

NOTES

Newtonian Liquid Hole Pressures at Small Reynolds Numbers

We consider shear flow of a liquid between parallel walls (at separation h). One wall contains a hole whose cross section can be either a circle (of diameter b) or a transverse slot (of narrow dimension b parallel to the main flow direction). There is no net flow through the hole. A transducer at the base of the hole records a pressure P_2 . A transducer mounted flush with the wall face opposite the hole records a pressure P_1 at a location on the hole centerline (produced). Current research interest concerns the relation between the "hole pressure" $P^* = P_1 - P_2$ and N_1 and N_2 , the first and second normal stress differences in steady shear flow.

For Newtonian liquids, $N_1 = N_2 = 0$, and negative values of P^* are generated by inertial forces. These values are of interest (a) for testing apparatus designed to measure P^* and (b) for subtracting inertial contributions to P^* values obtained for non-Newtonian liquids.

Table I summarizes values for P^* for values of Reynolds number $Re = \rho b h \dot{s}^2 / 4\sigma$ less than 10 (ρ = density; \dot{s} and σ denote wall values of shear rate and shear stress at locations remote from the hole perturbation). At these low Reynolds numbers, P^* is approximately proportional to the product $Re\sigma$. As Re increases, computations for transverse slots⁷ and measurements for circular holes⁴ show that the ratio $-P^*/\sigma$ reaches maximum values less than 1 near $Re = 100$. For comparison, Table I also includes results of Thom and Apelt¹ for a slightly different but related flow—past two opposite identical

TABLE I
 Values of Hole Pressure P^* Generated by Newtonian Liquids (of Density ρ and Viscosity η) Flowing Past Holes of Depth Greater Than Their Width b at Low Values of Reynolds Number $Re = \rho b h \dot{s} / 4 \eta^a$

$P^*/(\sigma Re)$	Hole shape	Re	b/h	Method	Year	Reference
-0.02	Two opposite transverse slots	≤ 1	0.5	Calculated	1958	Thom and Apelt ¹
+17 to +437	Circular	≤ 0.1	1	Measured with Indopol L-50	1973	Han and Kim ²
-0.010	Circular	≤ 3	1.85	Measured with glycerine	1976	Baird ³
-0.024	Circular	≤ 10	1	Measured with diethyl and dioctyl phthalate, and oil base stock S100N	1980	Tong ⁴
-0.043	Transverse slot	≤ 4	0.4	Calculated	1981	Richards and Townsend ⁵
-0.033 ₃	Transverse slot	≤ 4	1	Calculated	1981	Crochet ⁶
-0.033 ₄	Transverse slot	$\leq 2b/h$	≤ 1	Calculated	1982	Jackson and Finlayson ⁷
-0.031	Transverse slot	1	0.5	Calculated	1982	Malkus ⁸

^a σ and \dot{s} denote wall values of shear stress and shear rate unperturbed by the holes.

transverse slots; here, P_1 is taken as the value at the centerline intersection with the plane of symmetry parallel to the two die walls.

It is clear from Table I that, with respect to sign and order of magnitude, all results except those of Han and Kim² are alike. A closer quantitative comparison between calculated and measured values can hardly be made at present because of the differences in hole cross-sectional shape. It is not unreasonable, however, to expect that, for a given value of b/h , the magnitude of $P^*/\sigma Re$ would be smaller for a circular hole than for a transverse slot. This expectation is borne out by the results which therefore give confidence in the validity of the measurements of Baird³ and Tong,⁴ at least as to order of magnitude and sign. The difference between their values could perhaps be due to the difference in their values of b/h .

Thus, for circular holes with $b = h$ and for $Re \leq 10$, we can have confidence in the approximate result $P^*/(\sigma Re) = -0.02$. The values measured by Han and Kim² differ from this value by factors ranging from -727 to $-18,200$. This suggests that their apparatus was influenced by some systematic source of error of which they were not aware. One possible source of error is leakage of test liquid past the hole-mounted transducers. It can be seen that this would explain the anomalous features of their data: the sign of the hole pressure, its dependence on hole diameter and location along the die, and independence of viscosity. Because the hole pressures are in any case only a very small fraction of local pressures, only a very small leakage (a few drops per hour⁴) would suffice to account for the size of their Newtonian anomaly.

Such a leakage would also affect their hole pressures measured with polymer solutions. Direct tests for leakage are easy to perform by closing the input end of the die and pressurizing the output end (or vice versa); leakage would then show up in values of hole pressure, so that a quantitative assessment, and, if necessary, a correction can be made. Han and Kim refer to no such tests, nor to the possibility of leakage errors. It is clear that, until the explanation of their anomalous results is published, one can have no confidence in any conclusions drawn from their paper about the validity of methods of using hole pressures (leak free) to determine normal stress differences. One important feature of one such method is that the equations used predict that the elastic contribution to the hole pressure should be

independent of hole size. Han⁹ recently has referred once again to the hole-diameter dependence of his polymer solution hole pressures as evidence that conflicts with such predictions, but he does not mention the large Newtonian liquid anomaly, described above, that was obtained with the same apparatus. Table I suggests that the evidence offered for this conflict should be re-examined critically.

More detailed discussions of hole pressures for Newtonian and non-Newtonian liquids, together with analysis of possible measurement errors, have been published elsewhere.^{4,10,11} The possibility of subtracting inertial contributions from measured hole pressures in order to give elastic contributions will be discussed in a future publication.

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