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PROGNOSTIC TESTING IN GEOMETRY

A PRELIMINARY REPORT

BY

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I am deeply grateful to Principal H. L. Miller of the Wisconsin High School for giving his hearty cooperation; to Professor A. B. Hart for his kindly cooperation and interest in the problem; and to Mrs. A. H. Voss for the various interruptions of class work which she allowed me to make in getting test results. Space does not permit me to mention many others who have given advice and helpful criticisms.

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Chapter I

PRELIMINARY INVESTIGATION

THE PROBLEM

In the field of vocational and educational guidance it has become increasingly important, within the last decade, to be able to give pupils competent guidance and advice as to their chances of success in scholastic and vocational lines. If our guidance is to be any better than a method of trial and error or a sampling of the job before the requisite information can be obtained, then we must find some means of telling a pupil what his chances of success will be within certain limits. It seems that the best means thus far devised for obtaining anything in the way of objective information in advance is the vocational or educational prognostic test.

The prognostic test is rather a new instrument and the statistical means of treating the results obtained from such tests are a very recent development. The vocational and educational fields offer excellent opportunities for the development and application of such tests. Interest in the problem of predicting ability in Geometry lead the writer of this thesis to attempt the problem of organizing a prognostic

test battery for that subject. This problem has been undertaken from the standpoints both of research and interest in practical school affairs. The problem involved a study of the work that has already been done in this line.

The problem of constructing a test battery for predicting ability in geometry involves the devising or the selection of a group of tests which, from preliminary study, seem to be usable. These tests must be carefully applied to a representative group of pupils. Having given the tests there must be some means of getting a reliable criterion for the ability in question, in this case geometry. This is done by the application, to the same group of pupils, of a good objective geometry test. The results of the various tests then are correlated with the criterion and, in case results are promising, a prediction formula is worked out by means of the regression equation. This gives the proper weighting of the various tests used in the prognostic battery for the best predicted score of any pupil in plane geometry. This preliminary study usually results in a refinement of the test battery and its practical value can then be tested in a new situation.

STANDARDS OF INSTRUCTION IN MATHEMATICS

In order properly to evaluate a criterion one must first have some knowledge of the standards of instruction and of achievement which have been set up in the subject. In order to determine what these standards are in the subject of mathematics in general and in Geometry in particular, the writer made a study of the standards as listed in the report of the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America, which was published in 1923. The findings of this committee and the thoroughness with which their work was done merits space here for the statement of the aims as given in their report. They are as follows:

I. Practical aims (By a practical aim, in the narrower sense, we mean then the immediate or direct usefulness in life of a fact, method, or process in mathematics.)

1. The immediate and undisputed utility of the fundamental processes of arithmetic in the life of every individual demands our first attention. . . By the end of the sixth grade the child should be able to carry out the four fundamental processes with integers and with common and decimal fractions accurately and with a fair degree of speed.

a. A progressive increase in the pupil's understanding

of the nature of the fundamental operations and power to apply them in new situations.

b. Exercise of common sense and judgment in computing from approximate data, familiarity with the effect of small errors in measurements, the determination of the number of figures to be used in computing and to be retained in the result and the like.

c. The development of self-reliance in the handling of numerical problems through the consistent use of checks on all numerical work.

2. Of almost equal importance to every educated person is an understanding of the language of algebra and the ability to use this language intelligently and readily in the expression of such simple quantitative relations as occur in every-day life and in the normal reading of the educated person. Appreciation of the significance of formulas and ability to work out simple problems by setting up and solving the necessary equations must nowadays be included among the minimum requirements of any program of universal education.

3. The development of the ability to understand and to use such elementary algebraic methods involves a study of the fundamental laws of algebra and at least a certain minimum of drill in algebraic technique, which, when

properly taught, will furnish the foundation for an understanding of the significance of the processes of arithmetic already referred to. The essence of algebra as distinguished from arithmetic lies in the fact that algebra concerns itself with the operations upon numbers in general, while arithmetic confines itself to operations on particular numbers.

4. The ability to understand and interpret correctly graphic representations of various kinds, such as nowadays abound in popular discussions of current scientific, social, industrial, and political problems, will also be recognized as one of the necessary aims in the education of every individual.
5. Finally, among the practical aims to be served by the study of mathematics should be listed familiarity with the geometric forms common in nature, industry, and life; the elementary properties and relations of these forms, including their mensuration; the development of space-perception; and the exercise of spatial imagination.

II. Disciplinary Aims

1. The acquisition, in precise form, of those ideas or concepts in terms of which the quantitative thinking of the world is done. Among these ideas and concepts may be mentioned ratio and measurement (lengths, areas,

volumes, weights, velocities, and rates in general, etc.), proportionality, and similarity, positive and negative numbers, and the dependence of one quantity upon another.

2. The development of the ability to think clearly in terms of such ideas and concepts. This ability involves training in -
   a. Analysis of a complex situation into simpler parts. This includes the recognition of essential factors and the rejection of the irrelevant.
   b. The recognition of logical relations between inter-dependent factors and the understanding and, if possible, the expression of such relations in precise form.
   c. Generalization; that is, the discovery and formulation of a general law and an understanding of its properties and applications.
3. The acquisition of mental habits and attitudes which will make the above training effective in the life of the individual. Among such habitual reactions are the following: a seeking for relations and their precise expression; an attitude of enquiry; a desire to understand, to get to the bottom of a situation; concentration and persistence; a love for precision, accuracy, thoroughness, and clearness, and a distaste for vagueness and incompleteness; a desire for orderly and logical organization as an aid to understanding and memory.

4. Many of these disciplinary aims are included in the broad sense of the idea of relationship or dependence - in what the mathematician in his technical vocabulary refers to as a function of one or more variables.

III. Cultural Aims

1. Appreciation of beauty in the geometrical forms of nature, art, and industry.
2. Ideals of perfection as to logical structure, precision of statement and of thought, logical reasoning (as exemplified in the geometric demonstration), discrimination between the true and the false, etc.
3. Appreciation of the power of mathematics - and the role that mathematics and abstract thinking, in general, have played in the development of civilization; in particular in science, in industry, and in philosphy.

Having concluded a summary of the aims as stated by the committee the next problem is to find out what has been done in the way of construction of achievement tests in the field of Geometry, in which this study is concerned.

STANDARDS OF ACHIEVEMENT IN PLANE GEOMETRY

Again we must refer to the report of the National Committee on Mathematical Requirements because they have made an exhaustive study of the subject. Propositions and theorems are here listed so that we may have at hand a ready means of evaluating any achievement test from this standpoint. We find the following summary in the report of the Committee: List of propositions which may be assumed or treated informally:

1. Through two distinct points it is possible to draw one straight line, and only one.
2. A line segment may be produced to any desired length.
3. The shortest path between two points is the line segment joining them.
4. One and only one perpendicular can be drawn through a given point to a given straight line.
5. The shortest distance from a point to a line is the perpendicular distance from the point to the line.
6. From a given center and with a given radius one and only one circle can be described in a plane.
7. A straight line intersects a circle, in at most two points.
8. Any figure may be moved from one place to another without changing its shape or size.
9. All right angles are equal.

10. If the sum of two adjacent angles equals a straight angle their exterior sides form a straight line.
11. Equal angles have equal complements and equal supplements.
12. Vertical angles are equal.
13. Two lines perpendicular to the same line are parallel.
14. Through a given point not on a given straight line, one straight line, and only one, can be drawn parallel to the given line.
15. Two lines parallel to the same line are parallel to each other.
16. The area of a rectangle is equal to its base times its altitude.

Fundamental Theorems and Constructions:

Theorems:

1. Two triangles are congruent if (a) two sides and the included angle of one are equal, respectively, to two sides and the included angle of the other; (b) two angles and a side of one are equal, respectively to two angles and the corresponding side of the other; (c) the three sides of one are equal, respectively, to the three sides of the other.
2. Two right triangles are congruent if the hypotenuse and

one other side of one are equal, respectively, to the hypotenuse and another side of the other.

3. If two sides of a triangle are equal, the angles opposite these sides are equal; and conversely.
4. The locus of a point (in a plane) equidistant from two given points is the perpendicular bisector of the line segment joining them.
5. The locus of a point equidistant from two given intersecting lines is the pair of lines bisecting the angles formed by these lines.
6. When a transversal cuts two parallel lines, the alternate interior angles are equal; and conversely.
7. The sum of the angles of a triangle is two right angles.
8. A parallelogram is divided into congruent triangles by either diagonal.
9. Any (convex) quadrilateral is a parallelogram (a) if the opposite sides are equal; (b) if two sides are equal and parallel.
10. If a series of parallel lines cut off equal segments on one transversal they cut off equal segments on any transversal.
11. (a) The area of a parallelogram is equal to the base times the altitude. (b) The area of a triangle is equal to one-half the base times the altitude. (c) The

area of a trapezoid is equal to half the sum of its bases times its altitude. (d) The area of a regular polygon is equal to half the product of its apothem and perimeter.

12. (a) If a straight line is drawn through two sides of a triangle parallel to the third side it divides these sides proportionally. (b) If a line divides two sides of a triangle proportionally it is parallel to the third side. (Proofs for commensurable cases only). (c) The segments cut off on two transversals by a series of parallels are proportional.

13. Two triangles are similar if (a) they have two angles of one equal, respectively, to two angles of the other; (b) they have an angle of one equal to an angle of the other and the including sides are proportional; (c) their sides are respectively proportional.

14. If two chords intersect in a circle, the product of the segments of one equal the product of the segments of the other.

15. The perimeters of two similar polygons have the same ratio as any two corresponding sides.

16. Polygons are similar, if they can be decomposed into triangles which are similar and similarly placed, and conversely.

17. The bisector of an interior or exterior angle of a triangle divides the opposite side (produced if necessary) into segments proportional to the adjacent sides.
18. The areas of two similar triangles (or polygons) are to each other as the squares of any two corresponding sides.
19. In any right triangle the perpendicular from the vertex of the right angle on the hypotenuse divides the triangle into two triangles each similar to the given triangle.
20. In a right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.
21. In the same circle or in equal circles, if two arcs are equal their central angles are equal, and conversely.
22. In any circle angles at the center are proportional to their intercepted arcs (Proof for commensurable cases only).
23. In the same circle or in equal circles, if two chords are equal their corresponding arcs are equal; and conversely.
24. (a) A diameter perpendicular to a chord bisects the chord and the arcs of the chord. (b) A diameter which bisects a chord (that is not a diameter) is perpendicular to it.

25. The tangent to a circle at a given point is perpendicular to the radius at that point. and conversely.
26. In the same circle or in equal circles, equal chords are equally distant from the center; and conversely.
27. Any angle inscribed in a circle is equal to half the central angle having the same arc.
28. Angles inscribed in the same segment are equal.
29. If a circle is divided into equal arcs, the chords of these arcs form a regular inscribed polygon and tangents at the points of division form a regular circumscribed polygon.
30. The circumference of a circle is equal to $2\pi r$ (Informal proof only).

Constructions:

1. Bisect a line segment and draw the perpendicular bisector.
2. Bisect an angle.
3. Construct a perpendicular to a given line through a given point.
4. Construct an angle equal to a given angle.
5. Through a given point draw a straight line parallel to a given straight line.

6. Construct a triangle, given (a) the three sides; (b) two sides and the included angle; (c) two angles and the included side..
7. Divide a line segment into parts proportional to given segments.
8. Given an arc of a circle; find its center.
9. Circumscribe a circle about a triangle.
10. Inscribe a circle in a triangle.
11. Construct a tangent to a circle through a given point.
12. Construct a fourth proportional to three given line segments.
13. Construct the mean proportional between two given line segments.
14. Construct a triangle (polygon) similar to a given triangle (polygon).
15. Construct a triangle equal to a given polygon.
16. Inscribe a square in a circle.
17. Inscribe a regular hexagon in a circle.

Besides the above which are deemed most important, the Committee has given a list of twenty-two subsidiary propositions. I shall take the liberty to omit that list from this account as the above list is thought to be inclusive enough to check any of the tests now on the market or about to be completed.

STANDARD TESTS IN PLANE GEOMETRY

I shall here simply make reference to the people who have engaged in the work of constructing geometry tests and to the chief accomplishments in this field to date.

In 1916, L. V. Stockard of the Texas State Department of Education and J. Carleton Bell of the Brooklyn Training School for Teachers made a study entitled "A Preliminary Study of the Measurement of Abilities in Geometry", and devised a test which is a kind of examination in geometry. The test includes problems demanding a variety of abilities, but it does not attempt to provide for examining a pupil's ability in geometry with the same detail found in the algebra tests.

J. H. Minnick, Dean of the School of Education of the University of Pennsylvania published in 1918 a series of geometry tests designed primarily to measure certain fundamental abilities needed in the formal demonstration of a theorem. The test is intended to be given after a pupil has studied the first two books of the usual high-school course in formal or deductive geometry. This test is one evidently designed for diagnostic purposes rather than an achievement test proper.

Another attempt at a geometry test which ranks among the earliest ones is that of Daniel Starch and H. S. Hemenway at the University of Wisconsin in 1918. This test was designed

as an achievement test to cover a year's work. Its construction and development are described in the thesis entitled "Testing Attainment in Geometry", written by Mr. H. S. Hemenway as a bachelors thesis in 1918.

The more recent developments in the field of Geometry testing are as follows: The Schorling-Sanford Achievement Test in Plane Geometry, devised by Raleigh Schorling, Professor of Education at the University of Michigan, and Vera Sanford of the Lincoln School of Teachers College, Columbia University. This test is well developed to test achievement at the close of a year's work in the subject. Another of the later tests is the one designed by Herbert E. Hawkes, Ph.D., Professor of Mathematics and Dean of Columbia College, and Ben D. Wood, Ph. D., Director of the Bureau of Collegiate Educational Research, Columbia College, Columbia University. This test is designed primarily as a placement test in the subject and is now being revised and improved by its authors.

The writer of this paper is now in touch with two other people who are interested in the problem of constructing tests for achievement in Geometry. Forms A and B of a test entitled "A Test For Ability in Plane Geometry" have just been received from Mr. H. J. Peterson of Ohio University. This is a test which is being developed for testing at the end of the course and is not yet standardized. The most recent is the

Geometry Test now being devised by Mr. George E. Jones at present Superintendent of the Plainfield, Wisconsin, Public Schools. This test is designed to be given at the close of the first semester of work in plane Geometry. In this respect it is different from any tests that have appeared on the market to date. It is the plan of the author of this test to devise a second test to be given at the close of the year's work in the subject.

Due to the fact that the writer of this thesis must have a criterion for the work of pupils at the close of one semester in Geometry it was decided to make use of the results obtained from the Jones test. Although the latter test is not yet standardized and there are no norms of achievement it seems to have decided merits. It has been constructed with full knowledge of the information contained in the report of the National Committee on Mathematical Requirements and after a rather careful study of what has already been done in the testing field in Geometry. It is composed of seven parts which test different phases of the work in Geometry. The responses called for are definite and should be scored without great difficulty. It calls for a type of response that does not necessitate much of the subjective element in scoring and this is one of the prime requisites of an objective test. The number of questions and the range of difficulty should give a very fair sample of the pupils' actual ability in the subject.

PROGNOSIS IN GEOMETRY

As far as the writer knows there have been no attempts to predict success or achievement in plane Geometry alone. There has been a comprehensive test for mathematical ability devised by Agnes L. Rogers and known as the Rogers Test of Mathematical Ability. This was worked out by Miss Agnes L. Rogers in 1918. She was then at Columbia University and did this work as part of the requirement toward the degree of Doctor of Philosophy. The test is primarily a means of determining the inborn power of the pupil along mathematical lines. A full description of the construction, administration and results of the tests designed by Miss Rogers may be found in a monograph entitled "Tests of Mathematical Ability and their Prognostic Value", New York: Bureau of Publications, Teachers College (1918).

Dr. Rogers gave seventeen tests in her preliminary work and after evaluating her results selected six tests which seemed to be the most valuable from the standpoint of their predictive value. These have been revised somewhat and are now available in printed form with directions for their administration.

Dr. L. L. Thurstone has devised a test of Geometry in connection with his vocational guidance test wotk. This is

really a test of the applications of Geometry rather than of geometric proof.

Dr. Rogers found during the course of her work that Geometry and algebra abilities were not any closer than English and Geometry abilities or English and algebra abilities. Quoting from the report of the National Committee on Mathematical Requirements "Much more scientific work remains to be done along the line of the Rogers test. We need tests to measure innate algebraic intelligence, before the child has had any algebraic training whatever. We should have a test or a group of tests of such a character applied the first day in the ninth-grade algebra course, so that we could from the outset group pupils more appropriately, in consonance with their powers and their promise as students of the subject. The same holds good of Geometry. For it, too, we need a team of tests or a single test of high diagnostic power, which presupposes no previous knowledge of the subject, and which could provide us with an adequate measure of innate ability in that field."

With the above survey of the work done and the possibilities in this field of testing, the problem of predicting success in Geometry and a study of the various abilities involved in success in the subject, has been undertaken by the writer.

Chapter II

SELECTING AND GIVING THE TESTS

The first test used in this investigation was a test devised by Mr. H. N. Irwin and published in the School Review for 1918. Mr. Irwin was at the time in the Fairmount Junior High School in Cleveland, Ohio. This test was called, by its author, a test in the Mental Manipulation of Space Relations. One form of the test was used in this investigation. A copy of this as well as of all other tests used will be found in Appendix A.

A test called the Plymouth Educational Test was next used. This consists of three forms, all of which are somewhat similar in character. The test sheet is divided by a horizontal line. Above the line are figures - rectangles in one form, triangles in another, and lines in another - which are numbered. Below the line are similar figures which the pupil is asked to number to correspond with figures above the line which he judges to be of the same size. This is really a test of discrimination or judgment of the size of figures.

The next test used was one in which the pupils were required to reproduce angles which were drawn on large cards

and held in plain view of all pupils in the room. There are in this test five angles to be reproduced each of which is drawn on a large manilla card about twelve inches square and only one card is exposed at a time. This test was devised by Dr. C. L. Hull of the Department of Psychology of the University of Wisconsin. It had been used previously in a battery for prognosis in freehand drawing and it was thought that it might have some value in this study.

The Woodworth and Wells Hard Directions Test was the next one used. This is a short test of directions which the pupil reads and follows out as read. This is probably most familiar to everyone who has had anything to do with tests and testing.

Thurstone's Error Checking test was used because it was thought that there might be some correlation between the ability to check through material of a mathematical nature and check errors in it, and ability in using facts in geometry. This test consists of a series of simple subtraction and addition combinations some of which are not correct and which the pupil is required to check.

Another Thurstone test was selected for use, the Thurstone Reasoning test, form A. This consists of a series of statements and conclusions some of which are false and others of which are true. The pupil is asked to place a

plus sign after those which are true and a minus sign after those which are false.

The Thurstone Spatial Relations test was the next test selected for use. This consists of an exercise in manipulation of figures without the use of pencils except in recording the final result. It can best be understood by reference to the copy of the test in the appendix.

The last group of tests selected for use was the Terman Group test of Mental Ability. These are familiar to all who are in any way interested in the problem of testing. They were given so that the test as a whole as well as the individual tests in the battery might be used in connection with this study.

The last test used, the criterion, was given at the end of the first semester of work in Plane Geometry. This is the Jones Plane Geometry test which is mentioned in the introduction to this paper.

The above tests were applied to a group of ninety pupils in the Wisconsin High School during the first semester of the school year 1925-1926. All the tests designed for the prognosis battery were given during the semester and the criterion or geometry test was applied at the close of the semester's work. Out of the group of pupils to whom the

tests were applied there were finally secured complete scores for seventy pupils. It is the results of the application of the tests to this group of seventy pupils which are given in the following pages. In order to clear up any question as to the time of the tests and the directions used in their administration they are given in full here.

DIRECTIONS GIVEN AND TIME OF THE TESTS:

Test 1- Mental Manipulation of Space - The following directions were printed on the front sheet and were read aloud by the examiner and silently by the pupils:

"The purpose of this test is to find out how well you can handle facts regarding space without using a pencil or any other means of figuring out the answer except in your mind.

This test covers several pages and you will begin work upon the signal to begin and keep right on until you have finished this test or until the signal to stop is given.

Read the question carefully.

Decide upon the answer in your mind and without using your pencil or hand in any way.

Write the answer in the blank space after the question or make the drawing which the question asks you to make.

Remember you are to decide upon the answer in your mind and without using your pencil; draw a figure only when the question asks you for one. Begin." At the end of thirty minutes pupils were stopped.

Test 2- Plymouth Educational - The following oral directions were given for this test. Pupils were asked to look at the sheet as the directions were given. They are as follows: "Notice that the sheet is divided by a horizontal

black line into two parts. For each figure below the heavy black line there is one of the same size above the line. Number each figure below the line with a number which is found in a figure above the line which you judge to be the same size. Go." Pupils were given five minutes.

Test 3- Reproduction of size of angles - The following oral directions were given. (Pupils were provided with a sheet upon which they could draw the angles as exposed to view.) "On these cardboards I have five angles. I will show them one at a time and you will have just one half minute to draw each angle. The legs may be of any length but the angle must be made exactly the same size as it is here. Draw the angle in the same position as shown and don't move your paper around. Draw angle."

Test 4- Hard Directions - The only direction used here was the following: "On the other side of this sheet are some directions which you are to read and carry out. Do just what they tell you to do." Time given was one minute and fifteen seconds.

Test 5- Error Checking - The direction used here is found on the sheet. It was read aloud by the examiner while the pupils read silently. "This is a test of concentration

in checking for errors. You will be shown a list of simple additions and subtractions. Put a small check mark opposite each mistake. Take the examples in the order given and work as rapidly as possible without leaving mistakes unmarked. Turn over the sheet and begin." Time given was five minutes.

Test 6- Thurstone Reasoning Test A - The following direction was read aloud by the examiner while the pupils read silently from the sheet. "This is a test of reasoning ability and concentration. You will be shown a list of arguments. Some of these arguments have true conclusions and some have false conclusions. In the square after each argument, make a plus sign if the conclusion is true; make a minus sign if the conclusion is false. Work as rapidly as you can without making any mistakes. Turn over the sheet and begin." Time given to this test was ten minutes.

Test 7 - Thurstone Spatial Relations Test A - The following direction was read aloud by the examiner while the pupils read silently from the sheet. "Assume that the lozenge shaped figure with a circle in it represents a small card with one of its edges printed black and with a hole in one corner. Imagine that this card is picked up, turned over, and placed face down with the black edge of the card touching the long heavy black line to the right. Imagine the card

moved along this black line until its edges fit the edges of one or the other of the lozenge shaped outlines. With your pencil draw a circle in the corner where the hole will be. Try the three following examples: Then work the examples on the other side of this blank. Begin." Time given to this test was eight minutes.

Test 8- Terman Group Test of Mental Ability - In giving this test the directions used and the time allowed were just as given in the manual of directions which accompanies this test.

The criterion - Jones Geometry Test - General directions as found on the front sheet of this test were read aloud by the examiner while the pupils read them silently. They are as follows: "Following are a number of tests in plane geometry. You are to do them in the order in which they occur. When you have quit working on one you are not to go back to it. You will be allowed the following lengths of time for each test: (Here the time for each test is inserted). When told to start on the next test, do so at once. If you finish a test before the signal to stop is given you may start work on the next test. Begin." Time for the group of tests given here was forty minutes.

Chapter III

TEST RESULTS

It is the aim here to give a brief summary of the test results secured from the group in the Wisconsin High School. Most of this will be found in tabular form in the following pages. Table I, below gives the mean scores on the various tests and the standard deviations.

TABLE I

- Table showing the means and the standard deviations of the test scores in the various tests used.

	Mean	S.D.
Manipulation of Space	8.649	4.318
Plymouth Educational	17.357	8.517
Reproduction of Angles	30.171	15.194
Hard Directions	7.214	3.175
Thurstone Error Checking	76.428	13.747
Thurstone Reasoning Test	14.957	2.940
Thurstone Spatial Relations . . .	13.914	8.149
Terman Information	13.842	4.241
Terman Best Answer	17.685	4.150
Terman Word Meaning	16.514	8.053
Terman Logical Selection	12.885	3.432

Table I (Concluded)

	Mean	S.D.
Terman Arithmetic	11.771	5.621
Terman Sentence Meaning . . .	10.571	5.327
Terman Analogies	13.714	3.802
Terman Mixed Sentences . . .	8.685	4.743
Terman Classification	13.571	1.989
Terman Number Series	14.571	6.415
Terman Group (total)	133.799	36.201
Jones Geometry Test (Criterion)	79.814	21.882

In order to check the tests against the achievement pupils as estimated by teachers it was decided to tabulate semester grades in plane geometry for the pupils who had taken the various tests. English grades were also obtained for all pupils who had taken the tests. The table below gives the mean and the standard deviation of these school grades.

TABLE II

[illegible] - Table showing the means and the standard deviations of the semester grades in Geometry and English.

	Mean	S.D.
Geometry	84.985	7.224
English	85.999	6.381

It was necessary to know the relationship between the various tests given and the criterion scores and also between the grades in geometry and in english and the scores obtained on the criterion. To determine this the various tests and the semester grades were correlated with the criterion and the results are given in Table III which follows.

TABLE III

- Table showing coefficients of correlation between the various tests and the criterion - Jones geometry test-; and between the semester grades in geometry and english and the criterion.

	r.
Manipulation of Space	.608
Plymouth Educational	-.041
Reproduction of Angles	-.069
Hard Directions	.501
Thurstone Error Checking	.260
Thurstone Reasoning	.569
Thurstone Spatial Relations . . .	.315
Terman Information	.562
Terman Best Answer	.477
Terman Word Meaning	.585
Terman Logical Selection	.582
Terman Arithmetic	.598
Terman Sentence Meaning	.473

Table III (Concluded)

	r.
Terman Analogies	.718
Terman Mixed Sentences	.579
Terman Classification	.342
Terman Number Series	.560
Terman Group Total	.738
English grades	.568
Geometry grades	.754

In order to test the agreement of the various tests with school marks in English and Geometry the correlations between each of the tests used and the school grades in these two subjects were worked out. They appear in the following table.

TABLE IV

- Table showing the coefficients of correlation between the various test scores and the semester grades in Geometry and English.

	English Grades	Geometry Grades
Manipulation of Space	.294	.445
Plymouth Educational	-.136	-.007
Reproduction of Angles	-.016	.017

Table IV (Concluded)

	English Grades	Geometry Grades
Hard Directions	.543	.534
Error Checking	.152	.163
Reasoning test	.417	.425
Thurstone Spatial Relations	.034	.198
Terman Information	.432	.382
Terman Best Answer	.471	.394
Terman Word Meaning	.581	.522
Terman Logical Selection .	.551	.631
Terman Arithmetic	.330	.471
Terman Sentence Meaning . .	.353	.377
Terman Analogies	.589	.596
Terman Mixed Sentences . .	.433	.570
Terman Classification . . .	.274	.373
Terman Number Series . . .	.443	.511
Terman Group Total	.601	.643

The Geometry grades correlated with the semester grades in English gave an r ~~equal to~~ of +.573. The Geometry grades correlated as highly with the English grades as the English grades did with the Geometry criterion.

We next wanted to find what the intercorrelations between the most promising members of the test battery were. In this

procedure ten of the tests were selected and correlations were worked out between each one of the ten and every other one. This gave us an array of coefficients showing at once which tests correlated highly with each other and which ones comparatively low. In choosing tests for a team or battery to yield the maximum prediction there must be not only as high a correlation as possible between each test and the ctiterion, but at the same time as low a correlation as possible among the individual tests.[1]

The tests selected for inter-correlation were the following: Manipulation of Space, Reproduction of Angles, Plymouth Educational, Hard Directions, Thurstone Error Checking, Thurstone Reasoning test, Thurstone Spatial Relations test, Terman Arithmetic, Terman Analogies, and Terman Number Series. The coefficients obtained appear on the following page in table V.

1. Hull, C. L. The Joint yield from teams of tests. Journal of Educational Psychology. 396-406. October, 1923.

TABLE V

- Table showing the inter-correlations between ten of the original tests. Roman numbers refer to the tests listed below.

	I	II	III	IV	V	VI	VII	VIII	IX
II	.093								
III	-.282	-.235							
IV	.366	-.023	-.079						
V	.150	-.046	-.143	.286					
VI	.476	-.042	.142	.394	.218				
VII	.310	.195	.001	.255	.068	.149			
VIII	.557	-.022	-.112	.371	.418	.345	.198		
IX	.494	-.164	-.131	.568	.216	.447	.157	.589	
X	.500	-.001	-.198	.482	.197	.374	.322	.536	.710

I. Manipulation of Space

II. Plymouth Educational

III. Reproduction of Angles

IV. Woodworth and Wells Hard Directions

V. Thurstone Error Checking

VI. Thurstone Reasoning Test

VII. Thurstone Spatial Relations

VIII. Terman Arithmetic

IX. Terman Analogies

X. Terman Number Series

Having the correlation of each test with the criterion and the intercorrelations between the various tests it was necessary to decide which tests might be used in an aptitude battery. In regard to the procedure used in selecting the tests to be used I quote here from an article published by Dr. C. L. Hull of the Department of Psychology of the Univeristy of Wisconsin and C. E. Limp, a former graduate student at the University of Wisconsin. "The mere fact that a number of tests are available, each yielding a fairly high correlation with the criterion, does not necessarily mean that all should be included in the battery. If two or more of them test practically the same type of behavior as shown by a high correlation between the tests themselves, it is largely a waste of time to use them all just as it would be to use the same test over and over again. It may be much better to choose a test with a distinctly lower correlation with the criterion provided it measures a trait not already represented on the battery as shown by a low correlation with the other tests to be used."[2]

Manipulation of Space was included in the tests used because it gave a fairly high correlation with the criterion

2. Hull, C. L. and Limp, C. E. Differentiation of the aptitudes of an individual by means of test batteries. Journal of Educational Psychology. February, 1925.

and quite low correlations with the other tests selected. The highest intercorrelation is that between this test and the Terman Analogies which gives us an r. of +.494. The Reproduction of Angles test was selected because it gives small negative or positive correlation with most of the other tests used. Thurstone's Error Checking test was used largely because it gave us very low correlations with the other tests selected and gave us a sample of behavior which, judged from this standpoint, differed quite markedly from that of the other tests of the battery. Thurstone's Spatial Relations test was used in the battery for practically the same reasons. Neither it nor the Error checking give very high correlation with the criterion but it was thought that the type of response required in these tests was sufficiently different from that of the ordinary test that it would be worth while to include them in the battery. The Terman Analogies test was included because it gave a very high correlation with the criterion and did not correlate very highly with any of the other tests used except as previously noted, with the Manipulation of Space test.

Having chosen the tests to be used in the battery it was next necessary to determine the proper weight to be assigned to each test score to give the highest prediction. This was done by means of the regression equation using the method of

calculation from the moments as devised by Tolley and Ezekiel.[3]

By this method it is not necessary to secure the complete correlation coefficient but only what is known as the moments. There are two kinds of moments. The first are simply the squares of the respective standard deviations. These values can be secured by the formula:

$$p_{1\,1} = M_{1}\,2 - (M)^{2}_{1}$$

$$\text{or} \quad p_{2\,2} = M_{2}\,2 - (M)^{2}_{2}, \text{ and so forth.}$$

The above formula is that for the standard deviation except that it has no radical over it.

The second type of moments is related to the correlation coefficient. In fact it is nothing more than the numerator of a formula for the coefficient:

$$p_{1\,2} = M_{1 \times 2} - M_{1} \times M_{2}$$

$$\text{or} \quad p_{2\,3} = M_{2 \times 3} - M_{2} \times M, \text{ and so on.}$$

3. Tolley, H. R. and Ezekiel, M. J. B. A method of handling multiple correlation problems. Quarterly Publication of the American Statistical Association. December, 1923.

The moments for the test scores used were computed, using the above formulae and a set of normal equations was written. From these equations, a series of five simultaneous equations, the values of the various weights to be assigned the first five tests were determined. This gave the primary form of the regression equation which is given below.

$$X_1 = W_2 X_2 + W_3 X_3 + W_4 X_4 + W_5 X_5 + W_6 X_6$$

Replacing the W's with the weights found by the above calculation we have the following form of the regression equation:

$$X_1 = .162 X_2 + .015 X_3 + .165 X_4 + .335 X_5 + 3.059 X_6$$

In the above equation the symbols represent the following tests previously referred to:

X_1 is the criterion (Jones Geometry Test).
X_2 is the Test of Manipulation of Space.
X_3 is the Reproduction of Angles.
X_4 is the Thurstone Error Checking Test.
X_5 is the Thurstone Spatial Relations Test.
X_6 is the Terman Analogies Test.

In order to transform this formula into one in which the actual test scores can be used each x above is considered

as the deviation of the actual score from the mean. Upon the substitution of values such as $X_2 = X_2 - M_2$, and so on for the various X's above we finally derive the useful multiple regression equation which requires simply the substitution of the actual test score of any individual in order to make a prediction of his most probable score in the criterion. The final form of the multiple regression equation as worked out by this method is given below:

$$X_1 = .162 X_2 + .015 X_3 + .165 X_4 + .335 X_5 + 3.059 X_6 + 18.654.$$

Using the above formula the actual scores made on the tests were substituted in the equation and a prediction made for each of the seventy subjects who had formed the basis of the previous work. Before this a coefficient had been worked out to determine how high the test battery would correlate with the criterion when weighted according to the multiple regression equation. This coefficient is symbolized by R and it is called the coefficient of multiple correlation. The formula used and the coefficient obtained are found below:

$$R = \sqrt{\frac{W_2 p_{12} + W_3 p_{13} + W_4 p_{14} + W_5 p_{15} + W_6 p_{16}}{p_{11}}}$$

Substituting for the various W's above the values obtained from our calculations and for the various moments their numerical values we obtain the following value for R: R = +.797.

Having made the predictions for each of the seventy pupils from their scores in the several tests of the battery the predicted scores were now correlated with the original criterion scores and with the semester grades in Geometry and English. The various means, sigmas and coefficients obtained are given in the table below.

TABLE VI

- Table showing the means and the standard deviations of the semester grades in Geometry, and in English; and of the original criterion and the predicted criterion scores. Part B shows the coefficients of correlation of the original criterion scores, the English and the Geometry semester grades with the predicted criterion scores.

A. Means and Sigmas	Mean	Sigma
Geometry grades (semester)	84.98	7.224
English grades (semester)	85.99	6.381
Criterion scores	79.8142	21.88
Predicted criterion scores	79.885	13.38

B. Coefficients obtained with predicted scores	r.
Criterion and prediction	+.810
English grades and prediction	+.558
Geometry grades and prediction	+.605

DISCUSSION AND SUMMARY:

The coefficient of correlation obtained between the predicted scores and the original criterion scores should correspond to that obtained by the formula for big R. It is used as a check for the accuracy of all computations involved in the regression equation. The coefficient given in table VI is +.810 which is slightly higher than that for big R which is given above, namely, + .797. To avoid using huge numbers in the final computation all predictions were treated as whole numbers. If the decimal part of the number exceeded 0.5 the number was given the value of the next higher whole number. It is quite certain that had we used all numbers as found to the second or third decimal place that our coefficient would have been practically the same as that found for big R. In fact we found that by treating some of the numbers as though they were just one unit lower than actually used in the final computation, that the coefficient of correlation was slightly lower than that for big R.

A coefficient of +.810 gives us a high prediction value as compared to the usual type of coefficient found thus far. When we apply the formula to determine the efficiency of such a coefficient we find that it gives us a prediction efficiency

of about 41.3 % which is very high as tests run. The formula used for E in this case is as follows:

$$\text{Efficiency} = 1 - \sqrt{1 - r^2}$$

E, in this case, represents the percentage reduction in the actual error of estimating, or forecasting, an aptitude over the amount of error or inaccuracy that would result from making such a forecast from a set of test scores correlating zero with the criterion; that is, a set having a purely chance relation and giving no information at all.[4]

We find that there is about the same degree of correlation between the semester grades in English and in Geometry for this particular group as there is between semester grades in English and the scores in the criterion, that is, the Jones Geometry test. The coefficient for Geometry grades and English grades is +.573 while that for the criterion and English grades is +.568. On the other hand the coefficient for Geometry grades and the criterion is +.754 which seems to indicate a closer relationship between them than in the case of English grades and the criterion scores. This would seem to indicate that semester

4. Hull, C. L. and Limp, C. E. The differentiation of the aptitudes of an individual by means of test batteries. Journal of Educational Psychology. February, 1925.

grades have to some extent differentiated the pupils abilities in the two subjects. The truth of this really depends upon the validity of the criterion used in this particular study. It is regretted that we have no objective measure of the work in English with which to compare the results of the tests used in this study. There really are no very serviceable scales constructed for use in high school work in English. It might be possible to construct a composite score based upon the achievement in composition, results of vocabulary testing, and some objective information as to the pupils' familiarity with required literature. A weakness in the criterion used in this study is the fact that it is a pencil and paper test and for that reason might tend to give us a higher correlation with the tests used which are themselves all pencil and paper tests. This is an objection which is not peculiar to the tests used in this study alone however, because practically all of the tests used in any school situation, whether they are of the type given here or simply the informal examination or test given by the teacher, are pencil and paper tests. We may find a method of avoiding this but thus far no one has suggested any very successful substitute for the measurement of school achievement.

From the results obtained with the Terman Group Test of Mental Ability it would seem that there is a high degree

of over-lapping in the Terman tests. In this particular study it seems that one could predict ability in Geometry about as well from a single test of the Terman Group, the Analogies Test, as from the results of the whole test. The Terman Analogies gives a coefficient of +.718 with the criterion while the Terman test as a whole gives a coefficient of only +.738. It is evident from the inter-correlations of Table V that there is a good deal of over-lapping in the Terman tests for which coefficients are given there. This table shows that the Terman Analogies and the Terman Number Series give a correlation of +.710 and Terman Arithmetic gives coefficients of +.589 with the Analogies and +.536 with the Number Series. It is evident from a study of the coefficients set forth in Table V that the other tests of the original group used at least have the merit of little over-lapping as evidenced by the size of the inter-correlations obtained. Some of them do correlate rather highly with the Terman tests but among themselves the correlations run comparatively low.

The only attempt to predict ability in mathematics so far is that of Miss Agnes Rogers which has been previously referred to in this thesis. The chief objection to the method employed there is that she used a composite of the scores of the original tests as a criterion with which to

measure each of the tests of the battery. In regard to her method of weighting the tests used she says in her account, "The measures for each function were weighted with reference to two main factors, the importance of the ability measured and the reliability of the test from which they were derived."[5]

From all other standpoints Miss Rogers' work was carefully done and it is unfortunate that her tests have not been evaluated against some good objective criteria in the subjects for which they are designed. The chief limitation in this respect is due to the fact that there are very few tests available in the field of high school mathematics at the present time. The work being done now in the construction of achievement tests and their evaluation will undoubtedly result in some fairly good objective standards of achievement in that particular branch of high school mathematics.

This thesis presents only the preliminary work toward the organization of a battery of tests which may give us a fair sampling of some of the abilities closely related to what we may term ability in Geometry. The tests used here

5. Rogers, Agnes L. Experimental tests of mathematical ability and their prognostic value. Teachers College, Columbia University, Contributions to Education, No. 89. 1918.

are undergoing revision and will be given to a new and larger group of subjects in the hope of a further improvement in technique and results. It is planned to use a larger number of pupils in the original application of the revised tests, to use some new tests which may give us a measure of certain abilities not touched by the other tests used, and to work out a new regression equation based upon this application of the tests. This work is already under way and it is our hope that we may be able to apply the revised tests to a group of pupils at the beginning of the semester in the fall of 1926 and make predictions as to their success in Geometry on the basis of the showing they make on these tests. Results of this work will be published later.

Our work up to the present points to the probability that tests may be found which will really differentiate between ability in Geometry and in other school subjects.

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APPENDIX A

Frank White
13
12 R
3 W

MENTAL MANIPULATION OF SPACE

The purpose of this test is to find out how well you can handle facts regarding space without using a pencil or any other means of figuring out the answer except in your mind.

This test covers several pages and you will begin work upon the signal to begin and keep right on until you have finished this test or until the signal to stop is given.

DIRECTIONS:

1. Read the question carefully.
2. Decide upon the answer in your mind and without using your pencil or hand in any way.
3. Write the answer in the blank space after the question or make the drawing which the question asks you to make.

Remember you are to decide upon the answer in your mind and without using your pencil; draw a figure only when the question asks you for one.

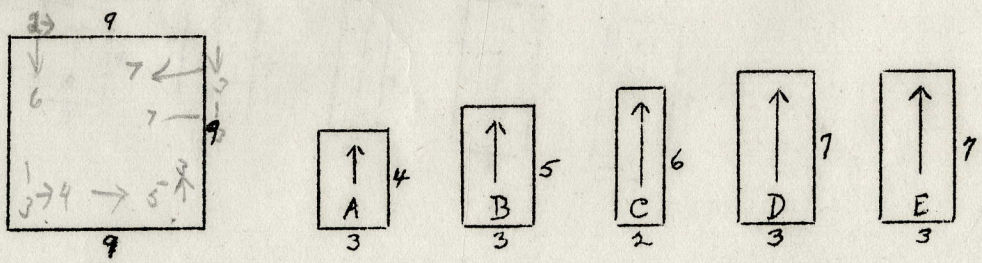
1. A pupil has a sheet of paper 14 inches long and 10 inches wide. On it he wishes to draw a map so that there will be a space of 2 inches left between each edge of the map and the edge of the paper. What must be the length and width of the map? Length 10" Width 6"

2. It is 5:12 o'clock. Suppose the hands change places so that the large hand takes the place of the small hand while the small hand takes the place of the large hand; what time would it then be? 2:25

3. A square sheet of paper eight inches long on each edge is to be divided by lines into small squares two inches long on each side. How many lines must be drawn across the sheet of paper to do this? Answer 6 lines

4. Four points are placed on the blackboard as if they represented the corners of a four-sided figure. How many different straight lines can be drawn between these points? 6 lines

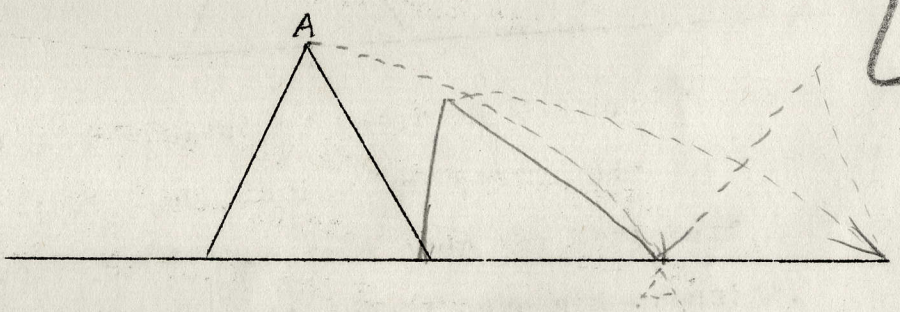
5. Using the letters and arrows only show how you would place A,B,C,D, and E, in the large square so as to fill it. The numbers show the size. A,B,C,D, and E, may be turned around or over in any way. Use the letters and arrows only to show where you would place them. Draw no lines in the large square.



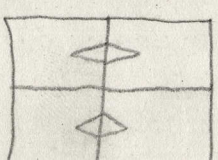
6. Three men, A, B, and C, start from the same point, A going north three miles, B going east three miles, and C going west three miles. If the straight line distance between them is measured, which men are farthest apart? B and C are farthest Which men are the same distance apart? A and B are the same distance away as A and C.

7. A boy has a square box lid in which he can lay just three rows of three marbles each. If he now places on top of this first layer of marbles a second layer in which each marble touches four marbles of the first layer, how many marbles are there in the second layer? Answer 4 marbles

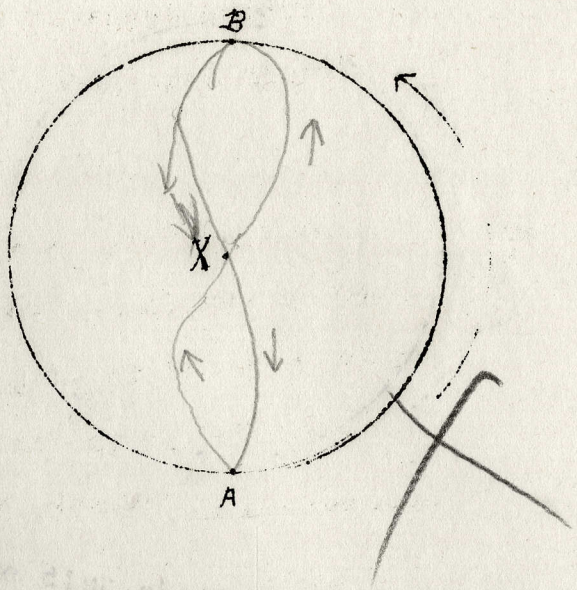
8. In the figure below make a drawing to show the path which point A will follow as the triangle is rolled over and over on the straight line until it reaches the other end of the line.



9. I have a square piece of paper on the desk in front of me. I fold it once placing the bottom edge on the top edge and creasing. I then fold over the left half of this placing the left edge on the right edge and creasing. I next cut a small piece out of the middle of the folded edge on my left. Make a drawing of the paper as it will appear to me when unfolded, showing the creases and what results from the cutting.



10. The circle at the right represents a round metal disk like a victrola record that is made to rotate or turn about its center X in the direction shown by the arrow. As it rotates a needle point moves from A across the disk through X to B, and back again to A, in the same time that it takes the disk to make half a complete turn. Draw a line on the disk showing the path followed by the needle point.



11. A pupil has a square sheet of paper 10 inches long on a side, in the middle of which he draws a square 4 inches long on a side, each side of the square being the same distance from the edge of the sheet of paper. What is this distance?

3 inches on each side

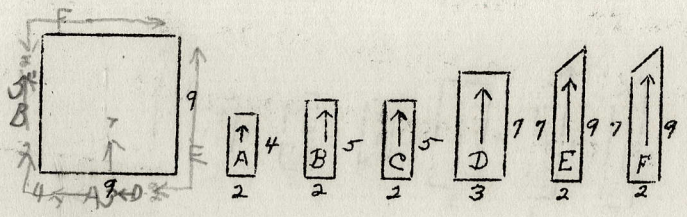
12. It is 6:48 o'clock. Suppose the hands change places so that the large hand takes the place of the small hand while the small hand takes the place of the large hand; what time would it then be? Answer About 9:35

13. Four lines are drawn on the blackboard each one inch apart. Four other lines also one inch apart are then drawn across these in such a way as to make a number of one inch squares. How many squares are thus made?

Answer 9 squares

14. A box is to be drawn on the blackboard, each edge being represented by either a solid or dotted line. Altogether, how many solid and dotted lines must be drawn? 12 lines altogether.

15. Using the letters and arrows only, show how you would place A,B,C,D,E, and F, in the large square so as to fill it. The numbers show the size. A,B,C,D,E, and F may be turned around or over in any way. Use the letters and arrows only, to show how you would place them. Draw no lines in the large square.

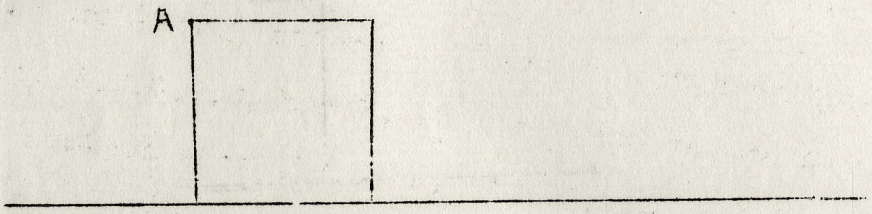


16. Three men A,B,C, start from the same point, A going west five miles, B going south five miles, and C going north five miles. If the straight line distance between them is measured, which men are the farthest apart? _____

Which men are the same distance apart? _____

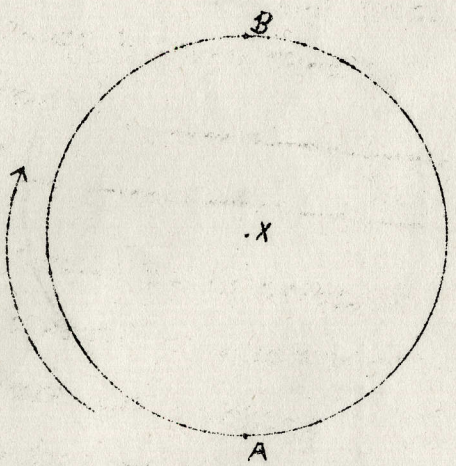
17. In a show case there is a pile of tennis balls arranged thus: At the top is a single ball which rests on a layer of four balls arranged in the form of a square and touching each other. Each of these four balls in turn rests on four balls in the next layer. How many balls are there in the layer directly under the four balls? Answer _____

18. In the figure below make a drawing to show the path which point A will follow as the square is rolled over and over on the straight line until it reaches the other end of the line.



19. I have a square piece of paper on the desk in front of me. I fold it once, placing the lower left hand corner on the upper right hand corner and creasing. I then fold this, placing the lower right hand corner on the upper left hand corner and creasing. I next cut a small piece out of the middle of the folded edge on my right. Make a drawing of the paper as it will appear to me when unfolded, showing the creases and what results from the cutting.

20. The circle at the right represents a round metal disk like a victrola record that is made to rotate or turn about its center X, in the direction shown by the arrow. As it rotates a needle point moves from A, across the disk through X to B, and back again to A, in the same time that it takes the disk to make one complete turn. Draw a line on the disk to show the path followed by the needle point.



21. A photographer has a picture 24 inches long, and 18 inches wide, which he wishes to mount on a piece of cardboard so that each edge of the picture will be 3 inches from the edge of the cardboard. What must be the length and width of the cardboard? Length _____

Width _____

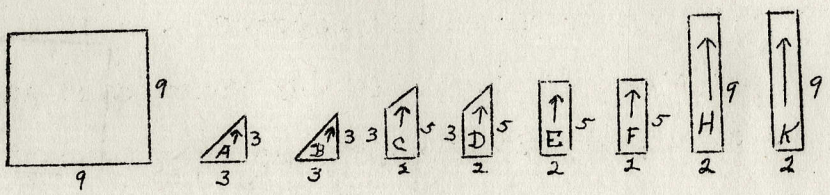
22. It is 2:36 o'clock. Suppose the hands change places so that the large hand takes the place of the small hand while the small hand takes the place of the large hand; what time would it then be? Answer _____

23. In making a square cushion buttons are sewed on top of the cushion as follows: The buttons are arranged in rows 5 inches apart and 5 inches from the edge of the cushion. The buttons in each row are also 5 inches apart and 5 inches from the edge of the cushion. How many buttons are needed for the top of a square cushion 25 inches long on each side? _____

24. A square cornered trunk with open lid is to be drawn, using a straight line to represent each edge of the top and each edge of the bottom except the edge of the top and the edge of the bottom along which the hinges are fastened. Since these two edges come together they are to be represented by a ~~separate~~ single straight line. How many lines must be drawn to represent the entire figure?

Answer _____

25. Using the letters and arrows only, show how you would place A,B,C,D,E,F,H,K, in the large square so as to fill it. The numbers show the size. A,B,C,D,E,F,H,K, may be turned around or over in any way. Use the letters and arrows only, to show where you would place them. Draw no lines in the large square.



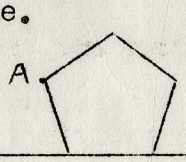
26. Three men, A,B, and C, start from the same point, A going northwest ten miles, B going southeast ten miles, and C going southwest ten miles. If the straight line distance between them is measured which men are the farthest apart? _____ Which men are the same distance apart?

_____

27. I have a small three cornered box along any one side of which I can just fit a single row of three tennis balls. Altogether, how many tennis balls can I place in the box?

_____

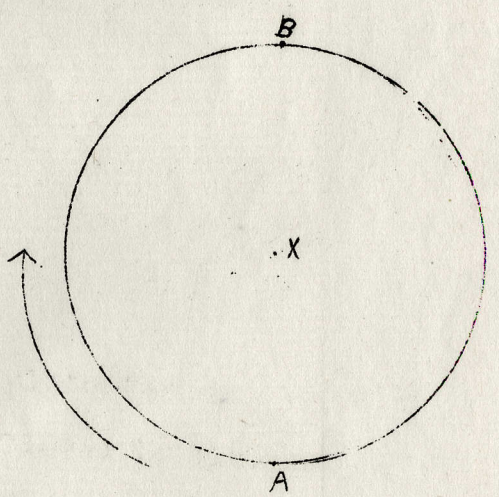
28. In the figure below make a drawing to show the path which point A will follow as the five-sided figure is rolled over and over on the straight line until it reaches the other end of the line.



_____

29. I have a square piece of paper on the desk in front of me. I fold it once placing the right edge on the left edge and creasing. I then fold up the lower half of this placing the bottom edges on the top edge and creasing. I next cut a hole through the middle of this while keeping it folded. Make a drawing of the paper as it will appear to me when unfolded showing the creases and what results from cutting the hole through the folded paper.

30. The circle at the right represents a round metal disk like a victrola record that is made to rotate or turn about its center X, in the direction shown by the arrow. As it rotates a needle point moves from A, across the disk through X to B, and back again to A, in the same time that it takes the disk to make two complete turns. Draw a line on the disk showing the path followed by the needle point.

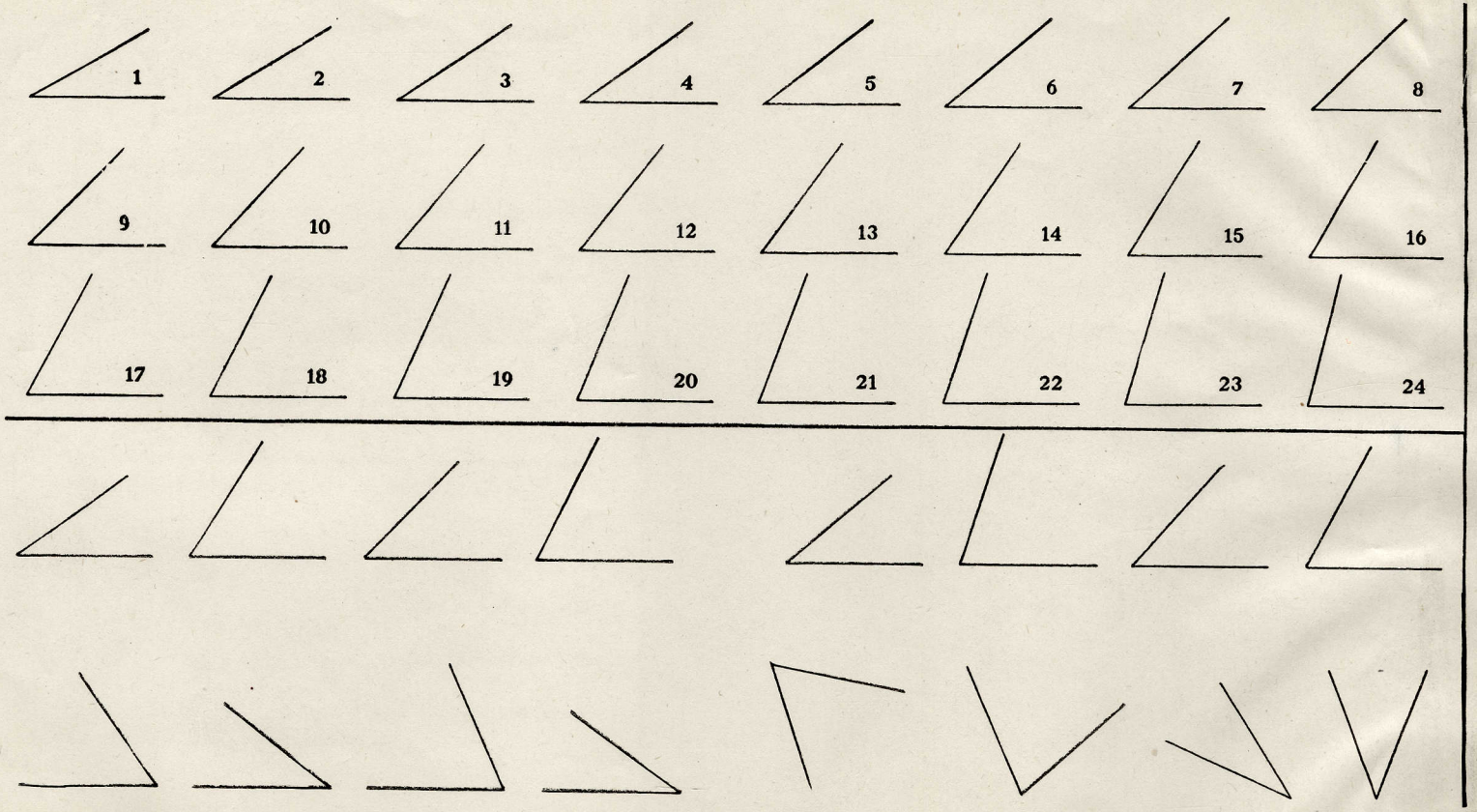


Test Number

111A

Equal angles.

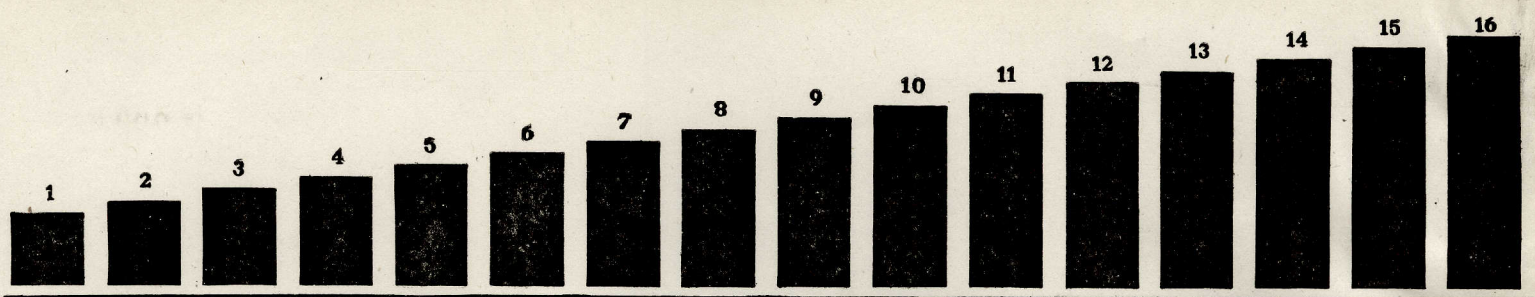
PLYMOUTH EDUCATIONAL TESTS

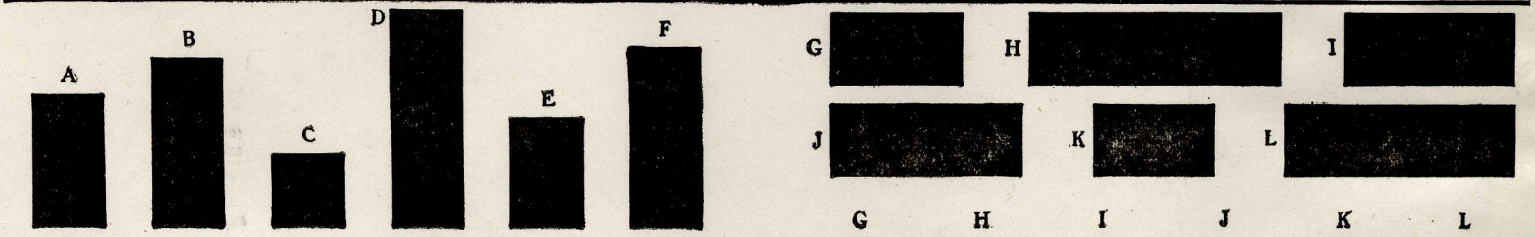


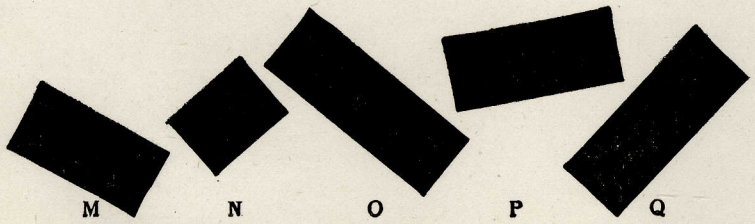
Test Number

112A

Equal rectangles.







Which equals twice No. 4?	Which equals half of No. 15?	Which equals one-half more than No. 7?	Which equals one-third of No. 16?	Which equals No. 3 + No. 5?

PLYMOUTH EDUCATIONAL TESTS

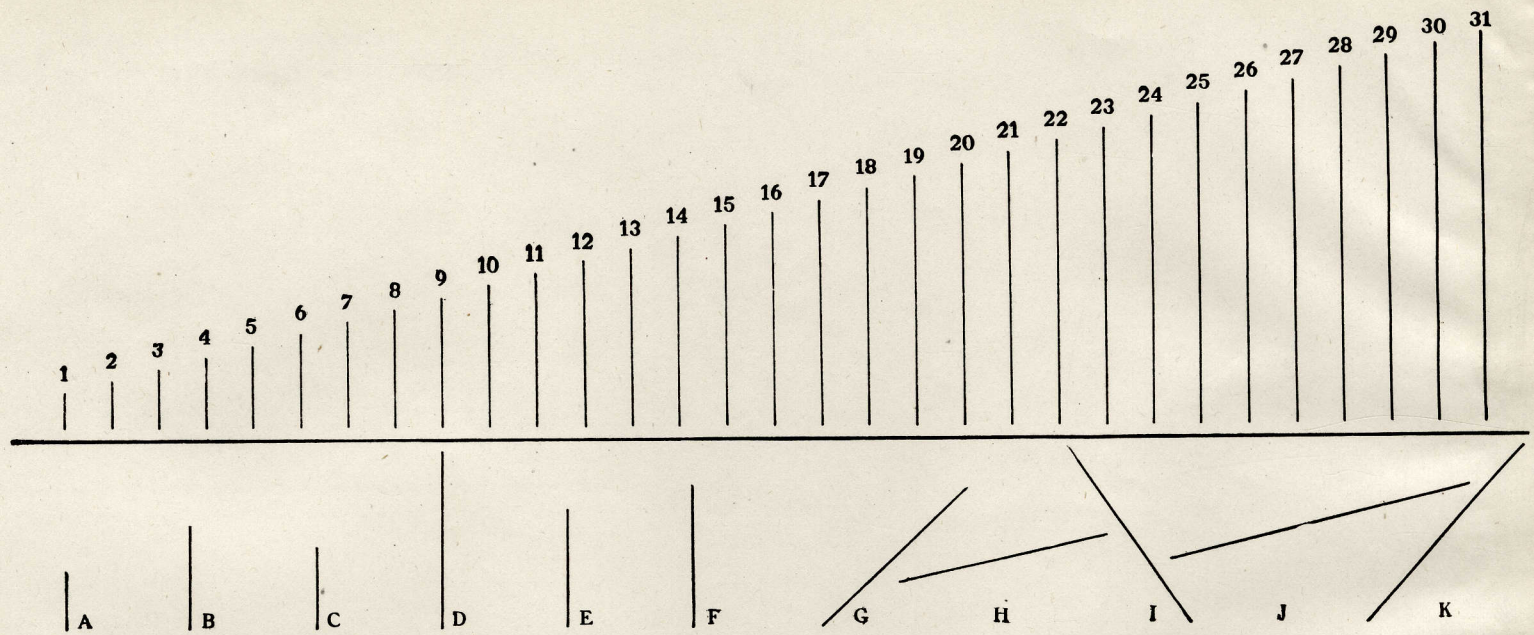
Test Number

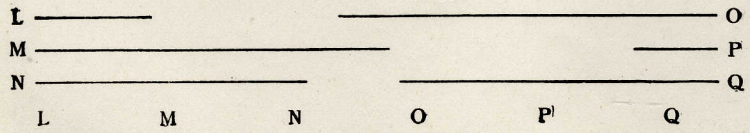
113A

Equal lines.

PLYMOUTH EDUCATIONAL TESTS

PATENT APPLIED FOR

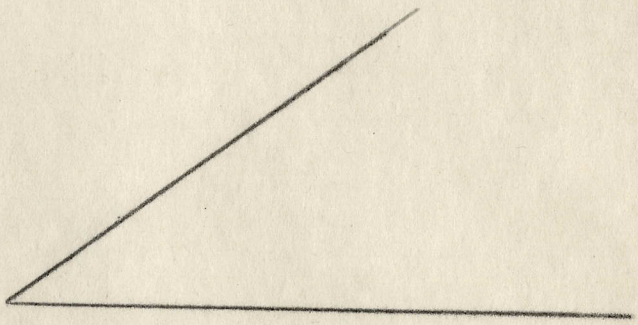




Which line is equal to twice No. 9?	Which equals half of No. 10?	Which equals one-half more than No. 14?	Which equals one-third of No. 28?	Which equals No. 8 + No. 12?

Name _____

1.



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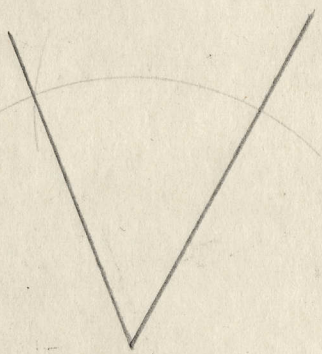

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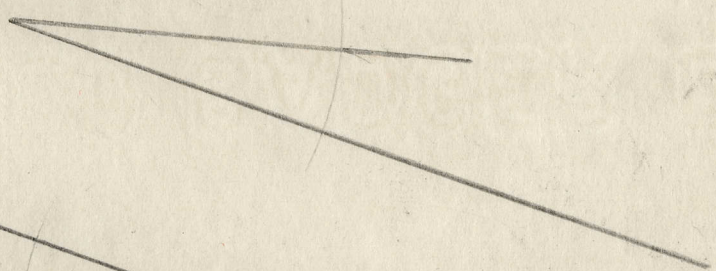

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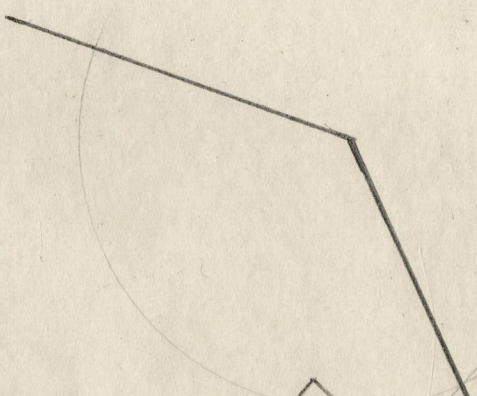

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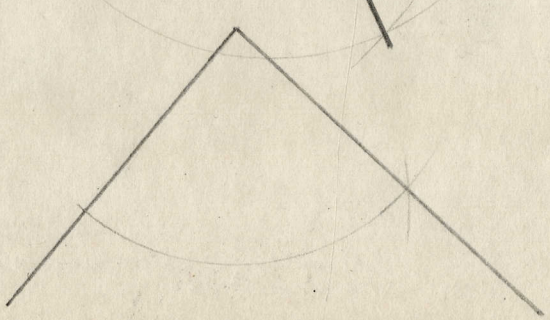


With your pencil make a dot over any one of these letters **F G H I J,** and a comma after the longest of these three words: **boy mother girl** Then, if Christmas comes in March, make a cross right here..... but if not, pass along to the next question, and tell where the sun rises..... If you believe that Edison discovered America, cross out what you just wrote, but if it was some one else, put in a number to complete this sentence: "A horse has.....feet." Write *yes*, no matter whether China is in Africa or not .....; and then give a wrong answer to this question: "How many days are there in the week?"..... Write any letter except *g* just after this comma, and then write *no* if 2 times 5 are 10..... Now, if Tuesday comes after Monday, make two crosses here.....; but if not, make a circle here.....or else a square here ..... Be sure to make three crosses between these two names of boys: George.....Henry. Notice these two numbers: **3, 5.** If iron is heavier than water, write the larger number here....., but if iron is lighter write the smaller number here..... Show by a cross when the nights are longer: in summer?..... in winter?..... Give the correct answer to this question: "Does water run uphill?"..... and repeat your answer here..... Do nothing here ($5 + 7 =$ .....), unless you skipped the preceding question; but write the first letter of your first name and the last letter of your last name at the ends of this line:

No. 27225—Woodworth & Wells Test C. H. Stoelting Co., Chicago, Ill., U. S. A.

THURSTONE ERROR CHECKING TEST

This is a test of concentration in checking for errors. You will be shown a list of simple additions and subtractions. Put a small check mark opposite each mistake. Take the examples in the order given and work as rapidly as you can without leaving mistakes unmarked.

C. H. Stoelting Co., Chicago, Ill., U. S. A.

ERROR CHECKING TEST

7+5=12	9+9=18	5+10=14	18−5=13	9+2=10
11−7=4	16−7=9	15−2=12	3+12=15	4+16=19
2+13=16	4+15=20	4+15=19	16−9=8	5+9=14
19−4=15	11−8=3	14−9=6	7+6=13	4+14=17
13−6=7	4+8=12	6+8=15	19−5=15	5+15=21
6+12=19	14−7=8	5+12=17	4+8=13	9+11=20
13−7=5	7+13=21	12−8=4	9+8=17	12−3=8
7+7=14	11−4=7	2+15=16	13−3=10	20−3=18
12−5=7	7+9=15	13−2=11	7+6=12	3+11=15
3+13=15	18−3=16	11−2=8	3+8=11	11−4=8
11−3=7	2+14=16	3+9=13	15−8=8	4+10=14
17−4=12	8+6=13	4+14=18	5+11=16	14−7=7
8+5=13	19-2=16	19−5=14	17−7=10	2+12=15
8+7=14	12−4=7	16−3=12	6+5=12	20−9=11
17−8=9	7+6=13	6+11=17	14−2=11	15−4=11
4+12=16	18−5=14	20−9=10	3+10=13	3+14=16
17−2=15	12−6=6	7+10=16	14−6=9	20−8=11
8+10=18	2+11=12	18−4=15	16−6=10	5+6=11
13−3=11	13−2=12	8+7=15	2+17=18	17−5=11
13−4=10	2+12=14	20−7=14	20−8=12	7+4=10
7+10=17	19−9=11	8+3=12	6+12=18	12−7=5
3+12=16	17−3=13	13−5=7	18−9=8	6+13=19
18−6=12	14−5=9	18−7=10	17−6=12	4+13=18
8+3=11	19−3=16	8+4=12	5+14=20	16−4=11
12−8=4	4+9=13	19−2=17	17−6=11	8+11=19
14−9=5	15−8=7	2+14=15	3+11=14	20−3=17
9+2=11	13−6=6	6+6=12	6+13=18	7+9=16
20−2=19	7+11=19	16−3=13	19−3=15	2+9=12
13−8=4	20−6=15	2+11=12	8+4=13	20−2=18
7+11=18	7+4=11	5+14=19	14−6=8	2+9=11
11−3=8	14−3=11	16−8=7	17−7=11	19−7=12
6+10=15	14−2=12	2+10=11	5+13=18	15−2=13
15−4=12	5+6=10	12−7=6	7+12=18	5+12=18
5+9=13	20−6=14	3+8=10	19−8=11	2+9=12

THURSTONE REASONING TEST A

This is a test of reasoning ability and concentration. You will be shown a list of arguments. Some of these arguments have true conclusions and some have false conclusions.

In the square after each argument, make a plus sign if the conclusion is true; make a minus sign if the conclusion is false.

Work as rapidly as you can without making any mistakes.

C. H. Stoelting Company, Chicago, Ill.

REASONING TEST A

Silver is heavier than iron; copper is lighter than silver; therefore copper is heavier than iron.

John has two sons, Henry and George. George has a son, James, and Henry has a son William. Therefore Henry is James' uncle.

Jones owes Smith one hundred dollars; Brown owes Jones one hundred dollars. The two debts will be settled if Smith pays one hundred dollars to Brown.

All the members of the Civic Club are members of the University Club; Smith is not a member of the University Club; therefore he is not a member of the Civic Club.

John's birthday is before Harry's and Tom's birthday is before John's. Therefore Tom's birthday is before Harry's.

Iron is lighter than copper; silver is heavier than copper; therefore silver is heavier than iron.

William's father, Henry, has a brother, George, who has a son James. Therefore William and James are cousins.

Jones owes Smith one hundred dollars; Brown owes Smith one hundred dollars. The two debts will be settled if Jones pays one hundred dollars to Brown.

All the members of the Civic Club are members of the University Club; Smith is a member of the University Club; therefore he is a member of the Civic Club.

Silver is heavier than copper; iron is lighter than silver; therefore copper is heavier than iron.

John's birthday is before Harry's and Tom's birthday is after Harry's. Therefore John's birthday is before Tom's.

William has a son, James, and a brother, George, who has a son Henry. Therefore James is Henry's uncle.

Jones owes Smith one hundred dollars; Brown owes Jones one hundred dollars. The two debts will be settled if Brown pays one hundred dollars to Smith.

All the members of the Civic Club are members of the University Club; Smith is not a member of the Civic Club; therefore he is not a member of the University Club.

John's birthday is before Harry's, and Harry's birthday is after Tom's. Therefore John's birthday is after Tom's.

Silver is heavier than copper; copper is heavier than iron; therefore silver is heavier than iron.

Henry's father, George, has a brother, William, who has a son, James. Therefore George is James' nephew.

Jones owes Smith one hundred dollars; Smith owes Brown one hundred dollars. The two debts will be settled if Jones pays one hundred dollars to Brown.

All the members of the Civic Club are members of the University Club; Smith is a member of the Civic Club; therefore he is a member of the University Club.

John's birthday is before Harry's, and Tom's birthday is after John's. Therefore Tom's birthday is after Harry's.

THURSTONE SPATIAL RELATIONS TEST A

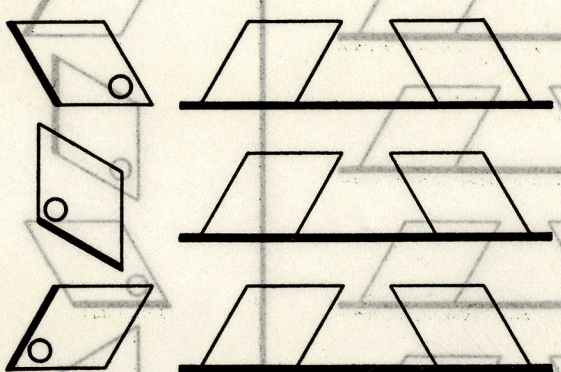
Instructions

Assume that the lozenge shaped figure with a circle in it represents a small card with one of its edges printed black and with a hole in one corner.

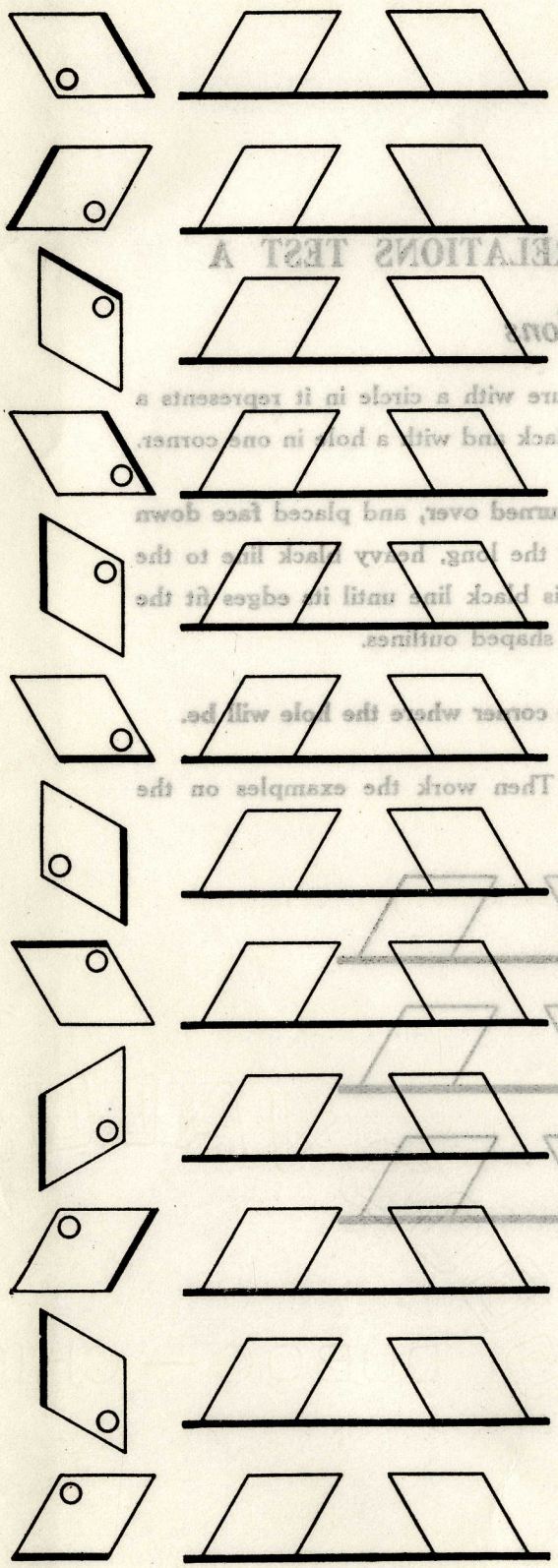
Imagine that this card is picked up, **turned over,** and placed **face down** with the black edge of the card touching the long, heavy black line to the right. Imagine the card moved along this black line until its edges fit the edges of one or the other of the lozenge shaped outlines.

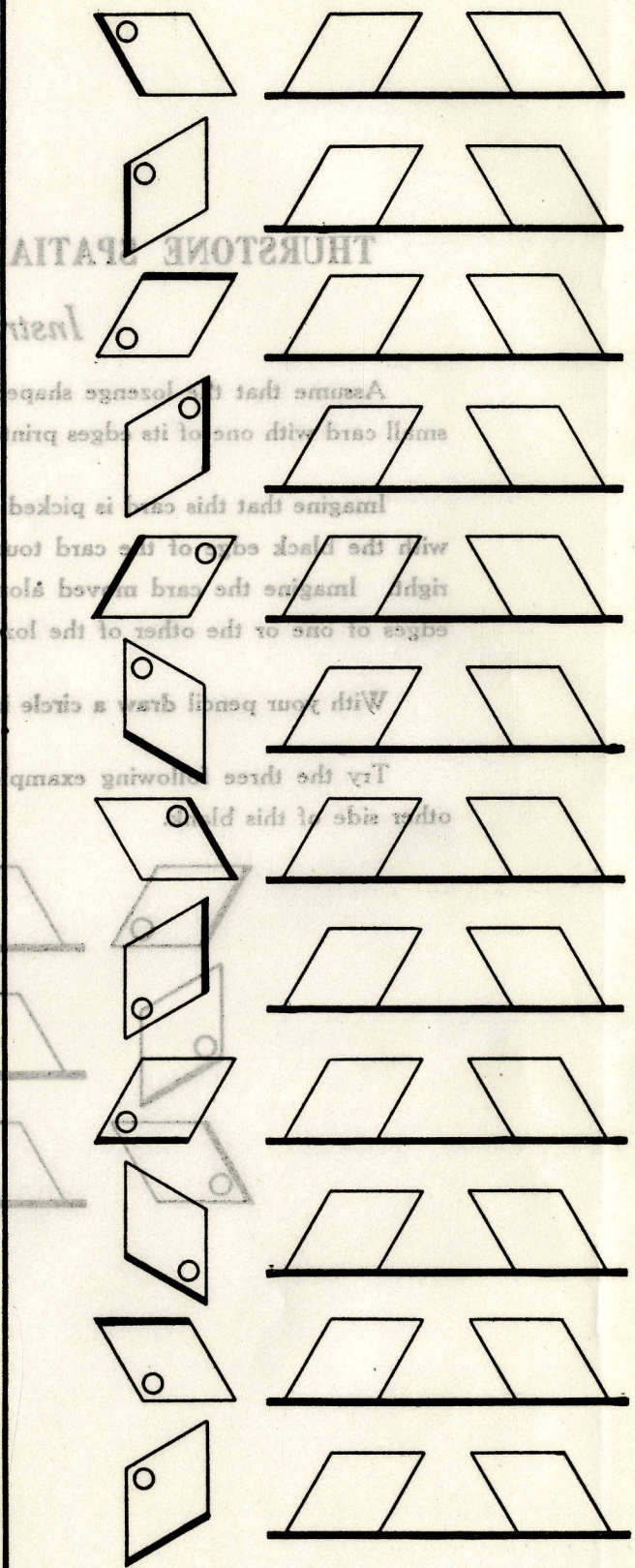
With your pencil **draw a circle in the corner where the hole will be.**

Try the three following examples. Then work the examples on the other side of this blank.



TOP





BOTTOM

TERMAN GROUP TEST OF MENTAL ABILITY

For Grades 7 to 12

Prepared by Lewis M. Terman, Stanford University, California

EDITION I

EXAMINATION: FORM A

1. Name ..... (First name / Last name)
2. Boy or girl ..... Grade ..... High or Low .....
3. Age last birthday ..... Date of birthday ..... (Month / Day)
4. Name of city (or county) .....
5. Name of school .....
6. Name of teacher .....
7. Date of this examination ..... (Month / Day) 19..... (Year)

Do not turn the page until you are told to.

Test	Score	Remarks or Further Data
1. Information		
2. Best Answer		
3. Word Meaning		
4. Logical Selection		
5. Arithmetic		
6. Sentence Meaning		
7. Analogies		
8. Mixed Sentences		
9. Classification		
10. Number Series		
Total		

TEST 1. INFORMATION

Draw a line under the ONE word that makes the sentence true, as shown in the sample.

SAMPLE. Our first President was
Adams Jefferson Lincoln <u>Washington</u>

1 Coffee is a kind of
bark berry leaf root 1

2 Sirloin is a cut of
beef mutton pork veal 2

3 Gasoline comes from
grains petroleum turpentine seeds 3

4 Most exports go from
Boston San Francisco New Orleans New York. 4

5 The number of pounds in a ton is
1000 2000 3000 4000 5

6 Napoleon was defeated at
Leipzig Paris Verdun Waterloo 6

7 Emeralds are usually
blue green red yellow 7

8 The optic nerve is for
seeing hearing tasting feeling 8

9 Larceny is a term used in
medicine theology law pedagogy 9

10 Sponges come from
animals farms forests mines 10

11 Confucius founded the religion of the
Persians Italians Chinese Indians 11

12 The larynx is in the
abdomen head throat shoulder 12

13 The piccolo is used in
farming music photography typewriting 13

14 The kilowatt measures
rainfall wind-power electricity water-power 14

15 The guillotine causes
death disease fever sickness 15

16 A character in " David Copperfield " is
Sindbad Uriah Heep Rebecca Hamlet 16

17 A windlass is used for
boring cutting lifting squeezing 17

18 A great law-giver of the Hebrews was
Abraham David Moses Saul 18

19 A six-sided figure is called a
scholium parallelogram hexagon trapezium 19

20 A meter is nearest in length to the
inch foot yard rod 20

Right .....

TEST 2. BEST ANSWER

Read each question or statement and make a cross before the BEST answer, as shown in the sample.

SAMPLE

Why do we buy clocks? Because

1 We like to hear them strike.
2 They have hands.
× 3 They tell us the time.

1 **Spokes of a wheel are often made of hickory because**
1 Hickory is tough.
2 It cuts easily.
3 It takes paint nicely.

2 **The saying, "A watched pot never boils," means**
1 We should never watch a pot on the fire.
2 Boiling takes a long time.
3 Time passes slowly when we are waiting for something.

3 **A train is harder to stop than an automobile because**
1 It has more wheels.
2 It is heavier.
3 Its brakes are not so good.

4 **The saying, "Make hay while the sun shines," means**
1 Hay is made in summer.
2 We should make the most of our opportunities.
3 Hay should not be cut at night.

5 **If the earth were nearer the sun**
1 The stars would disappear.
2 Our months would be longer.
3 The earth would be warmer.

6 **The saying, "If wishes were horses, beggars would ride," means**
1 Wishing doesn't get us very far.
2 Beggars often wish for horses to ride.
3 Beggars are always asking for something.

7 **The saying, "Little strokes fell great oaks," means**
1 Oak trees are weak.
2 Little strokes are best.
3 Continued effort brings results.

8 **A steel battleship floats because**
1 The engines hold it up.
2 It has much air space inside.
3 It contains some wood.

9 **The feathers on a bird's wings help him to fly because**
1 They make a wide, light surface.
2 They keep the air off his body.
3 They decrease the bird's weight.

10 **The saying, "A carpenter should stick to his bench," means**
1 Carpenters should not work without benches.
2 Carpenters should not be idle.
3 One should work at the thing he can do best.

11 **The saying, "One swallow does not make a summer," means**
1 Swallows come back for the summer.
2 A single sign is not sufficient proof.
3 Many birds add to the pleasures of summer.

Right..... × 2 = *Score*.....

TEST 3. WORD MEANING

When two words mean the SAME, draw a line under "SAME."
When they mean the OPPOSITE, draw a line under "OPPOSITE."

SAMPLES	fall — drop	same — opposite	
	north — south	same — opposite	
1	expel — retain	same — opposite	1
2	comfort — console	same — opposite	2
3	waste — conserve	same — opposite	3
4	monotony — variety	same — opposite	4
5	quell — subdue	same — opposite	5
6	major — minor	same — opposite	6
7	boldness — audacity	same — opposite	7
8	exult — rejoice	same — opposite	8
9	prohibit — allow	same — opposite	9
10	debase — degrade	same — opposite	10
11	recline — stand	same — opposite	11
12	approve — veto	same — opposite	12
13	amateur — expert	same — opposite	13
14	evade — shun	same — opposite	14
15	tart — acid	same — opposite	15
16	concede — deny	same — opposite	16
17	tonic — stimulant	same — opposite	17
18	incite — quell	same — opposite	18
19	economy — frugality	same — opposite	19
20	rash — prudent	same — opposite	20
21	obtuse — acute	same — opposite	21
22	transient — permanent	same — opposite	22
23	expel — eject	same — opposite	23
24	hoax — deception	same — opposite	24
25	docile — submissive	same — opposite	25
26	wax — wane	same — opposite	26
27	incite — instigate	same — opposite	27
28	reverence — veneration	same — opposite	28
29	asset — liability	same — opposite	29
30	appease — placate	same — opposite	30

Right.....*Wrong*.....*Score*.....

TEST 4. LOGICAL SELECTION

In each sentence draw a line under the TWO words that tell what the thing ALWAYS has. Underline TWO, and ONLY TWO, in each line.

SAMPLE. A man always has
body cap gloves mouth money

1 A horse always has
harness hoofs shoes stable tail 1
2 A circle always has
altitude circumference latitude longitude radius 2
3 A bird always has
bones eggs beak nest song 3
4 Music always has
listener piano rhythm sound violin 4
5 An object always has
smell size taste value weight 5

6 Conversation always has
agreement persons questions wit speech 6
7 A banquet always has
food music persons speeches toastmaster 7
8 A pistol always has
barrel bullet cartridge sights trigger 8
9 A ship always has
engine guns keel rudder sails 9
10 A debt always involves
creditor debtor interest mortgage payment 10

11 A game always has
cards contestants forfeits penalties rules 11
12 A magazine always has
advertisements paper pictures print stories 12
13 A museum always has
animals arrangement collections minerals visitors 13
14 A forest always has
animals flowers shade underbrush trees 14
15 A citizen always has
country occupation privileges property vote 15

16 Controversy always involves
claims disagreement dislike enmity hatred 16
17 War always has
airplanes cannons combat rifles soldiers 17
18 Obstacles always bring
difficulty discouragement failure hindrance stimulation .. 18
19 Abhorrence always involves
aversion dislike fear rage timidity 19
20 Compromise always involves
adjustment agreement friendship respect satisfaction ... 20

Right .....

TEST 5. ARITHMETIC

Find the answers as quickly as you can.
Write the answers on the dotted lines.
Use the bottom of the page to figure on.

---

1 How many hours will it take a person to go 66 miles at the rate of 6 miles an hour? *Answer*.....

2 At the rate of 2 for 5 cents, how many pencils can you buy for 50 cents? *Answer*.....

3 If a man earns $20 a week and spends $14, how long will it take him to save $300? *Answer*.....

4 $2 \times 3 \times 4 \times 6$ is how many times as much as 3×4? *Answer*.....

5 If two pies cost 66 cents, what does a sixth of a pie cost? *Answer*.....

6 What is $16\frac{2}{3}$ per cent of $120? *Answer*.....

7 4 per cent of $1000 is the same as 8 per cent of what amount? *Answer*.....

8 A has $180, B has $\frac{2}{3}$ as much as A, and C has $\frac{1}{2}$ as much as B. How much have all together? *Answer*.....

9 The capacity of a rectangular bin is 48 cubic feet. If the bin is 6 feet long and 4 feet wide, how deep is it? *Answer*.....

10 If it takes 7 men 2 days to dig a 140-foot ditch, how many men are needed to dig it in half a day? *Answer*.....

11 A man spends $\frac{1}{4}$ of his salary for board and room, and $\frac{3}{8}$ for all other expenses. What per cent of his salary does he save? *Answer*.....

12 If a man runs 100 yards in 10 seconds, how many feet does he run in $\frac{1}{5}$ of a second? *Answer*.....

Right.....$\times 2 =$ *Score*.....

TEST 6. SENTENCE MEANING

Draw a line under the right answer, as shown in the samples.

SAMPLES	Is coal obtained from mines?	Yes	No	
	Are all men six feet tall?	Yes	No	
1	Does a conscientious person ever make mistakes?	Yes	No	1
2	Is an alloy a kind of musical instrument?	Yes	No	2
3	Is scurvy a kind of medicine?	Yes	No	3
4	Are mysterious things often uncanny?	Yes	No	4
5	Are destitute persons often subjects of charity?	Yes	No	5
6	Are anonymous letters ever properly signed?	Yes	No	6
7	Is the mimeograph sometimes used by stenographers?	Yes	No	7
8	Is a curriculum intended for horses?	Yes	No	8
9	Are proteids essential to health?	Yes	No	9
10	Does "perfunctory" mean the same as "careful"?	Yes	No	10
11	Are premeditated deeds always wicked?	Yes	No	11
12	Do alleged facts often require verification?	Yes	No	12
13	Are sheep carnivorous?	Yes	No	13
14	Are aristocrats subservient to their inferiors?	Yes	No	14
15	Are venerable people usually respected?	Yes	No	15
16	Is clematis sometimes cultivated?	Yes	No	16
17	Are ultimate results the last to appear?	Yes	No	17
18	Are cerebral hemorrhages helpful to thinking?	Yes	No	18
19	Are all people religious who have hallucinations?	Yes	No	19
20	Are intermittent sounds discontinuous?	Yes	No	20
21	Are sable colors preferred for nations' flags?	Yes	No	21
22	Does social contact tend to reduce eccentricities?	Yes	No	22
23	Are tentative decisions usually final?	Yes	No	23
24	Is rancor usually characterized by persistence?	Yes	No	24

Right.....*Wrong*.....*Score*.....

TEST 7. ANALOGIES

SAMPLES

Ear is to hear as eye is to
table <u>see</u> hand play

Hat is to head as shoe is to
arm coat <u>foot</u> leg

Do them all like samples.

---

1 Coat is to wear as bread is to
eat starve water cook 1

2 Week is to month as month is to
year hour minute century 2

3 Monday is to Tuesday as Friday is to
week Thursday day Saturday 3

4 Tell is to told as speak is to
sing spoke speaking sang 4

5 Lion is to animal as rose is to
smell leaf plant thorn 5

6 Cat is to tiger as dog is to
wolf bark bite snap 6

7 Success is to joy as failure is to
sadness luck fail work 7

8 Liberty is to freedom as bondage is to
negro slavery free suffer 8

9 Cry is to laugh as sadness is to
death joy coffin doctor 9

10 Tiger is to hair as trout is to
water fish scales swims 10

11 1 is to 3 as 9 is to
18 27 36 45 11

12 Lead is to heavy as cork is to
bottle weight light float 12

13 Poison is to death as food is to
eat bird life bad 13

14 4 is to 16 as 5 is to
7 45 35 25 14

15 Food is to hunger as water is to
drink clear thirst pure 15

16 b is to d as second is to
third later fourth last 16

17 City is to mayor as army is to
navy soldier general private 17

18 Here is to there as this is to
these those that then 18

19 Subject is to predicate as noun is to
pronoun adverb verb adjective 19

20 Corrupt is to depraved as sacred is to
Bible hallowed prayer Sunday 20

Right .....

TEST 8. MIXED SENTENCES

The words in each sentence below are mixed up. If what a sentence means is TRUE, draw a line under "TRUE." If what it means is FALSE, draw a line under "FALSE."

SAMPLES	hear are with to ears	true	false	
	eat gunpowder to good is	true	false	
1	true bought cannot friendship be	true	false	1
2	good sea drink to is water	true	false	2
3	of is the peace war opposite.....	true	false	3
4	get grow they as children taller older.....	true	false	4
5	horses automobile an are than slower.....	true	false	5
6	never deeds rewarded be should good	true	false	6
7	four hundred all pages contain books.....	true	false	7
8	to advice sometimes is good follow hard	true	false	8
9	envy bad greed traits are and	true	false	9
10	grow an than strawberries oak tree higher	true	false	10
11	external deceive never appearances us	true	false	11
12	never is man what show a deeds.....	true	false	12
13	hatred bad unfriendliness traits are and	true	false	13
14	often judge can we actions man his by a	true	false	14
15	in are always American cities born presidents	true	false	15
16	certain always death of cause kinds sickness	true	false	16
17	are sheet blankets as as a never warm.....	true	false	17
18	never who heedless those stumble are	true	false	18

Right.....*Wrong*.....*Score*.....

TEST 9. CLASSIFICATION

SAMPLES
1 bullet cannon gun sword ~~pencil~~
2 Canada ~~Chicago~~ China India France

In each line cross out the word that does not belong there.
Cross out JUST ONE WORD in each line.

1	Frank James John Sarah William	1
2	Baptist Catholic Methodist Presbyterian Republican	2
3	automobile bicycle buggy telegraph train	3
4	Collie Holstein Shepherd Spitz Terrier	4
5	hop run skip stand walk	5
6	death grief picnic poverty sadness	6
7	bed chair dish sofa table	7
8	hard rough smooth soft sweet	8
9	mechanic doctor lawyer preacher teacher	9
10	Christ Confucius Mohammed Moses Cæsar	10
11	butterfly hawk ostrich robin swallow	11
12	cloth cotton flax hemp wool	12
13	digestion hearing sight smell touch	13
14	down hither recent up yonder	14
15	anger hatred joy pity reasoning	15
16	Australia Cuba Iceland Ireland Spain	16
17	Dewey Farragut Grant Paul Jones Schley	17
18	give lend lose keep waste	18

Right.....

TEST 10. NUMBER SERIES

SAMPLES	5	10	15	20	25	30	35
	20	18	16	14	12	10	8

In each row try to find out how the numbers are made up, then on the two dotted lines write the TWO numbers that should come next.

1st Row	8	7	6	5	4	3	....	....
2d Row	3	8	13	18	23	28	....	....
3d Row	$11\frac{3}{4}$	12	$12\frac{1}{4}$	$12\frac{1}{2}$	$12\frac{3}{4}$	....	....	
4th Row	8	8	6	6	4	4	....	....
5th Row	1	2	4	8	16	32	....	....
6th Row	4	3	5	4	6	5	7	....
7th Row	16	8	4	2	1	$\frac{1}{2}$	....	....
8th Row	8	9	12	13	16	17	....	....
9th Row	7	11	15	16	20	24	25	29 ....
10th Row	31.3	40.3	49.3	58.3	67.3	76.3	....	....
11th Row	$\frac{1}{25}$	$\frac{1}{5}$	1	5	....	....		
12th Row	3	4	6	9	13	18	....	....

Right ..... × 2 = *Score* .....

THE JONES PLANE GEOMETRY TEST
FORM I FIRST SEMESTER

Name..... Date.....

School..... Teacher.....

Town..... State.....

Class..... Score.....

GENERAL DIRECTIONS

Following are a number of tests in plane geometry. You are to do them in the order in which they occur. When you have quit working on one you are not to go back to it. You will be allowed the following lengths of time for each test:

TEST I	12 minutes
TEST II	2 minutes
TEST III	5 minutes
TEST IV	8 minutes
TEST V	6 minutes
TEST VI	5 minutes
TEST VII	2 minutes

When told to start on the next test, do so at once. If you finish a test before the signal to stop is given you may start to work on the next test.

DIRECTIONS TO PERSON ADMINISTERING THE TEST

At the end of twelve minutes say "Begin Test II". At the end of two more minutes say "Begin Test III", etc. The directions for the separate tests are not to be read to the pupils but see that each one follows them. Read the "General Directions" to those taking the test and see that all start Test I together.

SCORING

TEST I	NUMBER RIGHT	_____	X ½	_____
TEST II	NUMBER RIGHT	_____	X 1	_____
TEST III	NUMBER RIGHT	_____	X 2	_____
TEST IV	NUMBER RIGHT	_____	X 2	_____
TEST V	NUMBER RIGHT	_____	X 2	_____
TEST VI	NUMBER RIGHT	_____	X 1	_____
TEST VII	NUMBER RIGHT	_____	X 3	_____

TOTAL SCORE _____

TEST I
COMPLETION TEST

DIRECTIONS: Following are statements of a number of geometric propositions from which certain words have been omitted. On the right margin of the page are some blanks with numbers corresponding to the numbers where the words have been omitted. You are to write in these blanks on the right margin the words which have been omitted.

(1) If two triangles have (1) and the (2) (3) of one equal, respectively, to two (4) and the (5) (6) of the other, they are (7) .

2. If two triangles have three (8) of one respectively equal to three (9) of the other they are (10) .

3. If two (11) have two (12) and a (13) of one respectively equal to two (14) and the corresponding (15) of the other they are (16) .

4. If two (17) of a triangle are equal, the (18) opposite these (19) are (20) .

5. Every point on the (21) (22) of a line is (23) from its extremities.

6. If two (24) lines are cut by a (25) the (26) (27) are equal.

7. A (28) is divided into (29) (30) by either diagonal.

8. Any quadrilateral is a (31) if two (32) are (33) and (34) .

9. In any circle angles at the center are (35) to their (36) (37) .

1. two
2.
3.
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27.
28.
29.
30.
31.
32.
33.
34.
35.
36.
37.

10. In any circle if two chords are equal the (38) which they intercept are (39) .

11. A diameter (40) to a chord (41) the chord and the (42) of the chord.

12. A tangent to a circle at a given point is (43) to the (44) at that point.

13. An angle inscribed in a circle is measured by (45) its (46) (47) .

14. Angles inscribed in the same (48) are (49) .

15. An (50) angle of a triangle is equal to the (51) of the (52) (53) (54) .

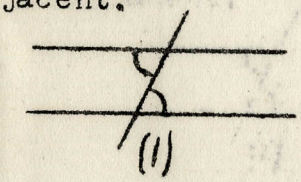
16. If two chords are unequal the (55) is at a less distance from the (56)

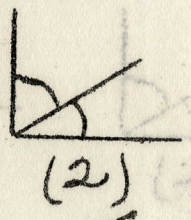
17. An angle formed by two secants is measured by (57) the (58) of the (59) (60) .

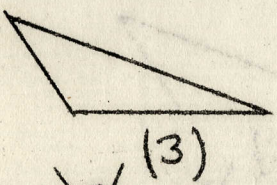
38. _____
39. _____
40. _____
41. _____
42. _____
43. _____
44. _____
45. _____
46. _____
47. _____
48. _____
49. _____
50. _____
51. _____
52. _____
53. _____
54. _____
55. _____
56. _____
57. _____
58. _____
59. _____
60. _____

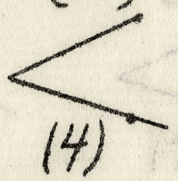
TEST II
GEOMETRIC FIGURES

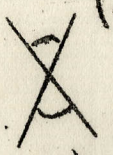
DIRECTIONS: On the left of this page are the diagrams of certain geometric figures. On the right of the page are the names of the figures. You are to place in the blank at the right of each name the correct number of the diagram which illustrates it. In illustrating adjacent angles use the diagram which shows them in no other relation than adjacent.

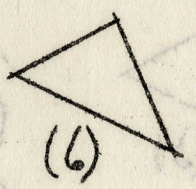


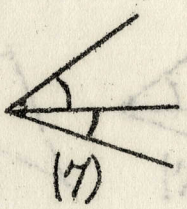


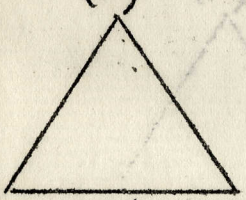


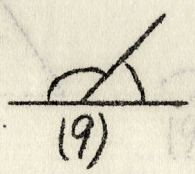


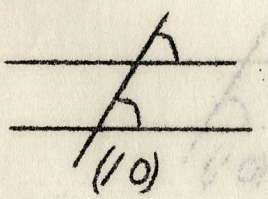


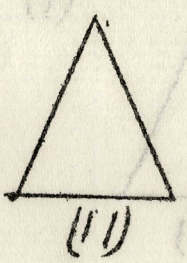












obtuse angle ___

right angle ___

acute angle ___

vertical angles ___

adjacent angles ___

complementary angles ___

supplementary angles ___

right triangle ___

obtuse triangle ___

acute triangle ___

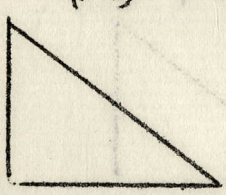
equilateral triangle ___

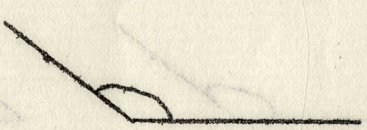
isosceles triangle ___

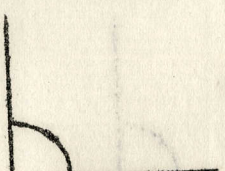
equiangular triangle ___

corresponding angles ___

alternate interior angles ___

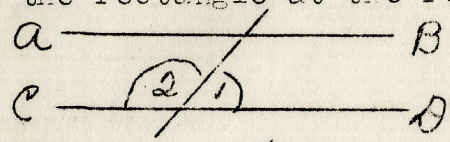




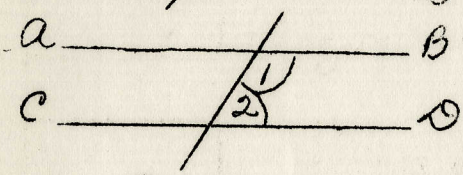


TEST III
TRUE-FALSE TEST

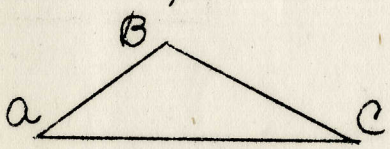
DIRECTIONS: Following are some figures. Some facts are given true about each of the figures and because these facts are true a conclusion about some other fact is made. In some cases these conclusions are true, in others they are not. You are to write true or false in the rectangle at the right of each figure.



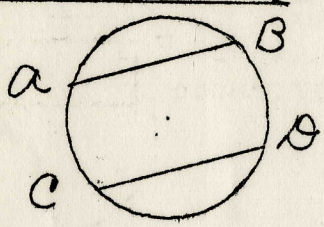
GIVEN: Angle 1 equals angle two.
CONCLUSION: AB is parallel to CD.



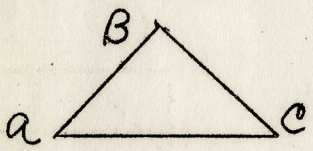
GIVEN: Angle 1 supplementary to angle 2.
CONCLUSION: AB is parallel to CD.



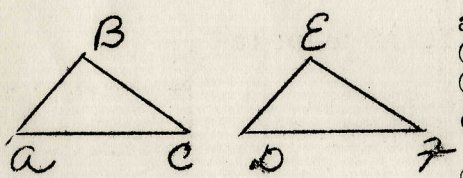
GIVEN: A ngle A' equals thirty degrees. Angle B equal 110 degrees.
CONCLUSION: Angle C equals 50 degrees.



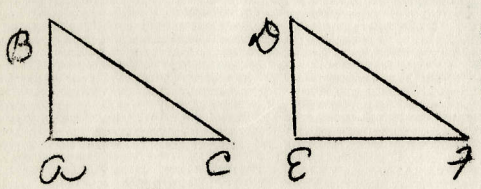
GIVEN: Chord AB less than chord CD.
CONCLUSION: Arc DC is less than arc AB.



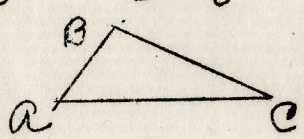
GIVEN: AB equals BC.
CONCLUSION: Angle A' equals angle C.



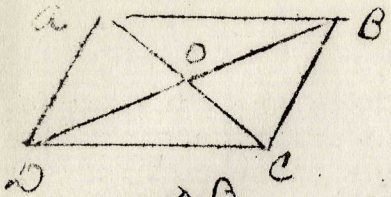
GIVEN: Angle A' equals angle D, angle B' equals angle E, angle C equals angle F.
CONCLUSION: Triangle ABC is congruent to triangle DEF.



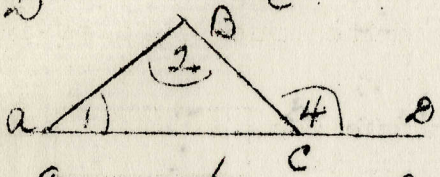
GIVEN: Angles A' and E are right angles. BC equals DF, angle C equals angle F.
CONCLUSION: Triangle ABC is congruent to triangle DEF.



GIVEN: AB less than BC.
CONCLUSION: Angle C is less than angle A.

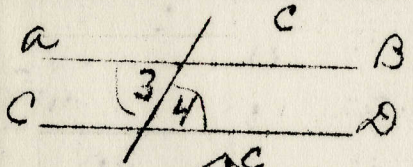


GIVEN: AB parallel to DC, and AD parallel to BC.
CONCLUSION: AO equals OC.



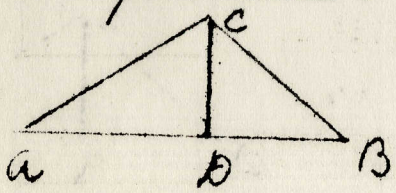
GIVEN: Triagnle ABC.

CONC LUSION: Angle 4 equals angle 1 plus angle 2.



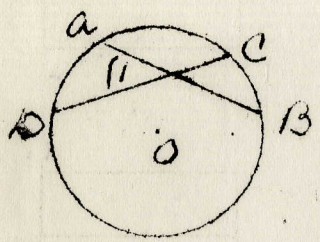
GIVEN: Angle 3 equals angle 4.

CONCLUSION: AB is parallel to CD

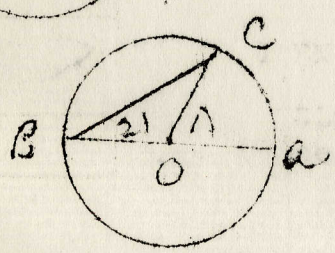


GIVEN: AC equals BC .

CONCLUSION: AD equals DB.

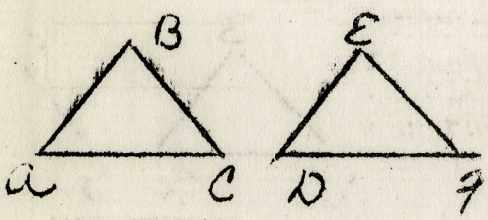


GIVEN: Circle O and chords AB and CD.
CONCLUSION: Angle 1 is measured by one half the difference of arcs AD and BC.



GIVEN: Circle O.

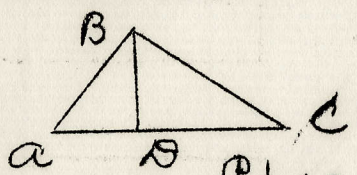
CONCLUSION: Angle 1 equals twice angle 2.



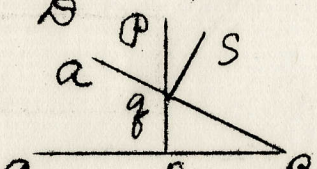
GIVEN: AB equals DE, AC equals DF and BC equals EF.
CONCLUSION: Triangle ABC is congruent to triangle DEF.

TEST IV
CORRECT CONCLUSION TEST

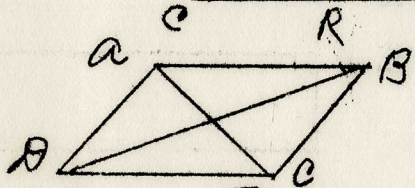
DIRECTIONS: For each of the following figures some fact or facts are given. Because these facts are given some other fact is true about the figure. Write this fact in the rectangle at the right. Write only one fact for each figure. Make use of the letters used in the diagram in telling the facts. Do not place more letters or figures on the diagrams; use only those given there. You are not to give reasons for your conclusions.



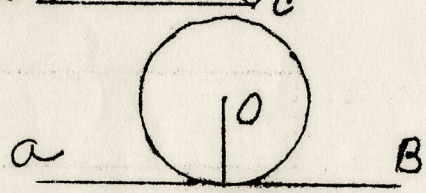
GIVEN:
Angle A equals angle C.



GIVEN:
PQ is perpendicular to BC and QS is perpendicular to AB.

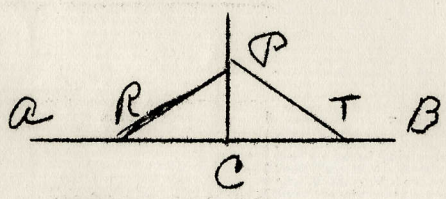


GIVEN:
AB equals DC,
A D equals BC.

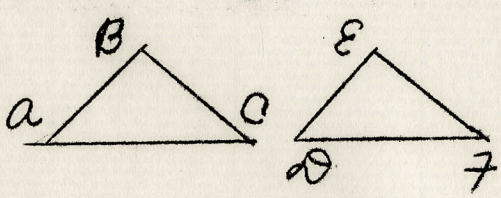


GIVEN:
OP PERpendicular to AB.

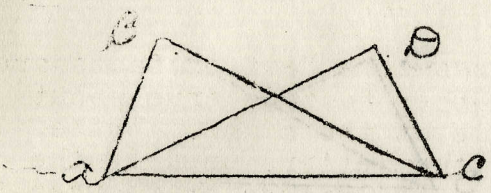
GIVEN:
Angle A is greater than angle C.



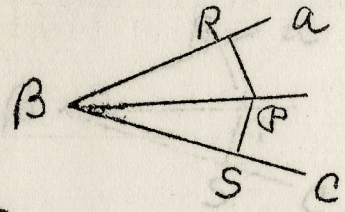
GIVEN:
PC is perpendicular to AB, RC equals CT.



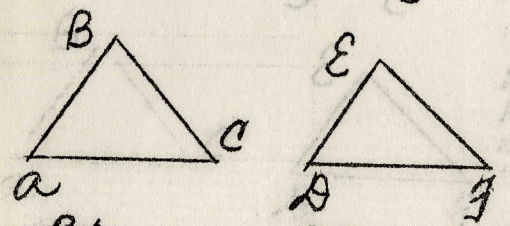
GIVEN:
AB equals DE,
AC equals DF,
angle A equals angle D.



GIVEN:
AB equals DC,
AD equals BC.

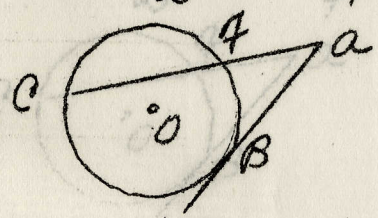


GIVEN:
BP bisector of angle ABC, PR perpendicular to AB, PS perpendicular to BC.

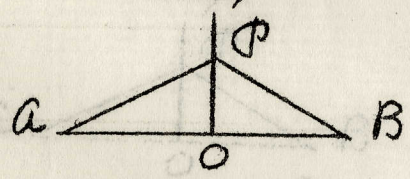


GIVEN:
AB equals DE,
AC equals DF,
angle A is greater than angle D.

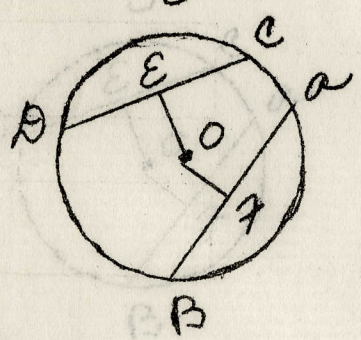
GIVEN:
Angle 1 equals angle 4. Angle 2 equals angle 5.



GIVEN:
AB tangent to circle O.



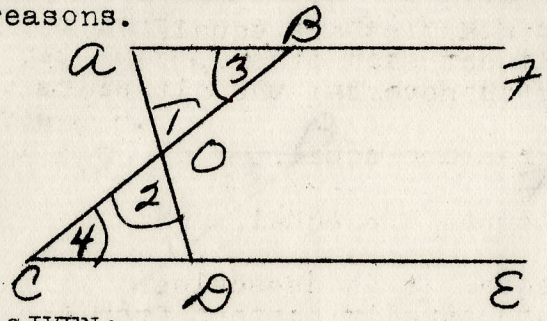
GIVEN:
PO perpendicular to AB, AO equals OB.



GIVEN:
OF perpendicular to AB, OE perpendicular to CD, OE greater than OF.

TEST V
CORRECT REASONS TEST on the opposite page are

DIRECTIONS: Following the diagrams are some statements and reasons. There are more reasons given than statements. After each statement is a rectangle in which you are to place the number of the correct reason that goes with the statement. Each reason is numbered. Read what is given about each figure before you begin to give the correct reasons.



GIVEN:
AF is parallel to CE,
BO equals CO.

Angle 1 equals angle 2

BO equals CO

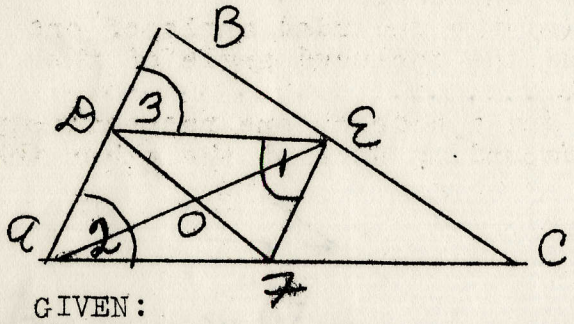
AF is parallel to CE

Angle 3 equals angle 4

Triangle A BO is congruent to triangle COD

AB equals CD

.....



GIVEN:
DE parallel to AC,
EF parallel to AD.

DO equals OF

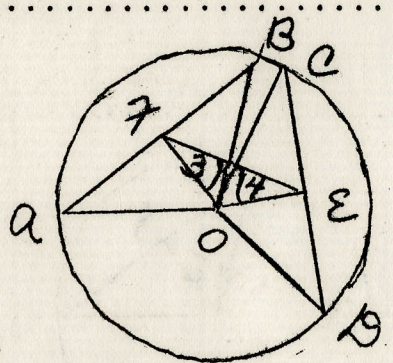
EF equals AD

Angle 1 equals angle 2

Angle 2 equals angle 3

Angle 1 equals angle 3

.....



GIVEN:
Circle O with angle AOB equal to angle COD.

Arc AB equals arc CD

AB equals CD

OF equals CE

Angle 3 equals angle 4

(1) If two parallel lines are cut by a transversal the interior angles on the same side of the transversal are supplementary.
(2) The angles opposite the equal sides of an isosceles triangle are equal.
(3) Things equal to the same thing are equal to each other.
(4) If two parallel lines are cut by a transversal the corresponding angles are equal.
(5) Corresponding parts of congruent triangles are equal.
(6) The diagonals of a parallelogram bisect each other.
(7) If two parallel lines are cut by a transversal the alternate interior angles are equal.
(8) The opposite sides of a parallelogram are equal.
(9) All vertical angles are equal.
(10) The opposite angles of a parallelogram are equal.
(11) All right angles are equal.
(12) If two angles of a triangle are equal it is isosceles.
(13) In the same circle equal chords are equally distant from the center.
(14) In the same circle equal arcs determine equal chords.
(15) In the same circle equal central angles intercept equal arcs.
(16) In the same circle equal chords determine equal arcs.
(17) If two triangles have three sides of one respectively equal to three sides of the other they are congruent.
(18) If two triangles have two sides and the included angle of one respectively equal to two sidesand the included angle of the other they are congruent.
(19) If two triangles have two angles and a side of one respectively equal to two angles and the corresponding side of the other they are congruent.
(20) Given.

TEST VI
LOGICAL ARRANGEMENT TEST

DIRECTIONS: Following are the steps of the proofs of several exercises. The statements are not in their proper order. You are to place the proper number after each statement to show the order in which it should come in the proof. Use "1" to designate the statement which should come first, "2" the statement which should come second, etc. Begin with the number "1" for the first step in each separate exercise.

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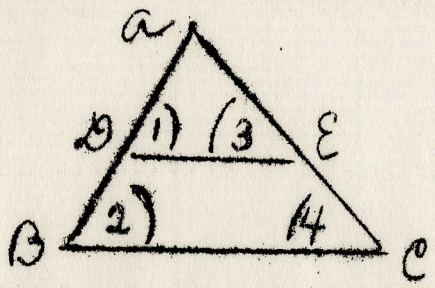
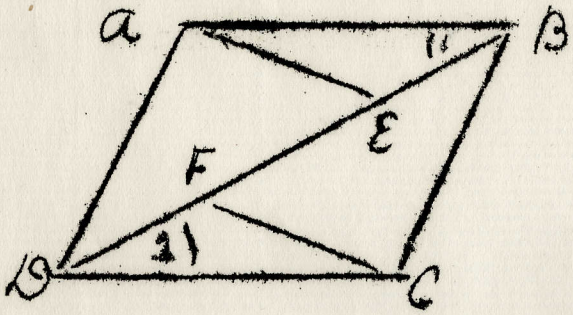


Given:
Triangle ABC in which AB equals AC and DE is parallel to BC.
To prove:
Triangle ADE is isosceles.

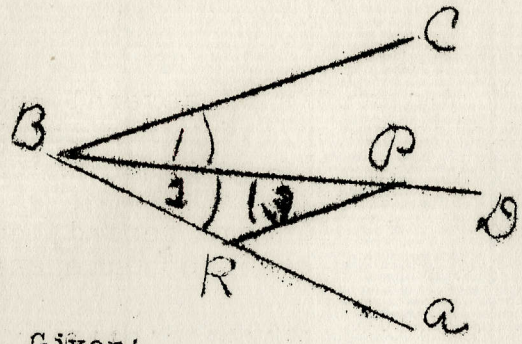
AD equals ~~DE~~ AE
Angle 1 equals angle 3
Angle 2 equals angle 4
DE is parallel to BC
Angle 3 equals angle 4
AB equals AC
Angle 1 equals angle 2

.....



Given:
ABCD a parallelogram,
BE equals DF.
To prove:
AE equals C F.

BE equals DF
Triangle ABE is congruent to triangle DFC
AE equals CF
AB is parallel to DC
AB equsls DC
Angle 1 equals angle 2



Given:
BD bisector of angle ABC; PR parallel to CB.
To prove:
Triangle PBR is isosceles.

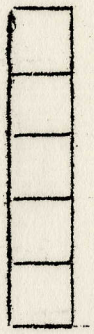
Angle 1 equals angle 2.

BR equals PR

Angle 2 equals angle 3

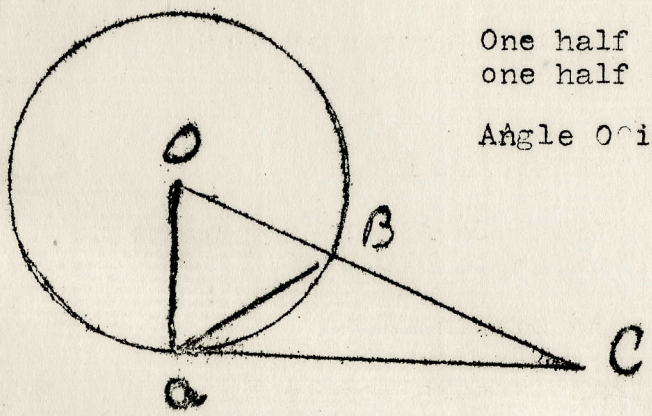
Angle 1 equals angle 3

BC is parallel to PR



.....

Given:
AC tangent to circle O.
To prove:
Angle BAC equal to one half of angle O.

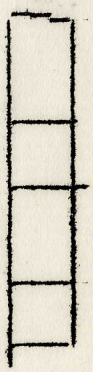


Angle BAC is measured by one half of arc AB

Angle BAC equals one half of angle O.

One half of angle O is measured by one half of arc AB

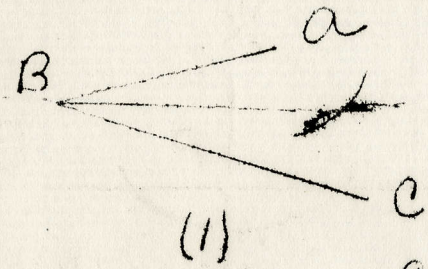
Angle O is measured by arc AB



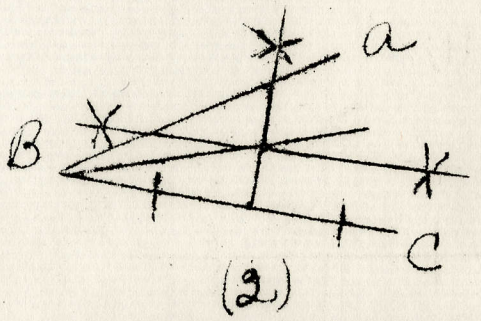
TES T VII
CONSTRUCTIONS

DIRECTIONS: Following are four figures showing how to make each of the following constructions. Three of each are wrong and one is correct. Write the number of the correct construction in the blank at the right of the page.

(1) To bisect the angle ABC.

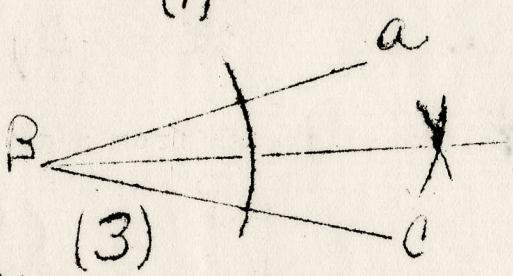


(1)

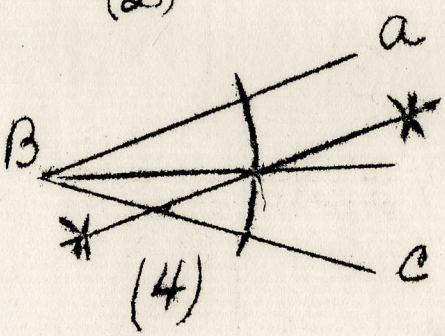


(2)

3

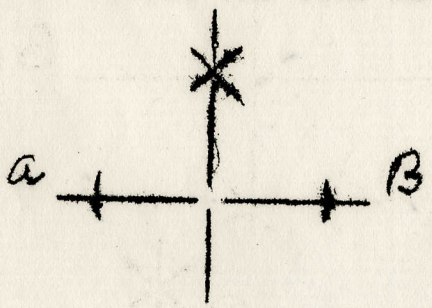


(3)

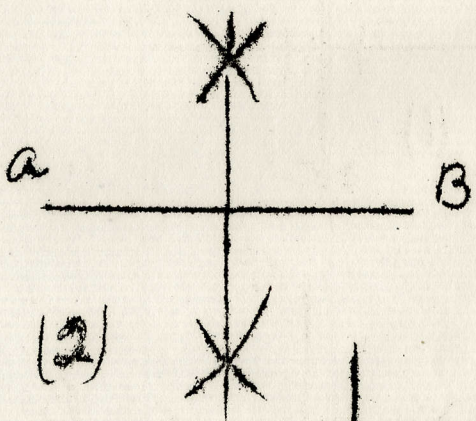


(4)

(2) To bisect the line AB.

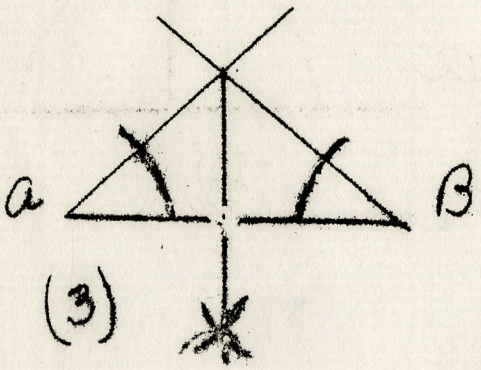


(1)

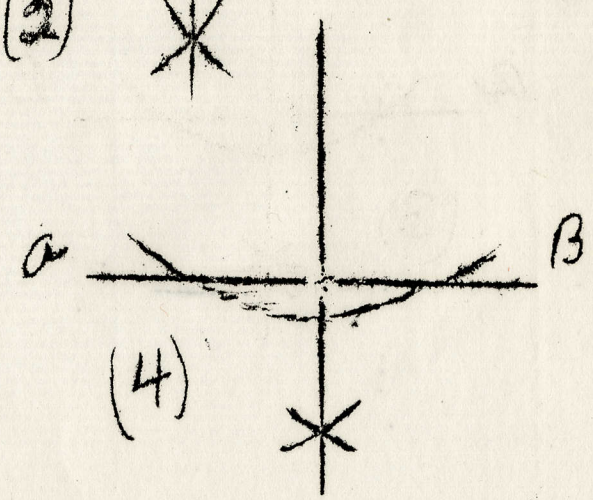


(2)

2

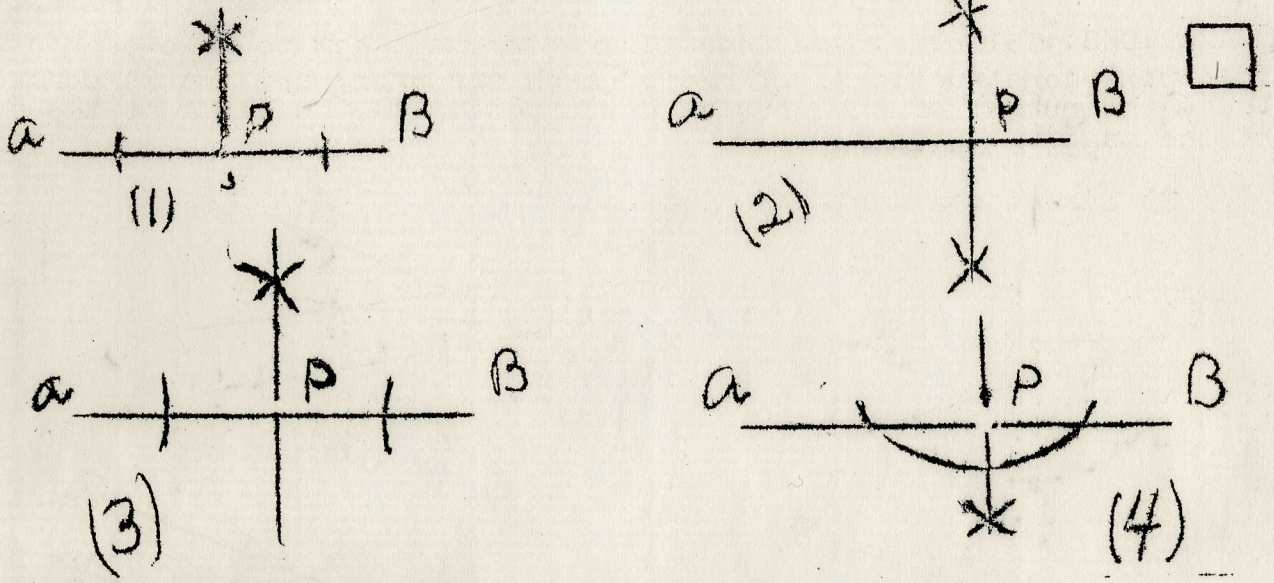


(3)

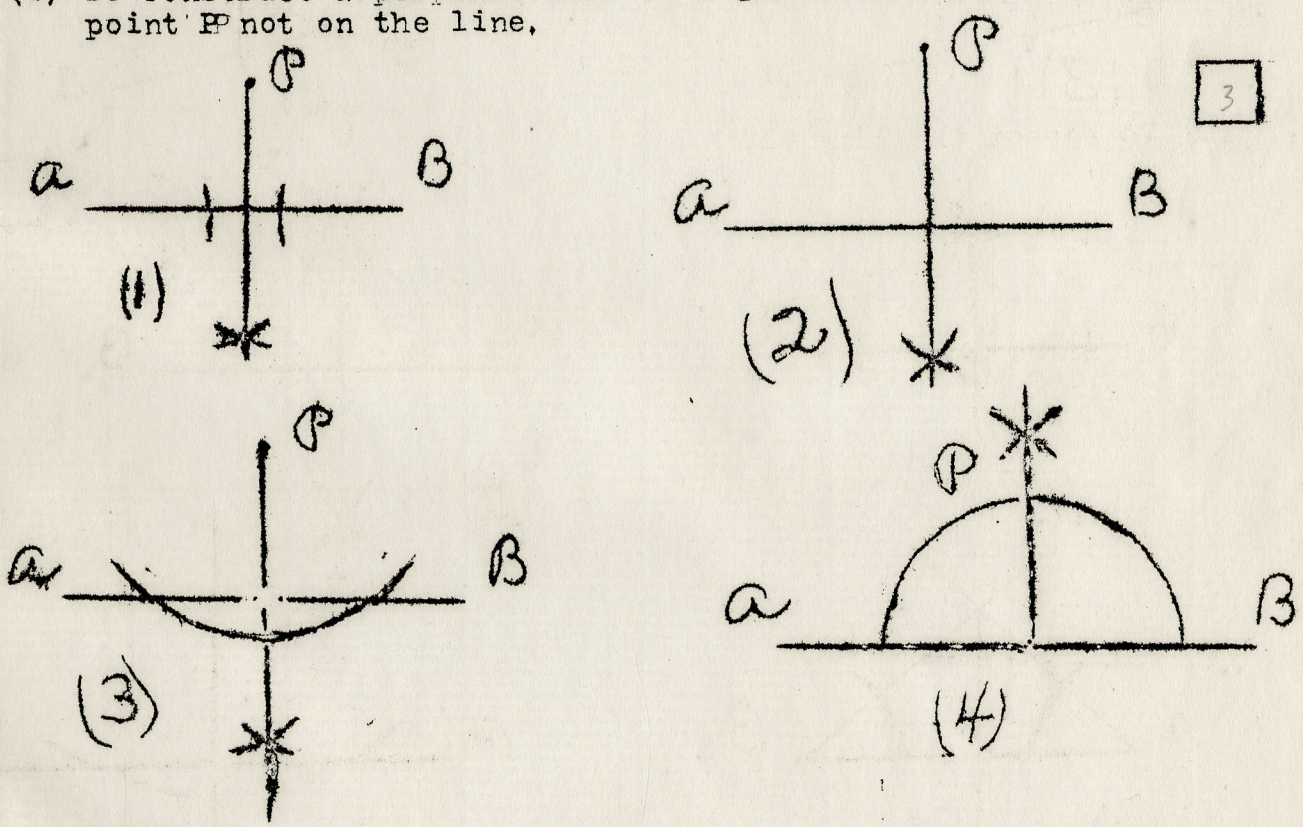


(4)

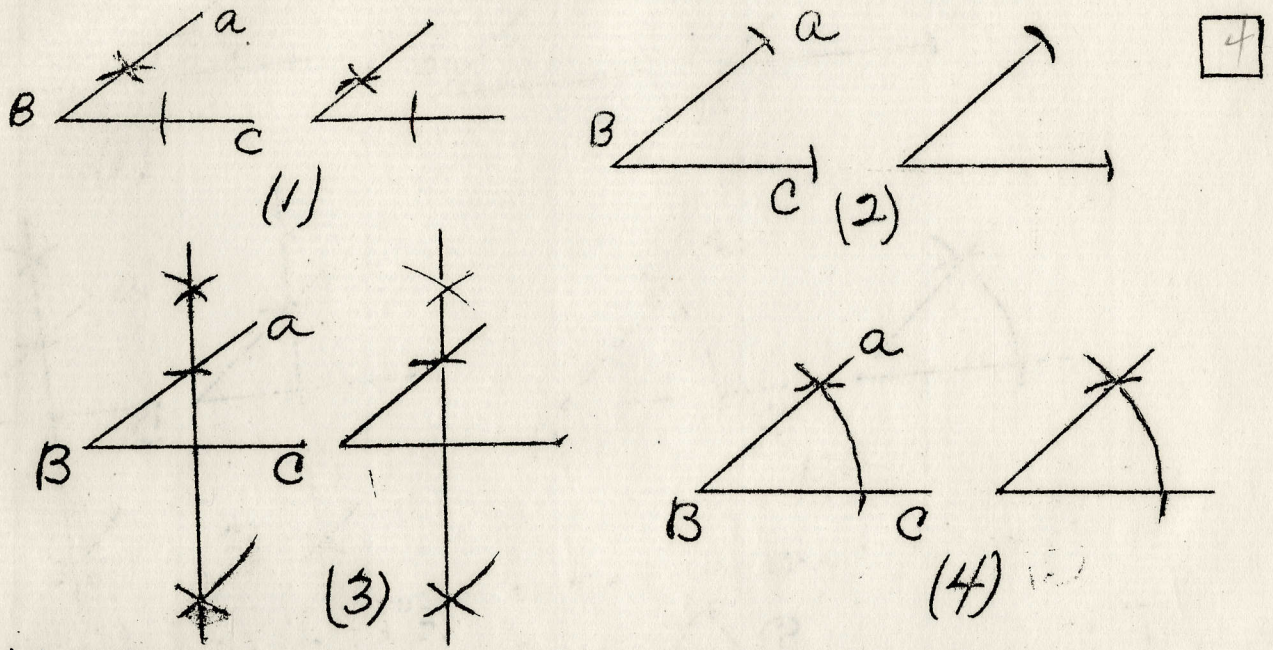
(3) To construct a perpendicular to a given line AB at a given point P on the line.



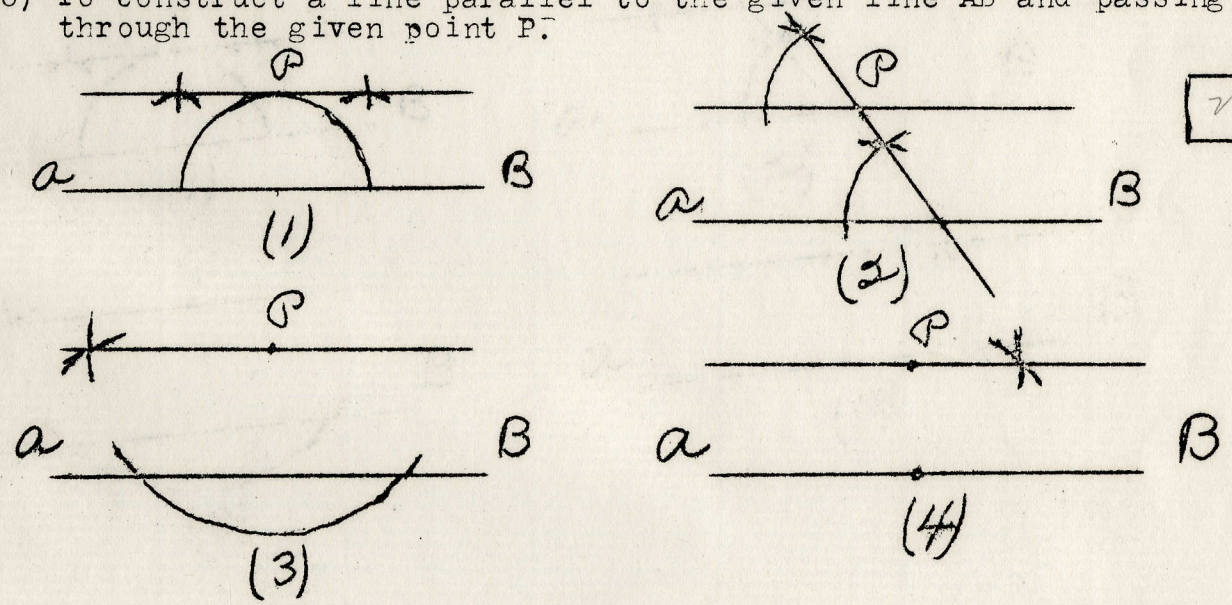
(4) To construct a perpendiculzr to a given line AB from a given point P not on the line.



(5) To construct an angle equal to the given angle ABC .



(6) To construct a line parallel to the given line AB and passing through the given point P.



Approved Clark L Hull Prof of Psychology

June 11, 1926