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Report No. 24

AN INVESTIGATION OF OA-BASED METHODS FOR PARAMETER DESIGN OPTIMIZATION

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PRACTICAL SIGNIFICANCE

Building quality into products during the design stages is an approach to quality improvement that has been practiced widely in Japan and has recently received great attention in North America. Use of experimental design methods for studying the effects of different parameters on a product's quality characteristics is central to this concept of moving quality upstream. Designing and running the experiments are relatively straightforward tasks; analysis of the results, however, is a matter of controversy. If analyzed correctly, the data can point to ways to optimize parameter values to the advantage of the product as a whole, e.g., by making it more robust to environmental variation or minimizing error transmission from components to assembled products. A method of analysis for choosing these optimal parameter values, proposed by Taguchi, is based on a technique called analysis of marginal means. Unfortunately, this technique has many deficiencies and often gives poor results, as discussed in this paper. An alternative method of analysis is based on sequential elimination of levels (SEL). Numerical and simulation studies on the design of a heat exchanger and two circuits show that SEL has several advantages, including the ability to search for optimal results over a wide range of parameter values.

Key words: Parameter design; orthogonal array; confirmation experiment; sequential elimination of levels; analysis of marginal means; control array; noise array.

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1. Introduction

An effective and economic approach to quality improvement is to "build-in" quality at the design stage. Central to this is the use of experimental design to efficiently search for desirable parameter settings for quality, productivity and other objectives. Planning of experiments usually consists of the following steps: (i) understanding of the problem, (ii) identification of important factors and choice of factor levels, (iii) choice of an experimental plan (e.g., fractional factorial design) and implementation of the experiment according to the plan, (iv) analysis of data and, based on it, choice of new factor levels for confirmation. These steps may be repeated until satisfactory results are obtained. Especially important is step (iv), the confirmatory stage. A method for (iv), advocated by Taguchi (1976,1986) and commonly used in certain sectors, is to determine the "optimum" level for each factor based on the marginal means (detail in Sections 2 and 3). A primary purpose of this paper is to point out some deficiencies of this method, which we call analysis of marginal means, and to propose a general class of methods for conducting confirmation experiments.

Comparisons of these methods are made on three examples involving the robust design of a heat exchanger, an OTL circuit and a voltage stabilizer circuit. The objective of robust product design, an original concept due to Taguchi (1976,1986), is to select the levels of the control factors so that the product at the chosen setting performs satisfactorily and stably over a wide range of conditions of the noise factors. In designing an electric circuit, the control factors may be the nominal values of the components (e.g., resistance, capacitance, gain) of the circuit and the noise factors may

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be the deviations of the components from their nominal values. In designing a steering wheel, the control factors may include the steering geometry and the hardness of spring, while the noise factors are environmental factors such a driving speed and road conditions. For robust product design, Taguchi advocates the use of an orthogonal array (OA) for the control factors and an orthogonal array for the noise factors. The former array is called control array (or inner array), the latter noise array (or outer array). Details of this approach can be found in Section 2.

Two common features of the three illustrative examples are the presence of noise factors and the existence of an explicit mathematical model (i.e., a transfer function) that relates the response to the control and noise factors. The relevance of our study is not limited to these situations. The proposed methods and the results of our study are also applicable to design problems in which no noise factors are explicitly stated and no noise arrays are used. Mathematical models are chosen only to facilitate objective comparisons of the various methods. Our findings also apply to physical experiments. For a given model it is possible to devise a more efficient design strategy than those considered in the paper by exploiting the special features of the model. In practice this situation-dependent strategy may be a useful supplement to good general strategies.

The paper is organized as follows. Deficiencies of the "analysis of marginal means" are studied in Sections 2 and 3. A new method, SEL (sequential elimination of levels), which includes the analysis of marginal means and the pick-the-winner rule as special cases, is proposed in Section 4. A careful study of its performance for two examples is given. The relative merits of two versions of SEL, SEL(mean) and SEL(mini), are compared in Section 5. When searching over a large space of factors, these methods can be more effective if applied iteratively to different regions of the factor space as determined by the results of the previous experiments. In section 6 several such iterative schemes are compared. Some concluding remarks are given in Section 7.

2. An example of the use of orthogonal arrays for parameter design optimization.

We will use the design of a heat exchanger from Cui, Fan and Zhang (1983) to illustrate the use of orthogonal arrays for obtaining better parameter designs.

Example A.

In the heat exchanger under consideration, the inlet temperature T_1 fluctuates in the range 670°C ± 30°C, and the flow rate V fluctuates in the range $(42000 \pm 2000) \times \frac{1}{21} \times (1 + \frac{T_1}{273})$. The three control factors are d (outside diameter of pipe), D (diameter of heat exchanger) and L/D, where L is the length of pipe. The objective of design is to select the levels of control factors so that the outlet temperature T_2 does not deviate from the target temperature 360°C by more than 15°C for values of T_1 and V in their respective ranges.

According to the derivations in Cui, Fan and Zhang (1983), the outlet temperature T_2 is related to the other factors by the following equation

$$T_2 = (T_1 - T_g)e^{-A} + T_g, \quad T_g = 222.7^\circ C ,$$

and

$$A = \frac{57.1(L/D)D^3\lambda_t}{Vd^2\rho_t C_{pm}} \left| 1.53 \times 10^{-3} \frac{d_i\rho_t}{\mu_t} \frac{V}{D^2} \left(\frac{d}{d_i}\right)^2 \right|^{0.8} \left| \frac{C_{pm}\mu_t}{\lambda_t} \right|^{0.4} , \tag{A}$$

where d_i = inside diameter of pipe (m), ρ_t = gaseous density (Kg/m^3), μ_t = gaseous viscosity, λ_t = thermal conductivity (K Cal/s·m·°C) = 3.335×10^{-5}, C_{pm} = specific heat (K Cal/Kg·°C).

Since T_1 and V cannot be fixed at the design stage but may vary during operation of the exchanger, they are the noise factors. Unlike other examples in the paper, the levels of noise factors here do not form a rectangular grid, i.e., the level of V increases as that of T_1 increases. The layouts of control factors and noise factors are given in

Tables A.1 and A.2. The parameter d_i in equation (A) is a deterministic function of d (Table A.3); ρ_t, C_{pm} and μ_t in equation (A) are deterministic functions of T_1 (Table A.4).

Table A.1: Levels of Control Factors

level	Factor d	D	L/D
1	.025	.8	3
2	.032	1.0	4
3	.038	1.2	5

Table A.2: Levels of Noise Factors

level	Factor T_1	V
1	640	$\frac{40000}{21} \times (1 + \frac{T_1}{273})$
2	670	$\frac{42000}{21} \times (1 + \frac{T_1}{273})$
3	700	$\frac{44000}{21} \times (1 + \frac{T_1}{273})$

Table A.3: d_i as Function of d

d	d_i
.025	.019
.032	.025
.038	.031

Table A.4: ρ_t, C_{pm} and μ_t as Function of T_1

T_1	ρ_t	C_{pm}	μ_t
640	5.286	1.024	2.83×10^{-5}
670	5.185	1.029	2.89×10^{-5}
700	5.089	1.031	2.93×10^{-5}

The levels of the control factors are chosen according to the orthogonal array $L_9(3^3)$ given in Table A.5. This orthogonal array is called the control array. For each setting (i.e. combination of levels) of the control factors, the T_2 value given in formula (A) is computed for the nine combinations of the T_1 and V values in Table A.2. These nine combinations form the noise array. Let $\Delta_i = 360 - T_2$ denote the difference between the target 360°C, and the outlet temperature T_2 for the i^{th} combination of T_1 and V. One such set of Δ_i values is given in Table A.6. Then the performance measure for the given combination of control factor levels is defined as

$$\Delta = \max_{1 \leq i \leq 9} |\Delta_i| \ .$$

Note that Δ is not a smooth function of the control factors. The objective of design is to make Δ less than 15. From the Δ values given in Table A.5, only the ninth run satisfies the requirement $\Delta < 15$.

Based on the Δ values of the nine runs in Table A.5, is it possible to find a better setting of the control factors? One method proposed and favored by Taguchi (1986) is described as follows. Compute the average of the Δ values, denoted by $\bar{\Delta}$, at each level of a control factor. For each control factor, select the level with the smallest $\bar{\Delta}$ value. Taguchi calls the resulting combination of levels "optimum." One more experiment is then conducted at the "optimum" setting of the control factors. We call this method the analysis of marginal means (AM). The word "optimum" is misleading; as shown later, it often gives less than desirable results.

Let us now examine its performance on the heat exchanger example. The $\bar{\Delta}$ value for the three levels of factors d, D, L/D are given in their respective columns in Table A.5. For example, the value 79.57 is the average of Δ values for $d = 1$. The levels selected by the AM method are $d = 3$, $D = 1$ and $L/D = 1$, which gives $\Delta = 107.71$ (see Table A.7) and is much worse than most runs in the L_9 array (Table A.5). On the other hand, the best Δ value from the nine runs is 14.97. This selection rule is called pick-the-winner rule (PW) since it picks the winner from among the existing runs. The PW method is a convenient benchmark for comparing methods for parameter design optimization.

In the next section we will examine the AM method more closely.

Table A.5: Layout of Control Factors and Δ Values

run	d	D	L/D	Δ
1	1	1	2	54.90
2	1	2	1	58.18
3	1	3	3	125.64
4	2	1	1	67.03
5	2	2	3	81.71
6	2	3	2	85.25
7	3	1	3	19.78
8	3	2	2	19.82
9	3	3	1	14.97
level		mean Δ		
1	79.57	47.23	46.72	
2	78.00	53.24	53.32	
3	18.19	75.28	75.71	
level		minimum Δ		
1	54.90	19.78	14.97	
2	67.03	19.82	19.82	
3	14.97	14.97	19.78	

Table A.6: Layout of Noise Factors and Δ_i Values (for run no. 1 in Table A.5)

run	T_1	V	Δ_i
1	640	6370.14	54.90
2	670	6579.45	46.97
3	700	6788.77	39.49
4	640	6688.65	53.59
5	670	6908.42	45.56
6	700	7128.21	37.97
7	640	7007.15	52.34
8	670	7237.40	44.20
9	700	7467.64	36.52

3. Does "analysis of marginal means" give better designs?

The above comparison of the analysis of marginal means (AM) and the pick-the-winner rule (PW) is based on results from the particular L_9 array given in Table A.5. There are altogether twelve distinct orthogonal arrays with nine runs and three factors. The best Δ values selected by AM and PW are summarized in Table A.7. It is obvious that PW is superior to AM. Disappointingly, AM never gives a design with $\Delta \leq 15$.

Our finding that the analysis of marginal means is much worse than the straight pick-the-winner rule is further supported by a collection of studies reported in the monograph "Three-Stage Design of Experiments with Known Transfer Functions" (in Chinese) edited by the Committee on Three-Stage Design, Chinese Applied Statistics Society. Direct comparisons of AM and PW are made for the nine experiments (with known transfer functions) reported in the monograph. Only four out of the nine experiments make use of noise arrays. The other five do not have noise factors. In experiments using sequential designs, a comparison is made for each stage of the experimentation. Altogether we have found twenty-two such comparisons. In each

Table A.7: Summary of Results Using the Analysis of Marginal Means and the Pick-the-Winner Rule

	design chosen	no. of times	Δ
AM	(3 1 1)	5	107.71
	(2 2 2)	4	54.11
	(2 1 1)	1	67.03
	(3 1 2)	1	58.88
	(3 2 1)	1	54.56
PW	(2 2 1)	4	12.80
	(1 1 1)	2	13.69
	(3 3 1)	2	14.97
	(2 1 2)	1	16.69
	(3 1 3)	1	19.78
	(2 1 3)	1	41.83
	(3 2 1)	1	54.56

comparison, we compute the ratio of performance measures of the two combinations of factor levels given by AM and PW. Performance measures vary from problem to problem. A smaller performance measure means a better combination. The results as reported in Table 1 lend further support to our previous finding that AM is much poorer than PW. Our study in the remaining part of the paper also supports this.

Table 1. Performance Measure of AM/Performance Measure of PW (based on 22 cases)

	0.32	0.98-1.2	1.7-2.0	4.0-8.2	10-65.8
no. of cases	1	6	3	4	8

In only one out of twenty-two cases does AM outperform PW, with the ratio equal to 0.32. This case turns out to be the often-quoted Wheatstone bridge example (Taguchi and Wu, 1982). In this example an additive model with main effects only is quite adequate. According to the analysis in (4) and (5), this explains why AM performs particularly well here. An analysis of this example by Box and Fung (1986) also supports this.

The severe limitations of the AM method may be better understood through the following analysis. Consider the problem of minimizing $K(\underset{\sim}{x})$, $\underset{\sim}{x} = (x_1,\ldots,x_p)$, over $a_i \leq x_i \leq b_i$. Instead of solving the p-dimensional minimization problem

$$\text{Min}\{K(\underset{\sim}{x}) : a_i \leq x_i \leq b_i,\ i = 1,\ldots,p\} \ , \tag{1}$$

the following p one-dimensional minimization problems are considered,

$$\text{Min}\{K_i(x_i) : a_i \leq x_i \leq b_i\}, \quad i = 1,\ldots,p \ , \tag{2}$$

where $K_i(x_i)$ is the i^{th} marginal function of $K(\underset{\sim}{x})$,

$$K_i(x_i) = \int K(\underset{\sim}{x}) \prod_{j \neq i} dx_j \tag{3}$$

and the integral is taken over the intervals $[a_j,b_j]$, $j \neq i$. If the x_i in (1) and (2) can take only a finite number of values (discrete x_i), the integral in (3) is replaced by a finite sum. Let x_i^* be a solution to the i^{th} problem in (2). The combination $\underset{\sim}{x}^* = (x_1^*,\ldots,x_p^*)$ may be proposed as an approximate solution to (1). For discrete x_i, $\underset{\sim}{x}^*$ is what one would get from the AM method. So a key issue regarding the adequacy of AM is whether $\underset{\sim}{x}^*$ solves (1). A sufficient condition for $\underset{\sim}{x}^*$ to be a solution of (1) is that $K(\underset{\sim}{x})$ can be represented as

$$K(\underset{\sim}{x}) = \psi(K_1(x_1),\ldots,K_p(x_p)) \tag{4}$$

and

$$\psi \text{ is nondecreasing in each } K_i \ .$$

A special case of (4), which may be of interest to statisticians, is

$$K(\underset{\sim}{x}) = \sum_i \alpha_i K_i(x_i) + \sum_{i,j} \lambda_{ij} K_i(x_i) K_j(x_j), \quad \alpha_i \geq 0, \lambda_{ij} \geq 0 . \tag{5}$$

Although (4) is not a necessary condition for $\underset{\sim}{x}^*$ to be a solution of (1), its rather restrictive nature suggests that $\underset{\sim}{x}^*$ does not often give a good result for (1), i.e., $K(\underset{\sim}{x}^*)$ can be much larger than $\min K(\underset{\sim}{x})$. This may explain why AM does not give improvement over PW.

The AM method is based on optimizing an additive linear model with main effect terms or a slightly more complicated model such as (5). Its failure might be due to the simplicity of the model. An improvement of AM can be made by including interactions in the analysis. If, for example, the interaction of factor A and factor B is considered, the additional step consists of computing the mean of the response values for each level combination of $A \times B$, and selecting the combination with the best mean. The confounding pattern of $A \times B$ should also be noted. This will work well if the resulting model provides a good approximation to the true response surface. However, if the surface is too rugged to permit a good approximation, even a more sophisticated model will not give good results as illustrated by the following.

A regression model based on the nine Δ values (in Table A.5) is selected by applying stepwise regression to the sixteen variables $\{x_i, x_i x_j, x_i x_j x_k\}$, $i,j,k = 1,2,3$ and x_1, x_2, x_3 are the three control factors properly standardized. The constant for F-to-enter and for F-to-remove is 2.3. The resulting model is

$$\hat{y} = 73.7 - 21.5\, x_1 + 14.0\, x_2 + 14.5\, x_3 - 21.7\, x_1^2 - 18.4\, x_2 x_3 , \tag{6}$$

$$y = \Delta, \qquad R^2 = .974 .$$

The fitted model will be judged by its ability in predicting the other low

Δ values not in the nine runs. There are indeed two more Δ values smaller than 15 (see Table A.7). The model (6) however misses both of them, i.e.,

$$\hat{\Delta} = 57.38 \quad \text{while} \quad \Delta = 12.8$$

$$\hat{\Delta} = 63.29 \quad \text{while} \quad \Delta = 13.69 \ .$$

We also transform Δ to $\ell n \ell n \Delta$ in order to get a model with a better fit. Applying the above procedure to $y = \ell n \ell n \Delta$ with constant 2.8 for F-to-enter and for F-to-remove, the following model is obtained

$$\hat{y} = 1.5 - 0.15\, x_1 - 0.25\, x_1^2 - 0.08\, x_1 x_2 + 0.04\, x_2 x_3^2 - 0.06\, x_1 x_2^2 \qquad (7)$$

$$y = \ell n \ell n \Delta, \qquad R^2 = .998 \ .$$

Again it misses the two low Δ values, i.e.,

$$\hat{\Delta} = \exp(\exp\, \hat{y}) = 82.73 \quad \text{while} \quad \Delta = 12.8$$

$$\hat{\Delta} = 46.36 \qquad\qquad \text{while} \quad \Delta = 13.69 \ .$$

Models obtained by using $y = \ell n \Delta$ and/or other constants for entering and removing variables lead to essentially the same conclusion.

When the response surface is too rugged or the levels chosen for the array are too wide apart, an empirical model that fits well the observed data may not be a good approximation to the true response surface. Prediction based on the fitted model may not be trustworthy. In this situation design optimization should be based on empirical models as well as the ranking of parameter settings given by the experimental data. This is embodied in the SEL method proposed in the next section.

The Δ values of the heat exchanger model (A) provide an example of a rugged surface. Applying ANOVA to the 27 observations in the full 3^3-design (data in Tables A.5, A.7-A.10), we find the percent contributions of the main-effect terms, the two-factor interaction terms, and the three-factor interaction to be respectively 40%, 40% and 20%. A large three-factor interaction may explain why the previous quadratic models do not predict well.

Another problem with the AM method is that, for widely spaced levels, the use of the mean as the criterion for comparison of different factor levels may be inappropriate. This will be further discussed in Section 4.

4. Sequential Elimination of Levels (SEL): a sequential method for parameter design optimization.

A general method, which includes the analysis of marginal means (AM) and the pick-the-winner (PW) rule as special cases, is proposed as follows:

(i) for each factor eliminate those level(s) with the worst mean value(s) computed from the current array,

(ii) choose an orthogonal array (typically of a smaller size) for the remaining levels, and replace the array in (i) by the new array,

(iii) conduct another experiment on the new array,

(iv) repeat (i)-(iii) if necessary.

This method is called the sequential elimination of levels (SEL). In step (i), if the mean is replaced by another descriptive statistic x, we call the method SEL(x). We consider x = mean and mini(minimum) in this paper. Another choice of x is the mean of the few smallest values. Comparison of SEL(mean) and SEL(mini) will be taken up in the next section. Note that more than one level may be eliminated each time.

SEL is a very general method as it includes both AM and PW as extreme cases. If in step (i) all but one level are eliminated, it is easy to see that SEL(mean) reduces to AM and SEL(mini) reduces to PW. Typically there is no need to retain more than three levels. If one level is distinctly better than the rest, only one is retained. If the best two or three levels are not significantly different, they are retained. The new array in step (ii) is of the type $2^m 3^n$ and typically does not exceed 18 runs, since $L_{18}(2^1 \times 3^7)$

can accommodate a fair number of 2-level and 3-level factors. SEL can also handle interactions. If the interaction of A and B is desired, factors A and B will be replaced by the new "factor" $A \times B$ in the procedure.

We now illustrate the SEL method on the heat-exchanger example. The mean and minimimum Δ values for each level of the factors d, D and L/D are given in the bottom part of Table A.5. According to SEL(mean), $d = 1$, $D = 3$, and $L/D = 3$ are eliminated (step (i)). Each factor now has two levels. Select the full factorial $L_8(2^3)$ for the remaining levels (step (ii)). The second array has two combinations already in the first array (compare Tables A.5 and A.8). Conduct six additional runs (step (iii)) with the results given in Table A.8, which include the smallest Δ value 12.8. Similar results from using SEL(mini) are given in Table A.9. A better Δ value 13.69 is found, although the best value 12.8 is missed. For the convenience of the readers we include in Table A.10 the Δ values of runs not given in the previous tables.

Table A.8: Second-round Design Using SEL(mean)

run	d	D	L/D	Δ
1*	2	1	1	67.03
2	2	1	2	16.69
3	2	2	1	12.80
4	2	2	2	54.11
5	3	1	1	107.71
6	3	1	2	58.88
7	3	2	1	54.56
8*	3	2	2	19.82

*Runs already in the first-round design (see Table A.5).

Table A.9: Second-round Design Using SEL (minimum)

run	d	D	L/D	Δ
1	1	1	1	13.69
2	1	1	3	82.37
3	1	3	1	85.51
4*	1	3	3	125.64
5	3	1	1	107.71
6*	3	1	3	19.78
7*	3	3	1	14.97
8	3	3	3	83.32

*Runs already in the first-round design

Table A.10: Δ Values of Other Runs

d	D	L/D	Δ
1	2	2	91.84
1	2	3	111.19
1	3	2	113.44
2	3	1	49.72
2	3	3	106.37
3	2	3	51.72
3	3	2	56.04

The performance of SEL is tested on all the twelve distinct L_9 orthogonal arrays (obtained by changing the L/D column in Table A.5). In all the eight cases in which the first-round design does not include the smallest value 12.8, SEL(mean) captures it in the second-round design. SEL(mini) does not do as well. In five cases, 12.8 is not captured. But in two of these five cases, it results in improvement. However, for more complicated problems studied later, SEL(mini) outperforms SEL(mean).

SEL can be applied to more complex situations such as the following example, which has more factors and levels than Example A.

Example B (from Chen et al., 1983).

In designing an OTL (output transformerless) pull-push circuit (see Figure 1), an objective is to make the midpoint voltage V_m stable around 6V in the presence of variations of the components of the circuit.

The midpoint voltage V_m is related to the other factors by the equation

$$V_m = (V_{b1} + V_{be1}) \frac{\beta R_0}{\beta R_0 + R_f} + (E_c - V_{be3}) \frac{R_f}{\beta R_0 + R_f} + \frac{V_{be2} R_f \beta R_0}{(\beta R_0 + R_f) R_{c1}} , \quad \text{(B)}$$

where $V_{b1} = E_c \dfrac{R_{b2}}{R_{b1}+R_{b2}}$, $R_0 = R_{c2} + R_L$, $R_L = 9\Omega$, $E_c = 12V$,

$$V_{be1} = V_{be3} = 0.65V, \; V_{be2} = 0.74V ,$$

R_{b1}, R_{b2}, R_f, R_{c1} and R_{c2} are resistances and β is the current gain. The ranges of these six factors are

$$\begin{aligned} &R_{b2}{:}25K \sim 70K, \quad R_{b1}{:}50K \sim 150K, \quad R_f{:}0.5K \sim 3K, \\ &R_{c2}{:}0.25K \sim 1.2K, \quad R_{c1}{:}1.2K \sim 2.5K, \quad \beta{:}50 \sim 300 . \end{aligned} \quad (8)$$

In the original treatment by Chen et al. (1983), the six factors in (8) are used as control factors and also as noise factors. By examining equation (B) more closely, it is found that V_m depends on R_{b1} and R_{b2} through R_{b2}/R_{b1}. Therefore in our approach, we will use the following five factors

$$A = R_{b2}/R_{b1}, \; B = R_f, \; C = R_{c2}, \; D = R_{c1}, \; E = \beta .$$

The levels of A to E as control factors are given in Table B.1. The levels of B to E are the same as in Chen et al. (1983). The levels of A are chosen to be close to the levels of R_{b1} and R_{b2} in Chen et al. (1983, Table 1). The resistors, being of first grade, have 5% variations. The transistor β, being of third grade, has a 50% variation. This determines the choice of the levels of the noise factors A to E (Table B.2).

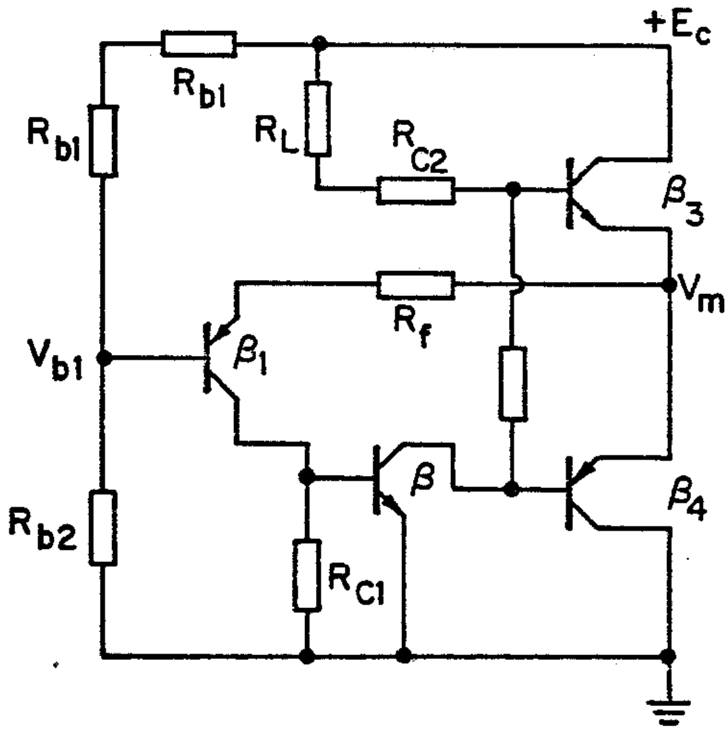

Figure 1. An OTL Circuit

Table B.1: Levels of Control Factors

level	Factor A	B	C	D	E
1	.215	649.38	237.14	1271.1	73
2	.316	865.96	316.23	1467.8	102
3	.464	1154.8	421.70	1695.0	143
4	.681	1539.9	562.34	1957.3	200
5	1.00	2053.5	749.89	2260.3	280

Table B.2: Levels of Noise Factors

level	Factor A to D	E
1	0.95 of level 2	0.5 of level 2
2	from control array	from control array
3	1.05 of level 2	1.5 of level 2

The purpose of parameter design is to choose the (nominal) values of A to E such that the variation of V_m (as A to E vary from their nominal values) is small. Since the total number of combinations of control factor levels is too large ($5^5 = 3125$), we select 25 combinations according to the array $L_{25}(5^6)$ given in Table B.5. This is the control array. For each combination in the control array, the levels of the factors A to E can vary by 5% or 50% of the nominal values (Table B.2). The total number of possibilities $3^5 = 243$ is too large. Instead we use the first five columns of the array $L_{18}(3^6)$ (see Appendix 1) to represent the variations of factors around the nominal value chosen in the control array. Here L_{18} is the noise array. For each combination of levels in the noise array $L_{18}(3^5)$, V_m can be determined from equation (B). The mean squared error of the 18 V_m values

$$v = \frac{1}{18} \sum_{1}^{18} (V_m - 6)^2$$

is used as a stability measure of the setting of the control factors. One such set of v values are given in Table B.5. Both arrays L_{25} and L_{18} are chosen in accordance with Chen et al. (1983).

Since each factor has five levels, SEL may be applied to eliminate one level at a time or, for reducing the number of iterations, to eliminate several levels at a time. Both SEL(mean) and SEL(mini) are considered. For each version, three sequential schemes are considered. See Table B.3.

Table B.3. Sequential Schemes Using SEL

round	scheme 1	cumulative sample size	scheme 2	cumulative sample size	scheme 3	cumulative sample size
1st	$L_{25}(5^5)$	25	$L_{25}(5^5)$	25	$L_{25}(5^5)$	25
2nd	$L_{16}(4^5)$	41	$L_{18}(3^5)$	43	$L_{16}(2^5)$	41
3rd	$L_{18}(3^5)$	59	$L_{16}(2^5)$	59		
4th	$L_{16}(2^5)$	75				

The six schemes are compared in a simulation study. The array $L_{25}(5^5)$ is obtained from randomly selecting 5 columns from the array $L_{25}(5^6)$ in Table B.5; $L_{16}(4^5)$ from randomly selecting (i.e. permuting) 5 columns from the array in Table B.6; $L_{18}(3^5)$ from randomly selecting 5 columns from the array $L_{18}(3^6)$ in Appendix 1; and $L_{16}(2^5)$ from randomly permuting columns 1, 2, 4, 8, 15 of the array $L_{16}(2^{15})$ in Taguchi and Wu (1982, Appendix). In each round, the best (smallest) v value among the runs in the control array is recorded. The frequencies of the best v values (based on 500 simulations) are summarized in Table B.4.

Table B.4. Frequencies* of Best v Values from Designs Using SEL (based on 500 simulations).

design		scheme 1							scheme 2							scheme 3						
		I	II	III	IV	V	VI	VII	I	II	III	IV	V	VI	VII	I	II	III	IV	V	VI	VII
$L_{25}(5^5)$		46	48	130	151	95	20	10														
$L_{16}(4^5)$	SEL(mean)	97	87	163	107	35	5	6														
	SEL(mini)	67	74	167	155	33	4	0														
$L_{18}(3^5)$	SEL(mean)	142	114	169	59	15	1	0	80	84	171	114	41	7	3							
	SEL(mini)	137	138	171	51	3	0	0	90	125	195	79	11	0	0							
$L_{16}(2^5)$	SEL(mean)	185	164	130	17	3	1	0	114	173	159	39	14	1	0	59	78	154	135	59	15	0
	SEL(mini)	210	188	94	8	0	0	0	148	177	152	21	2	0	0	110	165	142	75	8	0	0

*Frequencies in the seven intervals: I = (.0148, .015], II = (.015, .016], III = (.016, .018], IV = (.018, .022], V = (.022, .03], VI = (.03, .04], VII = (.04, .0664]

Summary of Table B.4

1. Except for $L_{16}(4^5)$ (2nd round), SEL(mini) outperforms SEL(mean). The difference is more significant when there are fewer levels (i.e. in later rounds). Comparison of SEL(mean) and SEL(mini) will be addressed in the next section.
2. When SEL(mean) is used, for approximately the same cumulative sample size, scheme 1 is better than scheme 2, which is better than scheme 3. That is, schemes that eliminate fewer levels in each round perform better.
3. When SEL(mini) is used, the situation in 2 is reversed.
4. Design schemes with larger cumulative sample sizes do better.
5. From 1 to 3, sequential schemes that use SEL(mini) and eliminate more than one level in the early rounds are recommended. In addition to producing good results for the same sample size, they require fewer iterations. (In practice the latter may be as important a consideration as sample size reduction.)

A limited study on the sensitivity of v to the choice of columns in the noise array $L_{18}(3^5)$ was conducted. It appears that there is little change in the v values and their relative order in the control array.

A more efficient way for choosing the control factor levels is to use an $L_{25}(5^4)$ for the levels of four factors, say A to D, and, for each level combination of A to D, to choose the level of E so that $V_m(A,B,C,D,E)$ as given in equation (B) equals the target 6. The resulting v values are much smaller than those in Table B.5. This improvement of Taguchi's approach to parameter design will be addressed elsewhere.

In this section we only consider the use of SEL on a fixed grid determined by a small number of levels. Better results can be obtained by combining the use of SEL, adaptive choice of grid, and search over a broader range of factor levels. This will be addressed in Section 6.

5. SEL(mean) or SEL(mini)?

In order to understand why SEL(mini) does substantially better than SEL(mean) in the simulation study for Example B, we will study one of the simulations more closely. Its control array is given by the A-E columns of Table B.5. The mean and minimum v values for the five levels of each factor are given at the bottom of the table. Using the analysis of marginal means (AM), the levels (4,5,1,1,5) or (4,5,1,2,5) are chosen for factors A to E with their respective v values 0.7805 and 0.5341, both much bigger than the smallest v value 0.066 from the array L_{25} in Table B.5. Again AM is worse than PW.

The application of SEL(mini) results in the elimination of the levels (1,4,5,1,3). The second-round design using an $L_{16}(4^5)$ is given in Table B.6. Its best v value is .018. On the other hand, SEL(mean) eliminates the levels (1,2,2,5,2). The second-round design using the same $L_{16}(4^5)$ is given in Table B.7 and has a bigger best v value .042. Applying SEL(mean) to the second-round design results in the elimination of level 3 of factor B. It turns out from a more extensive study that all combinations with low v values have levels 2 or 3 for factor B. Therefore SEL(mean) cannot give good results in this case.

So, where does SEL(mean) go wrong? Let us find out how level 2 of factor B is eliminated. The five values for $B = 2$ are

$$.066,\ 1.085,\ 1.464,\ 4.189,\ 8.00\ ,$$

including the smallest value 0.066 of the first-round design. The mean 2.96 of the five values is influenced by the two large values 4.189 and 8.00. The small values play little role in pulling down the mean. The same reason explains why level 3 of B is eliminated after the second round. The behavior of SEL(mini) is quite different. Level 2 of B is kept because its minimum value 0.066 is small.

Table B.5. Layout of Control Factors and v Values

run	A	B	C	D	E		v
1	4	2	1	3	1	2	.066
2	5	4	2	5	1	5	2.153
3	1	1	3	4	1	4	7.704
4	3	3	4	1	1	1	.471
5	2	5	5	2	1	3	1.326
6	3	4	1	4	2	3	.326
7	2	1	2	1	2	2	3.685
8	4	3	3	2	2	5	.085
9	5	5	4	3	2	4	2.917
10	1	2	5	5	2	1	8.005
11	5	1	1	2	3	1	1.164
12	1	3	2	3	3	3	6.177
13	3	5	3	5	3	2	.423
14	2	2	4	4	3	5	4.189
15	4	4	5	1	3	4	.260
16	2	3	1	5	4	4	3.560
17	4	5	2	4	4	1	.248
18	5	2	3	1	4	3	1.464
19	1	4	4	2	4	2	5.391
20	3	1	5	3	4	5	1.522
21	1	5	1	1	5	5	3.150
22	3	2	2	2	5	4	1.085
23	2	4	3	3	5	1	2.850
24	4	1	4	5	5	3	.076
25	5	3	5	4	5	2	1.254

level			mean v		
1	6.085	2.830	1.654	1.806	2.344
2	3.122	2.962	2.670	1.811	3.003
3	.765	2.310	2.505	2.706	2.443
4	.147	2.196	2.610	2.744	2.437
5	1.791	1.613	2.473	2.843	1.683

level			minimum v		
1	3.150	.076	.066	.260	.066
2	1.326	.066	.248	.085	.085
3	.326	.085	.085	.066	.260
4	.066	.260	.076	.248	.248
5	1.165	.248	.260	.076	.076

Table B.6: Second-round Design Using SEL (minimum)

run	A	B	C	D	E	v
1	2	1	1	2	1	3.359
2	3	1	2	3	2	1.246
3	4	1	3	4	4	.051
4	5	1	4	5	5	.794
5	3	2	1	4	5	1.268
6	2	2	2	5	4	4.269
7	5	2	3	2	2	1.421
8	4	2	4	3	1	.018
9	4	3	1	5	2	.060
10	5	3	2	4	1	1.772
11	2	3	3	3	5	3.538
12	3	3	4	2	4	.800
13	5	5	1	3	4	3.011
14	4	5	2	2	5	.477
15	3	5	3	5	1	.226
16	2	5	4	4	2	1.990

Table B.7: Second-round Design Using SEL (mean)

run	A	B	C	D	E	v
1	2	1	1	1	1	3.184
2	3	1	3	2	3	1.303
3	4	1	4	3	4	.042
4	5	1	5	4	5	.843
5	3	3	1	3	5	.851
6	2	3	3	4	4	3.666
7	5	3	4	1	3	1.947
8	4	3	5	2	1	.068
9	4	4	1	4	3	.156
10	5	4	3	3	1	2.416
11	2	4	4	2	5	2.598
12	3	4	5	1	4	.346
13	5	5	1	2	4	3.486
14	4	5	3	1	5	.664
15	3	5	4	4	1	.216
16	2	5	5	3	3	2.015

Example B should not lead the readers to believe that SEL(mini) is always better than SEL(mean). Their comparison will be discussed below using typical but hypothetical situations.

The sequential design problem may be viewed as a problem of predicting factor levels with low v values from the observed data. If a low (observed) value is a clue to where lower values can be found, SEL(mini) is a good method since the levels of this low value will be retained. Similarly, if a low mean (observed) value is a clue to where lower values can be found, SEL(mean) is good. One such situation, in which both SEL(mean) and SEL(mini) correctly select level 2, is described in Table 2. Here the values in the "low" category are not observed.

Table 2. SEL(mean) and SEL(mini) both do well.

level	low (unobserved)	medium value	high
1	X	X	X
2	X	X	X

Table 3 typifies situations in which SEL(mini) outperforms SEL(mean). Based on the observed values in the "medium" and "high" categories, the level containing the lower low value is to be predicted. (For Table 3 it is level 2.) SEL(mini) correctly selects level 2, because the observed minimum value (in the "medium" category) at level 1 is higher. On the other hand, SEL(mean) incorrectly selects level 1 because the mean value at level 2, being influenced by the value in the "high" category, is larger. In Table 3, the data pattern in the "low" category is similar to that in the "medium" category, but not to that in the "high" category. SEL(mini) bases its prediction on values in "medium," while SEL(mean) bases its prediction on mean values.

Table 3. SEL(mini) better than SEL(mean).

level	low (unobserved)	medium value	high
1	X	X	X
2	X	X	X

Let us now go back to Example A to see why SEL(mean) outperforms SEL(mini) there. The data patterns for Example A may be described as in Table 4. In this situation, SEL(mean) correctly selects level 2, while SEL(mini) incorrectly eliminates level 2. The reason is that the data pattern in "low" is similar to that in "high" and to the average of "medium" and "high," but not to that in "medium."

Table 4. SEL(mean) better than SEL(mini).

level	low (unobserved)	medium value	high
1	X	X	X
2	X	X	X

For the comparison of SEL(mean) and SEL(mini), one central question is which of the scenarios in Tables 2 to 4 is more likely to occur. This needs further investigation. From our limited experience including a simulation study of a more complex problem reported in Section 6, it appears that SEL(mini) is quite effective for situations in which substantially better values are yet to be found.

6. Iterative search using orthogonal arrays.

So far we have only considered the use of orthogonal array (OA) and SEL over a fixed grid of factor levels. The technique can be made more powerful if applied iteratively to different regions of the factor space as determined by the results of the previous experiments. This will be illustrated with the following example taken from Zhang and Zheng (1983). It is closely related to the TV power circuit example of Taguchi and Wu (1982, Section 5.3). We do not consider the latter example since some of the figures reported there are not reproducible.

Example C.

The function of a stabilizer circuit in a color TV is to convert a 100V AC input into a 115V DC output. The output voltage E_0 is determined by equation (C). The layout of the circuit can be found in Zhang and Zheng (1983) and closely resembles the one in Taguchi and Wu (1982, Figure 5.5). There are 13 factors that can influence E_0,

$$z_1 \text{ to } z_3 \text{ and } z_5 \text{ to } z_{10} = \text{resistances},$$

$$z_{11} = \text{Zener voltage}, \quad z_{13} = h_{FE} \text{ of } TR_3,$$

$$z_{15} = h_{FE} \text{ of } TR_1, \quad z_{17} = h_{FE} \text{ of } TR_2.$$

The equation is

$$E_0 = \frac{136.67(a + b/z_9) + d(c + e)g/f - 1.2}{1 + (de)/f + b(0.006 + 1.08202/z_9) + 0.08202 \times a}, \qquad \text{(C)}$$

where

$$a = \frac{z_2}{z_1 + z_2}, \qquad b = \frac{1}{z_{15}z_{17}}\left(\frac{z_1z_2}{z_1 + z_2} + z_3\right) + z_{10},$$

$$c = z_5 + \frac{1}{2}z_7, \qquad d = \frac{z_1z_2}{z_1 + z_2}z_{13}, \qquad e = z_6 + \frac{1}{2}z_7,$$

$$f = (c + e)(1 + z_{13})z_8 + ce, \qquad g = z_{11} + 0.6.$$

(In Zhang and Zheng, 1983, the formula for d has a typographical error.)

The objective of design is to choose the nominal values of z_i (treated as control factors) so that the output E_0 is stable around the target 115V in the presence of the variations of z_i (treated as noise factors) around their nominal values. It is known that the nine resistors (z_1 to z_3, z_5 to z_{10}) have 10% variations and the four rectifier tubes ($z_{11}, z_{13}, z_{15}, z_{17}$) have 50% variations. For the noise array, we choose three levels for each noise factor, with the mid-level coming from the control array, and the high and low levels equal to 1.1 and 0.9 of the mid-level for z_1 to z_{10}, and equal to 1.5 and 0.5 of the mid-level for $z_{11}, z_{13}, z_{15}, z_{17}$. We use $L_{27}(3^{13})$ (in Appendix 2) for the noise array. This choice of the noise array is the same as in Zhang and Zheng (1983). For each setting of the control factors (i.e. nominal values of z_i), 27 combinations of noise factor levels are determined by $L_{27}(3^{13})$. For each of the 27 combinations, the E_0 value is computed by equation (C). We then use the mean squared error

$$v = \frac{1}{27} \sum_{1}^{27} (E_0 - 115)^2$$

as a measure of stability of the given setting of the control factors.

We rewrite the factors as

$$A = z_1,\ B = z_2,\ C = z_3,\ D = z_5,\ E = z_6,\ F = z_7,\ G = z_8,$$

$$H = z_9,\ I = z_{10},\ J = z_{11},\ K = z_{13},\ L = z_{15},\ M = z_{17}\ .$$

The layout of the control factor levels is given in Table C.1. We choose the three levels given in Table C.2 for the initial design in any iterative scheme. Before describing the simulation study, we study two iterative schemes more closely. In scheme I of Table C.3, the $L_{27}(3^{13})$ array (Appendix 2) is used for the control array at the initial stage. For the

Table C.1: Layout of Factor Levels

factor	51	52	53	54	55	56	incremental* constant c
A	162	237	348	511	750	1100	10
B	147	215	316	464	681	1000	10
C			same as B				10
D	562	825	1210	1780	2610	3830	10
E			same as D				10
F			same as B				10
G			same as D				10
H			same as D				10
I	1	1.1	1.21	1.33	1.47	1.62	1.78
J	12	12.455	12.927	13.416	13.925	14.452	1.25
K	110	114.168	118.494	122.984	127.644	132.480	1.25
L	45	46.705	48.475	50.312	52.218	54.196	1.25
M			same as K				1.25

*The next six higher (lower) levels are obtained from multiplying (dividing) the six levels displayed in the table by c. Other levels are obtained by using the constant c^2, c^3 and so forth.

Table C.2: Factor Levels of Initial Design

factor	low	level medium	high
A	57	63	69
B	64	70	76
C	50	56	62
D	57	63	69
E	51	57	63
F	53	59	65
G	39	45	51
H	41	47	53
I	45	51	57
J,K,L,M		same as I	

second and subsequent stages, the two combinations with the best v values among all the previous runs are selected. Two control arrays both using $L_{27}(3^{13})$ are chosen for the above two combinations. The mid-levels of each array, denoted by m in Table C.3, are equal to the levels of the corresponding combination. The low and high levels are chosen to be $m - i$ and $m + i$ for the $(7 - i)$th stage (see Table C.3). The gap between levels is smaller for the later stages. The only change for scheme V is that, at each stage, after the experiment with $L_{27}(3^{13})$, SEL(mean) is used to eliminate one level of each factor, and then for the remaining two levels, 16 more runs are conducted according to the array $L_{16}(2^{13})$ (the first 13 columns of the $L_{16}(2^{15})$ array in Taguchi and Wu, 1982). The best two v values for each stage are given in Table C.3. They are consistently better than the v value given by the analysis of marginal means. For each v value we note the control array it belongs to. If the mid-levels of this parental array give the best (resp. second best) v value among the previous runs, we say that this v value has "parent index" 1 (resp. 2).

Three points are observed from Table C.3.

1. Use of SEL in scheme V is effective. This is confirmed by the simulations reported later.
2. As v becomes smaller, some factor levels become more extreme. Consider, for example, the history of runs (Table C.4) that lead to the smallest value .684 in scheme V. The levels of factors A,B,C move down while those of E,F,G,I,K move up.
3. For iterative search, it is good to keep two best combinations, not just one. Consider again the situation in Table C.4. The first and the fifth generations that lead to the smallest value $v = .684$ are the second best in their respective generations. This practice is especially beneficial at the beginning stages when optimum values are yet to be found.

Table C.3: Best v Values from Designs at Different Stages

	stage	design	cumulative sample size	levels	best two values	parent index
I	1st	$L_{27}(3^{13})$	27	(m-6,m,m+6)	133.023, 179.074	
	2nd	2 × same	81	(m-5,m,m+5)	2.622, 5.954	2,1
	3rd	2 × same	135	(m-4,m,m+4)	1.843, 1.890	1,1
	4th	2 × same	189	(m-3,m,m+3)	1.037, 1.340	2,1
	5th	2 × same	243	(m-2,m,m+2)	.857, 1.019	1,1
	6th	2 × same	297	(m-1,m,m+1)	.833, .847	1,1
V	1st	$L_{27}(3^{13}) + L_{16}(2^{13})$	43	(m-6,m,m+6)	133.023, 179.074	
	2nd	2 × same	129	(m-5,m,m+5)	1.751, 2.622	2,2
	3rd	2 × same	215	(m-4,m,m+4)	.845, 1.270	1,1
	4th	2 × same	301	(m-3,m,m+3)	.759, .776	1,1
	5th	2 × same	387	(m-2,m,m+2)	.736, .749	2,1
	6th	2 × same	473	(m-1,m,m+1)	.684, .690	2,2

Five design schemes (see Table C.5) are compared in the simulation study. Schemes I and V were described before. In Table C.5, "$L_{27} + L_{16}$" means that "$L_{27}(3^{13})$ followed by $L_{16}(2^{13})$ with the latter's levels chosen by SEL"; the multiplier 2 denotes the two best combinations chosen at each stage. Both SEL(mean) and SEL(mini) are considered. In the simulations, the columns of $L_{27}(3^{13})$ (resp. $L_{16}(2^{13})$) are generated from random permutations of the 13 columns of the $L_{27}(3^{13})$ array in Appendix 2 (resp. the first 13 columns of the array $L_{16}(2^{15})$ in Taguchi and Wu, 1982, Appendix).

Table C.4: History of Runs Leading to $v = .684$

generation	A	B	C	D	E	F	G	H	I	J	K	L	M	v	parent index
1st	57	64	56	69	63	53	39	47	51	51	51	57	45	179.074	
2nd	52	59	56	74	68	53	44	52	56	56	51	52	40	1.751	2
3rd	48	55	52	70	68	57	48	48	60	56	55	56	40	.845	1
4th	45	52	49	67	68	60	51	45	63	56	58	59	40	.759	1
5th	47	54	47	69	68	58	51	45	65	54	56	61	42	.749	1
6th	46	53	46	68	68	59	52	44	66	54	57	62	42	.684	2

Columns A–M are factor levels.

Based on 100 simulations, the percentiles of the 100 values of v are given in Table C.6 for each design scheme. In the original treatment by Zhang and Zheng (1983), two iterative schemes are considered. One requires 676 runs and yields the best v value 1.54. The other requires 648 runs and yields the best v value 1.34. The percentages of simulations with $v \leq 1.34$ are also reported in the table.

Summary of Table C.6

1. The greatest gain comes from the use of SEL at the initial stage. See the improvement of scheme II over I. Use of SEL at later stages (e.g., in schemes III to V) continue to give improvement, though of a smaller magnitude.
2. SEL(mini) outperforms SEL(mean). The difference is more significant at the beginning stages.

3. Designs using SEL such as schemes II to V are superior to the schemes considered by Zhang and Zheng (1983). For SEL(mini), it takes 183 runs (scheme III) to 215 runs (schemes IV and V) to have at least 50% of $v \leq 1.34$ in the simulations. For SEL(mean), it takes 205 runs (scheme II) to 301 runs (scheme V) to have at least 50% of $v \leq 1.34$ in the simulations. Recall that 1.34 is the best v value out of 648 runs in their study.
4. Our overall recommendation is schemes II and III with SEL(mini).

A more limited study of an iterative search procedure for Example B also confirms the above conclusions. To save space, the results are not given.

Table C.5. Design Schemes and Their Cumulative Sample Sizes (C.S.S.)

	design scheme									
stage	I	C.S.S.	II	C.S.S.	III	C.S.S.	IV	C.S.S.	V	C.S.S.
1st	L_{27}	27	$L_{27} + L_{16}$	43	Same as II					
2nd	$2 \times L_{27}$	81	$2 \times L_{27}$	97	$2 \times (L_{27} + L_{16})$	129	Same as III		Same as IV	
3rd	$2 \times L_{27}$	135	$2 \times L_{27}$	151	$2 \times L_{27}$	183	$2 \times (L_{27} + L_{16})$	215		
4th	$2 \times L_{27}$	189	$2 \times L_{27}$	205	$2 \times L_{27}$	237	$2 \times L_{27}$	269	$2 \times (L_{27} + L_{16})$	301
5th	$2 \times L_{27}$	243	$2 \times L_{27}$	259	$2 \times L_{27}$	291	$2 \times L_{27}$	323	$2 \times (L_{27} + L_{16})$	387
6th	$2 \times L_{27}$	297	$2 \times L_{27}$	313	$2 \times L_{27}$	345	$2 \times L_{27}$	377	$2 \times (L_{27} + L_{16})$	473

Table C.6. Selected Percentiles (5th, 25th, 50th, 75th, 95th) of Simulation Distributions and Percentages (in parenthesis) of Simulations with $v < 1.34$ for Nine Design Schemes (based on 100 simulations).

	design scheme (SEL(mini) for $L_{27} + L_{16}$)				
stage	I	II	III	IV	V
1st	418, 470, 514, 583, 834	19, 38, 63, 81, 158			
2nd	20.0,29.9,38.4,51.9,69.7	.88,1.8, 3.8, 6.6,19.5	.82,1.1, 2.3, 4.5,8.4		
3rd	2.4, 4.0, 6.3, 8.0,17.3	.79,1.00,1.4, 3.1, 7.8	.77, .93,1.2, 2.8,5.7	.63,.86,1.0, 2.5, 5.3	
	(0%)	(44%)	(54%)	(64%)	
4th	1.3, 2.1, 3.3, 4.7,13.8	.70, .85,1.1, 1.7, 4.7	.60, .83, .97,1.5,4.4	.58,.80, .93,1.5, 4.5	.56,.78, .92,1.4, 3.7
	(7%)	(69%)	(72%)	(74%)	(74%)
5th	1.1, 1.5, 2.2, 3.5,12.8	.61, .80, .96,1.2, 3.6	.60, .78, .93,1.3,3.8	.55,.78, .90,1.2, 3.1	.53,.74, .86,1.1, 3.1
	(23%)	(80%)	(75%)	(78%)	(81%)
6th	1.0, 1.2, 1.9, 2.8,12.2	.55, .77, .92,1.1, 2.8	.55, .76, .88,1.0,2.7	.50,.75, .85, .99,2.5	.50,.68, .81, .98,2.0
	(27%)	(84%)	(85%)	(87%)	(87%)

	design scheme (SEL(mean) for $L_{27} + L_{16}$)			
stage	II	III	IV	V
1st	26, 60, 90, 133, 275			
2nd	.88,2.1, 4.4, 8.0,16.9	.84,1.5, 3.5, 4.8,9.3		
3rd	.81,1.0, 1.9, 3.5, 5.7	.75, .97,1.8, 3.5,4.8	.73,.91,1.5, 3.1, 4.6	
	(37%)	(40%)	(44%)	
4th	.77, .88,1.2, 1.9, 4.0	.67, .83,1.1, 2.0,4.0	.59,.82,1.1, 2.3, 4.2	.55,.81,1.1, 2.2, 4.2
	(63%)	(58%)	(59%)	(60%)
5th	.61, .81,1.0, 1.4, 3.2	.61, .80, .98,1.5,3.3	.53,.80, .96,1.6, 3.5	.53,.80, .94,1.5, 3.5
	(73%)	(70%)	(68%)	(71%)
6th	.56, .79, .93,1.2, 2.7	.53, .78, .91,1.3,2.8	.53,.79, .90,1.3, 2.7	.44,.76, .88,1.3, 2.6
	(82%)	(78%)	(77%)	(79%)

7. Concluding remarks.

SEL (sequential elimination of levels) is a method that enables the experimenters to quickly focus on regions of promise by eliminating factor levels with poor results. Its general utility is supported by our study of the three examples. When the current guess is far from optimum and search over a wide grid is desired, SEL(mini) is especially effective in accelerating the convergence to optimum. It is usually used at the beginning stages of the experimentation. To reduce the number of iterations and the size of arrays at subsequent stages, several levels may be eliminated at one time. SEL(x) with the descriptive statistic x other than mean or minimum may be promising and deserves further study.

Our study does not mean to suggest that the "analysis of marginal means" (AM) method should be totally rejected. Its simplicity appeals to users especially when presented graphically (see, for example, the many case studies reported at the Supplier Symposiums on "Taguchi Method" organized by the American Supplier Institute). However, the method should be used with prudence. Interactions should be considered and confounding patterns noted. The setting determined by the AM method is often inappropriate as shown by our study. Further experimentation may be conducted in its neighborhood in the hope of getting better settings. Its strategy is based on information from the analysis of experimental data as well as engineering judgement. SEL provides a convenient way of doing this by retaining only one to three levels (for each factor) around the setting chosen by AM.

Perhaps the most striking feature of SEL is its deviation from traditional design strategies based on the concept of averaging (e.g., main effects and interactions) and on empirical model building (e.g., response surface methodology, Box and Draper, 1986). When the response surface is too rugged or search over a wide grid is desired, an empirical model that fits the data well may not be useful for predicting where better settings can be found. SEL works by eliminating poor factor levels and focusing in the neighborhood of the best current settings for iterations. Comparisons of these strategies deserve further investigation.

Other possibilities should also be considered. For example, all the iterative schemes in Section 6 select the best two combinations for the next iteration. Other methods for iterations such as directional search and the simplex method (Nelder and Mead, 1964) may be compared or incorporated. For mathematical models there are better ways for choosing the control factor levels than the strict use of orthogonal arrays. See, for example, the discussion at the end of Section 4. This will be reported elsewhere.

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Appendix 1: $L_{18}(3^6)$

run	I	II	III	IV	V	VI
1	1	1	3	2	2	1
2	2	1	1	1	1	2
3	3	1	2	3	3	3
4	1	2	2	1	2	3
5	2	2	3	3	1	1
6	3	2	1	2	3	2
7	1	3	1	3	1	3
8	2	3	2	2	3	1
9	3	3	3	1	2	2
10	1	1	1	1	3	1
11	2	1	2	3	2	2
12	3	1	3	2	1	3
13	1	2	3	3	3	2
14	2	2	1	2	2	3
15	3	2	2	1	1	1
16	1	3	2	2	1	2
17	2	3	3	1	3	3
18	3	3	1	3	2	1

Appendix 2: $L_{27}(3^{13})$

level	A	B	C	D	E	F	G (factor)	H	I	J	K	L	M
1	1	1	3	2	1	2	2	3	1	2	1	3	3
2	2	1	1	1	1	1	3	3	2	1	1	2	1
3	3	1	2	3	1	3	1	3	3	3	1	1	2
4	1	2	2	1	1	2	2	2	3	1	3	1	1
5	2	2	3	3	1	1	3	2	1	3	3	3	2
6	3	2	1	2	1	3	1	2	2	2	3	2	3
7	1	3	1	3	1	2	2	1	2	3	2	2	2
8	2	3	2	2	1	1	3	1	3	2	2	3	3
9	3	3	3	1	1	3	1	1	1	1	2	1	1
10	1	1	1	1	2	3	3	1	3	2	3	3	2
11	2	1	2	3	2	2	1	1	1	1	3	2	3
12	3	1	3	2	2	1	2	1	2	3	3	1	1
13	1	2	3	3	2	3	3	3	2	1	2	1	3
14	2	2	1	2	2	2	1	3	3	3	2	3	1
15	3	2	2	1	2	1	2	3	1	2	2	2	2
16	1	3	2	2	2	3	3	2	1	3	1	2	1
17	2	3	3	1	2	2	1	2	2	2	1	1	2
18	3	3	1	3	2	1	2	2	3	1	1	3	3
19	1	1	2	3	3	1	1	2	2	2	2	3	1
20	2	1	3	2	3	3	2	2	3	1	2	2	2
21	3	1	1	1	3	2	3	2	1	3	2	1	3
22	1	2	1	2	3	1	1	1	1	1	1	1	2
23	2	2	2	1	3	3	2	1	2	3	1	3	3
24	3	2	3	3	3	2	3	1	3	2	1	2	1
25	1	3	3	1	3	1	1	3	3	3	3	2	3
26	2	3	1	3	3	3	2	3	1	2	3	1	1
27	3	3	2	2	3	2	3	3	2	1	3	3	2

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