Electron heating by sheaths in radio frequency discharges

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The detailed interaction of individual electrons with oscillating radio frequency (rf) sheaths is considered in this reexamination of the sheath heating process. We develop an analytic expression for the energy delivered to the discharge through sheath heating and that lost due to electron escape to the electrode. Using a simple model of the time-dependent sheath electric field, we find that electron energy gain due to this process, averaged over an rf period is roughly proportional to the square of the maximum sheath speed and drops slightly with increasing electron temperature. Energy loss due to escaping electrons on the other hand, rises with temperature and ultimately outweighs gain when the average electron velocity exceeds the maximum sheath velocity. The requirement of a net power input to the plasma places upper bounds on bulk electron temperature and density attainable by plasmas to be maintained solely by this mechanism, independent of other aspects of container geometry. However, by increasing discharge frequency, power input can be increased for a fixed sheath voltage while reducing electron escape.

I. INTRODUCTION

Investigation of electrical discharges maintained through capacitive coupling of radio frequency (rf) power is an activity which has enjoyed a great deal of attention in recent years due to the surge of industrial use of such discharges in a variety of materials processing applications. Three distinct heating mechanisms have been identified which couple power from electric fields to the plasma. Ohmic heating due to rf currents across the bulk of the plasma may occur in capacitively coupled rf discharges, and is thought to be important at higher pressures (>500 mTorr). In the second mechanism, important at higher pressures and higher voltages, ion bombardment produces secondary electrons which can, through avalanching in the sheath, cause significant energy input into the plasma. However, for low pressure discharges (1–100 mTorr), the primary mechanism for power coupling for typical discharge voltages (100–1000 V) is the dynamic interaction of discharge electrons with oscillating sheaths.[1,2] This mechanism, called sheath heating (and sometimes referred to as stochastic heating or wave-riding), is treated in detail in this paper. For an electropositive gas under these conditions, almost all of the applied rf voltage appears across oscillating sheaths at the electrodes. Electrons entering the sheath from the bulk are accelerated back toward the plasma by the sheath electric field, and either escape to the electrode or return to the plasma, in the latter case gaining or losing energy in the interaction, depending on the direction of motion of the sheath edge. This direction is a function of the rf phase, and over the cycle, many electrons reach the sheath at different phases and with different velocities normal to the sheath edge. Summing up gains and losses, in order for sheath heating to be important, the net effect must be power input into the plasma.

In order to predict parameter scaling in rf discharges, a power balance approach is attractive as a quick and simple tool. This is particularly true for discharge geometries which do not lend themselves to the one-dimensional treatments which have been developed using fully self-consistent kinetic or particle-in-cell models.[3,4] Several researchers have employed such a macroscopic approach for cylindrical[5] and spherical,[6] as well as planar discharges.[1,7,8] Power balance calculations are simple and quick because they use many simplifying assumptions. In using such an approach, one must take care that the discharge physics is accurately represented over the entire range of parameters under consideration. For example, recent experiments[9] and simulations[3,4] show non-Maxwellian electron energy distributions, bringing into question the use of a single electron temperature in power balance calculations. Here we focus on one aspect of the power balance: collisionless electron interaction with electric fields at the powered electrode. In low pressure discharges (1–100 mTorr), the time spent in the sheath is short compared to a collision time, so to lowest order collisional effects can be neglected. We develop a model which gives the mapping of the electron velocity distribution resulting from interaction with the sheath. This represents an extension of the work of Goedde *et al.*[10] Then, for a given density and velocity distribution in the bulk plasma, the velocity distribution and flux of electrons returning to the bulk from the sheath can be determined. From this distribution, we will find the scaling with sheath parameters of power input due to sheath heating.

II. VELOCITY MAPPING

In this section we describe a numerical model of the sheath heating process. The model is in the form of a mapping which may be used to alter an electron velocity distribution function according to its interaction with an oscillating sheath with given parameters. For each incoming (from the plasma into the sheath) electron velocity and phase, the mapping returns the appropriate outgoing electron (from the sheath back out into the plasma) velocity.

Then, for a given set of sheath conditions, we examine the evolution of several electron velocity distribution functions in a single interaction with the sheath, averaged over one rf cycle.

A. Model description

The model solves the equations of motion for electrons in the sheath region, to find their change in velocity through collisionless interaction with the sheath, as a function of incoming velocity and phase. We use a simple model of the sheath electric field which is more or less consistent with known results.[3,11] In this model, the sheath electric field varies linearly in space and sinusoidally in time:

$$E(x,t)=\begin{cases}\frac{E_0}{2}\left(1+\cos(\omega t)-2\frac{x}{s_{\max}}\right), & x< s(t)\\ 0, & x>s(t),\end{cases} \quad (1)$$

where ω is the angular frequency of the rf source, x is the spatial coordinate measured normal to the electrode, and $s_{\max} = 2s_0$ is the maximum sheath thickness. The results will show that the spatial dependence of the electric field strength is not of critical importance to the electron heating rate. The maximum sheath voltage for this field is

$$V_0=E_0 s_{\max}/2. \quad (2)$$

The position of the edge of the sheath (defined as the first position at which $E=0$) is

$$s(t)=s_0[1+\cos(\omega t)], \quad (3)$$

and the velocity of the sheath edge is

$$v_{sh}(t) = -s_0\omega\sin(\omega t). \quad (4)$$

Following the work of Goedde *et al.*[10] we use the following definitions: θ is the phase ωt at which an electron passes $x = s_{\max}$ and enters the sheath region, and ϕ is the phase at which the electron actually enters the electric field, at $x=s(t)$.

The equations of motion:

$$m\frac{d^2x}{dt^2} = -eE(x,t) \quad (5)$$

are solved analytically. The constants appearing in the analytic solution must be evaluated numerically. The equations are solved separately for the two regions mentioned above (in some cases repeatedly, as there may be multiple interactions for a single electron), and the constants are determined according to initial electron velocity v_{in} and phase θ and by matching boundary conditions between the regions.

B. Mapping results

For fixed sheath thickness $s_{\max}$ and maximum sheath voltage V_0, we seek, as a function of incoming phase and velocity, θ and v_{in}, respectively, the exit velocity, v_{out}, of an electron returning to the bulk plasma from the sheath. Analytic solutions to the electron equations of motion are obtained for the two regions corresponding to $s_{\max} > x > s(t)$ (electron out of the electric field) and $0<x<s(t)$ (electron in the field), and the times t at which the electron passes from one region to the other are found numerically, from the roots of the analytic expression. Contours of v_{out}, the electron velocity upon exit from the sheath, for a sample case with maximum sheath voltage $V_0 = 500$ V and maximum sheath width $s_{\max}=1$ cm are shown in Fig. 1. Electrons which reach the electrode surface at $x=0$ recombine and are considered lost from the system. These cases are indicated in the figure by $v_{out}=0$, and occupy a large V-shaped region in Fig. 1(a) roughly centered at $\theta=\pi$. For this set of conditions, the maximum sheath

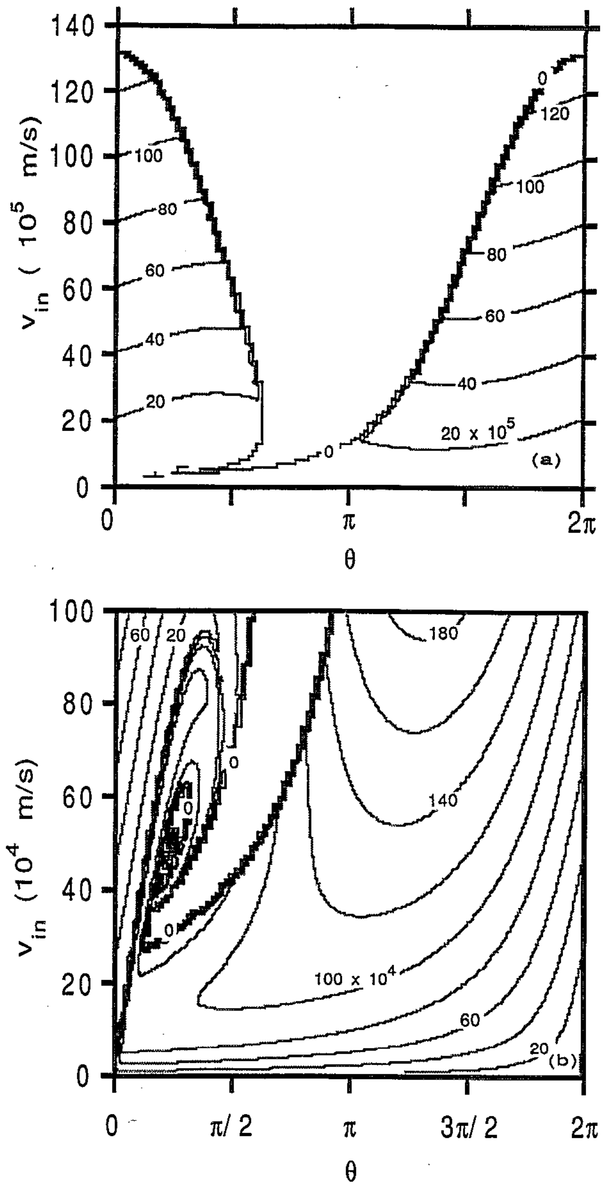

FIG. 1. Contours of exit velocity v_{out} through sheath interaction as a function of v_{in} and θ. For this case, $V_0 = 500$ V and $s_{\max}=1$ cm. (a) shows the complete mapping (contour levels 10^5 m/s). The contour for $v_{out}=0$ represents a threshold boundary for electron escape through the sheath to the electrode, so the V-shaped region without contours in the center of the plot corresponds to initial conditions which lead to electron loss. (b) is a blow up of the same information over a reduced range in initial velocity, $0 < v_{in} < 1.0\times 10^6$ m/s (contour levels 10^4 m/s).

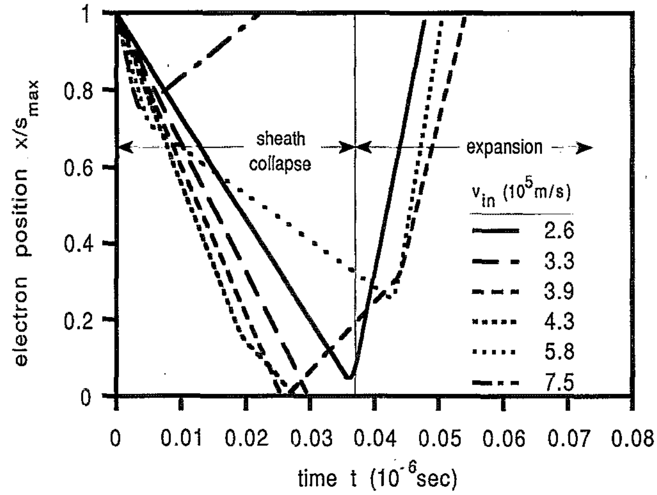

FIG. 2. Electron trajectories in the sheath region for $V_0 = 500$ V, $s_{max} = 1$ cm and $\theta = \omega t_0 = 0.2\pi$. The different trajectories correspond to: $v_{in} = 2.6\times10^5$ m/s (—), $v_{in} = 3.3\times10^5$ m/s (——), $v_{in} = 3.9\times10^5$ m/s (—), $v_{in} = 4.3\times10^5$ m/s (---), $v_{in} = 5.8\times10^5$ m/s (···), and $v_{in} = 7.5\times10^5$ m/s (——).

speed is $s_0\omega = 4.26 \times 10^5$ m/s. The mapping has several notable features:

(1) For $v_{in} > s_0\omega$, ϕ approaches θ, and for $0<\theta<\pi$ we find $|v_{out}| < |v_{in}|$, and for $\pi<\theta<2\pi$, $|v_{out}| > |v_{in}|$. In other words, electrons which enter a collapsing sheath are slowed down by the interaction, and electrons which enter an expanding sheath are sped up, and those reaching the sheath in an interval centered on $\phi = \pi$ are sufficiently energetic to overcome the remaining potential barrier and escape the system by hitting the electrode. All of these cases can be seen in Fig. 2, which shows electron trajectories for a group of electrons starting with the same initial phase but different initial velocities. The straight segments correspond to a constant velocity, when the electron is in a part of the sheath region where $E = 0$. The interaction time with the electric field is brief compared to the time spent in the sheath region for these conditions, and the penetration depth into the electric field is small compared to the sheath thickness.

(2) For intermediate velocities, $|v_{in}| \sim s_0\omega$, several different classes of electron trajectories are possible, giving rise to the structure visible in Fig. 1(b). When an electron reaches a retreating sheath with a velocity close to that of the sheath, it may have multiple interactions with the electric field during its time spent in the sheath region. As soon as the electron enters the electric field it starts slowing down, while at the same time the sheath edge is accelerating toward the electrode. Thus, the sheath edge moves out ahead of the electron while both are still directed toward the electrode. The electron then reenters the electric field either as the sheath edge slows down or during sheath expansion. Examples of these types of trajectories are shown in Fig. 2. The timing of the interactions depends on θ, the phase of entry into the sheath region, as well as initial velocity.

(3) Loss to the electrode: For a large V-shaped range of initial velocity and phase shown in Fig. 1(a), the electrons reach the sheath field at a phase ϕ such that the remaining potential barrier before the electrode is low enough to be overcome. Electrons with initial energy above eV_0 escape regardless of initial phase. There is also a minimum velocity, somewhat lower than $s_0\omega$, below which no electrons escape to the electrode regardless of initial phase θ. This is because slower electrons cannot catch up with the sheath edge because it retreats toward the electrode with a speed greater than the electron speed. These electrons are not hit until the sheath edge comes back around and reaches them on the expanding half of the cycle, and they always gain energy in the process. Thus, for a group of electrons incident on the sheath uniformly distributed in θ, the phase at which they cross $x = s_{max}$, there is a bunching to a narrower range of ϕ, the phase when they enter the sheath field. The slower the initial velocity v_{in}, the shallower the penetration of the electron into the sheath region at the point of interaction with the sheath, and the closer ϕ approaches 2π. In the limit as ϕ approaches 2π, the velocity of the sheath edge approaches zero, as does the change in speed of the electron.

C. Mapping of the electron velocity distribution

The velocity map described above determines the effect of the sheath on the electron velocity distribution function. Looking at the velocity dependent flux of electrons incident on the sheath region normal to the electrode, we consider initially Maxwellian distributions. Here, for a given distribution of electron velocities crossing the boundary into the sheath region, the map returns the distribution of electrons crossing back into the plasma region, averaged over one rf cycle. Figure 3(a) shows that for this set of sheath conditions and an initial distribution with $T_e = 1$ eV, the velocity dependence of the electron flux is significantly altered by interaction with the sheath. Heating has occurred, and the speed for which the flux is a maximum has shifted by about $2s_0\omega$. For higher temperatures [Figs. 3(b) and 3(c)] there is still a heating of electrons with low initial velocity, but higher velocities are depleted through electron loss to the electrode. In general, substantial heating takes place for any distribution for which the flux is peaked at a velocity less than $2s_0\omega$.

III. ENERGY GAIN

The results of the velocity mapping described in the previous section are used here to examine the magnitude of the energy gain in the sheath heating process. We develop an analytic model to estimate the energy gain per electron with a given initial velocity, averaged over initial phase θ. The analytic model gives the scaling of sheath heating power with discharge parameters, and exposes limits on T_e and n_e above which sheath heating cannot maintain a discharge.

A. Mapping

The velocity mapping is expressed in terms of the energy gain per electron as a function of initial conditions in

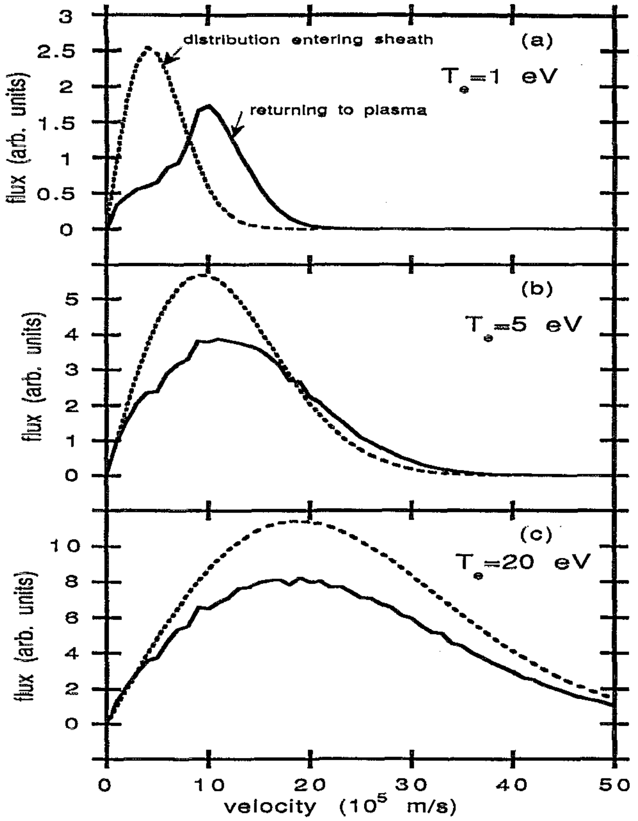

FIG. 3. Mapping of the velocity dependent electron flux through a single interaction with the sheath ($V_0 = 500$ V, $s_{max} = 1$ cm), for initially Maxwellian distributions with electron temperatures: (a) $T_e = 1$ eV, (b) $T_e = 5$ eV, and (c) $T_e = 20$ eV. The dashed lines represent the flux crossing $x = s_{max}$ into the sheath region, and the solid lines represent the flux back across the same boundary into the plasma.

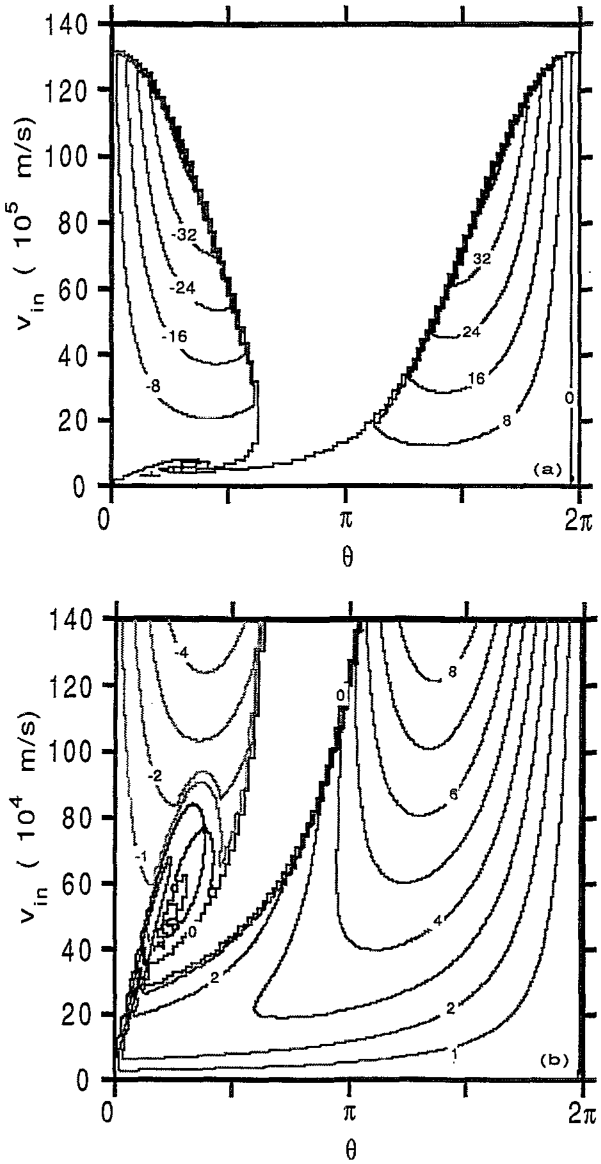

FIG. 4. Contours of electron energy change (eV) through sheath interaction as a function of initial velocity and phase of entry into the sheath region at $x = s_{max}$. For this case, $V_0 = 500$ V and $s_{max} = 1$ cm. (a) shows the complete mapping and (b) is a blow up of the range $0 < v_{in} < 1.4 \times 10^6$ m/s.

Fig. 4. The energy gain is shown as zero for electrons that reach the electrode. Below $v_{in} = s_0\omega$ we see energy gain over most of the rf cycle of a magnitude roughly $2ms_0^2\omega^2$, corresponding to a velocity change of $2s_0\omega$. The magnitude of the energy gain for those electrons that do not hit the electrode increases with v_{in}, but over the rf cycle, gains are offset by losses and escape. Assuming the electron flux is constant over an rf cycle and averaging over phase θ, the net energy gain per electron is determined by this mapping.

B. Analytic approximation

The energy gain can also be estimated using an "impulse" approximation. That is, the interaction is assumed to take place instantaneously at the moment the electron enters the sheath electric field, as though the sheath edge were a hard wall. We shall include a correction to this model below to account for the noninstantaneous nature of the interaction. This approach, at the expense of information about the electron velocity distribution, gives the scaling of sheath heating power with discharge parameters. In this case, the velocity change of the electron is just twice the velocity of the sheath edge: $\Delta v = 2v_{sh}(\phi)$. The corresponding change in electron energy, shown in Fig. 5, is

$$\Delta E_e = 2m(v_{sh}^2 - v_{sh}v_{in}). \tag{6}$$

The agreement with the mapping is very good both for $v_{in} \ll s_0\omega$ and $v_{in} \gg s_0\omega$; however, it does not reproduce the detail associated with the more complex trajectories in the range $v_{in} \sim s_0\omega$. In addition, for $v_{in} > s_0\omega$, the estimate for energy gain is a little higher than that returned by the map for those initial conditions for which the electron gains energy, and the estimate of energy loss is always a little lower.

To find the energy gain per unit flux, we must average the energy gain

$$\Delta E_e = 2ms_0\omega \sin\phi(s_0\omega \sin\phi - v_{in}) \tag{7}$$

over θ, which is related to ϕ through

$$\theta = \phi - \frac{s_0\omega}{|v_{in}|} + \frac{s_0\omega}{|v_{in}|}\cos\phi. \tag{8}$$

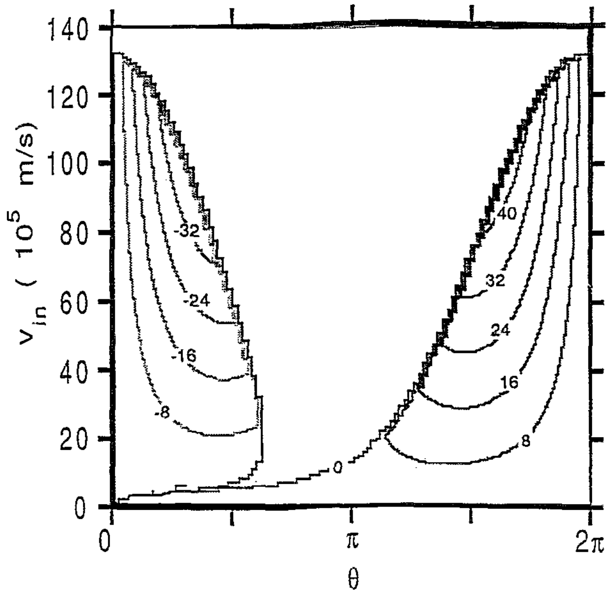

FIG. 5. "Hard wall" approximation for electron energy change through sheath interaction assuming $v_{\text{out}} = -v_{\text{in}} + 2v_{\text{sh}}$ for $V_0 = 500$ V and $s_{\max} = 1$ cm. Contours agree qualitatively with the results of the mapping shown in Fig. 4, but energy gains are larger than predicted by the mapping, and losses are lower.

A plot of this function is shown in Fig. 6 for several values of the ratio $s_0\omega/v_{\text{in}}$. For large values of v_{in}, the curve approaches a straight line with unity slope, as the phase change during electron transit across the $E=0$ region diminishes. The figure also illustrates that slow electrons cannot overtake the edge of the sheath when it is retreating at its maximum speed. Note that for the $v_{\text{in}} = \frac{1}{2}s_0\omega$ case, ϕ is multivalued for small values of θ. Of the three values, only the lowest one is physically important. The next highest value corresponds to the electron exiting the sheath as the velocity of the sheath edge increases, and the third

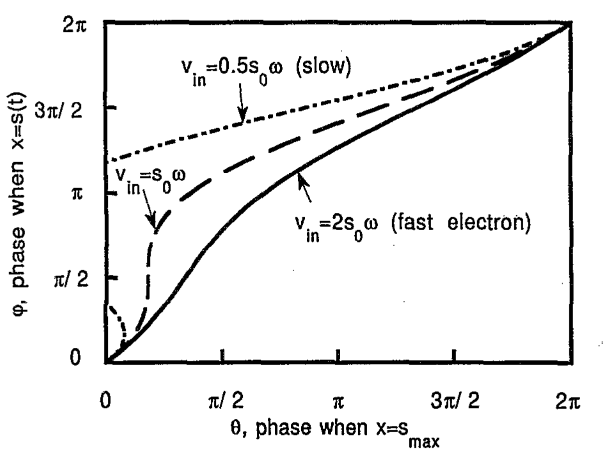

FIG. 6. Relationship between the phase in the rf cycle that an electron crosses $x = s_{\max}$ into the sheath region, $\theta =$, and ϕ, the phase at which it enters the sheath field. Three cases are shown, for $|v_{\text{in}}| = 2s_0\omega$ (—), $|v_{\text{in}}| = s_0\omega$ (— — —), and $|v_{\text{in}}| = 0.5s_0\omega$ (——), illustrating the enhanced probability that a slow electron will hit an expanding rather than retreating sheath edge.

corresponds to a second entry into the sheath. The latter two are not meaningful because once the electron enters the sheath the first time its velocity changes so that the specified v_{in} is no longer valid. Some values of ϕ are therefore inaccessible for $|v_{\text{in}}| < s_0\omega$, as was seen in the energy mapping, so an integral over all ϕ from 0 to 2π is incorrect for electrons slower than the maximum sheath speed. In the mapping, the minimum velocity for escape was somewhat lower than $s_0\omega$, even though electrons with velocities below this value cannot reach the sheath edge at $\phi=\pi$, when the sheath is fully collapsed. They can reach the sheath edge, however, at values of ϕ near π, so that the remaining potential barrier to reach the electrode is low enough to be overcome. Thus, the minimum velocity for escape is $s_0\omega$ with a correction which depends on V_0.

In averaging over the initial phase, the integral must exclude electrons which make it all the way through the sheath, reaching the electrode. The criterion used to determine the cutoff velocity is that the electron's kinetic energy be e times V_s, the total sheath voltage at the instant the electron reaches the sheath, $\frac{1}{2}mv^2 = eV_0(1 + \cos\phi)^2$, or in terms of ϕ:

$$\cos\phi = \sqrt{2mv^2/eV_0} - 1. \tag{9}$$

We now define two cutoff phases, ϕ_1 and ϕ_2 to be the two roots such that $0<\phi_1<\pi$ and $\pi<\phi_2<2\pi$. The energy gain per electron of initial velocity v_{in}, averaged over θ is

$$\langle \Delta E_e \rangle = \frac{2ms_0^2\omega^2}{\pi}(\phi_1 - \cos\phi_1 \sin\phi_1). \tag{10}$$

Excellent agreement between our analytic estimate and the full numerical mapping for the sheath heating rate is possible only when we include a correction to the analytic model to account for the fact that the interaction of an electron with the sheath is not instantaneous, but takes place over a finite amount of time. This correction manifests itself in two different aspects of the calculation. First, in determining the cutoff phase for escape to the electrode, the electron must have just enough kinetic energy to overcome the potential barrier at the electrode *at the time the electron reaches the electrode*, rather than when it enters the sheath field. This can be accounted for by shifting the cutoff phase for escape to the appropriate point earlier in the cycle. Assuming the time spent in the sheath field is short compared to the rf period, we neglect the time dependence of the electric field in estimating the duration. Then, the electron equation of motion is that of a simple harmonic oscillator with frequency

$$\omega_0 = \sqrt{eV_0/2ms_0^2}. \tag{11}$$

Note that this frequency is independent of the initial velocity of the electron. The cutoff phase for electron escape to the electrode is shifted back a quarter period of the simple harmonic oscillator, and the time taken to traverse the sheath by an electron which just grazes the electrode at its turning point is:

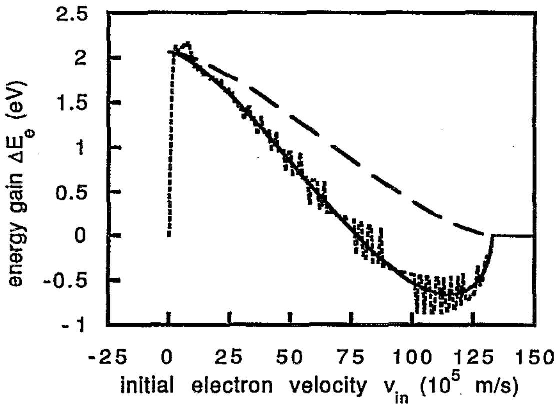

FIG. 7. Energy gain per electron averaged over initial phase θ, as a function of velocity at sheath entry. Plotted are results for the average of the numerical mapping (---), the simple "hard wall" approximation (— —), and the hard wall model corrected for the finite residence time of the electron in the sheath (—).

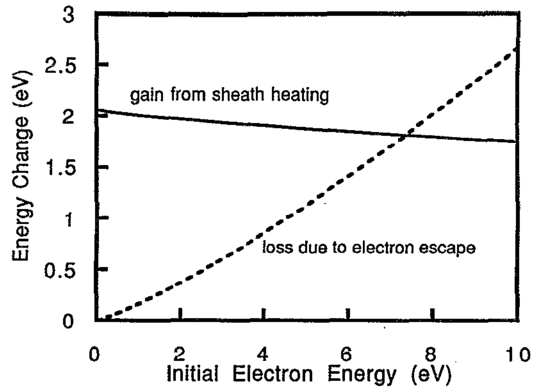

FIG. 8. Average electron energy gain over a rf cycle $\langle \Delta E_e \rangle$ through sheath heating (—) and loss (---) due to escape to the electrode for $V_0 = 500$ V and $s_{max} = 1$ cm.

$$\Delta\phi = -\pi s_0\omega/2\sqrt{2m/eV_0}, \quad (12)$$

accurate when $\Delta\phi \ll 2\pi$ or

$$s_0\omega \ll 4\sqrt{eV_0/2m}. \quad (13)$$

Second, the velocity of the sheath is changing during the time the electron is in the sheath field, and this must be evaluated at a phase later than ϕ. In fact, for faster electrons that turn around in the sheath (i.e., do not make it to the electrode), the velocity of the sheath always decreases during the period of interaction. What seems to really matter is the velocity of the sheath experienced by the electron after it has turned around and is headed back toward the sheath edge and plasma. We get best agreement with the numerical mapping by evaluating the sheath velocity at 13/32 of the harmonic oscillator period after the electron reaches the sheath field, yielding a value slightly greater than the average velocity of the sheath in the time period between when the electron turns around and when it exits the electric field (which occurs at 3/8 of an oscillator period after entering the sheath field). This phase shift partially cancels the shift back in the cutoff phase, giving a net forward phase shift of 5/32 of the oscillator period, or

$$\Delta\phi_{net} = 5\pi s_0\omega/16\sqrt{2m/eV_0}. \quad (14)$$

The corrected energy gain is

$$\langle \Delta E_e \rangle = \frac{2ms_0^2\omega^2}{\pi}(\phi_1 - \cos\phi_1 \sin\phi_1) - \frac{2ms_0\omega}{\pi} v_{in}\Delta\phi_{net}\sin\phi_1. \quad (15)$$

This function is plotted as a function of v_{in} for $s_{max} = 1$ cm and $V_0 = 500$ V in Fig. 7, along with the uncorrected energy gain and the energy gain determined by averaging the mapping results over phase. The agreement between this new analytic model and the mapping result is excellent, and shows that slowing of the sheath during interaction leads to a average "sheath cooling" for initially energetic electrons. The analytic model deviates from the numerical mapping for initial velocities below $v_{in} \sim s_0\omega$ because we have integrated over the full range of ϕ from 0 to 2π, when, as discussed earlier, not all values of ϕ are possible in this range. The error is substantial, however, only for approximately $0 < v_{in} < 0.5s_0\omega$, corresponding to slow electrons that interact with a nearly fully expanded (and therefore slowly moving) sheath. This will lead to a significant overestimate of heating only for very "cold" initial distributions (for our sample case, $T_e < 0.5$ eV). This model allows us to examine the dependence of the sheath heating rate on discharge parameters and determines power input due to sheath heating for power balance calculations.

This analytic prediction agrees with the results $\langle \Delta E_e \rangle = 2ms_0^2\omega^2$ of a similar analysis by Goedde *et al.*[10] only in the limit $v_{in} \to 0$. It retains the strong dependence on s_0 and ω, but shows an additional weak dependence on V_0 for two reasons: some electrons escape to the electrode and our "hard wall" approximation has been corrected for finite time spent in the sheath. The strong dependence on the rf driving frequency ω is suggestive of the neglected importance of this parameter in discharge operation for materials processing applications as discussed recently by Surendra and Graves.[13] Variation in sheath heating power with frequency for a fixed sheath voltage would allow control of ion flux (by controlling density with power input) independently from ion energy (determined by sheath voltage) at a substrate surface.

It is important here to also consider the energy loss from the plasma due to electron escape to the electrode. If sheath heating is the dominant mechanism for energy input into the discharge, then the corresponding energy gain at the electrode must be greater than the loss. This local balance of power places some limitations on allowable operating conditions for discharges powered by sheath heating, regardless of discharge geometry.

The average energy per electron incident at the sheath edge carried away from the plasma by electrons escaping to the electrode is

$$\langle \Delta E_e \rangle = 1/2\pi[mv_{in}^2(\phi_1 - \pi) - mv_{in}s_0\omega(1 + \cos\phi_1)]. \quad (16)$$

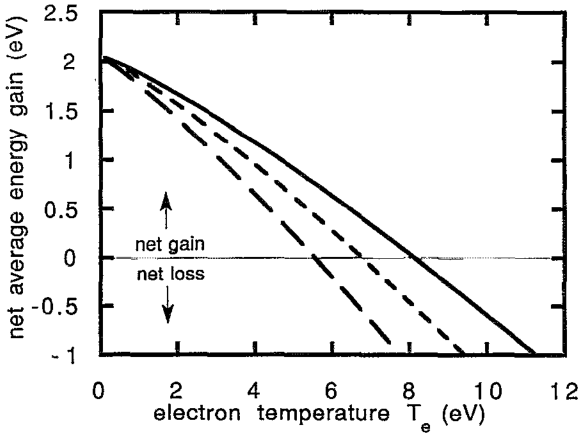

FIG. 9. Average net electron energy gain per unit flux as a function of temperature for a single Maxwellian distribution and sheath velocities $V_0 = 250$ V (—), 500 V (- -), and 1000 V (— —).

Both gain and loss are shown in Fig. 8 as a function of initial electron energy for the $V_0 = 500$ eV, $s_{max} = 1$ cm case. For higher energy electrons, energy gains are offset by even greater losses as electrons carry their energy off to the electrode. However, although sheath interaction of the higher energy electrons contributes a net loss of energy from the plasma, some fraction of electrons with initial energy up to eV_0 will be heated and returned to the bulk plasma with substantial energy gain, and these energetic electrons may play an important role in ionization.

C. Heating rate versus electron temperature

Even though the main point of this exercise is to look at the evolution of non-Maxwellian electron temperature distributions, we now consider the effect of the sheath on a Maxwellian distribution, using the analytic analysis developed in the previous section. This will show scaling of the sheath heating rate and allow us to place some qualitative limits on discharge operation.

In order to find the net energy gain per unit flux of a known electron energy distribution function, the distribution must be convolved with the energy dependent energy gain per electron. Here we consider the convolution of Maxwellian electron energy distributions having varying electron temperatures for the case of a 1 cm maximum sheath width and maximum sheath voltages $V_0 = 250, 500$, and 1000 V, as shown in Fig. 9. In the low-temperature limit, the net energy gain (gains through sheath heating minus losses due to electron escape) per unit flux for a fixed sheath thickness approaches Goedde's estimate[10] of $2ms_0^2\omega^2$, regardless of the sheath voltage. The energy gain drops with increasing electron temperature until a temperature is reached above which there is a net loss of energy from the plasma at the sheath rather than a net gain. This loss is due to the increasing proportion of escaping energetic electrons at higher temperatures, and is more substantial at lower voltages, when the escape barrier is lower. This has an extremely important implication. For any set of sheath conditions there is a maximum electron temperature determined by the sheath velocity above which sheath heating will cease to contribute to net power input into the discharge.

Macroscopic discharge models often use a particle balance argument to, at least qualitatively, successfully predict electron temperature scaling with gas pressure. By equating electron production rate with electron loss rate to the walls, the temperature of a single Maxwellian distribution is uniquely determined, making the above additional condition on the electron temperature appear to be an overconstraint on the problem. However, if the distribution is other than a single Maxwellian,[9,14] the particle balance by itself is no longer sufficient, and electron dynamics at the sheath are likely to play a role in establishing as equilibrium electron energy distribution.

We would like to consider the effect of varying plasma conditions on the sheath heating process. The present model is not, however, a self consistent model. The values of n_0 and T_e in the discharge will be sensitive not only to the sheath heating rate, but also to other external parameters, including pressure and geometrical factors, the latter of which will vary greatly from experiment to experiment (making a *direct* comparison of scaling of power with quantities such as discharge voltage questionable). In order to examine the sensitivity of sheath heating rate to various discharge parameters, and to decouple that sensitivity from these other factors, in the following cases, n_0, T_e, and V_0 are specified as input parameters. The sheath thickness s_0 is computed using these parameters and the Child–Langmuir law, assuming the ions enter the sheath edge at the Bohm velocity, u_B,

$$s_0^2 = \frac{4\epsilon_0}{9}\left(\frac{2e}{M}\right)^{1/2}\frac{|V_0|^{3/2}}{J}. \quad (17)$$

The current density is

$$J = \frac{n_{sh}}{2}u_B = \frac{n_{sh}}{2}\sqrt{\frac{T_e}{M}}, \quad (18)$$

where n_{sh} is the plasma density at the sheath edge (typically $n_{sh} \approx 0.6n_0$) and M is the mass of the ion, which in this case we have chosen to be argon.

We assume that the flux of electrons to the sheath, Γ_e, is proportional to $n_0\sqrt{T_e}$, so that power scaling can be determined by computing the product of the energy gain with the flux. Now a plot (Fig. 10) of heating power shows significant variation with sheath potential, through the associated change in sheath width at fixed discharge density. In this crude model of the sheath thickness, the scaling of s_0^2 with n_0 and T_e is exactly the inverse of the scaling of the electron flux to the sheath with these parameters. Thus, in taking the product of Γ_e and $\langle\Delta E_e\rangle$ to determine the total heating rate, the direct dependence on n_0 and T_e drops out at low temperatures. However, an indirect dependence on density comes out at higher temperatures, as illustrated in Fig. 11. Here we plot net heating rate versus discharge voltage for a fixed electron temperature of $T_e = 5$ eV, and several values of electron density. The heating rate here is almost independent of electron density up to $n_0 = 10^9$

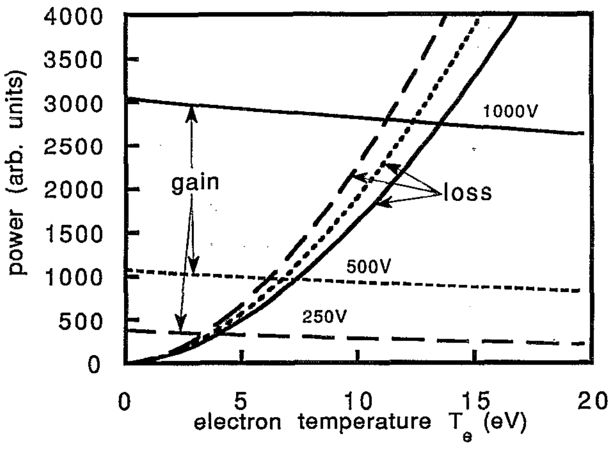

FIG. 10. Total heating rate vs electron temperature using the analytic model for sheath voltages $V_0 = 250$ V (—), 500 V (---), and 1000 V (— —). Electron density is $n_0 = 3 \times 10^9$ cm^{-3} and sheath thickness s_0 is found from the Child–Langmuir law. As in Fig. 8, heating gain and escape loss are plotted separately.

cm^{-3}, but then begins to drop with electron density for a given sheath voltage. This happens because even though changes in heating rate due to change in sheath thickness are nominally cancelled out by a concommitant change in flux, the sheath thickness (and therefore sheath velocity) becomes low enough that losses due to electron escape begin to dominate the process. This threshold occurs when the maximum sheath speed $s_0\omega$ becomes of the order of the electron thermal velocity. Thus we find that independent of other external aspects of discharge geometry, there is a limit on the electron density which can be supported (for a given sheath voltage) through the sheath heating process.

D. Discussion

Although the qualitative results of the previous sections are sound, the accuracy of this model in predicting actual heating rates precisely may be limited in some cases by simplifying assumptions used in its development. The following considerations may be important in comparison between the model and experiment.

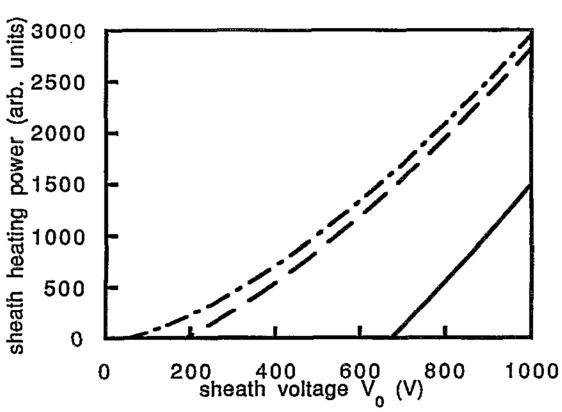

FIG. 11. Total heating rate for $T_e = 5$ eV vs sheath voltage for electron densities $n_0 = 10^8$ (———), 10^9 (— —), and 10^{10} cm^{-3} (—).

Simulations[3,14] and analytic models of rf sheaths[11,12] show the motion of the sheath edge not to be purely sinusoidal. This will have two effects: (1) the maximum sheath velocity may be slightly higher than that used here, which will lead to a greater energy gain for slower electrons, and (2) the sheath probably spends a smaller fraction of its time near the collapsed position than we have allowed for, so fewer electrons actually escape from the system. Perhaps more importantly, the sheath probably does not collapse completely, again improving confinement.[15] Further, sheath heating of electrons born in the sheath will amplify the power input.

The phase θ, at which the electrons pass into the sheath region at $x = s_{max}$, is not truly a random variable. There is an oscillating presheath which will sweep electrons into the sheath region preferentially at certain phases of the cycle.[3] This also makes it difficult to accurately estimate the electron flux (both the total and the distribution in velocity) at the sheath edge for a given bulk density.

Other heating mechanisms including bulk heating and plasma instabilities may (depending on discharge conditions) play an important role in the overall power balance. Sheath heating at the grounded electrode may also be relevant, depending on its area relative to the powered electrode.

Probably the greatest difficulty is in obtaining an accurate estimate of the time-averaged sheath thickness. The energy gain per unit flux is sensitive to the sheath thickness, varying as a power greater than 2. Use of the Child–Langmuir Law gives very good agreement with some reported self-consistent simulations of rf sheaths,[14] but not with others,[3] and use of more sophisticated models of the rf sheath[16] may be required. In addition, the more collisional the sheath (higher pressures), the less justified its use will be.

Last, the assumed spatial variation of the electric field may in some limits affect the accuracy of the heating rate prediction. If the interaction time is insignificant compared to the rf period, then the energy gained or lost by an electron depends only on the instantaneous velocity of the sheath edge. In that case, the magnitude and spatial dependence of the electric field in the sheath is important only in determining that the amount of time spent by the electron in the field is short. For the typical discharge parameters considered in this study, the interaction time is short and penetration depth is shallow, yet significant enough to require the correction mentioned above. We chose an electric field which varies linearly in space (in agreement with results reported for self-consistent numerical simulations of rf discharges) to estimate the magnitude of this correction. Since the penetration depth into the sheath is shallow for most electrons, only the electric field at the sheath edge is important. In light of the limitations mentioned above, higher-order improvement of this correction, which would require a better model of the electric field at the sheath edge (arguably the most difficult region to model), is not warranted. The associated error will increase with whatever parameter changes increase the electron residence time in the sheath field relative to the rf period, including reduction of the sheath voltage and increase of the rf driving frequency.

IV. CONCLUSIONS

We have developed a velocity and phase dependent mapping for interaction of the electron velocity distribution with an oscillating sheath. This mapping is generated by exact calculation of electron trajectories in a model sheath field. A "hard wall" approximation of the interaction gives an analytic expression for the time-averaged energy gain as a function of incoming electron energy. This analytic expression agrees extremely well with the mapping when a correction is added to account for the change in sheath velocity during the finite duration of the interaction.

The mapping shows significantly different results for electrons with incoming velocities slower as opposed to faster than the maximum velocity of the sheath edge. The slower ones are accelerated by the sheath regardless of phase ϕ, and typically reenter the plasma with speed close to $2s_0\omega$. Energetic electrons may gain energy, lose energy, or escape to the electrode. Those which do gain energy pick up more energy in a single interaction than a corresponding slower electron. Therefore, although there is a net loss of hotter electrons averaged over time, some fraction of hot electrons will be elevated above the ionization threshold in this process, and may play a very important role in discharge maintenance.

Our model assumes a sinusoidal oscillation of the sheath thickness with an average value of s_0 and a minimum value of zero. The results on the scaling of the sheath heating rate based on this model are summarized as follows: Changing the sheath voltage in and of itself has a relatively minor, but nonnegligible effect on the overall heating rate. This role comes not in determining the boost in energy for electrons heated by the sheath, but in setting the height of the potential barrier they must overcome to reach the electrode. However, the width of the sheath is strongly dependent on the voltage through ion sheath dynamics, and the heating rate is very sensitive to the width. Therefore, the primary role of the sheath potential is an indirect one which cannot be decoupled from other plasma parameters. Higher rf voltages mean thicker sheaths, and that means greater power input, and a higher cutoff energy for loss from the system. The equilibrium electron energy distribution will be established through a balance of this and other energy input mechanisms with loss processes.

If sheath voltage and sheath thickness were held fixed, power input would increase almost with the square of the driving frequency. There are two reasons for this: first, higher frequency means higher sheath velocity and therefore a greater energy gain for electrons which return to the plasma, and second, the cutoff energy for escape from the system is increased.

The electron temperature and density play an important role in the sheath heating process in establishing the sheath thickness. When conditions are such that the electron thermal velocity is of the order of the maximum sheath velocity, further increases in n_0 or T_e result in significant increases in energy loss due to electron escape to the electrode. To the extent that we require a net power input through electron-sheath interaction, this places upper bounds on n_0 and T_e, above which sheath heating cannot maintain the discharge.

ACKNOWLEDGMENT

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