

Modeling Deer-Vehicle Collisions

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Abstract

Deer-vehicle collisions are occurring in larger and larger numbers. One way to better understand why they are happening is through mathematical modeling. Using a Markov Chain to model deer movement, we can factor in information of deer movement and traffic patterns to create a sound model that can plausibly be applied to real life. Through this, we can find the probability of driving down a given Wisconsin road in 2002 and colliding with a deer and potentially see where roads can be built to minimize deer-vehicle collisions.

1 Introduction

Many studies focus on methods to prevent deer-vehicle collisions and perform experiments to determine whether their methods are successful. We wanted to find a more predictive, as opposed to empirical, method of analyzing deer and traffic movement. This way, certain locations could be targeted as having a high probability of deer-vehicle collisions. Additionally, before a road is constructed, logistics planners could use this method to predict how likely deer-vehicle collisions will be and plan accordingly.

We define a *deer-vehicle collision* to be contact occurring between a deer and vehicle that results in damage to the deer or vehicle involved. According to the Deer Vehicle Crash Information Clearinghouse, “A *reportable crash* is defined as a crash resulting in injury or death of any person, any damage to government-owned non-vehicle property to an apparent extent of \$200 or more, or total damage to property owned by any one person to an apparent extent of \$1000 (\$500 before 1/1/96) or more” [7]. According to the research of Dr. Keith Knapp of UW-Madison, assistant professor of engineering, some people report the crash and others do not [19]. The Wisconsin Department of Natural Resources (DNR) also keeps track of what is called *salvaged and unsalvaged deer carcasses*, which we will call the *carcass count*. The Wisconsin DNR defines this as “deer carcasses removed by contractors of WI DNR and salvaged with free permits” and reports it as “Fiscal year data, from the beginning of July of previous year to the end of June in the year recorded” [7].

For purposes of this study, the following definitions will be used. A *deer management unit* (DMU) is defined as a specified area of land within a state. The DNR defines the boundaries of each DMU and keeps track of deer population in each given deer management unit. In 2002, there were approximately 120 deer management units in Wisconsin ranging in size from approximately 25 square miles to approximately 1270 square miles [18].

Average annual daily traffic is defined as the expected number of cars passing a given point on a road on a given day of the year.

Through the use of this data, and with the use of the terms defined above, our intent is to build a Markov Chain to model deer movement, including mortality factors from roads.

2 Review of Literature

Various studies can be found on aspects of typical deer behavior surrounding deer-vehicle collisions. Nielsen, Anderson, and Grund focused on reducing deer-vehicle collisions in urban settings in

southern Minnesota [1]. They found that areas with fewer buildings and areas with more public land have more deer-vehicle collisions. Another study, by Ujvári, Baagøe, and Madsen, found that fallow deers' reactions to light reflections changed over time [2]. The authors discovered that the deer reacted strongly to them at first, but over a few days became used to the light reflections and did not react as much, if at all. Sullivan, Williams, Messmer, Hellinga, and Kyrychenko conducted a study titled “*Effectiveness of temporary warning signs in reducing deer-vehicle collisions during mule deer migrations*” [4]. Their literature review discovered that deer-whistles were ineffective and there is conflicting literature about the effectiveness of reflectors. They therefore focused their study on temporary signs during migration. They found they could reduce deer mortality by approximately 51%.

The need to find effective devices for reducing deer-vehicle collisions is apparent; one study found that deer-vehicle collisions are on the rise [3]. According to Paige, deer populations are growing, and states are now looking into other options, such as various expansions of hunting, to help control the population. The article also states that deer are over browsing areas because of their abundance in numbers, and affecting the rest of the species in the area that rely on the vegetation.

Since deer-vehicle collisions occur at such a high rate, different departments and agencies publish “tips” to help avoid accidents. One such article came from the State of Wisconsin Office of the Commissioner of Insurance [6]. It covers basic strategies drivers can use to help avert deer-vehicle collisions, such as avoiding swerving, which often can cause more severe accidents than deer-vehicle collisions. It also reminds drivers that not all insurance policies cover deer-vehicle collisions. The Minnesota Department of Transportation also puts out news releases on how to avoid deer-vehicle collisions [9]. It concurs that drivers should not swerve when trying to avoid deer; doing so would possibly cause more dangerous collisions with stationary objects or other vehicles. The news release also mentions that deer-vehicle collisions are more and more apparent as traffic flux increases, cities grow, and the deer population expands. It also states that deer-vehicle collisions most often occur between 6 p.m. and 11 p.m. with deer being crepuscular.

Unfortunately, not everyone reports when they collide with a deer [5]. In fact, Behm states that fewer than half of all deer-vehicle collisions are reported and that the carcass count from the DNR is closer to the actual amount of collisions that happen in a year. This fact helped further motivate our current study. Since people are not reporting all deer-vehicle collisions, an accurate model of what is actually occurring would be a useful tool. As of now, we are unaware of anyone else performing such a study.

3 Data

The Wisconsin DNR tracks the white-tail deer herd population within deer management units in the state, so we know approximately how many deer there are in each deer management unit. The Wisconsin DNR also tracks the number of deer killed every year by hunters. To find herd populations, the Wisconsin DNR uses what is called the Sex-Age-Kill (SAK) formula. To use the SAK formula, the Wisconsin DNR divides the “registered buck harvest by the estimate of the buck harvest rate ... 1) multiplying the buck population estimate by the adult sex-ratio to estimate the size of the adult doe population, and 2) multiplying the doe population estimate by the fall fawn:doe ratio” [10]. As an example of the SAK formula, we will look at deer management unit 4 with 2002 data. The registered buck harvest was 443 and the estimated buck harvest rate was 0.31535 for the year. Thus the prehunt buck population was $\frac{443}{0.31535} = 1405$. Multiplying by the Adult Sex Ratio we have a total doe prehunt population of $(1405)(1.7455) = 2452$. Multiplying the

doe prehunt total by the fawn:doe ratio we have $(2452)(0.9) = 2207$ fawns. Adding these prehunt totals, we have $1405 + 2452 + 2207 = 6064$ total deer. Subtracting the total deer killed (including a 15% factor for poaching and other causes of death), we have $6064 - (1.15)(1028) = 4882$ post hunt population for deer management unit 4. The DNR then rounds to the nearest hundred when the population is 1000 or greater [15], [18].

The Wisconsin DNR also tracks carcass counts. This gives a more accurate count for deer-vehicle collisions because many people do not report deer-vehicle collisions, as previously stated.

The Wisconsin Department of Transportation (DOT) keeps track of the average annual daily traffic. The Wisconsin DOT also records the number of reported deer-vehicle collisions in each county each year. Carcass counts and reported deer-vehicle collisions were compiled on a website by Dr. Keith Knapp working with the Department of Transportation [7].

4 Methodology/Model

The main mathematical tool for this project will be generalized Markov Chains. We first have to define what a directed graph and weighted directed graph are. A directed graph is defined as a pair (V, E) where V is a set that represents vertices and E is a set that represents the edges that connect the vertices. In other words, $E \subseteq V \times V$, or E is a set of 2-permutations of V . A weighted directed graph is defined as a directed graph that also has a weight function $W : E \rightarrow \mathfrak{R}$, the set of real numbers.

A Markov Chain is defined formally as a countable set S of “states” with a collection of transition probabilities $T : S \times S \rightarrow \mathfrak{R}$ such that

$$\forall x \in S, \sum_{y \in S} T(x, y) = 1$$

and

$$T(x, y) \geq 0 \text{ always.}$$

A Markov Chain can be translated to a weighted directed graph with $V = S$, $E = \{(x, y) : T(x, y) > 0\}$, and T is the weight function. A Markov Chain is an iterated process (i.e. proceeds in discrete time steps). The set of transition probabilities can be formed into a matrix where the sum of all the entries of a given column will add up to one. In our case, we will be losing deer in the population due to death from vehicles, so our columns will add up to slightly less than one. For this reason, we are actually using a modified Markov Process.

The goal of the model is to discover the probability of hitting a deer while driving down a certain road. We will begin with creating a very basic model and introduce different parameters as we progress. We do not have any categorical (i.e. non-numeric) parameters at this time.

Our quantitative (i.e. numeric) factors include three items. The first is the population density of deer; its units are deer per square mile in each given deer management unit. Initial values of the deer population are based on data from the Wisconsin DNR and then will be time dependent for the rest of the model. This is due to the fact that deer will be moving from one management unit to the next, so the population in each management unit is subject to change based on the current, and eventually previous, movement patterns of the deer.

The second parameter will be deer movement transition probabilities; we use two types, namely pre-traffic transition probabilities and post-traffic transition probabilities. We label pre-traffic transition probabilities $p_{i,j}$ which represent the probabilities of deer moving from unit j to unit i in one time step without any traffic information factored in. In the Markov Chain each $p_{i,j}$ corresponds

to the edge with i as its ending vertex and j as its starting vertex. For the case where $i = j$, which we will call *loop probabilities*, we use the expected distance a deer travels in a day to find how likely a deer is to stay within the boundaries of the unit. To do this, we estimate each management unit as a circle, a simplified approximation of land shape. To find the radius r of the circle, we find the perimeter of the unit and divide by 2π since the circumference of a circle is equal to $2\pi r$.

Now, say a deer travels x miles per day and the radius of the section is r . For simplicity, we assume deer travel in a straight line, equiprobably in any direction, and their locations are scattered evenly throughout the entire deer management unit. To find the loop probabilities, we assume every deer will choose to walk in the same direction. This movement over one day represents a vector translation of the circular deer management unit x miles in the direction of deer movement. Thus, the only deer left within the deer management unit at the end of the day will be those deer within the intersection of the pre-image and the image of the unit under the translation. Let α be this area of intersection. Then the probability of a deer staying within a given management unit is the area of the intersection (the deer that are still within the unit at the end of the day) divided by the entire area of the unit, namely

$$\beta = \frac{\alpha}{\pi r^2}$$

To find α , we label the center of one of the circles A and the center of the other circle B . The length of their connecting segment is x as previously stated. Label as in Figure 1. Then

Figure 1: Loop Probability Calculation

$AC = BC = BD = AD = r$ since all are radii of the circles. Segment CD is the perpendicular bisector of AB , with intersection M . Thus $MB = \frac{1}{2}x$. Label $\angle EBD = \theta$. The area of the circular-sector mapped by E, B , and D that includes θ , which we will label as K , is equal to $\frac{\theta}{2\pi} \cdot \pi r^2 = \frac{\theta r^2}{2}$. Triangle MBD makes up one part of K . The other part is the area of the circle enclosed by E, M , and D ; label this area L . Then the area of L is $\frac{\theta r^2}{2} - (\text{area of triangle } MBD)$. Area of MBD is $\frac{1}{2} \cdot MD \cdot MB = \frac{1}{2} \cdot MD \cdot \frac{1}{2}x = \frac{1}{4}x\sqrt{r^2 - \frac{1}{4}x^2}$ since triangle MBD is a right triangle. Using triangle MBD , we can also find the value of θ . Since $\cos \theta = \frac{\frac{1}{2}x}{r}$, we have $\theta = \arccos\left(\frac{x}{2r}\right)$. We now have the area of L , but we have four such areas within the intersection of the circles so

$$\alpha = 4L = 4 \left(\frac{\arccos\left(\frac{x}{2r}\right)r^2}{2} - \frac{1}{4}x\sqrt{r^2 - \frac{1}{4}x^2} \right) = 2r^2 \arccos\left(\frac{x}{2r}\right) - x\sqrt{r^2 - \frac{1}{4}x^2}$$

All that is left is to divide by πr^2 and we have β .

$$\beta = \frac{2r^2 \arccos\left(\frac{x}{2r}\right) - x\sqrt{r^2 - \frac{1}{4}x^2}}{\pi r^2}$$

For the case where $i \neq j$, we base the pre-traffic transition probabilities on two values: 1) the ratio of the length of the boundary i and j share to the perimeter of j and 2) how likely the deer is to leave unit j , namely $1 - \beta$. Then

$$p_{i,j} = \frac{a_j}{b_j} \cdot (1 - \beta_j)$$

where a_j is the shared boundary of i and j , and b_j is the perimeter of section j . This is a logical step due to the following reasons. If a deer is to leave a unit, say unit j , then it is more likely to

leave from the side of j that has a longer boundary than one of its shorter boundaries, assuming there are no other factors affecting the deer's decision. Thus to incorporate this, we use the ratio between that particular boundary and the entire perimeter of j .

The third quantitative parameter is traffic flux; its units are cars per day passing by a certain location on the road. Let c be the probability of a deer getting hit by a vehicle. This c will be proportional to traffic flux since a deer cannot be hit by a vehicle unless a vehicle happens to be traveling down the road at the same time the deer is crossing. Also, c will be related to how long it takes the deer to cross the road; the longer the deer is on the road, the more likely a vehicle will drive by. The time it takes a deer to cross a road will be based on lane width and the speed the deer is traveling. We consider each car as an independent event and assume we can only have one event at a time. Therefore, we do not take into consideration how many lanes a road has. Our estimate is

$$c = \frac{(\text{traffic flux}) \cdot (\text{lane width})}{(\text{deer speed})}$$

As can be seen, c has cars as its units. In practice, c seems to be less than one and can therefore be interpreted as a probability. Namely, it can be viewed as the probability of an event occurring, that is, a car coming by, as its units suggest, but this equates into the event of a deer-vehicle collision occurring due to the fact its calculations are based on how likely it is that a deer and vehicle will be in the same location.

Let us start with the simplest model. Let there be two units of land; unit 1 is on the left and unit 2 is on the right. Unit 1 represents one vertex in a Markov Chain, and unit two represents another vertex. Thus $p_{1,2}$ is the pre-traffic transition probability of a deer moving to unit 1 from unit 2, $p_{2,1}$ is the probability of a deer moving to unit 2 from unit 1, $p_{1,1}$ is the probability a deer stays in unit 1, and $p_{2,2}$ is the probability a deer stays in unit 2. Now, suppose there is a road located between the two units, as can be seen in Figure 2.

Figure 2: Basic Model

From $p_{1,2}$, $p_{2,1}$, and c , we can also find the probability of a deer surviving crossing the road and moving to the other unit, which we will call post-traffic transition probabilities and label as $m_{i,j}$. Let $m_{1,2}$ be the post-traffic transition probability of a deer surviving crossing the road to unit 1 from unit 2, and let $m_{2,1}$ be the probability of a deer surviving crossing the road to unit 2 from unit 1. Then

$$m_{1,2} = p_{1,2} \cdot (1 - c_{1,2})$$

and

$$m_{2,1} = p_{2,1} \cdot (1 - c_{2,1}),$$

since c is the probability of a deer being hit by a vehicle, and $1 - c$ is the probability of the deer not being hit by a vehicle. Also, $p_{i,j}$ for every $i, j \in \{1, 2\}$, is the probability of a deer crossing the road; multiplying this by the probability it survives crossing the road gives the probability a deer chooses to cross the road and survives; in other words, the probability it survives crossing the road and is now part of the population in the other unit.

Using linear algebra, we can analyze where the deer population is over time. To do this we set up a matrix where the entries are the post-traffic transition probabilities, that is, $m_{i,j}$ for every $i, j \in \{1, 2\}$, and are placed in the cells of the matrix according to their subscripts i and j in the usual manner. We label this matrix T . Now, say a deer starts at vertex 1. After n time steps in the Markov Chain ($n = 0, 1, 2, \dots$), the deer may be at a different vertex than it started on. Let \vec{v}_n

be the vector of the populations of deer at each vertex after n steps. So, if 500 deer start at vertex 1 and 800 deer start at vertex 2, we would have $\vec{v}_0 = \begin{bmatrix} 500 \\ 800 \end{bmatrix}$. Then $\vec{v}_1 = T\vec{v}_0$ will be the state vector after 1 step since \vec{v}_0 is where the population starts and T is how likely the deer are to move and survive in a given direction. So $\forall n \in \{0, 1, 2, \dots\}$

$$\vec{v}_{n+1} = T\vec{v}_n,$$

and $\forall n \in \{1, 2, \dots\}$,

$$\vec{v}_n = T^n \vec{v}_0$$

This model allows us to accomplish our primary goal — to determine how likely it is that a driver on a certain road will hit a deer. To do this, we take how many deer were killed on a certain section of road and divide by the number of vehicles expected to drive through, and how many miles long the stretch of road is. Thus for a certain section of road we have

$$\text{probability of hitting a deer (per mile traveled)} = \frac{\text{deer killed}}{\text{cars} \cdot \text{miles}}$$

This is due to the fact that we know how many deer are killed; some vehicle hit them. We therefore just need to figure out how likely an individual vehicle is to hit the deer being killed. We know how many vehicles are driving that stretch of road from the traffic flux, and we know how long a stretch of road these collisions are occurring in.

The data for the number of deer being killed can be extracted from our model. We create a new transition matrix, label it D , that is similar to T . It differs in two ways. First, instead of using $1 - c$, we use c since we are keeping track of deer deaths instead of survivals. Second, all of the loop probabilities are 0 since a deer has to cross a road for it to possibly die in our model. Thus each entry in D is either 0 or $p_{i,j} \cdot c_{i,j}$ where $c_{i,j}$ corresponds to the probability of being struck by a vehicle on the road between unit i and j .

Let v_j be the population of deer in unit j at some time. Then $p_{i,j} \cdot c_{i,j} \cdot v_j$ is how many deer will die in a given time step leaving unit j and crossing to unit i during this time step. To find the total number of deer dying on the road between units i and j in a given time step, we need to also find how many deer are dying leaving unit i and crossing to unit j in a given time step. Thus the total number of deer dying on the road between units i and j in a given time step is

$$p_{i,j} \cdot c_{i,j} \cdot v_j + p_{j,i} \cdot c_{j,i} \cdot v_i$$

This formula can be obtained as a matrix multiplication of D and the population vector. The output we desire is in the form of an $n \times n$ matrix, assuming we have n deer management units. This way we could look at the deaths occurring between two certain DMU. Multiplying D by the population vector gives us an $n \times 1$ vector, an incorrect size. Therefore, we write the population vector as an $n \times n$ matrix with the diagonal consisting of the entries of the population vector; all other entries are 0. Adding the entry of the i th row and j th column to the entry of the j th row and i th column gives us our desired output.

We now have the basic framework for a model, so we can expand beyond two vertices. Let us consider a case in Douglas county using four deer management units, namely 1M, 1, 2, and 4. Their layout is as in Figure 3. To keep track of the deer population, we assume the deer cannot travel anywhere outside of units 1M, 1, 2, and 4, as if the units were contained inside of a brick wall. For mathematical purposes, we order the DMU in the following sequence and index our vectors and matrices accordingly: 1M, 1, 2, 4. For example, in Table 1, the entry in the third row and fourth column, 0.01657709, corresponds to the probability of starting in DMU 4 and ending in DMU 2.

Figure 3: Douglas County DMU ([17])

Table 1: Matrix T

0.86738703	0.02165914	0.00481039	0.0
0.11633758	0.92738467	0.02621822	0.05041380
0.01559533	0.01582481	0.94947083	0.01657709
0.0	0.03478288	0.01894912	0.93264154

Using the above outlined methodology, we create the matrix T (see Table 1). Note that the pre- and post-traffic transition probabilities between units 1M and 4 will be zero since they do not share a boundary; every other transition probability follows the outline above. Similarly, we have matrix D , as seen in Table 2, which also follows the above outlined methodology.

Table 2: Matrix D

0.0	0.00012546	0.00000191	0.0
0.00067386	0.0	0.00024945	0.00010504
0.00000619	0.00015056	0.0	0.00026252
0.0	0.00007247	0.00030009	0.0

Using data from the DNR, we have $\vec{v}_0 = (240, 4100, 11700, 4900)$ [10]. The time unit for this model is one day; thus $n = 365$ corresponds to one year. We use a computer program to perform the calculations for $n = 365$; the source code can be found in the Appendix, and samples of the output for that program can be found in the Analysis and Output section below.

5 Analysis and Output

As can be seen in Table 3, for $n = 1$, about ten deer are killed on the roads on the boundaries between the deer management units. For $n = 365$ our model shows that a total of 3,041 deer are killed over the course of one year. This number is very high considering the entire county had only 98 reported deer-vehicle collisions for 2002 [7]. Furthermore, ten deer-vehicle collisions in one day along 60 miles of road seems high as well. In 2002, the total Wisconsin carcass count was 45,278 [7]. If one assumes that each deer management unit has an equal amount, then that leaves approximately 377 carcasses per unit, a number far below 3,041.

Several possible reasons exist for why our model may show more deer-vehicle collisions than are actually happening. One reason is that drivers can slow down when they see a deer and avoid hitting them in real life. Our model does not take that choice into consideration; plus some deer stand on the side of the road and pick their moment to cross, and our model does not capture that either. A second reason is that deer are crepuscular (i.e. most active at dawn and dusk). Traffic tends to be heavier at other times of the day when deer are less active, thus creating fewer opportunities for deer-vehicle collisions to occur. A third reason our calculations may be high is that we have yet to include environmental factors. Right now, deer are flocking to DMU 1M, which is mostly comprised of the city of Superior. In reality, this heavy migration is not happening and is most likely due to the environmental factors of the abundance of buildings and people. This

Table 3: Population Totals After n Days

n	pop. total	$1M$	1	2	4
0	20940.00	240.00	4100.00	11700.00	4900.00
1	20930.16	353.26	4383.98	11258.66	4934.26
10	20843.75	996.86	6250.55	8390.61	5205.73
100	20048.82	1502.23	8087.15	4877.78	5581.65
200	19208.71	1440.03	7751.95	4666.88	5349.85
300	18403.82	1379.69	7427.13	4471.32	5125.68
365	17898.81	1341.83	7223.32	4348.62	4985.03

concept is linked to the fact that some barrier, such as a building, may exist next to a road and restrict a deer’s ability to cross the road.

If we could correct for all the factors stated above, our model might show fewer deer-vehicle collisions than are occurring in real life. First, the carcass count is for all of the roads within each unit and not just the ones on the border, which are presently the only roads in our model. Though the roads included are the ones most heavily traveled, the model fails to factor in the deer-vehicle collisions that occur on the back roads, where traffic is rare but deer may be abundant. Also non-fatal accidents occur and sometimes there may be a lack of evidence that an accident even occurred.

Table 4: Boundaries Deer are Killed On Cumulatively

n	$1M : 1$	$1M : 2$	$1 : 2$	$1 : 4$	$2 : 4$
10	10.82	0.23	32.82	9.06	43.33
100	187.90	1.99	263.88	113.28	324.13
200	386.44	3.81	502.14	228.11	610.80
300	576.67	5.55	730.40	338.13	885.43
365	696.02	6.65	873.61	407.16	1057.74

Using the data from Table 4, we found the probability of driving down a certain road in our model on a certain day and hitting a deer. This probability is small as can be seen in Table 5. Over the course of one year, there is a chance of approximately one collision every 18,000 miles.

Table 5: Probability of Hitting a Deer (per mile traveled)

n	$1M : 1$	$1M : 2$	$1 : 2$	$1 : 4$	$2 : 4$
1	0.00001941	0.00007446	0.00008459	0.00004035	0.00009540
365	0.00005475	0.00005693	0.00005726	0.00005544	0.00005763

6 Conclusions

In the future, we would like to change the time unit of the model to one hour. In this way, we can have the traffic flux and deer movement depend on the hour of the day to accommodate the heavier

traffic at certain times and the crepuscular nature of deer. We also would like to include environmental factors. Deer are less likely to be in unit 1M since it primarily contains houses, businesses, and people. This tendency to stay away from this area will affect the transition probabilities. The trick will be to properly include such categorical factors in our numerical calculations.

Right now, the model focuses on the loss of deer life through deer-vehicle collisions. In other words, deer are disappearing, so to speak, from the model. Without a source of life in our model, the deer population would eventually become extinct. To represent a source of life, we need to have deer added to our model as time progresses, namely in the event of does having fawns.

Through these modifications, we believe our model will be a more accurate indicator of deer movement patterns.

7 Acknowledgements

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8 Appendix: Source Code in Java

```
package matrixmult;

public class Main
{
    /** Creates a new instance of Main */
    public Main()
    {
    }

    public static void main(String[] args)
    {
        //number of DMU using
        int numDMU = 4;

        /*in arrays of length 4:
        *element 0 corresponds to DMU 1M
        *element 1 corresponds to DMU 1
        *element 2 corresponds to DMU 2
        *element 3 corresponds to DMU 4
        *
        *in double arrays:
        *Same idea as above applied to both columns and rows
        */

        //creating population vector, deer per square mile
```

```

double V[] = {240, 4100, 11700, 4900};

//creating vector to keep track of how many deer killed each iteration
double KillV[] = new double[numDMU];

//creating variable to keep track of how many deer killed cumulatively
double TotalKill = 0;

//creating vector to keep track of how many deer killed between two DMU
double DeerKillij[][] = new double[numDMU][numDMU];

//creating vector to keep track of probability driver will hit deer
//          driving on road
double ProbHit[][] = new double[numDMU][numDMU];

//entire perimeters of each DMU in miles
double P[] = {35, 64, 92, 69};

//total perimeter deer can leave from each DMU in miles
//          (take away brick wall perimeters)
double SubPerim[] = {17, 50, 21, 32};

//perimeters two DMU share in miles
double SharePerim[][] = {{0, 15, 2, 0}, {15, 0, 11, 24}, {2, 11, 0, 8},
                          {0, 24, 8, 0}};

//approximate lane width in miles
double w = 11./5280.;

//approximate top speed of deer in miles per day
double s = 840.;

//traffic flux in cars per day
double Flux[][] = {{0, 2322, 160, 0}, {2322, 0, 3800, 2515./3.},
                  {160, 3800, 0, 44000./7.}, {0, 2515./3., 44000./7., 0}};

//probability of a deer getting hit by a car while crossing
//          borders to another DMU, in cars
double C[][];
C = new double[numDMU][numDMU];
for(int row = 0; row < C.length; row++)
{
    for(int column = 0; column < C.length; column++)
    {
        C[row][column] = Flux[row][column] * w / s;
    }
}

```

```

//radii of DMU circle approximation in miles
double R[];
R = new double[numDMU];
for(int i = 0; i < R.length; i++)
{
    R[i] = P[i] / (2*Math.PI);
}

//estimate of home range length of deer in miles per day
//found by homerange of approx. 350 hectare, converted to square
//miles, take square root
double x = 5. * Math.sqrt(74.) / 37.;

//loop probabilities
double B[];
B = new double[numDMU];
for(int i = 0; i < B.length; i++)
{
    B[i] = (2*R[i]*R[i]*Math.acos(x/(2*R[i]))-x*
           Math.sqrt(R[i]*R[i]-x*x/4)) / (Math.PI*R[i]*R[i]));
}

//creating transition prob matrix, tracking deer survival,
//          how likely to live
double T[] [];
T = new double[numDMU][numDMU]; //size of matrix is how many
//          DMU we have
for(int row = 0; row < R.length; row++)
{
    for(int column = 0; column < R.length; column++)
    {
        if(row == column)
        {
            T[row][column] = B[row];
        }
        else
        {
            T[row][column] = (1 - B[column]) *
                (SharePerim[row][column] /
                 SubPerim[column]) * (1 - C[row][column]);
        }
    }
} //close column loop
} //close row loop

//creating transition matrix for counting deer deaths,
//          how likely to die
double KillT[] [];
KillT = new double[numDMU][numDMU]; //size of matrix is

```

```

//                                     how many DMU have
for(int row = 0; row < R.length; row++)
{
    for(int column = 0; column < R.length; column++)
    {
        if(row == column)
        {
            KillT[row][column] = 0.;
        }
        else
        {
            KillT[row][column] = (1 - B[column]) * (
                SharePerim[row][column] /
                SubPerim[column]) * C[row][column];
        }

    }

}

} //close column loop
} //close row loop
System.out.println("T = ");
printmatrix(T);
System.out.println("KillT = ");
printmatrix(KillT);
System.out.println();

//tracking population of deer, output is # of deer in each DMU,
//                                     entry sum is how many deer still alive
for(int i = 1; i < 367; i++) //i is time increment of 1 day
{
    System.out.println("Total # deer=" + entrySum(V));
    System.out.println("Total # deer killed this iteration="
        + entrySum(KillV));
    TotalKill += entrySum(KillV);
    System.out.println("Cumulative # deer killed=" + TotalKill);
    System.out.println();

    for(int row = 0; row < KillT.length; row++)
    {
        for(int column = 0; column < KillT.length; column++)
        {
            DeerKillij[row][column] += deerkill(KillT, V,
                row, column);
        }
    }
}
if(i == 365)
{
    System.out.println("Which boundaries deer are killed,
        cumulative:");
    printmatrix(DeerKillij);
}

```

```

        for(int row = 0; row < DeerKillij.length; row++)
        {
            for(int column = 0; column < DeerKillij.length; column++)
            {
                ProbHit[row][column] = findprobhit(DeerKillij, row,
                    column, i, Flux, SharePerim);
            }
        }
        System.out.println("Probability of hitting a deer
                            on boundary: ");

        printmatrix(ProbHit);
    } //close if statement

    System.out.print("KillT*V" + i + "=");
    KillV = multiplymatrices(KillT, V);
    System.out.println();
    System.out.print("T^" + i + "V0=");
    V = multiplymatrices(T, V);

    System.out.println();
}

} //close public static void main

//finding how many deer are killed between two DMU, i and j
public static double deerkill(double[][] array1, double[] array2, int row,
    int column)
{
    double total = 0;
    total = array1[row][column]*array2[column] +
        array1[column][row]*array2[row];
    return total;
}

public static double findprobhit(double deerkilled[][] , int row, int column,
    int day, double cars[][] , double miles[][] )
{
    double prob = 0;
    prob = (deerkilled[row][column] / (cars[row][column] *
        miles[row][column])) / day;
    return prob;
}

public static double entrySum(double[] vector)
{
    double total = 0.0;
    for(int i = 0; i < vector.length; i++) total += vector[i];
}

```

```

        return total;
    }

    public static void printmatrix(double array[] [])
    {
        for(int row = 0; row < array.length; row++)
        {
            for(int column = 0; column < array[row].length; column++)
            {
                System.out.print(array[row][column] + " \t");
            }
            System.out.println();
        }

    } //close printmatrix method

    public static double[] multiplymatrices(double array1[] [], double array2[])
    {
        double E[];
        E = new double[array1.length];
        for(int row = 0; row < array1.length; row++)
        {
            for(int column = 0; column < array1[row].length; column++)
            {
                E[row] += array1[row][column] * array2[column];
            }
            System.out.print(E[row]+" ");
        }
        return E;
    } //close multiplymatrix method

} //close public class Main

```

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