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MINNESOTA STATE MATH STANDARDS FOR GRADE 2 NUMBERS AND  
OPERATIONS: ALIGNMENT AND TESTING**

# **Montessori Math Manipulatives and the Minnesota State Math Standards for Grade 2 Numbers and Operations: Alignment and Testing**

An Action Research Report  
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**Abstract**

The Montessori methodology is becoming more prevalent in public elementary schools. Within a public school, there are many standard requirements that need to be met for students at each grade level. Today, many students are being tested to assess if they are learning what is required by the State Standards. Because the Montessori methodology teaches students with differentiated and individualized instruction, many wonder if the Montessori methodology covers all State Standard requirements. Does the Montessori Math Curriculum meet the Minnesota State Math Standard requirements? This qualitative study was performed in a public Montessori school with nine-second grade students learning mathematical skills that aligned with the Minnesota State Math Standard. Instruction took place through the use of Montessori manipulatives. The assessments given to the students were the same assessments that other second grade students in this district were given for this standard. Students were given Montessori lessons and manipulatives aligned with the Minnesota State Math Standard: Numbers and Operations for Grade 2. In the example of the learning target “I can use fact families to understand addition and subtraction,” students pre-test score was 59.3% and increased to 77.7% in the post-instruction test. We can show if the Montessori methodology teaches students what is required by the State Standards, by further aligning Montessori lessons and use of manipulatives to those standards, while using assessments that traditional school use.

*Keywords:* Montessori, mathematics, manipulative, State standard, alignment

## **Literature Review**

### **Introduction**

In my current school district, we have been focusing on aligning the Montessori math curriculum to the Minnesota State Math Standards for 1st, 2nd, and 3rd grade. This has been very interesting to me because I want to know if the Montessori math curriculum meets the requirements for the Minnesota State Math Standards. I personally feel that the Montessori math curriculum covers many of the requirements for the Minnesota State Math Standards for those grade levels. I have chosen to do an action research project on alignment and testing of the Montessori math curriculum to the Minnesota State Math Standards for Grade 2, and more specifically, to the first strand of those standards, which addresses Numbers and Operations.

The Montessori math curriculum is based on the importance and use of specifically designed manipulatives. Children are introduced to the foundation of the Montessori math manipulatives in Children's House and use them throughout their experience in a lower elementary (E1) classroom. As children learn and discover more in their E1 classroom they are introduced to other Montessori math manipulatives that help support their understanding of math concepts. My research has underlined the importance of using manipulatives to increase understanding when teaching mathematics.

### **Manipulatives in Mathematics**

There have been many discussions about the importance of using manipulatives when teaching mathematics and whether they support or hinder it (Laski, Jor'dan,

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Daoust, and Murray, 2015). According to Laski, Jor'dan, Daoust, and Murray (2015), manipulatives may benefit learning but only under certain conditions. A manipulative should: 1) be used over a consistent period of time, 2) start with concrete representations and move to abstract representations, 3) avoid resemblance of everyday objects, 4) teachers need to explain the relation of the math concept being taught to the manipulative (Laski, Jor'dan, Daoust, and Murray, 2015).

DeGeorge and Santoro (2004, p.28) found that, "Students are better able to visualize math concepts and gain insights into necessary fundamentals when they use rods, cubes, and other tools." There is a wide range of math manipulatives students can use when learning math concepts, but when students are able to actively use concrete manipulatives it can help deepen their understanding of a math concept.

Johnson (2015) found that using manipulatives in the right way can help students gain a more abstract understanding of a math concept. In an action research study, Johnson (2015) had both students that were labeled as abstract learners, who also enjoyed using manipulatives to learn concepts, and concrete learners who needed more guidance when using manipulatives. The concrete learners made bigger gains after they used the manipulatives for a math concept.

In a Montessori Children's House classroom - which has students aged 3-6 – children are introduced to many concrete math manipulatives, which they will continue to use throughout their years in elementary. An example is the Golden Bead manipulative, which is a concrete representation of the base-10 number system. Children's House students are first introduced to quantity and numerals with the Golden Beads and

continue their practice with base-10 and operations in an E1 classroom (Laski, Jor'dan, Daoust, and Murray, 2015).

Laski, Jor'dan, Daoust, and Murray (2015) found that using the same or similar manipulatives to repeatedly solve problems leads to a deeper understanding of the relation between the physical math manipulative and the abstract concept. It is very helpful to children to be allowed to use the same manipulative to support their understanding of more complex mathematical concepts.

In a Montessori classroom, children are taught the different operations with the same manipulatives: Golden Beads, Stamp Game and Bead Frames to reinforce their understanding. Throughout the progression of the use of each manipulative, as well as the progression from one manipulative to the next, and via a spiral sequential approach, the math concepts become more abstract, leading the children to a more abstract understanding. Children will learn how to use the math manipulatives starting with addition, then learn how to use the same manipulative for multiplication, subtraction, and then division. The same concepts and progression are then revisited via the use of a slightly more abstract manipulative. The Montessori Method repeatedly uses the same manipulatives when teaching different math concepts.

### **Montessori Math Manipulatives**

Maria Montessori designed her manipulatives after scientifically observing children. She interpreted her observations, and created materials she thought would suit the developmental needs of the observed children. She then observed the children's interaction with these materials, and modified them until she thought she had a material

that would meet one or more of the specific needs (Lillard, 2008). The Montessori manipulatives have a specific design and purpose for meeting the various developmental needs of children.

The Montessori math materials are a great representation of manipulatives that do not represent real objects (Woessner, 1995). These materials isolate the learning target by keeping all parameters other than the targeted one constant. They also utilize a color coding system that remains consistent throughout all progressive steps of concrete and representational stages of instruction. For example, each colored bead bar is of a certain color which correlates to the amount of beads on each bead bar. The 1-bead bar is red, the 2-bead bar is green, the 3-bead bar is pink, and so on. The color of the colored bead bars remains the same for the beads on the Bead Cabinet. Children use the colored bead bars and the Bead Cabinet to explore similar and different math concepts.

Another example is using the colored bead bars for multiplication with the checkerboard and then using the similar colored bead bars from the Bead Cabinet for skip counting. The Montessori materials are a wonderful example of manipulatives that do not resemble real objects and are unified throughout the curriculum. This supports children's understanding of the math concepts because they are already familiar with the manipulative when they begin using it for different math concepts.

The numeral cards are a Montessori manipulative that supports children in representing numbers. Children can easily represent a number from 0-9,999 using the Montessori numeral cards (Gough, 2008). The numeral cards can be used for many different math concepts.

Montessori math manipulatives are designed with a precise purpose and meaning to help support children's focus and attention to their learning. The length of time children are allowed to use a math manipulative will affect their learning and understanding of the concept (Woessner, 1995). Woessner (1995, p. 40) found that, "using a manipulative helps establish a basic understanding of the math concept that in turn promotes deeper insights into how the material relates to the concept that in turn leads to better understanding of the concept."

### **Montessori method compared to Traditional method**

A Montessori classroom should have a prepared environment of multi-sensory manipulatives in a specific sequence that will both interest and support children's learning (Cohen, 1989). The method is based on hands-on learning with the use of many different types of manipulatives for mathematics, geometry, language, practical life skills, culture and botany/zoology.

Children are given lessons in small groups or individually, based on their developmental stages (Cohen, 1989). A Montessori classroom has multi-ages in the classroom. For example, in the lower elementary, children from grades 1, 2, and 3 are together. Children receive lessons by their developmental stage rather than their grade level so it is possible that a lesson may have children from 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> grade grouped together in a flexible group.

The Montessori method is designed to encourage children to independently choose work which they find interesting and is at their developmental level. Montessori teachers observe their students to identify their level of academic readiness. They assess

that readiness through a system that supports each individual learner (Reed, 2008). The teacher determines what lessons to give based on the child's overall development.

Montessori teachers "follow the child" through his developmental stages.

Children in a Montessori setting receive lessons that are developmentally appropriate for their level, and are then given time to work freely on other subject areas of their choosing. During a "work period" children are free to repeat a work as often as they would like to. Repetitive use of the materials is key in children learning and understanding mathematics (McNamara, 1994). Parents and teachers should not be concerned with moving children to abstraction quickly (McNamara, 1994).

Ryniker, Shoho (2001, p.45) found that, "The Montessori approach to education is sensory based and child centered, meaning that children learn at their own rate through the manipulation of materials created by Dr. Maria Montessori to help internalize concepts rather than just learn facts." Children in a Montessori classroom are able to learn and work at their own pace rather than wait for a teacher to direct them (Ryniker, Shoho, 2001).

Maria Montessori created manipulatives, more commonly known as materials in a Montessori classroom, to help children develop a deep understanding of mathematics (Bauch & Hsu, 1988). The manipulatives were specifically designed to support children's multi-sensory development. An example of this are the sandpaper numerals (Bauch & Hsu, 1988).

Montessori believed that children first learn and understand math concepts through concrete representation and then abstraction. The manipulatives first introduced

to children are very concrete and as more manipulatives are introduced they slowly become more abstract.

An example of this concept is the progression from the use of the Golden Beads to the Stamp Game when teaching the math operations. The Golden Beads are a very concrete representation of the decimal system. A unit of 1 is represented by a single bead, a 10 is represented by 10 beads connected into a bar, a 100 is represented by ten bars connected into a square, and a 1000 is represented by ten 100 squares, connected into a cube. When children transition to the Stamp Game the units, tens, hundreds, and thousands are all represented by a small wooden square tile of identical size. The unit tile is green, the ten tile is blue, the hundred tile is red, and the thousand tile is also green, as it represents a unit of thousands.

Lopata, C., Wallace, N. V., & Finn, K. V. (2005) found that Montessori programs help children develop attention, organization/order, written language skills, fine and gross motor skills and mathematical skills. Children learn self-control, self-direction, spontaneity and creativity through their independent practices (Lopata, C., Wallace, N. V., & Finn, K. V., 2005).

Traditional schools have single grade classrooms. Children are given whole class lessons directed by the teacher. During these lessons, all children are working on the same unit, regardless if they are ready for it or not. Children are introduced to a math concept, practice it for a couple of weeks and then move on to a different concept. To the contrary, Montessori schools use a three year cycle model, with children attending the same classroom from ages 3-6, 6-9, 9-12, 12-15, and 15-18.

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Children in a traditional school typically do not have the same access to concrete manipulatives as children in a Montessori classroom. In some instances, concrete manipulatives may be used in a traditional classroom, but they are put away after the particular math or other concept is taught and practiced for a short period of time. In a Montessori classroom, the manipulatives remain available for the children throughout their three year attendance in their multiage classroom. In a traditional school, children tend to work with textbooks, workbooks and worksheets solving by abstraction rather than with concrete manipulatives (Woessner, 1995).

**Conclusion**

The literature I have researched supports the use of manipulatives when teaching math concepts to children. However, it is important that children are given a long period of time to learn how to use and practice with one type of manipulative to really support their understanding. Children should also be using manipulatives that don't resemble real life objects.

A Montessori classroom better supports children to learn at their developmental level versus the grade level they are in, because of the multi-age classroom, differentiation and individualization of instruction, and wide range of manipulatives available to children. Children are given more opportunities in a Montessori classroom to practice math concepts with the same manipulatives over long periods of time. This supports their understanding as they move from concrete to abstraction.

## **Research Design and Methodology**

### **Purpose**

The purpose of this action research study is to examine the Montessori math curriculum and compare it to the Minnesota State Math Standards. In our district we have been focusing on aligning the Montessori math curriculum to the Minnesota State Math Standards. There are many lessons within the Montessori math curriculum that cover many of the requirements of the Minnesota State Standards for Mathematics but there are also some areas that the Montessori math curriculum does not cover, or it covers at an earlier or later time than traditional schools. Through alignment and district testing, we will be able to prove if students made gains in their test scores by practicing with Montessori math manipulatives.

### **Central Question**

Does the Montessori math curriculum meet the Minnesota State Math Standards for Grade 2: Numbers and Operations?

### **Topical Question**

- 1) Do the Montessori math manipulatives help support children's understanding of math concepts?
- 2) Are there gaps in the Montessori math curriculum compared to the Minnesota State Math Standards?

- 3) How can children represent mathematics while using the Montessori math manipulatives?

### **Participants**

Nine-second grade students participated in this study. Two of them receive English Language Learner services. Students were enrolled in an urban public Montessori elementary school in Minnesota.

### **Setting**

The study took place in my E1 classroom, in an urban public Montessori elementary school. Students are ages 6 – 9. The classroom has a full complement of Montessori manipulatives that support students' understanding of mathematics.

### **Materials**

- The Golden Beads
- Numeral Cards
- Stamp Game
- Bead Frame
- Subtraction Snake Game
- Geometry Cabinet: Equilateral Triangle
- Numeral and Operation Cards
- Addition and Subtraction Word Problems
- Student Math Notebooks

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- Fact Family practice page
- District Math Assessments

**Procedure**

The focus of this action research project is aligning and testing the Montessori math curriculum to the Minnesota State Math Standards for Grade 2: Numbers and Operations. Within the Strand Numbers and Operations, there are nine High Priority Benchmarks. The benchmarks I focused on are:

- 1) 2.1.1.1. a – Learning Target: I can read a number from 0-1000.
- 2) 2.1.1.1. b – Learning Target : I can write a number from 0-1000.
- 3) 2.1.1.1. c – Learning Target: I can represent a number from 0-1000 in a variety of ways.
- 4) 2.1.1.1. d – Learning Target: I can describe whole numbers 10-1000 in terms of ones, tens, and hundreds.
- 5) 2.1.2.2. a – Learning Target: I can use fact families to understand addition and subtraction.
- 6) 2.1.2.5. a – Learning Target: I can solve one and two digit addition problems.
- 7) 2.1.2.5. b – Learning Target: I can solve one and two digit subtraction problems.
- 8) 2.1.2.5. c - Learning Target: I can solve real world addition problems with one and two digit numbers.
- 9) 2.1.2.5. d – Learning Target: I can solve real world subtraction problems with one and two digit numbers.

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For each of the learning targets, Montessori lessons and manipulatives were aligned to the skills that were being taught. The teaching manuals being used for alignment were from *Montessori Research and Development*. Each lesson presented from the Montessori manuals has direct and indirect aims. Each learning target was aligned to the aims of Montessori lessons. If the learning target did not align with any Montessori lessons, this was either labeled as a gap in the Montessori math curriculum or a Montessori manipulative was used as a supplement to teach the learning target. Before each benchmark was taught, students were given a pre-test provided by the school district. The students were then given a Montessori lesson and manipulatives to practice the presented benchmark. After a week of practice, students were given a post-test provided by the school district to examine if students mastered the benchmarks.

**2.1.1.1. a – Learning Target: I can read a number from 0-1000**

Students must be able to read aloud a two digit number made out of base-10 blocks along with reading aloud the following: 1- two digit number, 4- three digit numbers, 1- four digit number, 1- five digit number and 1-six digit number.

Montessori manipulatives used and available for practice in this Montessori classroom were:

**Golden Beads** (See Appendix A)

Direct Aims:

- Development of order, concentration, coordination, independence and exactness.
- Putting together of quantities to produce a larger quantity.
- “Exchanging” or “regrouping” numbers.
- Preparation for dynamic addition and multiplication.

**Formation of Quantity in Stamp Game** (See Appendix B)

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- Composition of quantities using the stamp material.

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- Preparation for the composition of stamps for use in operations.

**Numerical Cards** (See Appendix C)

**Results:**

**I can read a number from 0-1000.** This assessment was only given once at the end of the research project. The assessment is given three times during the year and the third time was during this research project. Students were able to read two, three and four digit numbers.

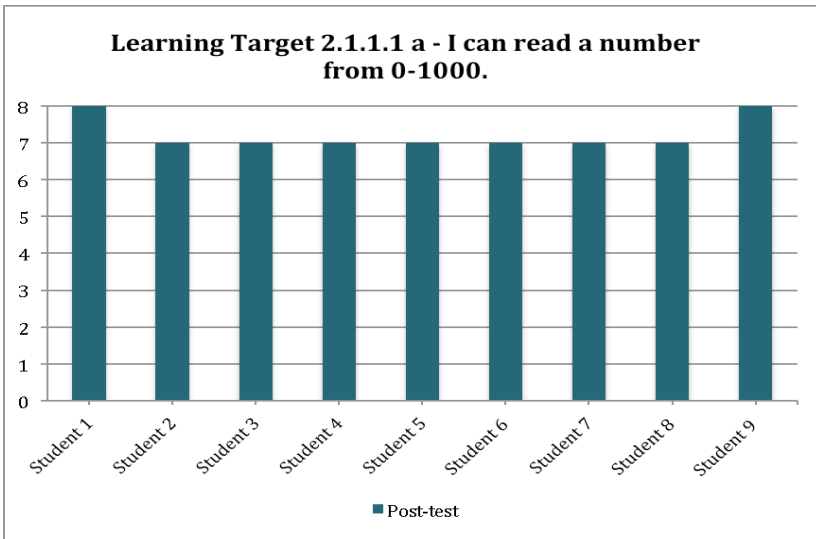


Figure 1: Results of Post-test Learning Target 2.1.1.1 a – I can read a number 0-1000.

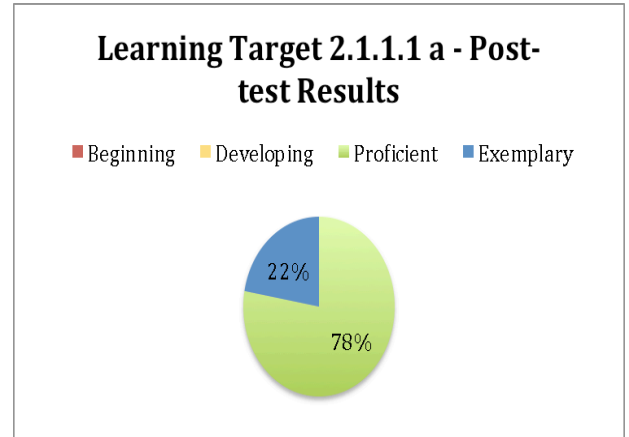


Figure 2: Results of Post-test Learning Target 2.1.1.1. a – I can read a number 0-1000.

**Impact of these results:**

*Based on the assessments given, few changes need to occur. All students were able to read a number from 0-1000 either at grade level or above grade level as shown in figure 2. This is an area of strength in our Montessori setting as students are introduced*

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to building and reading three and four-digit numbers early on in their Children's House classroom (ages 4-6) which is then continued in their E1 classroom.

**2.1.1.1. b – Learning Target : I can write a number from 0-1000.**

Students must be able to fill in one missing number from a two-digit number line, read and then write a three-digit number, record a three-digit number their teacher says to them and record a five-digit number their teacher says to them.

Montessori lessons and manipulatives used and available for practice in this Montessori classroom were:

**Golden Beads (See Appendix A)**

Direct Aims:

- Development of order, concentration, coordination, independence and exactness.
- Putting together of quantities to produce a larger quantity.
- “Exchanging” or “regrouping” numbers.
- Preparation for dynamic addition and multiplication.

**Formation of Quantity in Stamp Game (See Appendix B)**

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- Composition of quantities using the stamp material.

Indirect Aim:

- Preparation for the composition of stamps for use in operations.

**Formation of Number with the Bead Frame (See Appendix D)**

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- To constantly bring to the child's consciousness the function of position, i.e., place value.

**Numeral Cards (See Appendix C)**

**Results:**

**I can write a number from 0-1000.** In this assessment the lowest score a student could get was 0 and the highest score 4. The results for this assessment show that 89% of students were exemplary and 11% of students were proficient at this skill.

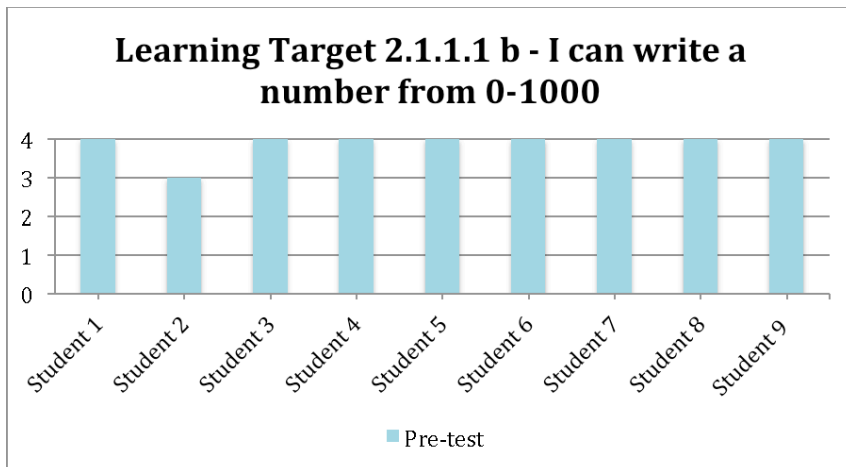


Figure 3: Comparison of Pre-test scores. Learning Target 2.1.1.1 b: I can write a number from 0 – 1000.

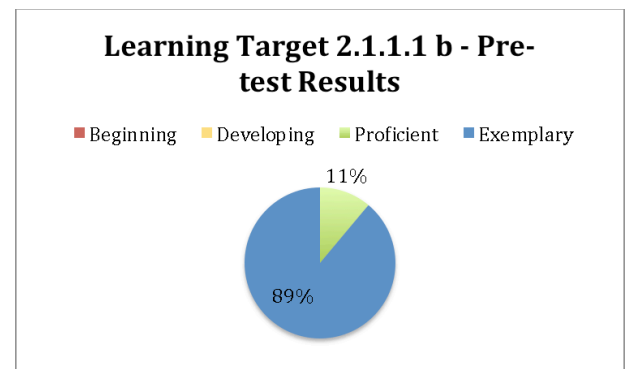


Figure 4: Results of Pre-test assessment – I can write a number from 0-1000.

**Impact of these results:**

Based on the assessments given, not many changes need to occur. The assessments in figure 4 shows that 89% of students tested at the exemplary level. The skill is effectively being taught in this Montessori setting. Through Montessori lessons and use of manipulatives, students are meeting the standard requirements for this skill. The results prove that students in this Montessori setting are learning how to write a number from 0-1000 through the use of Montessori mathematics manipulatives. The manipulatives and repetitive use of them are helping students master the skill of writing numbers from 0-1000.

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**2.1.1.1. c – Learning Target: I can represent a number from 0-1000 in a variety of ways.**

Students must represent a three-digit number in two different ways, write the number of an amount of tally marks, write a four-digit number in expanded form and write a number that is described by their place value. *An example is, 2 in the ones place, 5 in the tens place, 1 in the hundreds place.* (See Appendix)

Montessori lessons and manipulatives that were used and available for practice in this Montessori setting were:

**Golden Beads** (See Appendix A)

Direct Aims:

- Development of order, concentration, coordination, independence and exactness.
- Putting together of quantities to produce a larger quantity.
- “Exchanging” or “regrouping” numbers.
- Preparation for dynamic addition and multiplication.

**Formation of Quantity in Stamp Game** (See Appendix B)

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- Composition quantities using the stamp material.

Indirect Aim:

- Preparation for the composition of stamps for use in operations.

**Formation of Number with the Bead Frame** (See Appendix D)

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- To constantly bring to the child’s consciousness the function of position, i.e., place value.

**Numeral Cards** (See Appendix C)

**Results:**

**I can represent a number from 0-1000 in a variety of ways.** In this assessment the lowest possible score was a 0 and the highest possible score was a 4. The majority of students made 1 - 2 point gains from their pre-test to post-test.

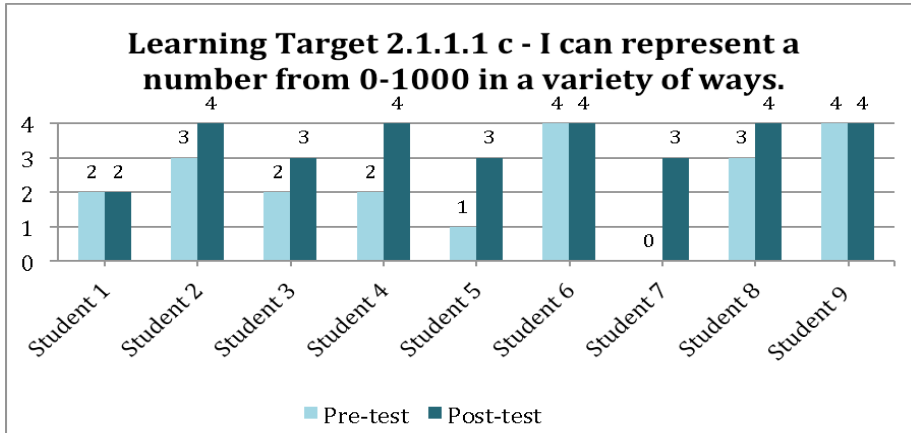


Figure 5: Learning Target 2.1.1.1 c – I can represent a number from 0-1000 in a variety of ways.

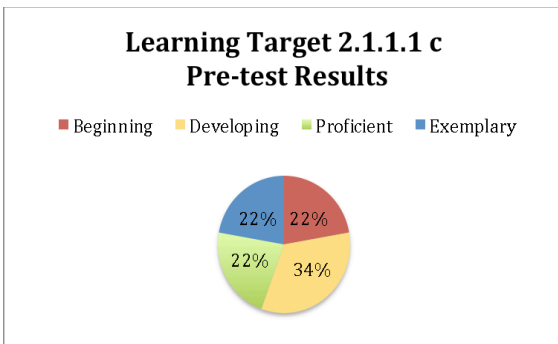


Figure 6: Results of the Pre-test assessment – I can represent a number from 9-1000 in a variety of ways.

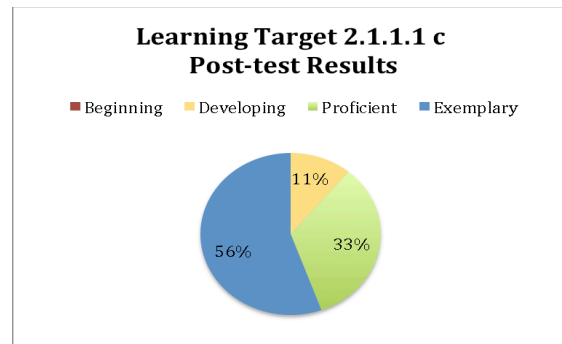


Figure 7: Results of the Post-test assessment – I can represent a number from 0-1000 in a variety of ways.

**Impact of results:**

Based on the assessments given, some changes need to occur. The results show in figure 7 that students can still make improvement. Figure 7 shows that 22% of students tested at exemplary in the pre-test and increased to 56% exemplary which is above grade level. Post-tests results show that 33% of students were proficient which is at grade level and one student = 11% were developing at the skill of representing a number from 0-1000 in a variety of ways. Students used a variety of Montessori manipulatives when practicing different ways to represent a number from 0-1000 in this Montessori setting.

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After reviewing the results compared to the assessments, the skill students need more practice with is learning how to draw and represent a number on paper without the Montessori math manipulatives.

**2.1.1.1 d – Learning Target: I can describe whole numbers 10-1000 in terms of ones, tens, and hundreds.**

Students must write the value of a number in the tens place value, draw a three-digit number with base-10 blocks (students substituted base-10 blocks for the Golden Beads, a Montessori manipulative), write a three-digit number in expanded notation and write a five-digit number in expanded notation.

Montessori lessons and manipulatives that were used and available for practice in this Montessori setting were:

**Golden Bead** (See Appendix A)

Direct Aims:

- Development of order, concentration, coordination, independence and exactness.
- Putting together of quantities to produce a larger quantity.
- “Exchanging” or “regrouping” numbers.
- Preparation for dynamic addition and multiplication.

**Formation of Quantity in Stamp Game** (See Appendix B)

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- Composition quantities using the stamp material.

Indirect Aim:

- Preparation for the composition of stamps for use in operations.

**Formation of Number with the Bead Frame** (See Appendix D)

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- To constantly bring to the child’s consciousness the function of position, i.e., place value.

**Numeral Cards** (See Appendix C)

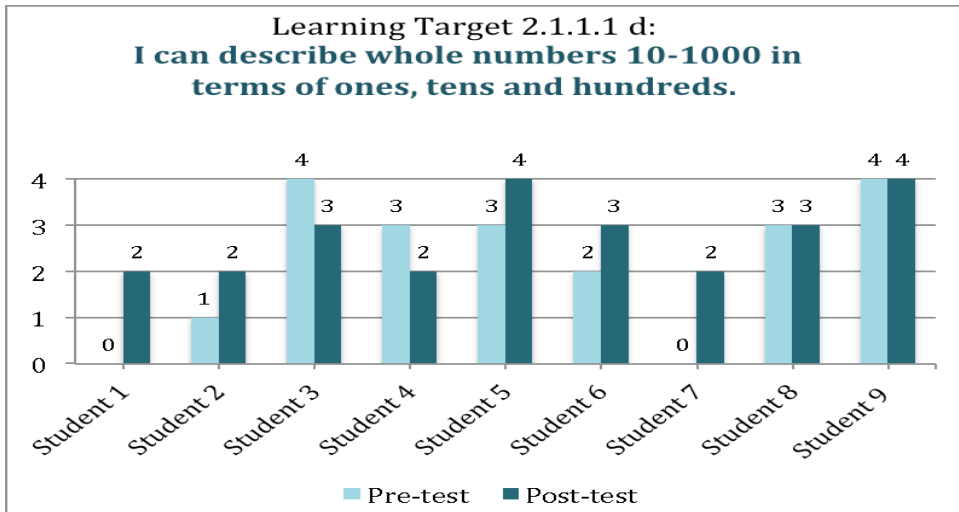
**Results:**

**I can describe whole numbers 10-1000 in terms of ones, tens and hundreds.**

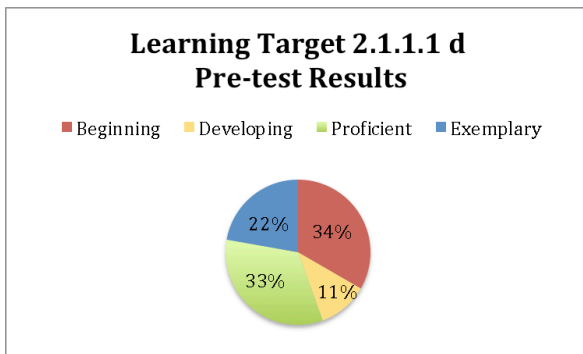
In this assessment the highest score possible was 4 and the lowest score possible was 0.

The assessment consisted of drawing a three-digit number using base-10 blocks

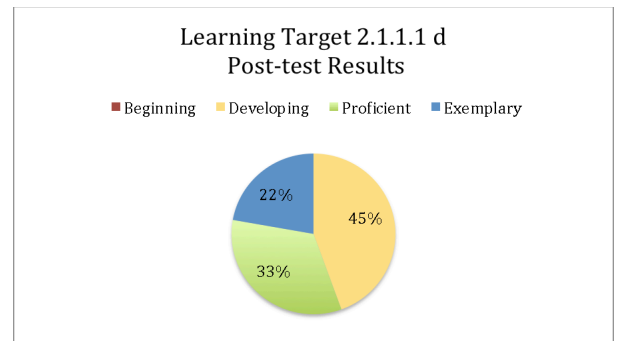
(\*Students were asked to draw the Golden Beads a Montessori manipulative that is similar to base-10 blocks), write a three-digit number in expanded notation and write a five-digit number in expanded notation.



*Figure 8: Learning Target 2.1.1.1 d: I can describe whole numbers 10-1000 in terms of ones, tens and hundreds.*



*Figure 9: Results of Pre-tests – I can describe whole number 10-1000 in terms of ones, tens and hundreds.*



*Figure 10: Results of Post-tests – I can describe whole numbers 10-1000 in terms of ones, tens and hundreds.*

**Impact of results:**

*Based on the results from the assessment, significant changes need to occur.* The results indicate that students need more support describing numbers from 10-1000 in this Montessori setting. *Figure 10* shows post-test results that 22% of students were exemplary at this skill which is above grade level, 33% of students were proficient at this skill which is at grade level and 45% of students were developing at this skill. A small group lesson would be an ideal opportunity for students to verbally describe the value of each number in a two, three or four-digit number using a Montessori manipulative of their choosing. An example for student practice could be to write  $362 = 3$  hundreds or  $300 + 6$  tens or  $60 + 2$  ones or 2. Students need practice describing each value of a three and four digit number.

**2.1.2.2. a – Learning Target: I can use fact families to understand addition and subtraction.**

Students had to complete four different addition number models that each equal 10, complete a fact family number model using the numbers 3, 6, 9 and choose the correct number model using the numbers 5, 12, 7.

Montessori lessons and manipulatives that were used and available for students in this Montessori setting were:

**Subtraction Snake Game** (See Appendix E)

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- Memorization of subtraction facts.

**Geometry Cabinet – Equilateral Triangle** (See Appendix E)

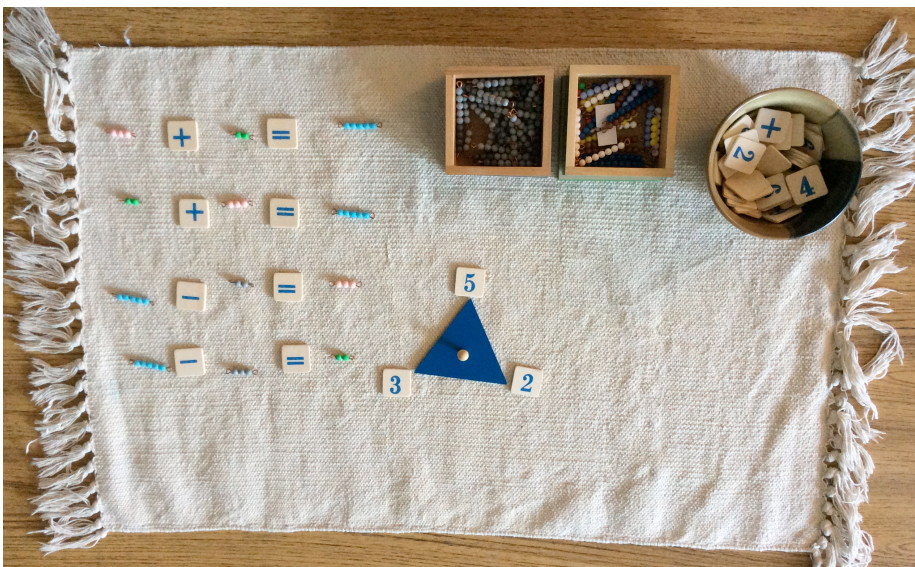
**Numeral and Operation Tiles** (See Appendix C)

Direct Aim:

- Numbers to create addition and subtraction problems with.
- Operation tiles to represent addition, subtraction and equal symbols.

**Montessori lesson:**

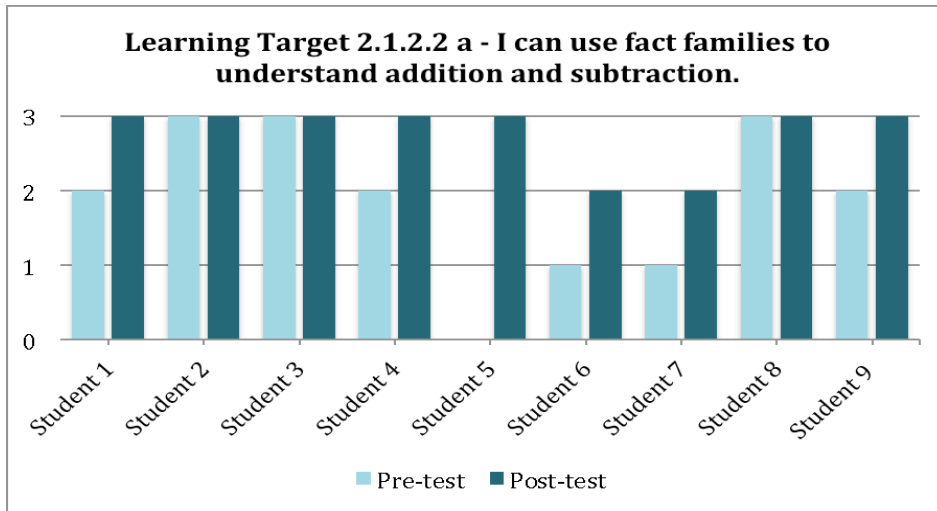
Students were presented a lesson on fact families using the colored bead bars and subtraction bead bars from the Subtraction Snake Game, number and operations tiles and the equilateral triangle from the Geometry Cabinet. Students were given a fact family practice page with three numbers given that create a fact family. Students then had to find the number and operation tiles for the given numbers and place them around the Equilateral Triangle. Next, students created addition problems using the colored bead bars, numeral and operation tiles. Students then created subtraction problems with the same three number tiles, colored bead bars and subtraction bead bars. *See Figure 11 below.*



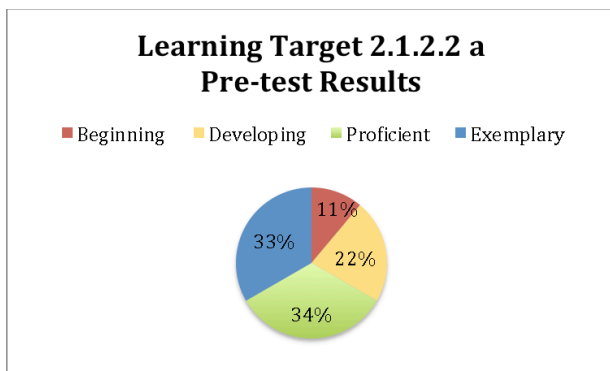
*Figure 11: Learning Target 2.1.2.2 a - I can use fact families to understand addition and subtraction.*

**Results:**

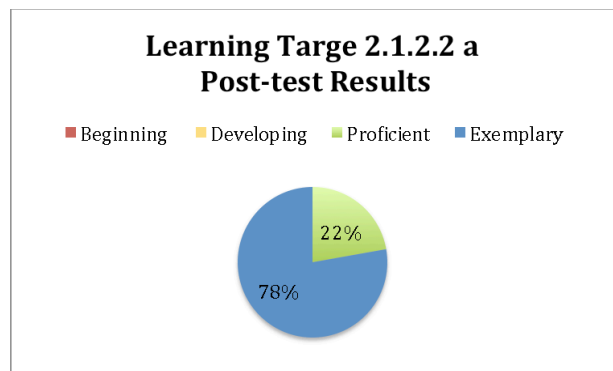
**I can use fact families to understand addition and subtraction.** Results show that majority of students made 1 – 2 point gains from their pre-test to their post-test. Post-test results show that after provided with instruction that utilized the Montessori mathematics manipulatives all students were either at grade level or above grade level for the skill of using fact families to understand addition and subtraction.



*Figure 12: Learning Target 2.1.2.2 a – I can use fact families to understand addition and subtraction.*



*Figure 13: Results of Pre-test – I can use fact families to understand addition and subtraction.*



*Figure 14: Results of Post-test - I can use fact families to understand addition and subtraction.*

**Impact of Results:**

*Based on the assessments given, not many changes need to be made. Figure 13 shows that 11% of students were beginning, 22% of students were developing, 34% of students were proficient and 33% of students were exemplary before Montessori instruction of this skill. Figure 14 shows that 22% of students tested at proficient which is at grade level and 78% of students were exemplary after Montessori instruction, which is testing above grade level for this skill. Although scores greatly improved after Montessori instruction with manipulatives, there is still room for growth. Students in this Montessori setting could use more practice with this skill of using fact families to understand addition and subtraction. The Subtraction Snake Game, number and operation tiles and Equilateral Triangle were a very sensorial example of creating a fact family to understand addition and subtraction. During practice with the Montessori mathematics manipulatives students were able to count the bead bars if necessary to find their sum and difference.*

**2.1.2.5. a – Learning Target: I can solve one and two digit addition problems.**

Students had to complete four two-digit addition problems.

Montessori lessons and manipulatives that were used and available for practice in this Montessori setting were:

**Addition with the Golden Beads (See Appendix A)**

Direct Aims:

- Development of order, concentration, coordination, independence and exactness.

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- “Exchanging” or “regrouping” numbers.
- Preparation for dynamic addition and multiplication.

**Addition with the Stamp Game** (See Appendix B)

Direct Aim:

- The operation of addition is explored with the representational material.

Indirect Aim:

- Preparation for the operation of addition with the more abstract material: the dot game.

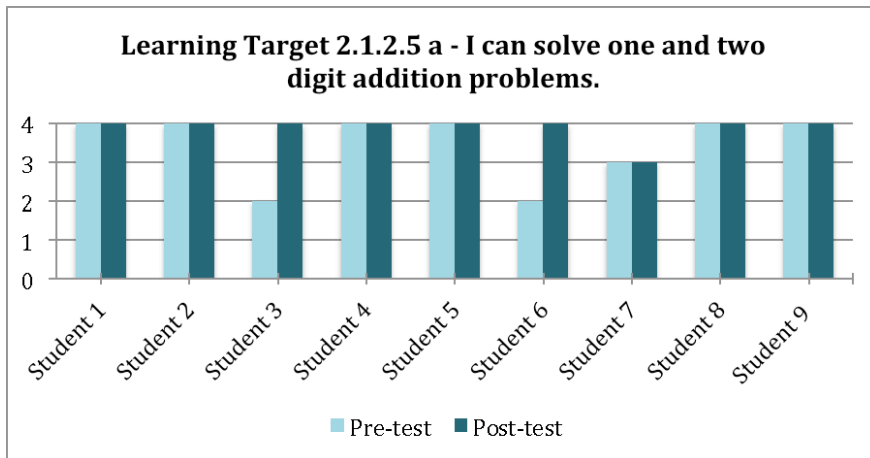
**Addition with the Bead Frame** (See Appendix D)

Direct Aim:

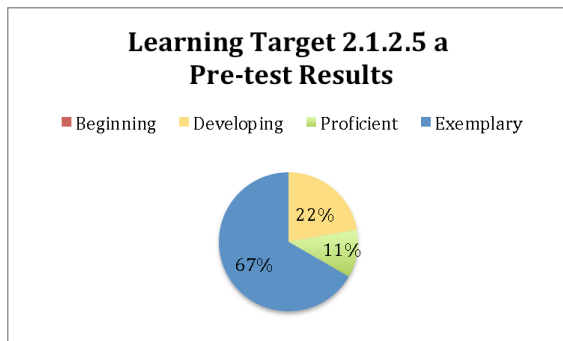
- The operation of addition is explored with this most abstract material.

**Results:**

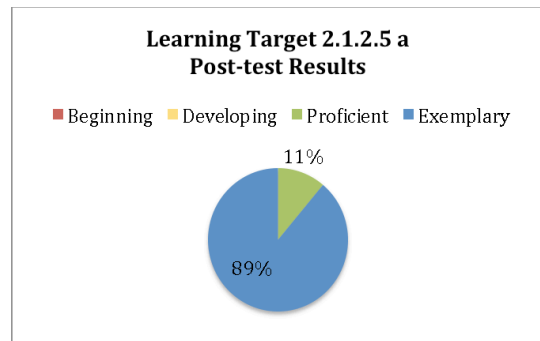
**I can solve one and two-digit addition problems.** Results show that 89% of students tested exemplary which is above grade level for this skill. 11% tested at proficient, which is at grade level for solving one and two-digit addition problems.



*Figure 15: Learning Target 2.1.2.5 a – I can solve one and two-digit addition problems.*



*Figure 16: Results of Pre-test - Learning Target 2.1.2.5 a – I can solve one and two-digit addition problems.*



*Figure 17: Results of Post-test – Learning Target 2.1.2.5 a – I can solve one and two-digit addition problems.*

**Impact of Results:**

*Based on the assessments given, few changes need to occur.* All students met or exceeded the skill of solving one and two-digit addition problems. *Figure 16* shows pre-test scores with 22% of students were developing, 11% of students were proficient and 67% of students were exemplary at this skill. Scores improved greatly after a Montessori lesson with manipulatives as shown in *figure 17* with 11% of students proficient and 89% of students exemplary. The Montessori math curriculum meets the requirements of this Minnesota State Standard. The Montessori manipulatives supported student’s understanding and skills of solving one and two-digit addition problems.

**2.1.2.5. b – Learning Target: I can solve one and two digit subtraction problems.**

Students had to complete four subtraction problems.

Montessori lessons and manipulatives that were used and available for students in this Montessori setting were:

**Subtraction with Golden Beads** (See Appendix A)

Direct Aim:

- To familiarize the child with the process of subtraction.

**Subtraction with Stamp Game** (See Appendix B)

Direct Aim:

- The operation of subtraction is explored with the representational material.

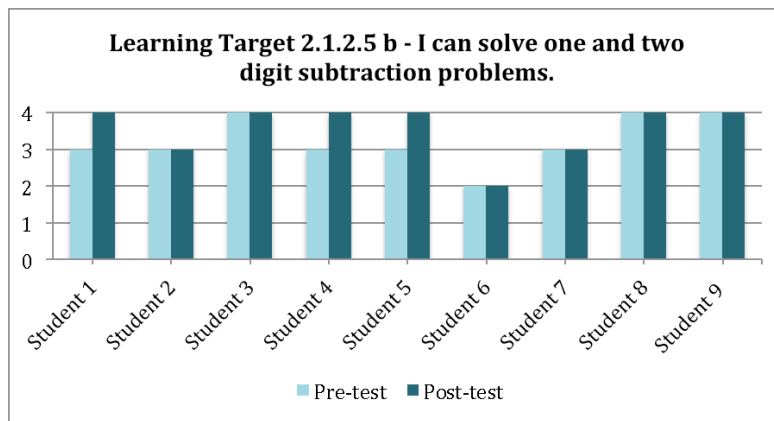
**Subtraction with Bead Frame** (See Appendix D)

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- The operation of subtraction is explored with this most abstract material.

**Results:**

**I can solve one and two-digit subtraction problems.** Results show that 67% of students were exemplary and 22% were proficient at the skill of solving one and two-digit subtraction problems. 11% of students were developing at this skill.



*Figure 18: Learning Target 2.1.2.5 b – I can solve one and two-digit subtraction problems.*

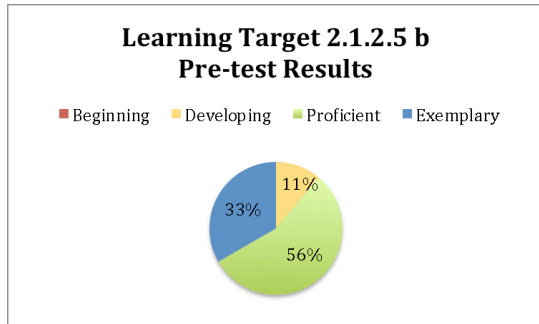


Figure 19: Results of Pre-test – Learning Target 2.1.2.5 b – I can solve one and two-digit subtraction

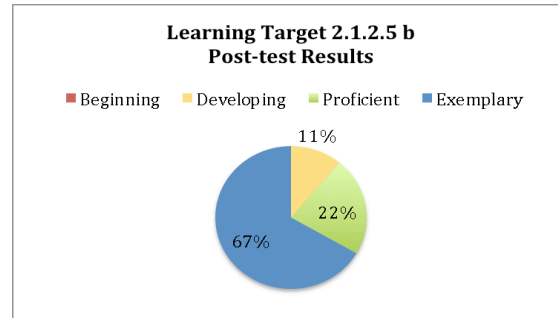


Figure 20: Results of Post-test – Learning Target 2.1.2.5 b – I can solve one and two-digit subtraction problems.

**Impact of Results:**

Based on the assessments given, some changes need to occur. The majority of the students met or exceeded the requirements for this skill as shown in figure 20. The student that needed more practice received further instructional lessons with the Stamp Game (See Appendix B) to reinforce a more concrete understanding of this skill. The Montessori math curriculum and manipulatives supported the majority of students understanding of one and two-digit subtraction problems.

**2.1.2.5. c - Learning Target: I can solve real world addition problems with one and two digit numbers.**

Students had to complete four real world addition problems.

Montessori lessons and manipulatives that were used and available for students in this Montessori setting were:

**Addition with Golden Beads (See Appendix A)**

Direct Aims:

- Development of order, concentration, coordination, independence and exactness.
- “Exchanging” or “regrouping” numbers.

- Preparation for dynamic addition and multiplication.

**Addition with Stamp Game** (See Appendix B)

Direct Aim:

- The operation of addition is explored with the representational material.

Indirect Aim:

- Preparation for the operation of addition with the more abstract material: the dot game.

**Addition with Bead Frame** (See Appendix D)

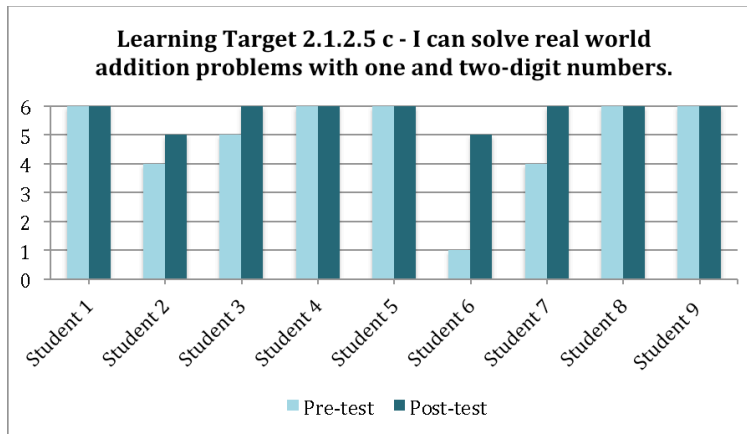
Direct Aim:

- The operation of addition is explored with this most abstract material.

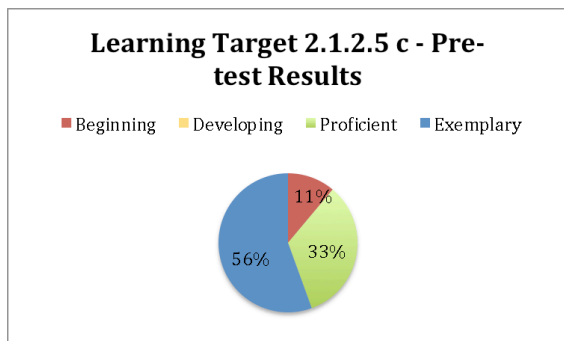
**Results:**

**I can solve real world addition problems with one and two-digit numbers.**

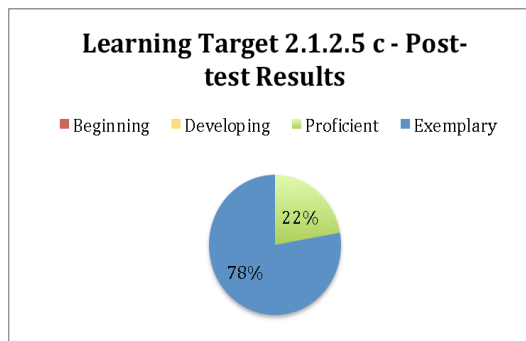
78% of students were exemplary and 22% of students were proficient at this skill. All students either met or exceed the requirements for this Minnesota State Standard.



*Figure 21: Learning Target 2.1.2.5 c – I can solve real world addition problems with one and two-digit numbers.*



*Figure 22: Results of Post-test – Learning Target 2.1.2.5 c – I can solve real world addition problems with one and two-digit numbers*



*Figure 23: Results of Post-test – Learning Target 2.1.2.5 c – I can solve real world addition problems with one and two-digit numbers.*

**Impact of Results:**

*Based on the assessments given, few changes need to occur. Figure 23 shows that all students succeeded at the skill of solving real world addition problems with one and two-digit numbers. Students were able to use a variety of Montessori mathematics manipulatives when working on their real world addition problems.*

**2.1.2.5. d – Learning Target: I can solve real world subtraction problems with one and two digit numbers.**

Students had to complete four real world subtraction problems.

Montessori lessons and manipulatives that were used and available for students in this Montessori setting were:

**Subtraction with Golden Beads (See Appendix A)**

Direct Aim:

- To familiarize the child with the process of subtraction.

**Subtraction with Stamp Game** (See Appendix B)

Direct Aim:

- The operation of subtraction is explored with the representational material.

**Subtraction with Bead Frame** (See Appendix D)

Direct Aim:

- Development of order, concentration, coordination, independence and exactness.
- The operation of subtraction is explored with this most abstract material.

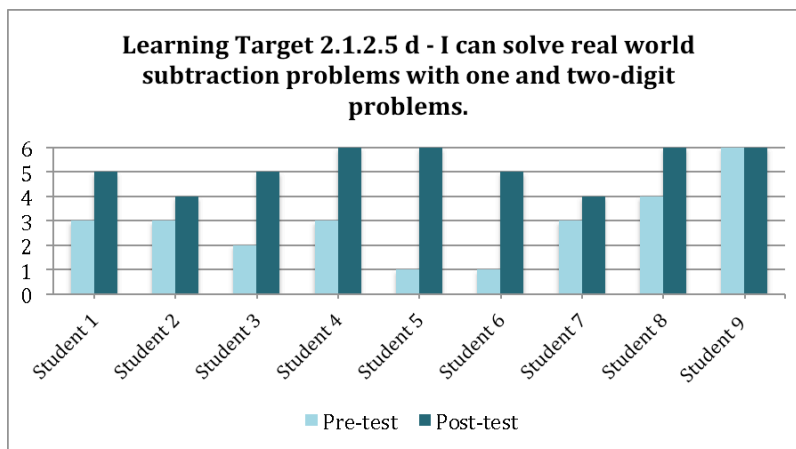
**Results:**

**I can solve real world subtraction problems with one and two-digit problems.**

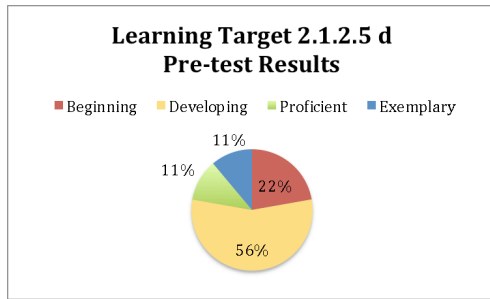
44% of students were exemplary and 56% of students were proficient at this skill.

Students tested either above grade level (exemplary) or at grade level (proficient) for this

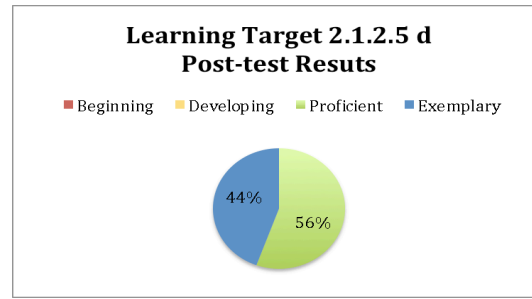
Minnesota State Standard.



*Figure 24: Learning Target 2.1.2.5 d – I can solve real world subtraction problems with one and two-digit problems.*



*Figure 25: Results of Pre-test – Learning Target 2.1.2.5 d – I can solve real world subtraction problems with one and two-digit problems*



*Figure 26: Results of Post-test - Learning Target 2.1.2.5 d – I can solve real world subtraction problems with one and two-digit problems.*

**Impact of Results:**

*Based on the assessments given, some changes need to occur. Figure 25 shows that the majority of students were developing (56%) at this skill. After instruction with a Montessori lesson and manipulatives, test scores increased to 56% of students being proficient which is at grade level. Figure 26 shows that all students tested either at grade level or above grade for this skill. Some students still need support with this skill, therefore, real-world subtraction problems have been added to this Montessori setting to support students who need additional practice. Students in this Montessori setting are receiving appropriate lessons and manipulatives to support their understating of real world subtraction problems with one and two-digit numbers.*

**Overall Results**

Through qualitative data collection in this study, it was shown that the Montessori math curriculum and manipulatives was very effective when teaching Grade 2 Numbers and Operation Strand for the Minnesota State Math Standards. Through alignment and testing of the Montessori math curriculum and the District’s math assessments, students

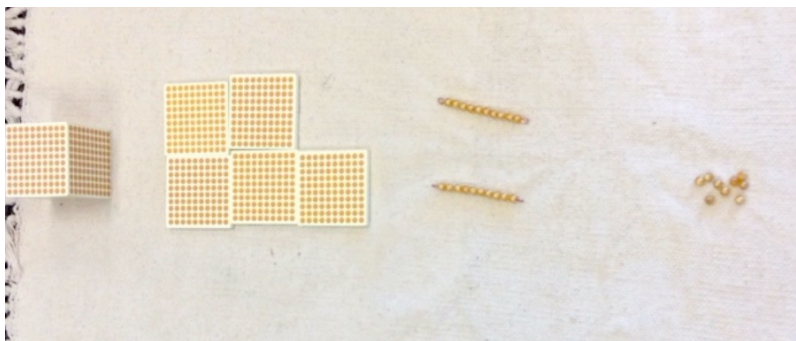
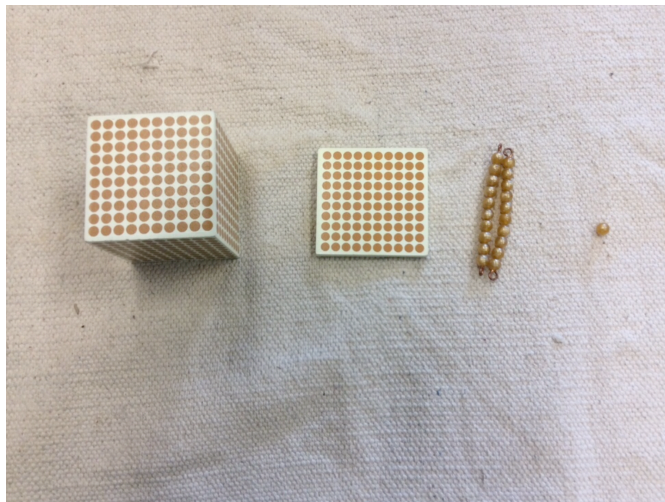
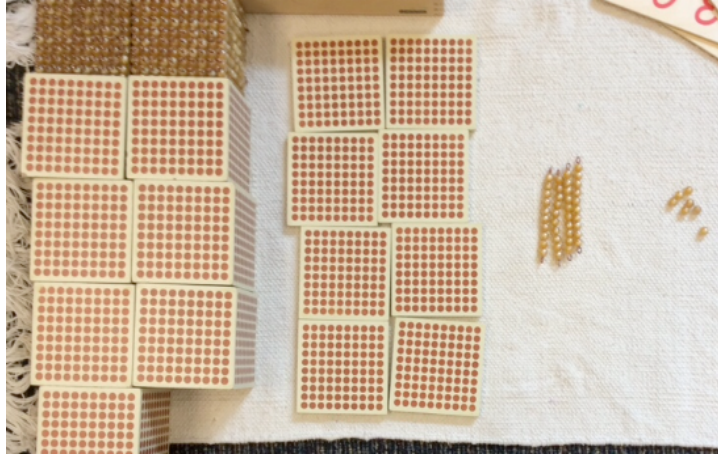
either tested at or above grade level in the majority of the learning targets within this strand. Students were given a Montessori math lesson utilizing a manipulative, to support their understanding of the presented math concept. The manipulatives were very effective in supporting the students' learning of different math concepts. Student's do need more practice and support describing the value of a number in four and five-digit numbers. Students received additional instructional Montessori mathematics lesson with the Large Bead Frame (See Appendix D). Students were presented the Formation of Quantity with the Large Bead Frame (See Appendix D) which has the place values vertically along the frame from units to millions. This lesson has supported the majority of students with the skill of describing the value of a number in four and five-digit numbers.

### **Impact of this Study**

This study revealed that the Montessori math curriculum meets most of the requirements for Grade 2: Numbers and Operations. Although there are some aspects of the district math assessments that need more support in the Montessori math curriculum, the Montessori math manipulatives substantially support students with their understanding of math concepts. This study will continue next year with further alignment and testing for the other Strands for Grade 2: Algebra, Geometry and Measurement. This study will also continue with Grades 1 and 3 by alignment and testing of Standards for those two grade levels.

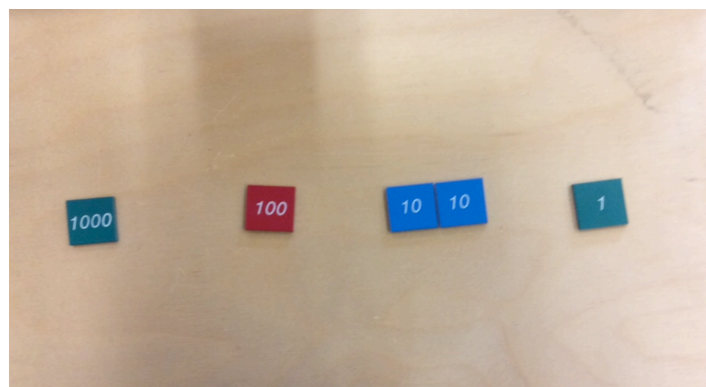
**Appendix A**

**The Golden Beads**



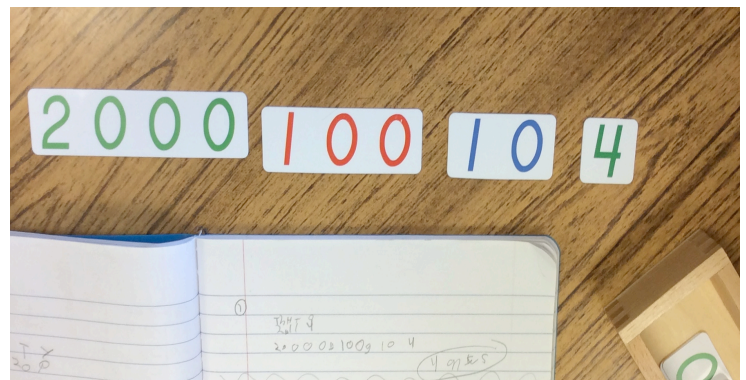
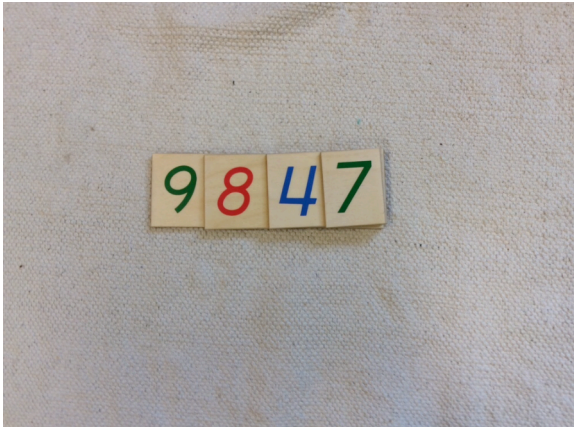
**Appendix B**

**Stamp Game**



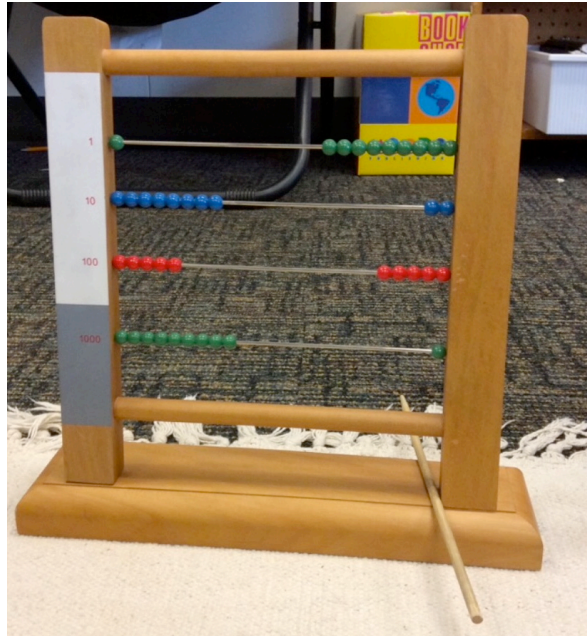
**Appendix C**

**Numerals Cards**

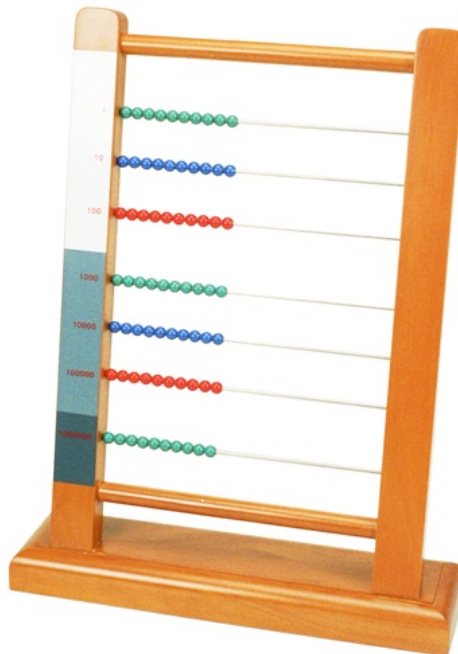


**Appendix D**

**Bead Frame**



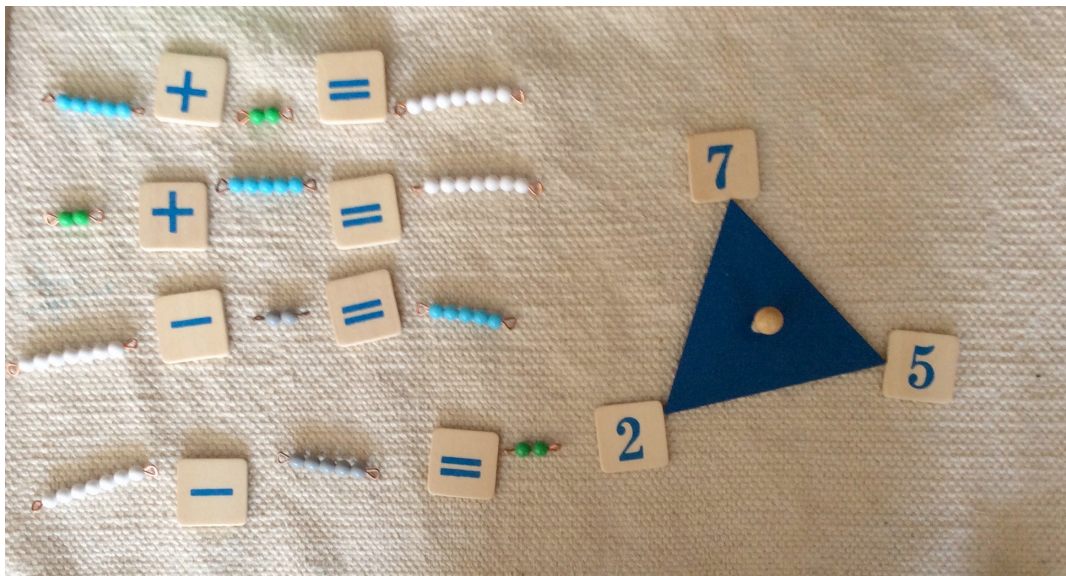
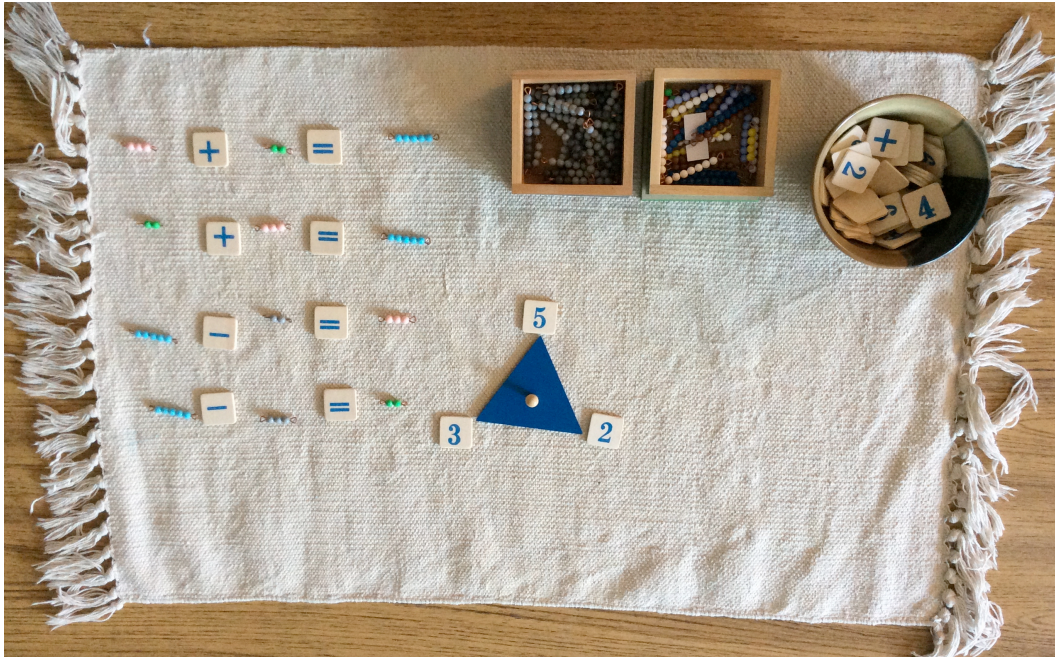
**Large Bead Frame**



Appendix E

Fact Families:

- Subtraction Snake Game
- Geometry Cabinet - Equilateral Triangle
  - Numeral and Operation Tiles



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