margin interest. Another reason is that the beginning value of statistical arbitrage portfolios is theoretically zero. This makes this formula impossible to use without some modification.

For example, the long and short portfolios in this study were formed by establishing equally weighted positions worth $100,000. To short securities worth $100,000, a collateral of roughly 34% is usually required to be posted by the broker. This means that this $34,000 would be blocked for the duration of the holding period. This money could only be held in treasuries or

in cash, which in turn means that the trader would be losing average market returns on this amount. Assuming a pre-inflation return on investment earned by the S&P 500 in the holding period to be the opportunity cost, a notional gross investment value equivalent to the holding period S&P 500 return calculated on collateral value of $34,000 per year was established. This annual gross investment was then viewed as the opportunity cost for the trader for each successive year of the holding period.

The beginning value for CAGR was then computed by aggregating these successive opportunity cost increments after discounting them to the present value as applicable on the first day of the holding period. The discount rate, r, used is the average annual return of the S&P 500. The Discount Cash Flow formula used was:

$$DCF = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}$$ ----------(15)

Where, DCF is Discounted Cashflow, CF_t is the opportunity cost of year t and r is the discount rate.

Portfolio Net Asset Value

Computation of portfolio volatility, daily return, and portfolio end value for TCAGR required the daily portfolio value. For this purpose, daily gross asset value (*GAV*) and net asset value (*NAV*) were calculated using the following formula:

Daily GAV = [(P₁ of Short LETF / Pₙ of Short LETF) * 0.5+ (P₁ of Long LETF /

Pₙ of Long LETF) * 0.5] * Total Amount Shorted ----------(16)

Where P_1 is the price of the security on day one of the holding period and P_n is the price of the security on day n of the holding period.

$$Daily\ NAV = Daily\ GAV - Accrued\ Cost\ of\ Borrowed\ Securities - Commission$$

$$Costs \qquad\qquad \text{---------(17)}$$

Based on information obtained by this author from Interactive Brokers, the cost of borrowing LETF securities was assumed as 6% per annum.

Tests for Statistical Inference

The *t* Test and *z* Test

The *t* test and *z* test were used to study the difference of performance metric values between back tested trading strategies and the benchmark trading strategy. The *t* test and *z* test were used for a sample count below and above 30, accordingly.

Number of Periods Test

The number of periods test is quoted in the results. It was used to calculate the win ratio of the active strategies over the benchmark strategy based on the number of periods under consideration. However, these figures are stated for informational purposes only. They played no role in drawing statistical inference in the hypotheses testing, which was solely based on the *t* test and *z* test.

Data

Daily price data was downloaded from the following sources: Sierra Charts Website (a premier industrial broker used by professional traders), S&P Dow Jones Indices Website, and CBOE Website. The study used the daily close prices to model the trade prices.

A detailed description LETFs, indexes, and the volatility model used to predict expected volatility is provided in Table 1.

Table 2

Leveraged Exchange Traded Funds Data in Detail

Security Number	Security Description[1]	Leverage Multiple[2]	Sector[3]	Index[4]	Volatility Prediction Model[5]
1.	NASDAQ[m]:DSLV - VelocityShares 3x Inverse Silver ETN[a]	-3x	Silver (Comex[b])	S&P GSCI Silver Index ER[c]	EWMA[d]
2.	NASDAQ:USLV - VelocityShares 3x Long Silver ETN	3x	Silver (Comex)	S&P GSCI Silver index ER	EWMA
3.	NASDAQ:DGLD - VelocityShares 3x Inverse Gold ETN	-3x	Gold (Comex)	S&P GSCI Gold Index ER[e]	Implied Volatility / CBOE[f]: GVZ[g]
4.	NASDAQ:UGLD - VelocityShares 3x Long Gold ETN	3x	Gold (Comex)	S&P GSCI Gold Index ER	Implied Volatility / CBOE: GVZ
5.	NYSE[n]:DUST- Direxion Daily Gold Miners Short 3X ETF	-3x	Gold (Mining)	NYSE Arca Gold Miners Index	EWMA
6.	NYSE:NUGT- Direxion Daily Gold Miners Long 3X ETF	3x	Gold (Mining)	NYSE Arca Gold Miners Index	EWMA
7.	NYSE:SPXL - Direxion Daily S&P500 Long 3X ETF	-3x	Equity	S&P 500 Index	Implied Volatility / CBOE:VIX[h]

Security Number	Security Description[1]	Leverage Multiple[2]	Sector[3]	Index[4]	Volatility Prediction Model[5]
8.	NYSE:SPXS- Direxion Daily S&P 500 Short 3X ETF	3x	Equity	S&P 500 Index	Implied Volatility / CBOE:VIX
9.	NYSE:SRTY- ProShares UltraPro Short Russell2000	-3x	Equity	Russell 2000 Index	Realized Volatility
10.	NYSE:URTY- ProShares UltraPro Russell2000	3x	Equity	Russell 2000 Index	Realized Volatility
11.	NYSE:SDOW- ProShares UltraPro Short Dow30	-3x	Equity	Dow Jones Industrial Average Index	Realized Volatility
12.	NYSE:UDOW- ProShares UltraPro Dow30	3x	Equity	Dow Jones Industrial Average Index	Realized Volatility
13.	NYSE:DUG- ProShares UltraShort Oil & Gas	2x	Energy	Dow Jones U.S. Oil & Gas[SM] Index	Implied Volatility / CBOE: VXXLE[i]
14.	NYSE:DIG- ProShares Ultra Oil & Gas	-2x	Energy	Dow Jones U.S. Oil & Gas[SM] Index	Implied Volatility / CBOE: VXXLE

Security Number	Security Description[1]	Leverage Multiple[2]	Sector[3]	Index[4]	Volatility Prediction Model[5]
15.	NYSE:SCO- ProShares UltraShort Bloomberg Crude Oil	-2x	Crude Oil	Bloomberg WTI[k] Crude Oil Subindex[SM]	GARCH(1,1)
16.	NYSE:UCO- ProShares Ultra Bloomberg Crude Oil	2x	Crude Oil	Bloomberg WTI Crude Oil Subindex[SM]	GARCH(1,1)
17.	TSE[o]:HND- BetaPro Natural Gas -2x Daily Bear ETF	-2x	Natural Gas	NYMEX Natural Gas Futures Contract	Implied Volatility / CBOE: VXXLE
18.	TSE:HNU- BetaPro Natural Gas 2x Daily Bull ETF	2x	Natural Gas	NYMEX Natural Gas Futures Contract	Implied Volatility / CBOE: VXXLE
19.	TSE:HGD- BetaPro Cdn Gold Miners -2x DlyBear ETF	-2x	Gold (Mining)	S&P/TSX Global Gold Index[p]	EWMA
20.	TSE:HGU- BetaPro Cdn Gold Miners 2x DlyBull ETF	2x	Gold (Mining)	S&P/TSX Global Gold Index	EWMA
21.	TSE:HZD- BetaPro Silver -2x Daily Bear ETF	-2x	Silver (Comex)	COMEX Silver Futures Contract	Implied Volatility / CBOE: GVZ
22.	TSE:HND- BetaPro Natural Gas -2x Daily Bear ETF	2x	Silver (Comex)	COMEX Silver Futures Contract	Implied Volatility / CBOE: GVZ

(continued)

Security Number	Security Description[1]	Leverage Multiple[2]	Sector[3]	Index[4]	Volatility Prediction Model[5]
23.	TSE:HOU- BetaPro Crude Oil 2x Daily Long ETF	-2x	Crude Oil	NYMEX[1] Light Sweet Crude Oil Futures Contract	GARCH(1,1)
24.	TSE: HOD- BetaPro Crude Oil -2x Daily Short ETF	2x	Crude Oil	NYMEX Light Sweet Crude Oil Futures Contract	GARCH(1,1)
25.	NYSE:JDST- Direxion Daily Jr Gld Mnrs Bear 3X ETF	-3x	Gold (Mining)	MVIS[q] Global Junior Gold Miners Index	Implied Volatility / CBOE: GVZ
26.	NYSE:JNUG- Direxion Daily Jr Gld Mnrs Bull 3X ETF	3x	Gold (Mining)	MVIS Global Junior Gold Miners Index	Implied Volatility / CBOE: GVZ
27.	NYSE:DGAZ- VelocityShares 3x Long Natural Gas ETN	-3x	Natural Gas	GSCI Natural Gas Index ER[r]	Implied Volatility / CBOE: VXXLE
28.	NYSE:UGAZ- VelocityShares 3x Inv Natural Gas ETN	3x	Natural Gas	GSCI Natural Gas Index ER	Implied Volatility / CBOE: VXXLE
29.	NYSE:KOLD- ProShares UltraShort Bloomberg Natural Gas	-2x	Natural Gas	Bloomberg Natural Gas Subindex[SM]	Implied Volatility / CBOE: VXXLE
30.	NYSE:BOIL- ProShares Ultra Bloomberg Natural Gas	2x	Natural Gas	Bloomberg Natural Gas Subindex[SM]	Implied Volatility / CBOE: VXXLE

Note. This table reports the LETF instruments used in this study. Following is the description of major headings:

1- Security Description: Gives a description of the security as listed on the exchange and name of the exchange on which the LETF is listed. For example, in 'NYSE: UGAZ- VelocityShares 3x Inv Natural Gas ETN', NYSE is reference to the New York Stock Exchange, UGAZ is the ticker symbol, and 'UGAZ- VelocityShares 3x Inv Natural Gas ETN' is the official description given for this security on exchange.

2- Leverage Multiple: Gives the leverage the LETF promises on the daily return. For example, 2x means two times the daily return and -2x means two times the inverse of daily return.

3- Sector: Gives industry sector for the LETF.

4- Index: The index tracked by the LETF.

5- Volatility Prediction Model: Gives details of the volatility model used to predict volatility. For example, Implied Volatility / CBOE: VXXLE means that the volatility model used was implied volatility. VXXLE is the ticker symbol for the energy VIX index and CBOE means that the implied volatility index for the energy sector is listed on Chicago Board of Option Exchange.

Significant abbreviations are expanded below:

a- ETN: Exchange Traded Note

b- Comex: Primary market for futures and options for trading metals.

c- S&P GSCI Silver Index ER: Standard & Poor's Goldman Sachs Composite Silver Index Excess Return

d- EWMA: Exponentially Weighted Moving Average

e- S&P GSCI Gold Index ER: Standard & Poor's Goldman Sachs Composite Gold Index Excess Return

f- CBOE: Chicago Board Options Exchange

g- GVZ: The Cboe Gold ETF Volatility Index

h- VIX: The Cboe Volatility Index

i- VXXLE: The Cboe Energy Sector ETF Volatility Index

j- OVX: The Cboe Crude Oil ETF Volatility Index

k- WTI: West Texas Intermediate

[l-] NYMEX: New York Mercantile Exchange

[m-] NASDAQ: National Association of Securities Dealers Automated Quotations (name of the exchange)

[n-] NYSE: New York Stock Exchange

[o-] TSE: Toronto Stock Exchange

[p-] S&P/TSX Global Gold Index: Standard & Poor's Toronto Stock Exchange Global Gold Index

[q-] MVIS: Market Vectors Index Solutions

[r-] GSCI Natural Gas Index ER: Goldman Sachs Composite Gas Index Excess Return

Table 3

Leveraged Exchange Traded Funds Pairs Sample Periods

Security Number	Leveraged Exchange Traded Fund Pairs[a]	Sample Period[b]
1.	Crude Oil Index LETF Pair (NYSE[c]: SCO and NYSE: UCO)	8 years (July 10, 2010–July 9, 2018)
2.	Crude Oil Index LETF Pair (TSE: HOD and NYSE: HOU)	9 years (February 17, 2009–February 16, 2018)
3.	Energy Index LETF Pair (NYSE: DIG and NYSE: DUG)	6 years (February 28, 2012–February 27, 2018)
4.	Nat Gas Index LETF Pair (NYSE: KOLD and NYSE: BOIL)	6 years (February 28, 2012–February 27, 2018)
5.	Nat Gas Index LETF Pair (NYSE: DGAZ and NYSE: UGAZ)	5 years (October 16, 2012–October 13, 2017)
6.	Nat Gas Index LETF Pair (TSE[e]: HND and TSE: HNU)	6 years (February 28, 2012–February 27, 2018)
7.	Gold Commodity Index LETF Pair (NASDAQ[d]: DGLD and NASDAQ: UGLD)	6 years (September 5, 2012–September 4, 2018)
8.	Gold Mining Index LETF Pair (TSE: HGD and TSE: HGU)	9 years (April 3, 2009–April 2, 2018)

Security Number	Leveraged Exchange Traded Fund Pairs[a]	Sample Period[b]
9.	Gold Mining Index LETF Pair (NYSE: DUST and NYSE: NUGT)	5 years (September 12, 2013–September 11, 2018)
10.	Junior Gold Mining Index LETF Pair (NYSE: JDST and NYSE: JNUG)	5 years (September 12, 2013–September 11, 2018)
11.	Silver Commodity Index LETF Pair (NASDAQ: DSLV and NASDAQ: USLV)	6 years (February 28, 2012–February 27, 2018)
12.	Silver Commodity Index LETF Pair (TSE: HZD and TSE: HXU)	6 years (February 28, 2012–February 27, 2018)
13.	S&P500 Index LETF Pair (NYSE: SPSX and NYSE: SPXL)	9 years (January 5, 2009–January 4, 2018)
		9 years (November 21, 2008–November 20, 2017)
		9 years (February 27, 2009–February 26, 2018)
		9 years (May 4, 2009–May 3, 2018)
		9 years (July 1, 2009–June 29 2018)
14.	DJIA Index LETF Pair (NYSE:SDOW and NYSE: UDOW)	8 years (February 17, 2010–February 16 2018)
		8 years (April 20, 2010–April 19 2018)
		8 years (June 23, 2010–June 22 2018)

(continued)

Security Number	Leveraged Exchange Traded Fund Pairs[a]	Sample Period[b]
		8 years (August 3, 2010–August 2 2018)
		8 years (September 15, 2010–September 14 2018)
15.	RUSSELL 2000 Index LETF Pair (NYSE: SRTY and NYSE: URTY)	8 years (September 15, 2010–September 14, 2018)
		8 years (February 16, 2010–February 14, 2018)
		8 years (April 19, 2010–April 18, 2018)
		8 years (June 23, 2010–June 22, 2018)
		8 years (July 27, 2010–July 26, 2018)

Note. This table reports the LETF pairs used in test trading strategies and sampling periods applicable to them. Given below is the description of major headings:

[a] Leveraged Exchange Traded Fund Pair: Long and short LETF pairs used to form market neutral portfolios. For example, in Nat Gas Index LETF Pair (NYSE: DGAZ and NYSE: UGAZ), NYSE is in reference to the New York Stock Exchange, UGAZ and DGAZ are the ticker symbols, and NAT GAS Index is the reference index tracked by LETF pair. Please refer to Security Description section of Table 2 for description of the symbols.

[b] Sample Period: The time range in which LETF data was sampled

[c] NYSE: New York Stock Exchange

[d] NASDAQ: National Association of Securities Dealers Automated Quotations (name of the exchange)

[e] TSE: Toronto Stock Exchange

Results

This section reviews the results of the study. First result tables are given. Thereafter, the results are interpreted to discuss evidence for hypotheses testing. Robustness checks are discussed next.

The *t* Value and *z* Score Tables

Given next are the *t* value and *z* score value tables for the excess return of each trading strategy, including the one for the robustness checks. All excess returns were calculated in reference to the benchmark strategy, USPTS, unless otherwise noted. Appendix A contains a few representative examples of charts giving visual insight into how the trading strategies behaved during their execution.

Table 4

*Long-Term Commodities Leveraged Exchange Trade Funds Return **t** Values*

Strategy[1]	Long-Term Mean Excess Return & SD[2]	Long-Term Excess Return t Value / z Score[3]	Test Type[4]	Win Ratio[5]	Sample Size[6]	Performance Measure[7]
VSMN1-6362	0.902/1.198	1.564* (0.073)	t value	58.33%	12	Sortino Ratio
VSMN1-5242	0.892/2.102	1.470* (0.085)	t value	58.33%	12	Sortino Ratio
VSMN2	2.146/3.149	2.361 *** (0.019)	t value	83.33%	12	Sortino Ratio
All Combined[a]	1.313/2.541	3.101*** (0.0096)	z-score	66.67%	36	Sortino Ratio
VSMN1-6362 / MDRC[8]	0.011/ 2.089	0.017 (0.493)	t value	41.67%	12	Sortino Ratio
VSMN1-5242 / MDRC[9]	0.001/1.643	0.002 (0.499)	t value	33.33%	12	Sortino Ratio
VSMN1-6362	0.070/0.462	0.527 (0.304)	t value	50.00%	12	Sharpe Ratio
VSMN1-5242	0.069/0.404	0.593 (0.283)	t value	58.33%	12	Sharpe Ratio
VSMN2	0.250/0.305	2.846 ***(0.008)	t value	83.33%	12	Sharpe Ratio
VSMN1-6362 / MDRC[8]	-0.050/0.337	-0.517 (0.692)	t value	41.67%	12	Sharpe Ratio
VSMN1-5242 / MDRC[9]	-0.051/0.295	-0.604 (0.721)	t value	50.00%	12	Sharpe Ratio
VSMN1-6362	-2.622%/ 9.911%	-0.083 (0.849)	t value	58.33%	12	Absolute Return

Strategy[1]	Long-Term Mean Excess Return & SD[2]	Long-Term Excess Return t Value / z Score[3]	Test Type[4]	Win Ratio[5]	Sample Size[6]	Performance Measure[7]
VSMN1-5242	-2.827% / 9.046%	-1.101 (0.853)	t value	66.67%	12	Absolute Return
VSMN2	-2.295%/4.834%	-1.645 (0.93)	t value	41.67%	12	Absolute Return
VSMN1-6362 / MDRC[8]	0.230%/9.208%	0.086 (0.466)	t value	75.00%	12	Absolute Return
VSMN1-5242 / MDRC[9]	27.930%/130.006%	-0.744 (0.764)	t value	41.67%	12	Absolute Return

Note. This table reports the t value and z score value statistics for each strategy. Following is the description of each heading:

1. Strategy: Name of the trading strategy. Following is the full expansions of strategy names used:

   a. VSMN1-6362:The description of the name of this strategy is Volatility Short Momentum Neutral-6362. Term 6362 indicates that the momentum oscillator was based on four moving averages each corresponding to 6 months, 3 months, 6 weeks and 2 weeks periods.

   b. VSMN1-5242: The description of the name of this strategy is Volatility Short Momentum Neutral-5242. Term 5242 indicates that the momentum oscillator was based on four moving averages each corresponding to 5 months, 2 months, 4 weeks and 2 weeks periods.

   c. VSMN2: The description of the name of this strategy is Volatility Short Momentum Neutral-2. This strategy does not use an explicit momentum oscillator to detect momentum. It capitalizes on the idea of implicit momentum based on the theory that momentum and volatility cannot exist simultaneously. The low volatility regime is hypothesized to encapsulate momentum regime as well.

   d. MDRC: The description of the name of this strategy is Momentum Detection Robustness Check. This is the modification of VSMN1 strategy. This was the robustness check developed to test that the momentum detector worked as theorized whereby a strategy that used it had to deliver superior returns as compared to the one that used a volatility-based signal only. In other words, this strategy was used as a performance benchmark to test the effectiveness of the variants of the VSMN1 strategy which used explicit momentum detector.

e. All Combined: This row aggregates the performance data of VSMN1-6362, VSMN1-5242, and VSMN2 to gain additional statistical power. This test value reinforced the robustness of the result obtained for the Sortino ratio using performance data for each strategy individually.

f. USPTS: Benchmark strategy Unconditionally Short Pair Trading Strategy

2. Long-Term Mean Excess Return & *SD*: Mean and standard deviation values for the excess return over the benchmark strategy, Unconditionally Short Pair Trading Strategy (USPTS) . In the last row for each performance measure, the excess return given is the excess return of VSMN1-6362 and VSMN1-5242 over MDRC strategy as a benchmark.

3. Long-Term Excess Return: *t* value or *z* score value for the excess return over the benchmark strategy, USPTS. In the last row, the excess return given is the excess return of VSMN1-6362 over MDRC as benchmark. Corresponding *p* values are given in the parentheses.

4. Win Ratio: Percentage of sample periods in which the strategy outperformed the benchmark.

5. Sample Size: Number of executions done for each trading strategy. For commodities, there was one strategy run for each LETF pair.

6. Test Type: Indicates whether T-Test or Z-Test was used based on sample size.

7. Performance Measure: Type of performance metric used to measure the return.

8. VSMN1-6362 / MDRC: This row gives the values for the excess return of VSMN1-6362 strategy over MDRC.

9. VSMN1-5242 / MDRC: This row gives the values for the excess return of VSMN1-6362 strategy over MDRC.

* Means the test value is significant at 90% confidence interval.

** Means the test value is significant at 95% confidence interval.

*** Means the test value is significant at 99% confidence interval.

Table 5

Annual Commodities Leveraged Exchange Trade Funds   Return z Scores

Strategy[1]	Annual Mean Excess Return & SD[2]	Annual Excess Return z Score[3]	Win Ratio[4]	Sample Size[5]	Performance Measure[6]
VSMN1-6362	18.432/93.646	1.738** (0.041)	41.03%	78	Sortino Ratio
VSMN1-5242	15.685/79.280	1.736 ** (0.041)	34.62%	78	Sortino Ratio
VSMN2	10.435/70.469	1.308* (0.095)	33.33%	78	Sortino Ratio
All Combined[a]	14.915/73.782	3.0923*** (0.001)	36.32%	234	Sortino Ratio
VSMN1-6362 / MDRC[7]	-5956.945/49573.338	-1.061 (0.856)	52.56%	78	Sortino Ratio
VSMN1-5242 / MDRC[8]	-5959.878/49572.787	-1.062 (0.856)	46.15%	78	Sortino Ratio
VSMN1-6362	0.049/0.637	0.677 (0.249)	57.69%	78	Sharpe Ratio

(continued)

Strategy[1]	Annual Mean Excess Return & SD[2]	Annual Excess Return z Score[3]	Win Ratio[4]	Sample Size[5]	Performance Measure[6]
VSMN1-5242	-0.009/0.532	-0.1436 (0.557)	53.85%	78	Sharpe Ratio
VSMN2	0.069/0.538	1.139 (0.127)	55.13%	78	Sharpe Ratio
VSMN1-6362 / MDRC[7]	184.470/682.346	2.388*** (0.008)	57.69%	78	Sharpe Ratio
VSMN1-5242 / MDRC[8]	184.413/682.348	2.387*** (0.008)	57.69%	78	Sharpe Ratio
VSMN1-6362	10.192%/110.899%	0.747 (0.228)	51.52%	66	Absolute Return
VSMN1-5242	12.132%/104.700%	0.941 (0.173)	50.00%	66	Absolute Return
VSMN2	-6.988%/51.063%	-1.112 (0.867)	39.74%	66	Absolute Return

Strategy[1]	Annual Mean Excess Return & SD[2]	Annual Excess Return z Score[3]	Win Ratio[4]	Sample Size[5]	Performance Measure[6]
VSMN1-6362 / MDRC[7]	305.505%/2717.300% 113.346%/416.246% 73.038%/381.650% [c]	- 0.913 (0.819) / 2.178**(0.015) / 1.457* (0.0725)[c]	52.56% / 57.69% / 53.03%[c]	66/64/58[c]	Absolute Return
VSMN1-5242/ MDRC[8]	-303.565%/956.588% 115.829%/420.488% 71.072%/375.669% [c]	- 0.2.578 (0.995) / 2.238** (0.013) / 1.441* (0.075)[c]	54.55% / 56.25% / 56.90%[c]	66/64/58[c]	Absolute Return

Note. This table reports the z score value statistics for each strategy. Following is the description of each heading:

1. Strategy: Name of the trading strategy. Following is the full expansions of strategy names used:

    a. VSMN1-6362:The description of the name of this strategy is Volatility Short Momentum Neutral-6362. Term 6362 indicates that the momentum oscillator was based on four moving averages each corresponding to 6 months, 3 months, 6 weeks and 2 weeks periods.

    b. VSMN1-5242: The description of the name of this strategy is Volatility Short Momentum Neutral-5242. Term 5242 indicates that the momentum oscillator was based on four moving averages each corresponding to 5 months, 2 months, 4 weeks and 2 weeks periods.

    c. VSMN2: The description of the name of this strategy is Volatility Short Momentum Neutral-2. This strategy does not use an explicit momentum oscillator to detect momentum. It capitalizes on the idea of implicit momentum based on the theory that momentum and volatility cannot exist simultaneously. The low volatility regime is hypothesized to encapsulate momentum regime as well.

    d. MDRC: The description of the name of this strategy is Momentum Detection Robustness Check. This is the modification of VSMN1 strategy. This was the robustness check developed to test that the momentum detector worked as theorized whereby a

      strategy that used it had to deliver superior returns as compared to the one that used a volatility-based signal only. In other words, this strategy was used as a performance benchmark to test the effectiveness of the variants of the VSMN1 strategy which used explicit momentum detector.

    e.   All Combined: This row aggregates the performance data of VSMN1-6362, VSMN1-5242, and VSMN2 to gain additional statistical power. This test value reinforced the robustness of the result obtained for the Sortino ratio using performance data for each strategy individually.

    f.   USPTS: Benchmark strategy Unconditionally Short Pair Trading Strategy

2.   Annual Mean Excess Return & *SD*: Mean and standard deviation values for the excess return over the benchmark strategy, Unconditionally Short Pair Trading Strategy (USPTS) . In the last row for each performance measure, the excess return given is the excess return of VSMN1-6362 and VSMN1-5242 over MDRC strategy as a benchmark.

3.   Annual Excess Return *z* Score: *z* score value for the excess return over the benchmark strategy, USPTS. In the last row, the excess return given is the excess return of VSMN1-6362 over MDRC as benchmark. Corresponding *p* values are given in the parentheses.

4.   Win Ratio: Percentage of sample periods in which the strategy outperformed the benchmark.

5.   Sample Size: Number of executions done for each trading strategy. For commodities, there was one strategy run for each Leveraged Exchange Traded Funds (LETF) pair.

6.   Performance Measure: Type of performance metric used to measure the return.

7.   VSMN1-6362 / MDRC: This row gives the values for the excess return of VSMN1-6362 strategy over MDRC.

8.   VSMN1-5242 / MDRC: This row gives the values for the excess return of VSMN1-6362 strategy over MDRC.

[*] Means the test value is significant at 90% confidence interval.

[**] Means the test value is significant at 95% confidence interval.

[***] Means the test value is significant at 99% confidence interval.

Table 6

Long-Term Non-Commodities Leveraged Exchange Trade Funds   Return t *Values*

Strategy[1]	Long-Term Mean Excess Return & SD[2]	Long-Term Excess Return *t* Value[3]	Win Ratio[4]	Sample Size[5]	Performance Measure[6]
VSMN2[a]	13.475/50.591	1.032 (0.160)	46.67%	15	Sortino Ratio
VSMN2[a]	-0.051/0.575	-0.341 (0.631)	40.00%	15	Sharpe Ratio
VSMN2[a]	0.633%/3.116%	0.787 (0.222)	60.00%	15	Absolute Return

Note. This table reports the *t* value statistics for each strategy. Following is the description of each heading:

1. Strategy: Name of the trading strategy. Following is the full expansions of strategy names used:

    a. VSMN2: The description of the name of this strategy is Volatility Short Momentum Neutral-2. This strategy does not use an explicit momentum oscillator to detect momentum. It capitalizes on the idea of implicit momentum based on the theory that momentum and volatility cannot exist simultaneously. The low volatility regime is hypothesized to encapsulate momentum regime as well.

    b. VSMN1-6362:The description of the name of this strategy is Volatility Short Momentum Neutral-6362. Term 6362 indicates that the momentum oscillator was based on four moving averages each corresponding to 6 months, 3 months, 6 weeks and 2 weeks periods. VSMN1 strategy variants for non-commodity LETF did not trade for multiple years due to low volatility and did not provide any meaningful statistics to compare and are hence not reported.

    c. VSMN1-5242: The description of the name of this strategy is Volatility Short Momentum Neutral-5242. Term 5242 indicates that the momentum oscillator was based on four moving averages each corresponding to 5 months, 2 months, 4 weeks and 2 weeks periods. VSMN1 strategy variants for non-commodity LETF did not trade for multiple years due to low volatility and did not provide any meaningful statistics to compare and are hence not reported.

      d.   USPTS: Benchmark strategy Unconditionally Short Pair Trading Strategy

2.   Long-Term Mean Excess Return & *SD*: Mean and standard deviation values for the excess return over the benchmark strategy, Unconditionally Short Pair Trading Strategy (USPTS) . In the last row for each performance measure, the excess return given is the excess return of VSMN1-6362 and VSMN1-5242 over MDRC strategy as a benchmark.

3.   Long-Term Excess Return *t* Value: *t* value for the excess return over the benchmark strategy, USPTS. In the last row, the excess return given is the excess return of VSMN1-6362 over MDRC as benchmark. Corresponding *p* values are given in the parentheses.

4.   Win Ratio: Percentage of sample periods in which the strategy outperformed the benchmark.

5.   Sample Size: Number of executions done for each trading strategy. For commodities, there was one strategy run for each Leveraged Exchange Traded Funds (LETF) pair.

6.   Performance Measure: Type of performance metric used to measure the return.

* Means the test value is significant at 90% confidence interval.

** Means the test value is significant at 95% confidence interval.

*** Means the test value is significant at 99% confidence interval.

Table 7

Annual Non-Commodities Leveraged Exchange Trade Funds   Return z Scores

Strategy[1]	Annual Mean Excess Return & SD[2]	Annual Excess Return z Score[3]	Win Ratio[4]	Sample Size[5]	Performance Measure[6]
VSMN2[a]	0.136/78.810	0.019 (0.492)	11.01%	125	Sortino Ratio
VSMN2[a]	-0.387/8.320	-0.521 (0.699)	56.88%	125	Sharpe Ratio
VSMN2[a]	3.544%/32.050%	-1.154 (0.876)	44.95%	109	Absolute Return

Note: This table reports the z score statistics for each strategy. Following is the description of each heading:

1. Strategy: Name of the trading strategy. Following is the full expansions of strategy names used:
    a. VSMN2: The description of the name of this strategy is Volatility Short Momentum Neutral-2. This strategy does not use an explicit momentum oscillator to detect momentum. It capitalizes on the idea of implicit momentum based on the theory that momentum and volatility cannot exist simultaneously. The low volatility regime is hypothesized to encapsulate momentum regime as well.
    b. VSMN1-6362:The description of the name of this strategy is Volatility Short Momentum Neutral-6362. Term 6362 indicates that the momentum oscillator was based on four moving averages each corresponding to 6 months, 3 months, 6 weeks and 2 weeks periods. VSMN1 strategy variants for non-commodity LETF did not trade for multiple years due to low volatility and did not provide any meaningful statistics to compare and are hence not reported.
    c. VSMN1-5242: The description of the name of this strategy is Volatility Short Momentum Neutral-5242. Term 5242 indicates that the momentum oscillator was based on four moving averages each corresponding to 5 months, 2 months, 4 weeks and 2 weeks periods. VSMN1 strategy variants for non-commodity LETF did not trade for multiple years due to low volatility and did not provide any meaningful statistics to compare and are hence not reported.
    d. USPTS: Benchmark strategy Unconditionally Short Pair Trading Strategy

2. Annual Mean Excess Return & *SD*: Mean and standard deviation values for the excess return over the benchmark strategy, Unconditionally Short Pair Trading Strategy (USPTS) . In the last row for each performance measure, the excess return given is the excess return of VSMN1-6362 and VSMN1-5242 over MDRC strategy as a benchmark.

3. Annual Excess Return z Score: z score value for the excess return over the benchmark strategy, USPTS. In the last row, the excess return given is the excess return of VSMN1-6362 over MDRC as benchmark. Corresponding p values are given in the parentheses.

4. Win Ratio: Percentage of sample periods in which the strategy outperformed the benchmark.

5. Sample Size: Number of executions done for each trading strategy. For commodities, there was one strategy run for each Leveraged Exchange Traded Funds (LETF) pair.

6. Performance Measure: Type of performance metric used to measure the return.

[*] Means the test value is significant at 90% confidence interval.

[**] Means the test value is significant at 95% confidence interval.

[***] Means the test value is significant at 99% confidence interval.

Results for Hypotheses Verification

This section reviews the empirical results in respect of each hypothesis introduced earlier in the study.

Hypothesis: H1a

H_{1a}: *An active short pair trading strategy conditioned on volatility and momentum in commodity LETFs outperforms a passive unconditioned short pair trading strategy on an absolute return basis.*

Empirical results: H_{1a}. This hypothesis explored whether an active short pair trading strategy conditioned on volatility and momentum in commodity LETFs outperforms a passive unconditioned short pair trading strategy on absolute returns basis. The study did not find any significant evidence which would support this hypothesis. As given in Table 4, out of the 12 separate commodity LETF pairs back tested independently using VSMN1-6362 and VSMN2 strategies for a period of 5 years or more, VSMN1-6362 outperformed USPTS in seven instances and VSMN2 outperformed USPTS in five instances. The number of periods metric did show VSMN1-6362 outperformed in 58.33% cases (i.e., had a win ratio of 58.33%), but none of the results were statistically significant with the t values for VSMN1-6362 and VSMN2 being -0.083 (p value = 0.849) and -1.645 (p value = 0.930), respectively. Therefore, null hypothesis could not be rejected. The annual results also did not generate any positive returns (see Table 5).

This result shows that the active short pair LETF trading strategies conditioned on momentum and volatility may not yield superior dollar returns. The passive unconditioned short trading strategy may have more risk, but it may also give better dollar returns. Whether these returns outweigh the risks is discussed next in the results of hypothesis H_{1b}. However, it also

needs to be said that the results of H_{1a} may also be sensitive to the regime switching strategy used in the trading strategy. Theoretically, a more sophisticated regime detection methodology may yield positive results.

Hypothesis: H1b

H1b: An active short pair trading strategy conditioned on volatility and momentum in commodity LETFs outperforms a passive unconditioned short pair trading strategy on risk-adjusted basis.

Empirical results: H_{1b}. This hypothesis presumed that an active short pair trading strategy conditioned on volatility and momentum in commodity LETFs outperforms a passive unconditioned short pair trading strategy on risk-adjusted basis. The evidence for this hypothesis is inclined towards its acceptance.

No evidence of outperformance over the USPTS strategy was found in the Sharpe ratio statistics of the VSMN1-6362 strategy. As given in Table 4, the t value for the excess value of the Sharpe ratio of VSMN1-6362 over USPTS was 0.527 (p value = 0.304), which suggests that support of H_{1b} is not even significant using a 90% confidence interval. However, the t value for the excess value of the Sortino ratio of VSMN1-6362 over USPTS was significant at the 90% level with a t value of 1.564 (p value = 0.073). The number of periods test resulted in a win ratio of 58.33% with outperformance in seven out of 12 periods. The results were no different for VSMN1-5242 variant of the trading strategy.

Regarding the outperformance of VSMN2 over USPTS, the excess value of the Sharpe ratio was 2.846 (p value = 0.008), which makes this result significant at a 99% confidence interval. To complement this, the t value for excess value of the VSMN2 Sortino ratio over that

of USPTS was 2.361 (*p* value = 0.019), which makes the result again significant at a 99%

confidence interval.

To further establish robustness of these results, a *z* test was performed on the 78 annual

results of these strategies (see Table 5). The results for the VSMN1-6362 excess value for the

Sharpe ratio were not statistically significant, with a *z* score of 0.677 (*p* value = 0.249). The

results for the VSMN2 excess value for the Sharpe ratio narrowly missed statistical significance

at a 90% confidence interval with a *z* score of 1.139 (*p* value = 0.127).

The results for the VSMN1-6362 excess value for the annual Sortino ratio were

statistically significant at a 95% confidence interval with *z* score of 1.738 (*p* value = 0.041). The

results for the VSMN2 excess value for the annual Sortino ratio also had statistical significance

at a 90% confidence interval with a *z* score of 1.308 (*p* value = 0.095).

As can be seen, the results for the Sharpe ratio excess returns were negative for the

VSMN1-6362 strategy but positive for the VSMN2 strategy. The results for the Sharpe ratio

were thus mixed, even though the number of periods test resulted in VSMN1-6362 and VSMN2

win ratios of 57.69% and 55.13%, respectively.

The results for Sortino ratio were, however, positive in each case. This was also

indicated when a *z* score for the excess Sortino ratio was calculated by combining the excess

return data from the runs of VSMN2, VSMN1-6362, and VSMN1-5242 vis-à-vis the benchmark

strategy. As given in Table 5, the *z* test conducted on the combined excess Sortino ratio value

combining three strategies yielded a sample of 234 returns, and it showed a *z* score of 3.092 (*p*

value = 0.001), which makes the result significant at a 99% confidence interval. The result for

combined long-term data for the three back tests containing 36 return periods yielded a *z* score of

3.101 (*p* value = 0096).

As already noted, the number of periods for the VSMN1-6362 test resulted in a win ratio

of 58.33% with outperformance in seven out of 12 periods (see Table 4). What is interesting is

that despite this highly positive result win ratio of the Sortino ratio for long-term volatility

conditioned strategies, VSMN1-6362 and VSMN2 outperformed USPTS in annual results only

in 41.03% and 33.33% of cases, respectively. This indicates that the superiority of the annual

Sortino ratio values for volatility conditioned strategies is not uniformly distributed across time.

This superior performance is concentrated in extreme values in certain years.

Hypothesis: H2

*H₂: An active short pair trading strategy conditioned on volatility and momentum in
LETFs outperforms a passive unconditioned short pair trading strategy only for commodity
LETFs and not for non-commodity LETFs as the latter tend to have less volatility.*

Empirical Results: *H₂*. This hypothesis, which stated that an active short pair trading

strategy conditioned on volatility and momentum in LETFs outperforms a passive unconditioned

short pair trading strategy only for commodity LETFs and not for non-commodity LETFs as the

latter tend to have less volatility, was supported by empirical results.

The results for the Sharpe ratio, Sortino ratio, and absolute excess returns were not

significant for non-commodities for trading strategy VSMN2 (see Table 6). The VSMN1

strategy did not even trade for multiple years and did not yield any meaningful numbers to

compare against the benchmark. This result was expected as none of the three reference indexes

used for this test, namely the S&P 500, DJIA, and Russell 2000, ever had average annual

volatility even close to 20%. The strategy for VSMN2 was based on LREV and binary

classification-based trading signals and was designed to overcome this drawback. Using a

relative expected volatility-based trading signal made it agnostic to any absolute volatility

regime.

The point of interest for this hypothesis was to see if non-commodities gave comparable

excess Sortino ratio values and long-term Sharpe ratio values as generated by commodity LETFs

using the VSMN2 strategy. The results showed non-commodities did not outperform the

benchmark USPTS strategy even in Sortino ratio values with a t value of 1.032 (p value =0.160).

Further, the long-term Sharpe ratio excess return value, which was highly significant at a 99%

confidence interval for commodity LETFs, with a t value of 2.846 ($p = 0.008$) and a win ratio of

83.33%, was not even significant at a 90% confidence interval for non-commodity LETFs. The

corresponding values for non-commodity LETFs were a t value of -0.341 (0.631) and a win ratio

of a mere 40% (see Table 6). This is a remarkable result.

Note that since there were only three non-commodity LETFs involved in the study, back

testing was repeated five times to allow for adequate statistical power, using different start and

end days for each LETF. This yielded a sample of 15 long-term returns and 125 annual returns.

These results are given in Table 7. As can be seen none of the annual excess returns were

significant either.

Hypothesis: H3

H₃: Underperformance of LETFs in reference to their stated leverage multiples in a

manner that yields any trading advantage is not just restricted to continuously high volatility

market states of the underlying indexes, but it also exhibits in low volatility market states

when there are abrupt upward shocks in expected volatility.

Empirical Results: H_3. This hypothesis, which stated that LETFs not only significantly underperform their stated leverage multiple in a way that gives any trading advantage in continuously high volatility market states but also in low volatility market states when there are abrupt upward shocks in expected volatility, was not supported by empirical results.

Excess values for absolute return for VSMN2 over those of USPTS had a t value of 0.787 (p value = 0.22). For the Sharpe ratio and Sortino ratio, these t values were -0.341 (p value = 0.631) and 1.032 (p value = 0.16), respectively. None of these results were statistically significant even at a 90% confidence interval. Tests done on annual values for these non-commodities LETFs yielded similar results, with excess return z score values for the Sortino ratio, Sharpe ratio, and absolute return being 0.019 (p value = 0.492), -0.5206 (p value = 0.699), and -1.154 (p value = 0.876), respectively. Note that only non-commodity LETFs were used in this test in order to meet the condition of low volatility. See Table 6 and Table 7 to review these results.

The implication of this result is that LETFs do a decent job in daily rebalancing in low-volatility regimes and can manage costs around abrupt volatility shocks reasonably well in such periods. Even if there are minor increases in rebalancing costs in such periods, these incremental costs are probably not significant enough to give any trading advantage over the passive benchmark strategy. In addition, in low volatility regimes LETF markets are relatively more efficient as predicted by the theory. This finding should reinforce the confidence of both investors and regulators in the efficiency of LETF products when there is low volatility in the

market. Further, it can reasonably be deduced that incremental deterioration in the efficiency of these products is mostly due to systemic jumps in the implied volatility rather than due to any incremental inefficiency in the daily rebalancing process. None of the equity indexes (DJIA, S&P 500, nor Russell 200) had average annual volatility of over 16.56% in the holding periods used in the back tests.

Results for Robustness Checks

Result of the Momentum Detection Robustness Check .

The MDRC strategy, discussed earlier in the methods section, appeared to provide enough evidence to confirm the effectiveness of the momentum detector used in the VSMN1 trading strategy. As given in Table 4, for a long-term period (5 year or more), both VSMN1-6362 and VSMN1-5242 returns showed no statistically significant superiority over MDRC returns with t value for excess returns of only 0.017 (p value = 0.493) and 0.002 (p value = 0.499) respectively.

However, as shown in Table 5, the annual results , ignoring Sortino ratios, showed a significant inclination towards the superiority of VSMN1-6362 and VSMN1-5242 returns over MDRC returns if reasonable constraints were applied on the return values by ignoring tail returns.

Based on 78 Sharpe ratio excess returns, with MDRC strategy as the benchmark, VSMN1-6362 and VSMN1-5242 strategies yielded z scores of 2.388 (0.008) and 2.387 (0.008) respectively. These results are thus significant at 99% confidence interval. The win ratio is also 57.69% in each case.

In regards to absolute returns, out of the 66 annual returns available for both strategies, starting from the second year, if two extreme values in annual returns for both 3x gold mining LETFs were ignored, the excess return of VSMN1-6362 and VSMN1-5242 strategies over MDRC gave a z scores of 2.178 (p value = 0.015) and 2.238 (p value = 0.013) respectively. This finding was statistically significant at a 95% confidence interval, almost bordering the 99% confidence interval. If the 3x gold mining LETFs were completely ignored and only the 58 returns applicable to the remaining 10 commodity LETFs were used in the robustness check then a statistical significance at a 90% confidence interval was shown with a z score of 1.457 (p value = 0.0725) and 1.441(p value = 0.075) respectively. In the number of periods test, the both variants of VSMN1 win ratios over MDRC was ranging between 53.03% and 56.90%.

The above results constitute reasonable evidence to justify the effectiveness of the moving averages-based momentum detector used in the VSMN1 trading strategy. It is pertinent to remember that Moskowitz, Ooi, and Pedersen (2012) showed that time series momentum can be predicted by an autoregressive TSMOM indicator, and Marshall et al. (2017) showed that same results can also be obtained by using moving average-based momentum indicators. In the present study if the moving average-based indicator had not worked, the trading strategy using it would not have outperformed the strategy using only the volatility indicator in any way. As discussed earlier, conditioning on momentum should give the trading strategy some advantage. The empirical finding appears to confirm the outcome predicted by the theory.

Result of Robustness Check for Variant 1 of Volatility Short Momentum Neutral Trading Strategy.

Returns generated by VSMN1-6362 and VSMN1-5242 were compared to see if there was any statistically significant difference between them due to variation moving average lag parameters. These results are given Table 8 and Table 9.

For a long-term period (5 year or more), the VSMN1-5242 returns, for 12 different LETF pairs, showed no statistically significant difference with the VSMN1-6362 returns, with a t value for excess return of -1.083 (p value = 0.849). Similar results were obtained for the long-term Sharpe ratio and Sortino ratio, with t values for each being -0.014 (p value = 0.505) and -0.023 (p value = 0.509). Annual Sharpe and Sortino results were no different, with excess values of VSMN1-6362 and VSMN1-5242 having t values of -1.056 (p value = 0.855) and -0.334 (p value = 0.631), respectively. Further, excess returns of VSMN1-6362 and VSMN1-5242 over the benchmark USPTS strategy showed no statistically significant variation in any of the tests. These results provided enough evidence against any selection bias in the choice of moving average parameters used for momentum detection.

Table 8

Long-Term Robustness Check t *Values for Variant 1 of Volatility Short Momentum Neutral Trading Strategy*

Strategy[1]	Long Term Excess Return t Value[2]	Win Ratio[3]	Sample Size[4]	Performance Measure[5]
VSMN1-6362	-0.023 (0.509)	N/A[6]	12	Sortino Ratio
VSMN1-6362	0.014 (0.505)	N/A[6]	12	Sharpe Ratio
VSMN1-6362	-1.083 (0.849)	N/A[6]	12	Absolute Return

Note: This table reports the t value statistics for each strategy. Following is the description of each heading:

1. Strategy: Name of the trading strategy. Following is the full expansions of strategy names used:
    a. VSMN1-6362:The description of the name of this strategy is Volatility Short Momentum Neutral-6362. Term 6362 indicates that the momentum oscillator was based on four moving averages each corresponding to 6 months, 3 months, 6 weeks and 2 weeks periods.
    b. VSMN1-5242: The description of the name of this strategy is Volatility Short Momentum Neutral-5242. Term 5242 indicates that the momentum oscillator was based on four moving averages each corresponding to 5 months, 2 months, 4 weeks and 2 weeks periods.
    c. USPTS: Benchmark strategy Unconditionally Short Pair Trading Strategy
2. Long-Term Excess Return t Value: t value for the excess return of VSMN1-6362 over VSMN1-5242 as benchmark. Corresponding p values are given in the parentheses.
3. Win Ratio: Percentage of sample periods in which the strategy outperformed the benchmark. None of the excess return were statistically significant.
4. Sample Size: Number of executions done for each trading strategy. For commodities, there was one strategy run for each LETF pair.

5. Performance Measure: Type of performance metric used to measure the return.

6. N/A: Not applicable. Since none of the excess return were statistically significant, Win Ratio values have been given as N/A

[*] Means the test value is significant at 90% confidence interval.

[**] Means the test value is significant at 95% confidence interval.

[***] Means the test value is significant at 99% confidence interval.

Table 9

Annual Robustness Check z *Scores for Variant 1 of Volatility Short Momentum Neutral Trading Strategy*

Strategy[1]	Annual Excess Return z Score[2]	Win Ratio[3]	Sample Size[4]	Performance Measure[5]
VSMN1-6362	-0.334 (0.631)	N/A[6]	125	Sortino Ratio
VSMN1-6362	-1.056 (0.855)	N/A[6]	125	Sharpe Ratio
VSMN1-6362	0.379(0.352)	N/A[6]	125	Absolute Return

Note: This table reports the z score statistics for each strategy. Following is the description of each heading:

1. Strategy: Name of the trading strategy. Following is the full expansions of strategy names used:
    a. VSMN1-6362:The description of the name of this strategy is Volatility Short Momentum Neutral-6362. Term 6362 indicates that the momentum oscillator was based on four moving averages each corresponding to 6 months, 3 months, 6 weeks and 2 weeks periods.
    b. VSMN1-5242: The description of the name of this strategy is Volatility Short Momentum Neutral-5242. Term 5242 indicates that the momentum oscillator was based on four moving averages each corresponding to 5 months, 2 months, 4 weeks and 2 weeks periods.
    c. USPTS: Benchmark strategy Unconditionally Short Pair Trading Strategy
2. Annual Excess Return z Score: z score for the excess return of VSMN1-6362 over VSMN1-5242 as benchmark. Corresponding *p* values are given in the parentheses.
3. Win Ratio: Percentage of sample periods in which the strategy outperformed the benchmark. None of the excess return were statistically significant.
4. Sample Size: Number of executions done for each trading strategy. For commodities, there was one strategy run for each LETF pair.

5. Performance Measure: Type of performance metric used to measure the return.

6. N/A: Not applicable. Since none of the excess return were statistically significant, Win Ratio values have been given as N/A

* Means the test value is significant at 90% confidence interval.

** Means the test value is significant at 95% confidence interval.

*** Means the test value is significant at 99% confidence interval.

Conclusion and Discussion

Passive unconditioned pair trading strategies based on the shorting of long and short LETFs have been shown to outperform the market in recent literature (Xinxin and Stanley, 2017). Existing literature has also shown path dependence and volatility decay as the behavior of LETFs (Guo and Leung , 2014). This study was undertaken to investigate if the foregoing properties of LETFs can be exploited to develop active short pair trading strategies conditioned on volatility and momentum to outperform the passive LETF pair trading strategies. For this purpose, a regime switching model, tailored for the LETF market, was developed and tested. This regime switching model used trading signals extracted from various volatility prediction models and time series momentum. The time series momentum detection leg of the regime switching model was developed using a moving average approach. The robustness checks confirmed its effectiveness in producing excess returns as predicted by the theory. This study also introduced the concept of LREV to model expected volatility shocks. The framework of LREV draws from an inductive learning used in machine learning.

The study found no evidence in support of the outperformance of the active LETF pair trading strategies over the passive ones when it comes to absolute returns. However, the study found results favorable for confirmation of the outperformance of active trading strategies over the passive ones on a risk-adjusted basis. This outperformance was exhibited in Sortino ratios only in both long-term and annual returns. Some evidence of outperformance on a risk-adjusted basis was also manifested in Sharpe ratios of long-term returns, but this result was not robust to annual return distribution. In addition, the outperformance of active trading strategies for commodity LETFs in Sortino ratios followed asymmetric distributions when reviewed strictly on

an annual return basis. Tests for outperformance in Sharpe ratios produced negative results in all cases.

The study also found evidence supporting the theory that the risk-adjusted outperformance of the active strategies is restricted to commodity LETFs only. This finding showed robustness in the checks performed using LREV-based trading signals deployed in active pair trading of equity LETF pairs. The main driver for this finding was lack of adequate volatility in non-commodity LETFs. The results based on LREV-based active trading strategy showed that even though non-commodity LETFs may empirically experience marginally higher rebalancing costs on upward shocks in expected volatility in low-volatility regimes, these costs do not appear to be high enough to offer any trading advantage or arbitrage opportunity. Another implication of this finding is that any incremental deterioration in the efficiency of LETF products in rapidly fluctuating markets appears to be mostly attributable to systemic jumps in the implied volatility and less to any incremental inefficiency in the daily rebalancing process. This observation may be of interest to regulators.

The results of this study were robust to be generally applicable to transaction costs in the form of commissions and borrowing costs for shorting of the securities. The study also developed a new theoretical framework based on the cost of capital and discounted cashflow approach to calculate the CAGR of statistical arbitrage portfolios. Last but not the least, the study also provided evidence from the LETF markets for the inverse relationship between volatility and momentum (Wang & Xu, 2015).

Limitations and Future Research Issues

The study did not explicitly investigate the influence of market liquidity as a separate factor in active trading strategies. To overcome this possible limitation, only the ratio-based performance metrics were used, and dollar returns were avoided to draw all of the conclusions. These ratio-based performance metrics should be replicable for all long and short pair trading portfolios of reasonable sizes. Further, ceteris paribus the results should remain robust to any liquidity issues because the same issues would be applicable to the passive benchmark strategy against which the active strategies were tested as well. To what extent liquidity issues would incrementally impact both active and passive trading strategies based on position sizing could itself be an independent research topic for future research.

Theoretically, path dependence and volatility decay properties of the LETFs should be enough to offer a trading advantage to an active short pair trading strategy that is conditioned on volatility and momentum. However, these may not be the only two variables responsible for outperformance. Another variable of theoretical interest is the borrowing cost for shorting securities which may be a contributor to the outperformance.

The reason is that the active strategy incurs security borrowing costs only on the days it goes short, whereas the passive strategy remains short throughout the holding period. This advantage in lower borrowing costs are, however, offset by higher commission costs due to active trading. Overall, security borrowing costs which incur at 6% annually should still give some net advantage even after adjusting for commissions. This contribution to risk-adjusted outperformance of lower net transaction costs was not studied independently and it could be a topic for future research.

The study assumed slippage to be around 0.30% of the order value. It is possible that due to many factors this estimate may not have been very accurate given the unique microstructure of commodity LETFs. However, even if realized slippage cost turns out to be an underestimation, it should not impact the main findings of this study in any significant manner. This is because absolute return of the active pair trading strategies never outperformed that of the passive one even when slippage was totally discounted. A significant negative estimation error would have made the absolute return underperform even more.

For risk-adjusted metrics of Sharpe ratio and Sortino ratio, daily returns were used. Since the trading strategies had long holding periods in either cash or in short position, any potential slippage cost related measurement errors would have been applicable only for the returns for limited number of days on which the strategy traded. For these reasons even if variances from the modeled slippage costs were realized due to some market factors, their impact on the Sharpe ratios and Sortino ratios would have been minimal to negligible. In other words, the findings of this study are reasonably robust to any errors in slippage costs estimation.

Due to data snooping concerns, this study used only a single set of moving average parameters and volatility threshold values for all LETF pairs. However, it is possible to use theoretical frameworks such as statistical learning theory to undertake this study as a classic machine learning project. This approach would enable a unique set of parameter values for each LETF pair to be used after learning them in out-of-sample data and then seeing if it further improved the performance of active strategies in the in-sample data after adjusting for the bias-variance tradeoffs. This approach could learn additional idiosyncratic properties of each LETF pair in the statistical learning theory framework to improve the performance of active trading

strategies. This optimization was not included in the present study but could be an interesting

topic for future research.

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Appendix A

Trading Strategy Performance Charts for Commodity LETFs

In this section, illustrative charts for trading strategy back tests for commodity LETFs are given

using ProShares Ultra Bloomberg Crude Oil LETF pair (NYSE:UCO and NYSE:SCO). The

performance of active strategies (VSMN1 and VSMN2) was compared against the passive

strategy (USPTS) which is mentioned as benchmark strategy ensuing charts. The illustrative

daily profit or loss chart is also given. Scaling was used and all charts assume an initial shorting

of $1,000.00 for easier formatting. As noted in Limitations and Future Research Issues section,

the choice of initial shorting amount does impact the relative performance of active strategies

against the passive strategy due to liquidity.

Figure A1

Performance Chart for WTI CLQ18 Future (Reference Index proxy for Crude Oil LETFs NYSE: SCO and NYSE: UCO), Holding Period: 8 years (July 10, 2010–July 9, 2018)
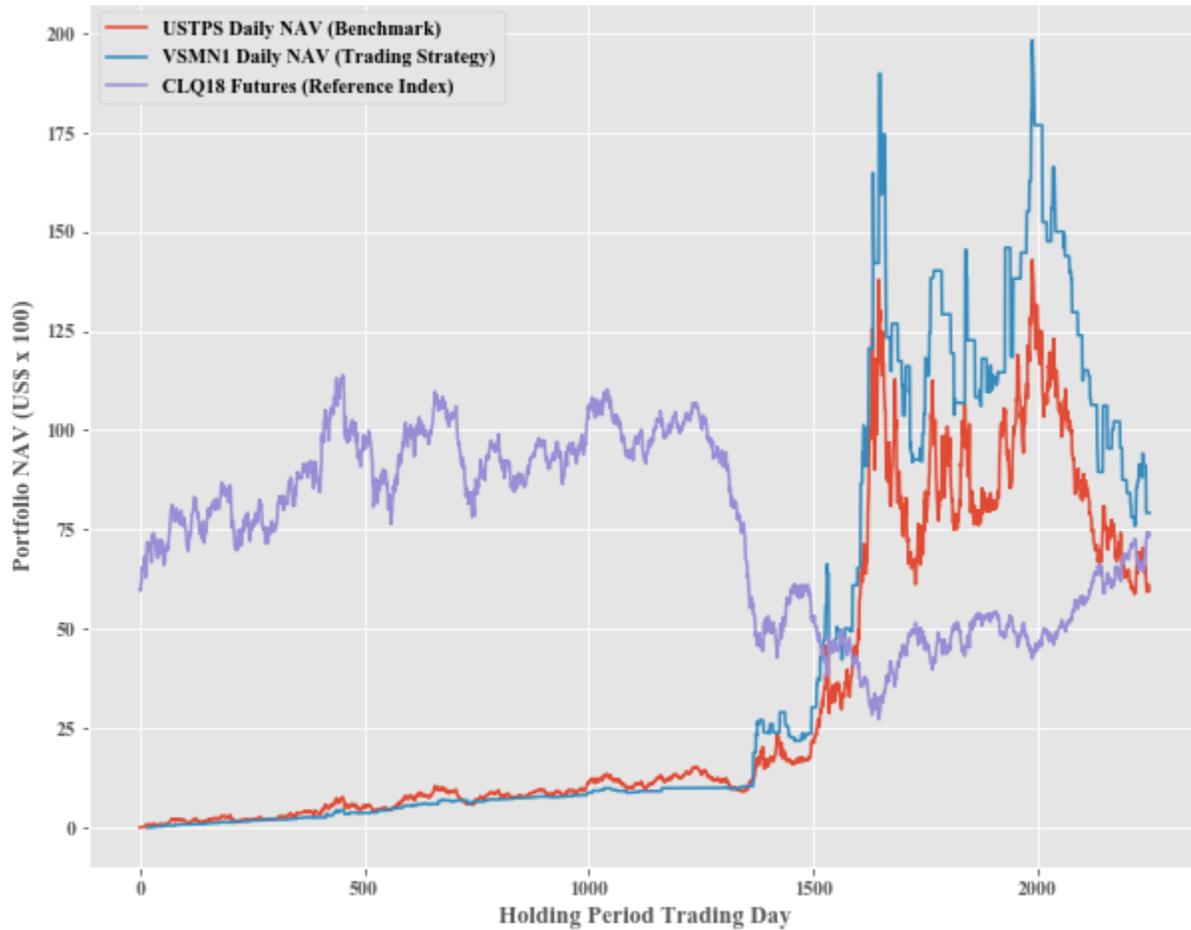
Figure A2

Performance Chart for Trading Strategy VSMN1-6362 Crude Oil LETF Pair (NYSE: SCO and NYSE: UCO), Holding Period: 8 years (July 10, 2010–July 9, 2018)
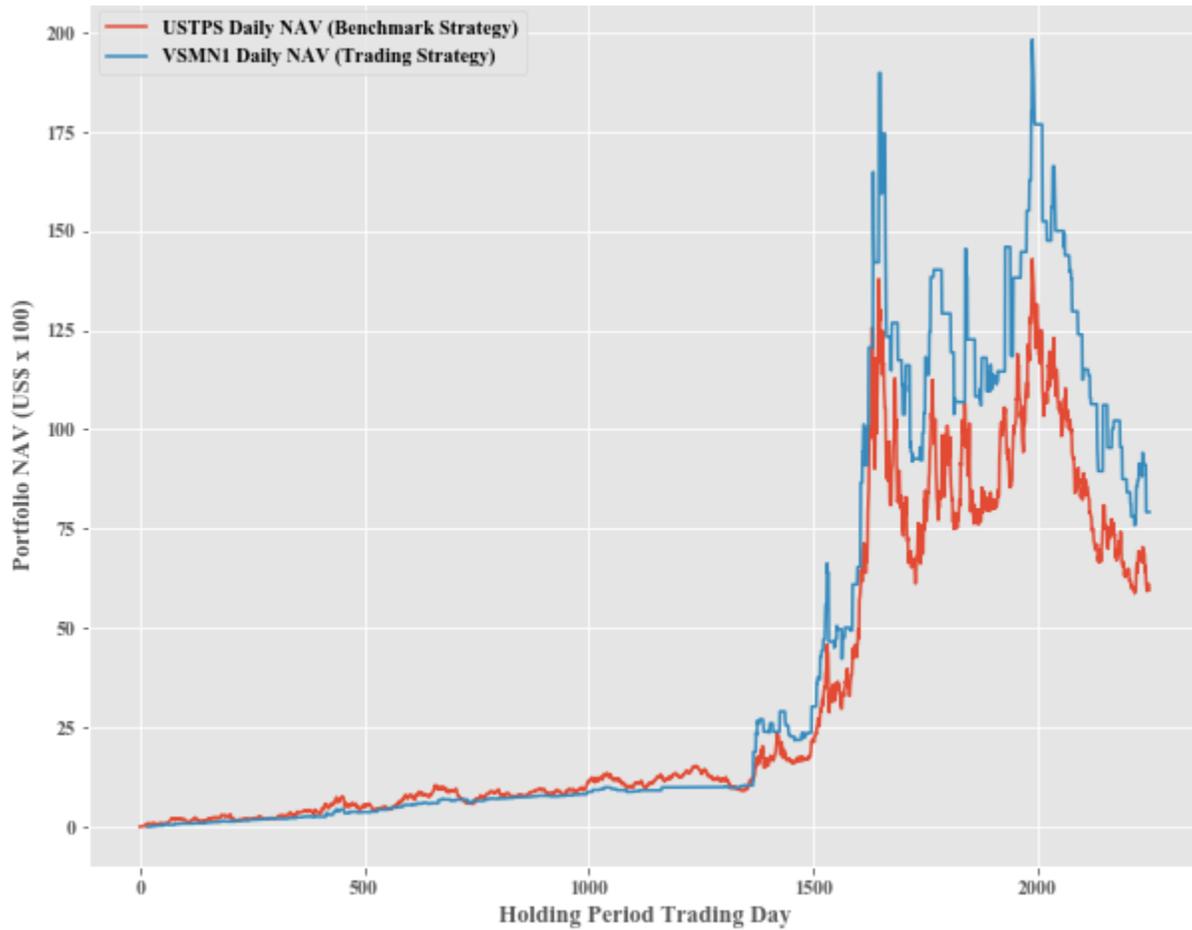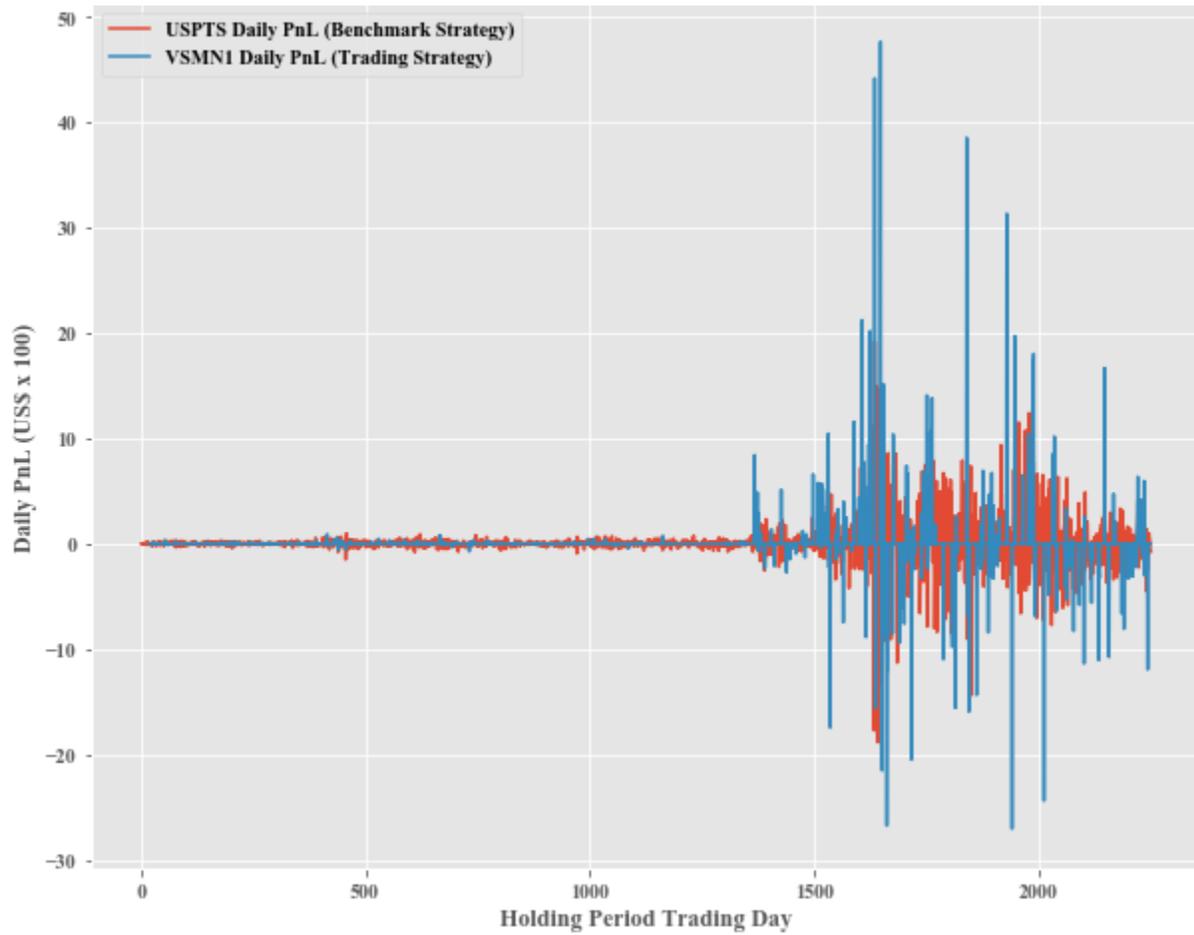
Figure A3

Performance Chart for Trading Strategy VSMN1-6362 Crude Oil LETF Pair (NYSE: SCO and NYSE: UCO), Holding Period: 8 years (July 10, 2010–July 9, 2018) – With Reference Index Plotted Together
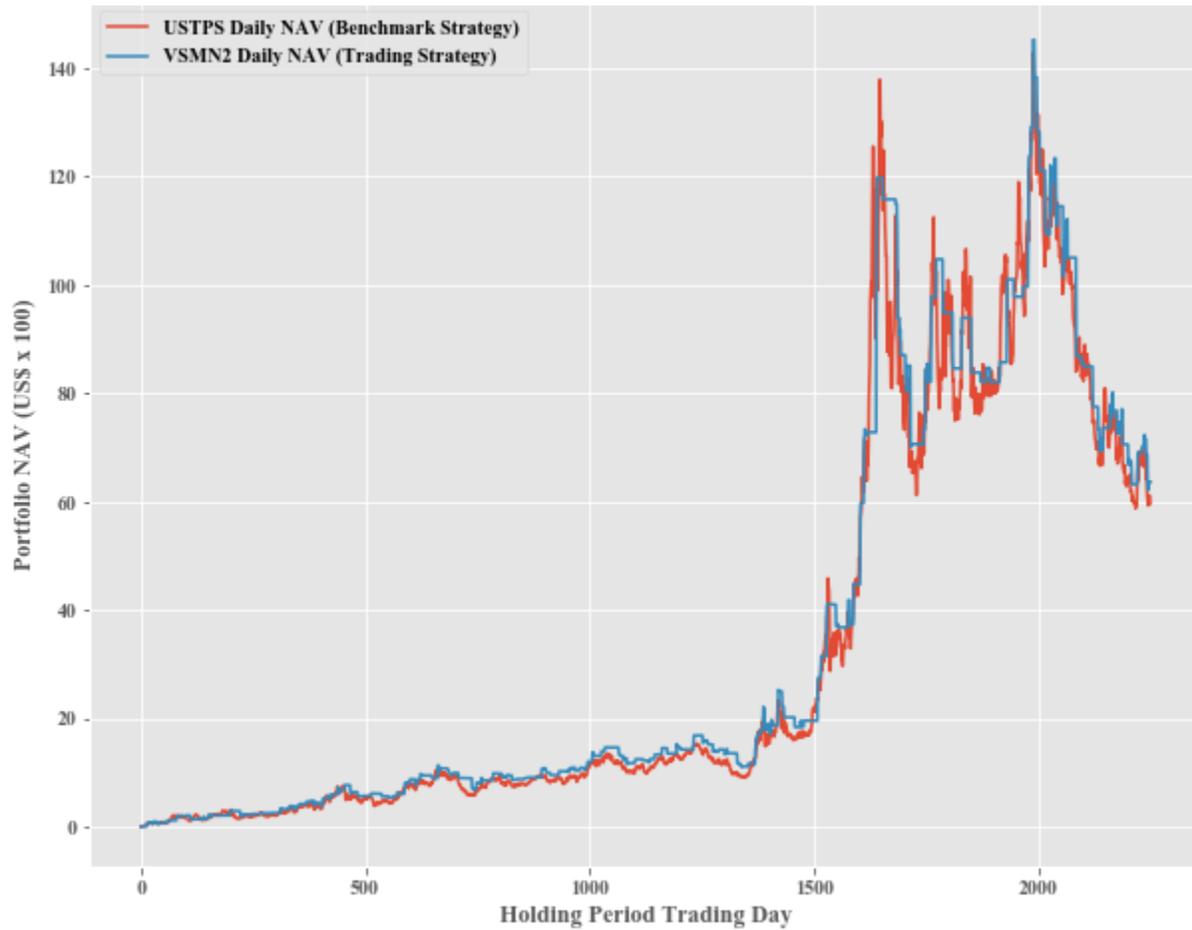
Figure A4

Daily Profit and Loss (PnL) for Trading Strategy VSMN1-6362 Crude Oil LETF Pair (NYSE: SCO and NYSE: UCO), Holding Period: 8 years (July 10, 2010–July 9, 2018)
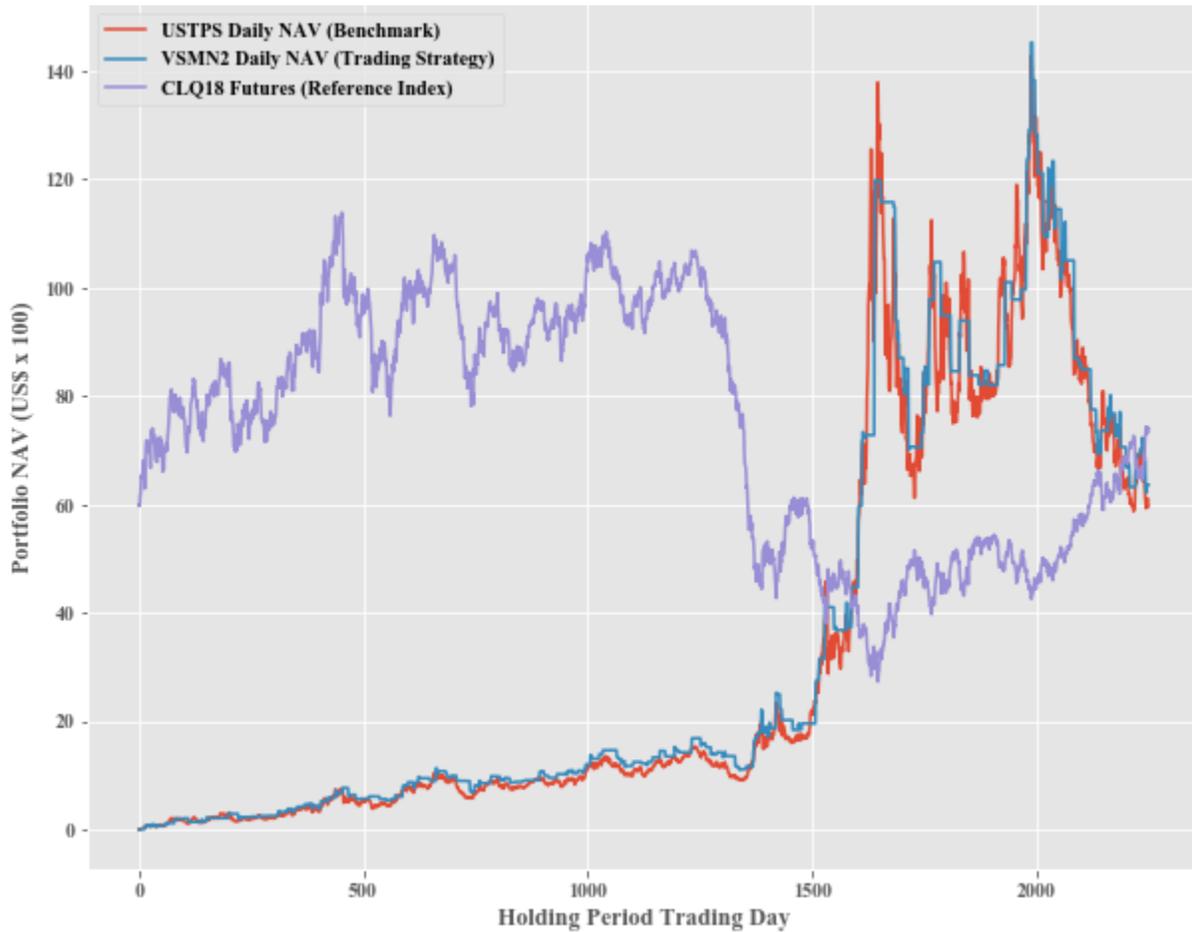
Figure A5

Performance Chart for Trading Strategy VSMN2 Crude Oil LETF Pair (NYSE: SCO and NYSE: UCO), Holding Period: 8 years (July 10, 2010–July 9, 2018)
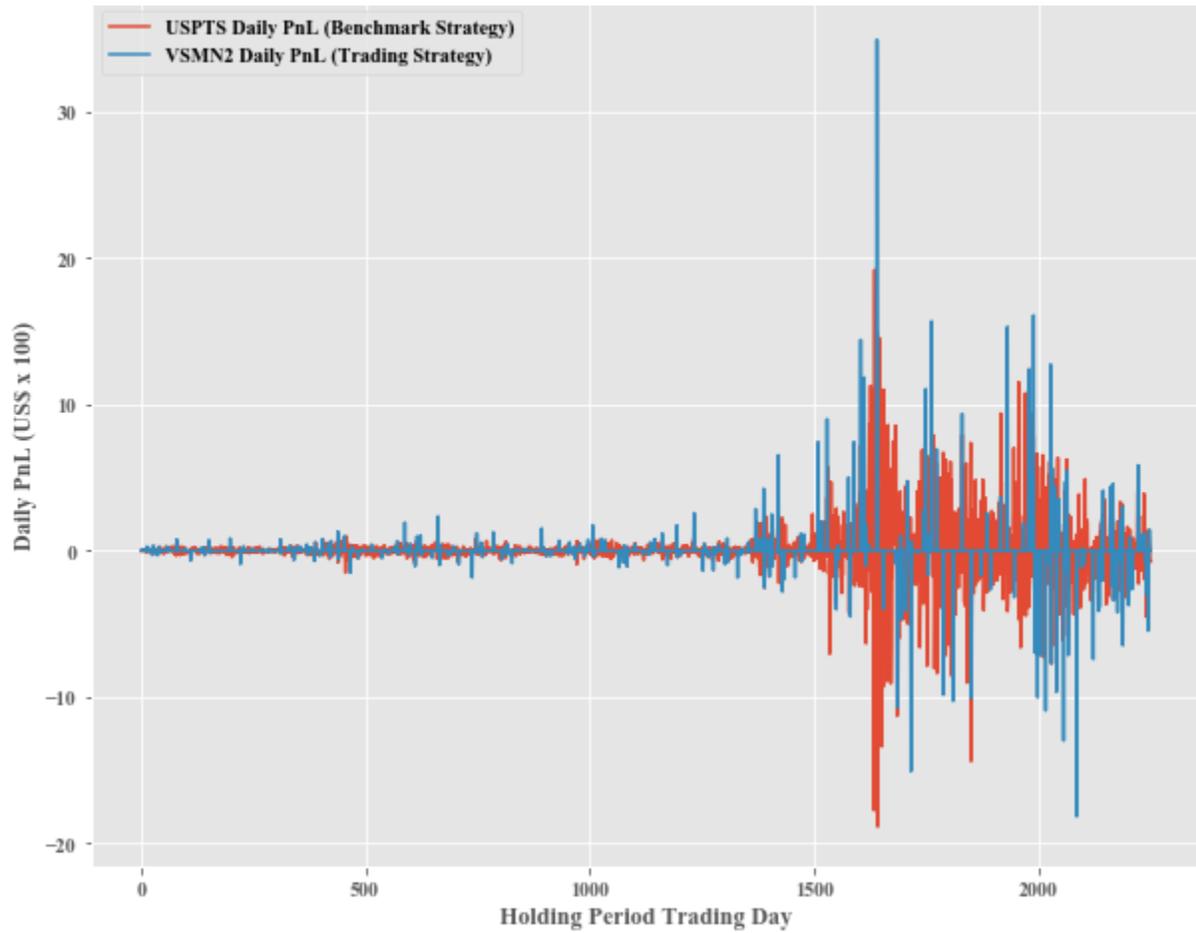
Figure A6

Performance Chart for Trading Strategy VSMN2 Crude Oil LETF Pair (NYSE: SCO and NYSE: UCO), Holding Period: 8 years (July 10, 2010–July 9, 2018) – With Reference Index Plotted Together

Figure A7

Daily Profit and Loss (PnL) for Trading Strategy VSMN2 Crude Oil LETF Pair (NYSE: SCO and NYSE: UCO), Holding Period: 8 years (July 10, 2010–July 9, 2018)

Appendix B

Software

The following software were used for back testing trading strategies and running data analytics:

a. Python Packages

- Pandas

- Matplotlib

- Numpy

- Statsmodels - statsmodels.tsa.arima_model (for calculating ARIMA models)

- Statsmodels- statsmodels.tools.eval_measures(for calculating forecasting errors)

- arch.univariate (for calculating ARCH and GARCH models)

Python version 3.6 was used for all analysis.

b. Microsoft Excel.