

MATHEMATICAL MODELING OF PHYSIOLOGICAL  
CHARACTERISTICS IN FEMALE SOCCER  
ATHLETES

by

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**ABSTRACT**  
**MATHEMATICAL MODELING OF PHYSIOLOGICAL**  
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The University of Wisconsin-Milwaukee, 2013  
Under the Supervision of Professor Bruce Wade

Intermittent sports create challenges regarding performance measurement. Quantification of various physiological characteristics can lead to increased performance and injury reduction throughout a season of competition. Currently, a variation of an athletes' heart rate is the primary physiological characteristic used for quantifying load on the athlete. With increasing technology, we have the ability to gather additional characteristics regarding the physicality of athletes during competition. This study statistically compares various models using these new characteristics as predictors to the athletes' lactate concentration in their blood. From this comparison, we determine which physiological characteristic(s) best represent the performance and fatigue of these athletes. In addition to the characteristics, this study evaluated the best method of quantifying the load on an athlete based on the best-fit model of the aforementioned variables. Finally, we use this model to calculate the load experienced by these athletes in a practice session to understand how this physical load quantity can be utilized in practices, matches and recovery.

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## ***Introduction***

Modeling performance and fatigue within sports of an intermittent nature (i.e. soccer, hockey, football etc.) poses different challenges compared to continuous sports (i.e. running, triathlon etc.). Intermittent sports generate a short period of elevated intensity, or load, followed by lower levels of load on an individual or team. Many attempts at modeling this type of performance have proven successful. The personnel that executed these previous studies possess strong backgrounds in physiological studies of the body. However, this study focuses on the mathematics and statistics behind these studies.

In order to understand the load on these athletes we need to introduce a metric used to quantify the heart rate of an individual during exercise, the Fractional elevation of heart rate (FEHR). FEHR is defined by equation 2 below:

$$FEHR = \frac{HR_{obs} - HR_{rest}}{HR_{max} - HR_{rest}} \quad (1)$$

Where  $HR_{obs}$  is heart rate at exercising,  $HR_{rest}$  is resting heart rate and  $HR_{max}$  is the maximum heart rate for an individual. FEHR provides a metric that compares the current heart rate of the athlete to their resting and maximum heart rates.

In an effort to understand fitness and fatigue of athletes, Banister, Calvert, Savage, & Bach (1975) introduced the idea of a Training Impulse, referred to as a TRIMP. A TRIMP is an arbitrary measurement that multiplies the training load (Stress) by the training intensity (Strain) (Bannister et al., 1975). Stress was defined as the amount of exercise in the bout, usually measured in minutes. Strain was calculated by

using the average heart rate of an exercise session to equate the FEHR. Equation 1 below describes this calculation.

$$TRIMP's = (Stress) \times (Strain) = t \times FEHR \quad (2)$$

Where  $t$  is the amount of exercise (duration of exercise) and FEHR is the fractional elevation of heart rate calculated using the average heart rate of the individual. Let's consider a soccer athlete that has a large range for their heart rate measurements (for example low of 120, high of 185) and an endurance athlete that has a more consistent heart rate (average of 150 bpm) during a 2 hour exercise bout. Both of these athletes could end up with the same TRIMP's, however, physiologically speaking the soccer athlete would experience more fatigue than the endurance athlete. For this reason, Foster, Flouhaug, Franklin, Gottschall, Hrovatin, & Parker (2001) introduced a new method for measuring TRIMP's developed for intermittent athletes.

Rather than use the average heart rate Foster et al. (2001) suggested to break the session into stages (or zones), each identified at a given heart rate to create a weighting factor based on the zone the athlete is experiencing (Foster et al., 2001). The amount of time an athlete spent in a zone would be multiplied by the weighting factor associated with each zone. The values were then summed to create the TRIMP value. One opportunity for improvement of this method was that the zones were modeled in a linear fashion; however, previous studies have shown that when an athlete reaches anaerobic threshold the lactate curve (an indicator of load on an athlete) does not follow a linear pattern (Wasserman, 1987).

Enter Stagno, Thatcher, & Van Someren (2007) who provided the groundwork for our study. They created a method of quantifying this load in intermittent sports by using a modified TRIMP method (Stagno et al., 2007). Stagno et al. (2007) believed that previous TRIMP methods were not efficacious indicators of work for intermittent sport athletes' performance because it used the mean heart rate. Using the mean heart rate is not relevant to intermittent athletes, since the load varies significantly during exercise (Stagno 2007). To combat this Stagno et al. (2007) used the lactate response of the athlete to determine the weighting factors. As we will see in this paper, the lactate curve follows an exponential pattern. The physiology behind this is that at higher intensities the weighting factor needs to indicate greater load on the athlete. These weighting factors were then multiplied by the time spent in each zone to get the modified TRIMP value. Even this weighting factor can be improved upon; this paper addresses such improvements. The weighting factors of Foster et al. (2001) and Stagno et al. (2007) are compared in Table 1 below.

**Table 1:** Comparison of weighting factors for Foster et al. (2001) and Stagno et al. (2007).

<u>Zone #</u>	<u>Foster et al. (2001)</u>	<u>Stagno et al. (2007)</u>
1	1	1.25
2	2	1.71
3	3	2.54
4	4	3.61
5	5	5.16

Rob Wilson most recently studied these improvements in his PhD dissertation at the University of Wisconsin – Milwaukee. Wilson examined female soccer athletes to understand the physical load quantity (PLQ), a new variation on the TRIMP (Wilson

2012). PLQ describes the load an athlete experiences by using a modeled equation for an athlete's load rather than breaking the exercise session into zones.

The goal of Wilson's study was to generate a model to predict the performance of National Collegiate Athletic Association (NCAA) Division I competitive women's soccer team to reduce injury and maximize performance. To understand this PLQ Wilson examined various characteristics (heart rate, breathing rate, body temperature etc) to generate an athlete's lactate curve. This lactate curve characterizes the work load experienced by an athlete (described in more detail in the next few paragraphs). This model would then provide a weighting factor for the intensity of an exercise session. Because of the heavy involvement of statistics in Wilson's dissertation, the Department of Mathematical Sciences at The University of Wisconsin – Milwaukee worked to support Wilson. By combining these two departments with a similar goal we were able to better understand how to model these intermittent sports and provide an end product that coaches can use to schedule their season.

The primary response variable used to indicate the PLQ an athlete has endured is their lactate concentration in their blood ([HLA]). Lactate is a by-product created during intense exercise that accumulates within the body tissue. The body then works to remove lactate from the system. However, if the intensity is high and the duration is long, the rate that lactate is created can exceed the rate the body can remove the lactate. As this concentration increases, it leads to more fatigued athletes. As athletes get more fatigued, recovery time is necessary to allow [HLA] to decline.

Due to the aforementioned physiological effects, [HLA] has become the preferred indicator for work load experienced by an athlete. Measuring [HLA] levels in individuals

is an intravenous process, creating difficulty in assessing the individuals' load. Rather than using [HLA], many instances exist where heart rate (or an iteration of heart rate) is used as a predictor for the [HLA] accumulation. By using heart rate to determine the [HLA] build-up, one can understand the load an athlete experienced during an exercise session. Stagno et al. (2007) introduced a model using heart rate stages (mentioned above) compared with the fractional elevation of heart rate (sometimes referred to the %HRR known as percent heart rate reserve). Stagno et al. (2007) used this for his independent variable for predicting [HLA] concentration. A primary concern in Stagno's model is the starting point of FEHR. Stagno et al. (2007) began their scaling at 65 % FEHR; however, with intermittent sports an athlete's FEHR can drop well below 65 % FEHR during practice or competition.

In addition to gathering data below the 65 % FEHR, technological advancements have allowed physiologists to gather additional data. Some additional variables (other than heart rate) that can now be measured include Skin Temperature (ST), Ventilation Rate (VR), Posture, Peak Acceleration (PA), % HR Max, and Activity (movement intensity). With the assistance of the University of Wisconsin Milwaukee Human Performance Lab, we were able to compile data from the women's soccer team. Physiological tests were performed in a range of intensities (less than 65 % FEHR) that allow us to expand beyond the intensities of previous models and incorporate these newly available variables. We intend to model [HLA] using various iterations of these variables.

## 1. Methods

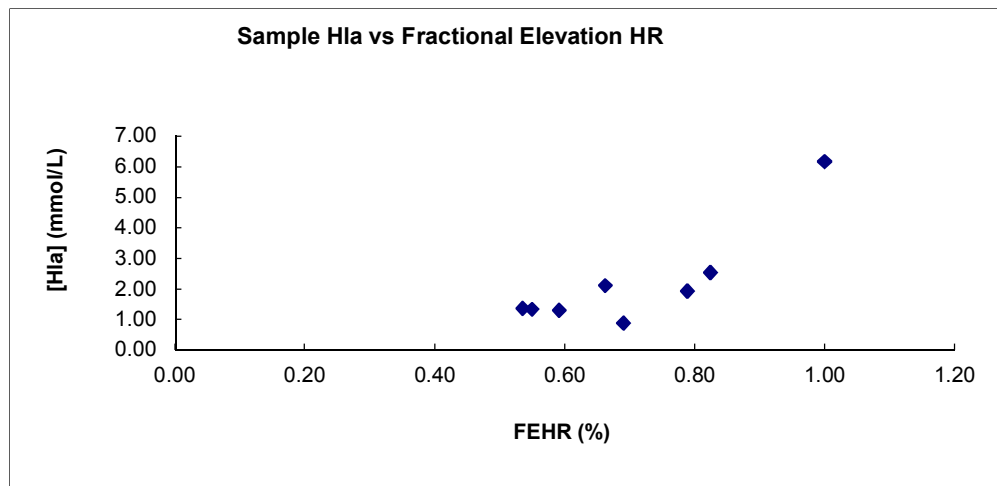
### *Background*

Data for 22 athletes (n=22) playing for a National Collegiate Athletic Association (NCAA) Division I competitive women's soccer team were provided from the Human Performance Lab at The University of Wisconsin - Milwaukee for analysis. Female athletes were studied due to familiarity of the research team with the female coaching staff. In addition, the men's team was in the midst of hiring a coach, therefore the men's coaches/team was not pursued immediately. Table 2 provides the demographics of the athletes tested. The necessary precautions were taken during testing, and consent was obtained from all individuals.

**Table 2:** Demographic data for the 22 female soccer athletes tested. BMI = Body Mass Index.

<u>Demographic</u>	<u>Mean</u>	<u>Std Dev</u>	<u>Max</u>	<u>Min</u>
Age (yr)	19.5	1.14	22	18
Height (in)	65.58	2.385	71	62
Weight (lbs)	135.57	13.51	158	110
BMI	22.19	1.729	25.2	19.3

Two types of physiological tests were conducted: a submaximal (submax) and maximal (max) test. Both tests were performed on treadmills. Submax tests were conducted in the Pre-season. The max test data was generated mid-season, during a latent period of the season. This was done to provide minimal fatigue in the athletes from a grueling season (end of season) or athletes not yet developed fully (pre-season). The max test provides the maximum value (single data point) for [HLa] of the individual. The submax test keeps the individual below a peak level of intensity. These two tests combined create the [Hla] curve of the individual. [HLa] is typically modeled using FEHR as the independent variable, which typically appears like Figure 1 below (exponential or polynomial-like).



**Figure 1:** Typical graph of [HLA] vs. FEHR for a single athlete. The first 7 data points represent the submax test and the last data point represents the max data point.

Prior to each test, athletes were fitted with a Zephyr<sup>TM</sup> Physiological Status Monitoring (PSM) device developed by Zephyr Technologies Inc., Auckland, New Zealand. This device provided the data necessary for this study: heart rate, skin temperature, breathing rate, posture, peak acceleration and activity. The Zephyr<sup>TM</sup> has the ability to take 59 data points in one minute. In addition, athletes wore the Polar Team1 Heart Rate Monitoring System (Polar Electro Oy, Kempele Finland) to capture heart rate data. The Polar captures 12 data points in one minute. These devices are worn inferior to the pectoral muscles of each individual. Table 3 describes each characteristic captured by the Zephyr<sup>TM</sup> and the Polar. At select times of the experiment (depending on the type of test being executed), blood samples were gathered from the fingertip of the individual to determine their lactate level.

**Table 3:** Variables gathered via the Zephyr™ (or Polar) device used for this study.

<u>Variable</u>	<u>Description</u>	<u>Units</u>
Hla	Blood Lactate Concentration	Millimole/L
HR_abs	Heart Rate measured by the Polar Team1 Heart Rate Monitor	beats/min
FEHR	$[\text{HR}(\text{observed}) - \text{HR}(\text{rest})] / [\text{HR}(\text{max}) - \text{HR}(\text{observed})]$	%
HR_Zephyr	Heart Rate measured by the zephyr device	beats/min
Skin Temp	Skin Temperature	Degrees Celsius
Breathing Rate	Breathing rate	Breaths/min
Posture	Posture taken as degrees from normal (vertical)	degrees
HR%	The HR calculated as a percent to maximum	%
Peak Accel	The largest acceleration value in any one axial direction	g force
Activity	Time integration (over 1 second intervals) of an average of three axial accelerations	g force

### *Submaximal Testing*

Submax testing is performed keeping the individual below their maximum lactate levels. This is required to ensure the athlete does not get fatigued during the test. In this test, athletes completed 6 minute bouts (called a stage) of running/walking on a treadmill at an elevation of 1% to simulate resistance comparable to running outdoors (Jones, 1996). Target heart rate for the start of the first stage was 140 bpm (~70% maximum heart rate and ~57 % FEHR). This heart rate was chosen because it should result in at least two stages before the participants reached their lactate threshold. After the first stage, the treadmill was increased in speed by 0.5 miles/hour for each subsequent stage. As the test progressed, each stage would demand more exertion from the athlete. At 30 seconds prior to the end of a stage the athlete would grab the treadmill front rail for stability to allow for the blood draw. Tests were stopped one stage after the participants reached their lactate threshold (measured as greater than 4 mmol/L and an increase of at least 1 mmol/L from the previous stage). Athletes were allowed to continue past this point if they consent. This would provide additional data for better modeling between lactate threshold and maximum lactate values (determined from the max test). Upon

completion of this test, the data from the Zephyr<sup>TM</sup> and Polar were downloaded for analysis in conjunction with the lactate measurements.

### *Maximal Testing*

Maximal testing obtains the maximal lactate value. This is a shorter test completed to exhaustion. In this test, each stage was 1 minute in length. For stage 1, the treadmill started with a speed of 6 km/hr and an incline of 0 %. At each even stage the speed was increased by 1 km/hr and the incline unchanged and at each odd stage the incline was increased by 2 % with the speed unchanged. One of the criteria below would terminate the test:

- Called by the research team for any safety concerns
- No change in oxygen uptake ( $O_2$ ) with an increase in workload from one stage to the next.
- Cessation by Athlete
- Respiration Exchange Ratio (RER) greater than or equal to 1.1 (vol expire  $CO_2$ /vol inspired  $O_2$ )

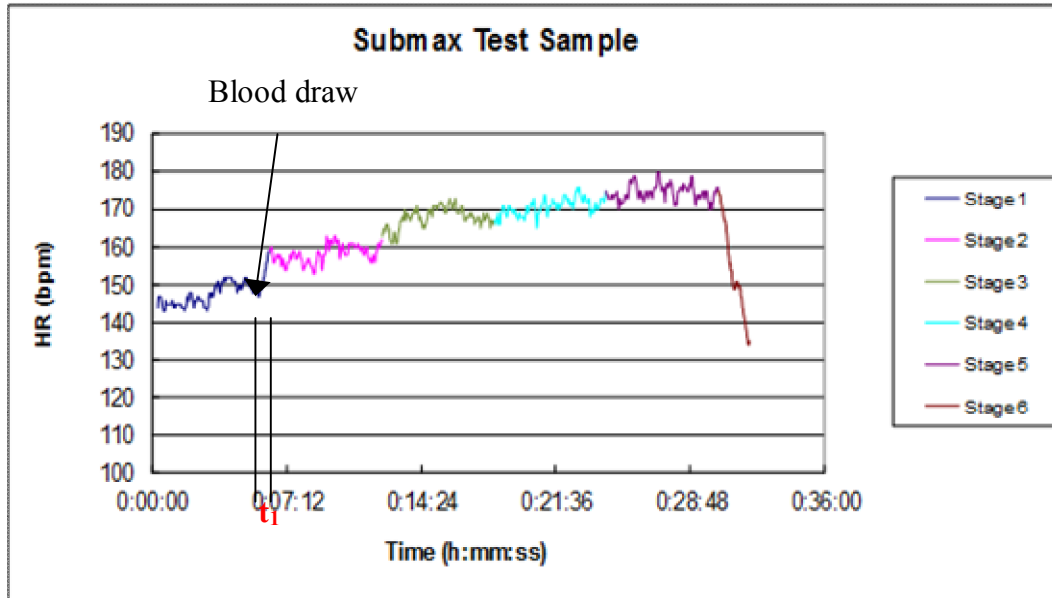
One minute after test completion the lactate of the individual was gathered and analyzed with the data from the PSM. Three physiological criteria for a test to be considered a successful maximal effort test are listed below:

- High  $O_2$
- High [HLA]
- Within 10 beats per minute of predicted  $HR_{max}$

### *Data Setup*

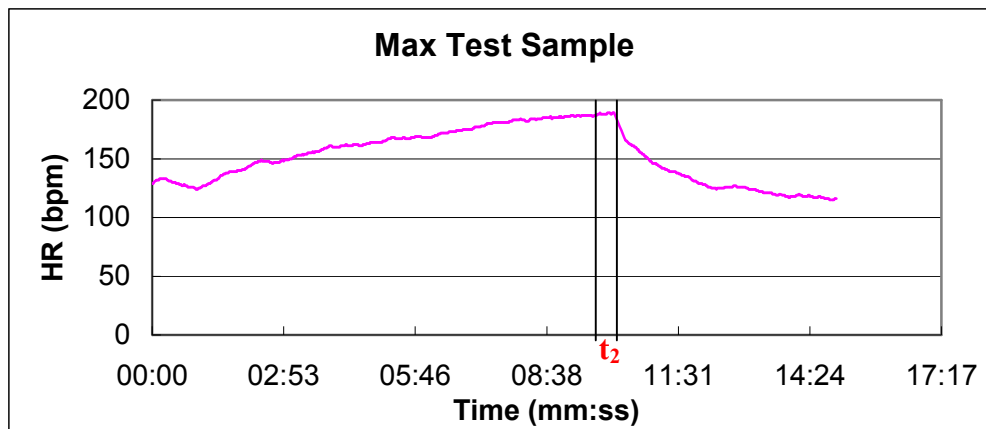
Gathering the variables previously discussed is straightforward by utilizing the Zephyr™ system. However, coordinating the blood draw with the Zephyr™ data provided some difficulties. The primary challenge was reconciling the blood draw with the time stamp of the data from the Zephyr™.

We will first discuss how the submax data was gathered. Since each athlete was providing the lactate measurement 30 seconds before the end of the stage we could use the start time of the test to pinpoint the time of the blood draw. Due to the nature of the submax test we would expect a plateau in the heart rate (submax test keeps same treadmill speed for 6 minutes). Depending on the nature of the heart rate curve, we would average data points near this blood draw to get the heart rate value corresponding to the blood draw. An example where the exact time may not be used could be due to a dip in the heart rate, as the athlete would grasp the treadmill bar for the sample. For this reason, we averaged 30 seconds of data prior to the sample draw time. As an example, stage 1 of an athlete begins at time  $t=0$  min and ends at  $t=6$  min. At 5 minutes 30 seconds the athlete grabbed the treadmill front rail for stability while the blood sample was drawn. At this 5 minute 30 second point we would average data points previous to this draw that show no erratic movement. Figure 2 below depicts this process graphically. With this process we now have a time interval that represents the heart rate of the individual compared to the blood draw. This time interval is then used to obtain the average of the other variables available via the Zephyr™ (see Table 3 above).



**Figure 2:** Submax test sample displaying the distinct stages (heart rate with time). Each stage we pinpointed the time of the blood draw and averaged the data points prior to the draw to get the time interval  $t_1$ .

Max test data was gathered as a single blood draw. With the single blood draw, it was less challenging to gather the heart rate at blood draw. Observing the heart rate we would see the value ramp up until it rapidly ramped down indicating the test was complete (heart rate drops rapidly when exertion is ceased), see Figure 3 below. The data points that are on this heart rate peak were averaged to provide the heart rate value. The same time interval was used to gather the other variables.



**Figure 3:** Notice the ramp of the heart rate until a peak is reached and the rapid drop in heart rate. This signifies the end of the test. The data leading up to the peak (indicated in the interval labeled  $t_2$ ) was analyzed to determine the length of the interval the heart rate was at the maximum. This interval was used to determine the value for the other physiological characteristics.

The initial plan was to obtain submax and max data from the same time of the season for each athlete. Factors that made this not possible include: varying training schedules, limited lab time, and sick athletes. With this in mind, we focused on the submax data from the pre-season and the max data from the mid-season. Of the 22 athletes tested (29 athletes total, 7 not tested), seven athletes were unable to provide submax data, and ten athletes were unable to provide max data. This left five individuals (2, 15, 16, 22, G) that were able to provide submax and max data, as described in Table 4 below. With the limited individual athletes' data, we chose to combine all data for a team analysis.

In order to determine entire teams' lactate curve we combined all data regardless of the time of the season it was gathered. In this method, of the 22 athletes that provided data we had submax data from 15 athletes and max data from 12 athletes. All data was combined to create a single plot to represent all athletes on the team (with the exception of the goaltenders). Table 4 describes what athletes' data was available for these graphs.

**Table 4:** All athletes tested and what data was available for analysis. An “X” indicates that the data was available.

Subject	Sub-Max	Max	Comments
1		X	Sub-max data not usable, very erratic
2	X	X	
3	X		Max data not available.
4	X		Max data not available.
5	X		Max data not available.
6			No data available
7	X		Max data not available.
8	X		Max data not available.
9			No data available
10		X	Sub-max data not available
11		X	Sub-max data not available
12		X	Sub-max data not usable
13	X		Max data not available.
14	X		Max data not available.
15	X	X	
16	X	X	
17		X	Sub-max data not available
18		X	Sub-max data not available
19	X		Max data not available.
20	X		Max data not available.
21		X	Sub-max data not available
22	X	X	
A			
B			
C	X		Max data not available.
D			Goaltender
E			Goaltender
F			Goaltender
G	X	X	

## 2. Mathematical & Statistical Modeling

### *Heart Rate Determination*

In addition to the Zephyr™ system, the Polar Team1 was used to gather Heart Rate data. The reason for both monitors was to draw conclusions between the two. It is not necessary to use the heart rate data from both the Zephyr™ and the Polar; however, we wanted to ensure the data was not statistically different. To do this a paired sample t-test was performed with the heart rate data from both devices. The paired t-test takes the

difference in each data point (from the Zephyr™ and Polar devices) then observes the mean of these differences. The mean of these differences is then analyzed to determine how it compares to the value of 0. The t-test was analyzed using Minitab version 16 developed by Minitab Inc., State College PA and results in the parameters shown below in Table 5.

**Table 5:** Paired t-test for heart rate values from the Zephyr and the Polar device shows (p-value > .05) we cannot discern between the two devices.

Paired T for HR_abs - HR_Zephyr				
	N	Mean	StDev	SE Mean
HR_abs	101	167.21	19.19	1.91
HR_Zephyr	101	166.36	18.69	1.86
Difference	101	0.847	7.333	0.730

95% CI for mean difference: (-0.600, 2.295)  
T-Test of mean difference = 0 (vs not = 0): T-Value = 1.16 P-Value = 0.248

For this t-test our hypotheses are below

Null hypothesis,  $H_0: \mu = 0$

Alternate hypothesis  $H_a: \mu \neq 0$

Where  $\mu$  is the mean value of the differences between each Zephyr™ and Polar value. From the data in Table 5 we see that the p-value (which provides the probability of making a Type 1 error, that is, incorrectly rejecting  $H_0$ ) for this test is .248. With  $\alpha = .05$  we see that our p-value is greater than  $\alpha$  so we would fail to reject our null hypothesis. In addition, we see the 95 % confidence interval for the mean of the differences is (-.600, 2.295), providing that we can be 95 % sure that the difference falls within this interval. Thus, our test value of zero is contained in this interval indicating that we cannot discern between the two measuring devices. With this evidence, it does not matter which device we use when analyzing our heart rate. We chose to use the data from the Polar as this

was also used to calculate their max heart rate in the calculation of the fractional elevation of heart rate and the Polar device was worn by individuals while in the field.

### *Single Variable Modeling*

Our first step in modeling was to understand how each variable behaved with respect to the [HLA] measurements. Scatter plots of the combined team [HLA] values versus each independent variable are available in Appendix A (8 independent variables). Of the eight plots, five (Skin Temp, Breathing Rate, Posture, Activity, Peak Acceleration) do not display a recognizable pattern in the data, thus there was no modeling performed on these variables. The data in the other 3, Heart Rate, FEHR, and HR % appear to have an exponential or polynomial behavior, however, we recognize for the Heart Rate the values are quite large, which is not possible to dampen the exponential curve, since for large values, the exponential grows rapidly. This fit was attempted, but led to a near linear fit, which does not depict the data. HR %, the % of maximum heart rate values, was not considered in final modeling as the range is limited compared to the FEHR. For example, an athlete with a resting heart rate of 60 beats per minute and maximum heart rate of 200 beats per minute would have their HR % range from .3 to 1.0 and their FEHR range would 0 to 1. Therefore we use the FEHR for our modeling. In addition, the FEHR allows better comparison between individuals as it gives an idea of where each athlete's heart rate is compared to their own resting and maximum heart rate. The remainder of the study was carried out analyzing the FEHR with exponential, polynomial and affine models.

Previous work (Stagno et al, 2007) performed exponential modeling in the form below (TRIMP mod model):

$$[HLa] = \theta_0 \exp(\theta_1 * (FEHR)) \quad (3)$$

where [HLa] is the lactate value,  $\theta_i$  ( $i=0, 1$ ) are unknown parameters determined by the best-fit model, and FEHR is the FEHR. Our data will be modeled using equation 3 above along with an enhanced exponential equation. Because our data exhibits a modified version of an exponential some form of shifting and scaling of the equation 3 is performed. To account for this scaling and shifting we modeled the data based on equation 4 below (affine transformation of equation 3):

$$[HLa] = \theta_0 \exp(\theta_1 * (FEHR)) + \theta_3 \quad (4)$$

Where [HLa] is the lactate value,  $\theta_i$  ( $i = 0, 1, 2$ ) are unknown parameters determined by the best-fit model, and FEHR is the FEHR.

To determine these unknown parameters we used Minitab 16 statistical software by Minitab Inc., State College, PA. We first obtain three approximate values for the  $\theta_i$ 's by taking three points and solving three equations with three unknowns. Minitab then uses the Gauss-Newton method which uses the least squares estimation to run through iterations of the  $\theta_i$ 's until it finds the best-fit equation by minimizing the sum of squares error (SSE). The Minitab output from this process can be found in Appendix C. The information provided by the Minitab output is detailed below:

#### 1. Method

- a. Gauss-Newton method previously described.
- b. Max iterations is the maximum iterations Minitab will attempt to find the best-fit equation.

*c.* Tolerance

2. Starting parameter values lists the starting values determined by solving three equations in three unknowns.
3. Equation provides the equation of the best-fit equation
4. Parameter estimates provides the estimates of the parameters from using the Gauss-Newton iterations minimizing the sum of the square errors in the model. This also provides the equation type used to fit the data.
5. Lack of fit provides statistics if the model contains replicates (FEHR values with varying Lactate values). We can use the p value in the lack of fit area to determine if there is a true lack of fit. This information only becomes useful for the team data, which provides replicates.
6. Summary provides the summary statistics of the model
  - a.* Iterations is the number of iterations the software used to minimize the SSE. Fewer iterations will indicate that the software was more easily able to identify the best-fit model.
  - b.* Final SSE is the sum of the square error for the model, the value the software worked to minimize.
  - c.* DF is the degrees of freedom for the model.
  - d.* MSE is the mean square error calculated by dividing the final SSE by the DF.
  - e.* S is the root mean square, the square root of MSE. S describes the deviation of the actual data to the fitted values. This is the primary

statistic used in comparing our models, as it provides the error in the same units as the data (mmol/L).

Now that we understand what the various statistics represent, the modeling will be presented in the following order.

1. Team Data – Polynomial
2. Team Data – Exponential
3. Team Data – Affine
4. Individual Data – Exponential
5. Individual Data – Affine

The above models' statistics are available in Appendix D with the analysis continuing below.

#### 4. Mathematical/Statistical Analysis

##### *Polynomial Data – Team Data*

The team raw data (as a scatterplot) can be seen in Figure A-2 (found in Appendix A). Based on the nature of the data, it was meaningless to model using a 2<sup>nd</sup> degree polynomial, as a parabolic shape is not evident. We then modeled this data as a 3<sup>rd</sup> degree polynomial in Figure D-1 (Appendix D). From this regression, we recognize an R-squared value of 77.9 % and an S (RMSE) of 1.32, which provide a reasonable fit. However, if we extend the curve to smaller values for the FEHR (Figure D-2) we notice that the given model of FEHR generates negative values for [HLA] as FEHR gets smaller. The value where this curve crosses the line [HLA]=0 is .213. Any FEHR values less than this value generates a negative lactate, which is not physically possible. For this reason, we did not model the individual data using a 3<sup>rd</sup> degree polynomial and performed exponential and affine regression from this point forward (for team and individual data).

### *Exponential vs. Affine Modeling*

Since the polynomial modeling proved ineffective, we modeled the team and individual data using exponential and affine regression. Appendix D contains the exponential and affine regressions along with the 3<sup>rd</sup> degree polynomial regression (team only) for the team and individual data. Table 6 (below) provides a summary of the plots in Appendix D.

**Table 6:** Glossary of figures in Appendix D. Figures D-1 and 2 have already been discussed and discarded.

<b>Figure #</b>	<b>Data Type</b>	<b>Regression</b>
AIV-1	3rd Degree Polynomial	Team
AIV-2	3rd Degree Polynomial Extended	Team
AIV-3	Exponential	Team
AIV-4	Affine	Team
AIV-5	Exponential	Athlete 2
AIV-6	Affine	Athlete 2
AIV-7	Exponential	Athlete 15
AIV-8	Affine	Athlete 15
AIV-9	Exponential	Athlete 16
AIV-10	Affine	Athlete 16
AIV-11	Exponential	Athlete 22
AIV-12	Affine	Athlete 22
AIV-13	Exponential	Athlete G
AIV-14	Affine	Athlete G

Appendix E contains a table of the key statistics generated by the models in Table 6.

We first compared the team models to the individual models (exponential and affine). We observe the final SSE values and notice the team models exhibit a much higher final SSE. This can be described by the degrees of freedom of the model. Since the DF is much higher for the team data we would expect a higher degree of deviation from the fitted line in the team data. The DF is much higher due to more data points. Also, there are replicates (different Lactate values for the same FEHR values) in this data due to combining multiple athletes in one model, which leads to a higher degree of

variation in the [HLA] values for particular FEHR values. For example, athlete 3 exhibits an [HLA] value of 1.44 at FEHR of .80 and athlete 5 has an [HLA] value of 3.90 at FEHR of .80. This athlete variation creates a much higher SSE compared to the individual data. These higher SSE values result in higher values for RMSE (for the team model). Individual data does not generate these replicates, creating a better RMSE value, indicating that the individual models have less variation from the model. With the combination of replicates and higher RMSE values, we conclude that an individual model will produce a more accurate description of an individuals' load in practice or a match. The remainder of the discussion will use the individual models only.

When comparing the individual models to each other we want to figure out which regression, exponential or affine, creates a better fit. On initial assessment we notice that athlete G has very high SE estimate values for the parameter estimates in the affine regression. This is understandable by looking at the plot in Figure D-14 (see Appendix D), which is nearly linear. These SE estimates indicate a poor fit, so this model was not used to determine a best-fit line for athlete G. A recommendation would be to validate the data with athlete G, or understand the physical fitness of athlete G in more detail. Based on the RMSE values for the remaining athletes we see that athletes 2 and 15 exhibit a better affine regression (based on the RMSE value) and athletes 16 and 22 have nearly the same RMSE values (exponential and affine). Since 16 and 22 have nearly the same RMSE values we continue to look at the best fit equation for both regressions for all athletes. We notice the differences between the exponential and affine equations by observing the each function evaluated at an FEHR of 0. Table 7 below shows these values. The average of this value for the individuals affine regression (excluding athlete

G) is 1.092 and for the exponential regression (excluding athlete G) is .079. Research suggests that a resting conditioned athlete (FEHR = 0) will exhibit [HLa] values around 1. For this reason, we conclude that the affine regression equations for the individuals (excluding athlete G) are a better model for predicting an athletes [HLa] values.

**Table 7:** This table provides the value of each function evaluated at an FEHR of zero. Notice the higher values for the affine regression (excluding athlete G).

<u>Regression</u>				
<u>Data Type</u>	<u>Type</u>	<u>Equation</u>	<u>f(0)</u>	<u>S</u>
Team	Exponential	[HLa] = 0.0419446 * exp(5.33871 * FEHR)	0.042	1.337
Team	Affine	[HLa] = 0.908886 + 0.00705673 * exp(7.03434 * FEHR)	0.916	1.321
Athlete 2	Exponential	[HLa] = 0.0887819 * exp(4.20084 * FEHR)	0.089	0.656
Athlete 2	Affine	[HLa] = 1.19761 + 0.00107693 * exp(8.43556 * FEHR)	1.199	0.428
Athlete 15	Exponential	[HLa] = 0.128738 * exp(3.7926 * FEHR)	0.129	0.656
Athlete 15	Affine	[HLa] = 1.56216 + 0.000244061 * exp(9.81577 * FEHR)	1.562	0.420
Athlete 16	Exponential	[HLa] = 0.088292 * exp(4.756 * FEHR)	0.088	0.960
Athlete 16	Affine	[HLa] = 0.942088 + 0.0270559 * exp(5.85742 * FEHR)	0.969	1.043
Athlete 22	Exponential	[HLa] = 0.0111148 * exp(6.8082 * FEHR)	0.011	0.803
Athlete 22	Affine	[HLa] = 0.63657 + 0.00239875 * exp(8.29292 * FEHR)	0.639	0.793
Athlete G	Exponential	[HLa] = 0.275954 * exp(3.41615 * FEHR)	0.276	1.065
Athlete G	Affine	[HLa] = -17.9874 + 12.3732 * exp(0.749261 * FEHR)	-4.975	1.084

### *Physical Load Quantity*

Now that we have determined the better models for predicting an athletes [HLa] values are generated by the individual, we can use this information to understand what load an athlete has experienced during a practice or match. From the UWM Human Performance Lab we were able to obtain practice data for the athletes. Data was gathered using the Polar Team1 heart rate monitor. This monitor gathers 12 data points in one minute (as opposed to the Zephyr™, which gathers 59 data points in one minute). The Polar device is the choice of the athletes because it is smaller and less cumbersome to wear. For each 5-second interval, we generate the FEHR value and plug this into the athlete specific [HLa] equation from Table 6 to get an [HLa] value for that interval. This value is then multiplied by 1/12, since that data point was gathered over 1/12 of a minute.

Over the course of the practice, we sum these values for each interval. The equation for this Physical Load Quantity is described in equation 5

$$PLQ = \frac{\sum_{i=1}^n f([FEHR]_i)}{12} \quad (5)$$

where PLQ is the physical load quantity for the practice/match, n is the number of 5-second intervals, f is the [HLa] function from Table 6 based on the athlete and  $[FEHR]_i$  is the FEHR for the  $i^{\text{th}}$  interval. We then divide this by 12 to create the PLQ value in minute increments.

Table 8 below summarizes the PLQ values for athletes 2, 15, and 16 (22 did not have data from practice) using the individual model.

**Table 8:** PLQ values for athletes 2, 15 and 16.

<b>Athlete</b>	<b>Equation</b>	<b>PLQ</b>
2	$Hla = 1.19761 + 0.00107693 * \exp(8.43556 * FEHR)$	211.516
15	$Hla = 1.56216 + 0.000244061 * \exp(9.81577 * FEHR)$	273.174
16	$Hla = 0.942088 + 0.0270559 * \exp(5.85742 * FEHR)$	262.908

The PLQ values between individuals are not comparable due to athlete-to-athlete variation. The use of the PLQ data is to understand what effort the athlete put forth in the practice session. The numbers above would need to be calculated for many different practice types as defined by coaches. Such practice types might be post match practice (recover practice), peak practice or somewhere in between. By varying the practice regime, we can then observe an individual's PLQ values to understand how the value varies based on the practice type. Once the coaching staff begins to understand these numbers, the practices can be better coordinated utilizing PLQ data, rather than intuition, resulting in fewer injuries and peak performance in match play.

## 5. Conclusion

Previous models of intermittent sports have paved the way for more in depth analysis of the load applied to an athlete. Previous modeling utilized the FEHR in an attempt to predict the [HLA]. New technology has created the opportunity to understand how other physiological characteristics may predict [HLA]. Additional variables considered in this study were breathing rate, acceleration, peak acceleration, body temperature, heart rate, activity (instrument calculation) and percent of maximum heart rate. These additional variables did not show as good correlation in the [HLA] of an individual as compared to FEHR. By determining that FEHR is the preferred variable for [HLA] modeling we were able to continue our study using this variable.

Coupled with FEHR being the preferred variable, it has been shown that affine modeling creates a better root means square error (RMSE) compared to an exponential model, indicating a better fit. This Affine behavior led to two types of modeling; individual and team based. The data suggests that modeling an individual will best predict the generated load on an athlete. The reason for this is that team data creates a higher degree of variation attributed to athlete-to-athlete variation. This allows us to conclude that individual models will better predict how an athlete has exerted himself or herself in practice or a match.

Now that we have identified the ideal variable for modeling and the chosen method of modeling we can use this to evaluate the load exerted on our athletes. This load is quantified by the PLQ (physical load quantity), which is the summation of [HLA] values for a given athlete at each measured FEHR during the session. The higher the

number the more fatigue an athlete will experience. This value can be utilized by coaches throughout a season to gauge the fitness and fatigue of a team, hence tailoring practice schedules according to obtain an optimum output for the team.

### *Next Steps*

To solidify this study, it is recommended to further test the athletes to confirm their lactate curves. This would include multiple tests for each athlete to generate an average [Hla] curve. In addition, more robustly designing the experiment to ensure all data is gathered (currently underway with men's data in the Human Performance Lab at UWM). After these above have been complete, then perform tests in practice to generate varying practice levels based on the models. Then utilize this data to determine what practice type is necessary to optimize fitness for the match play.

## References

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## Appendix A

This appendix contains the single variable modeling for each of the variables in Table 2 using Method 2 (team data).

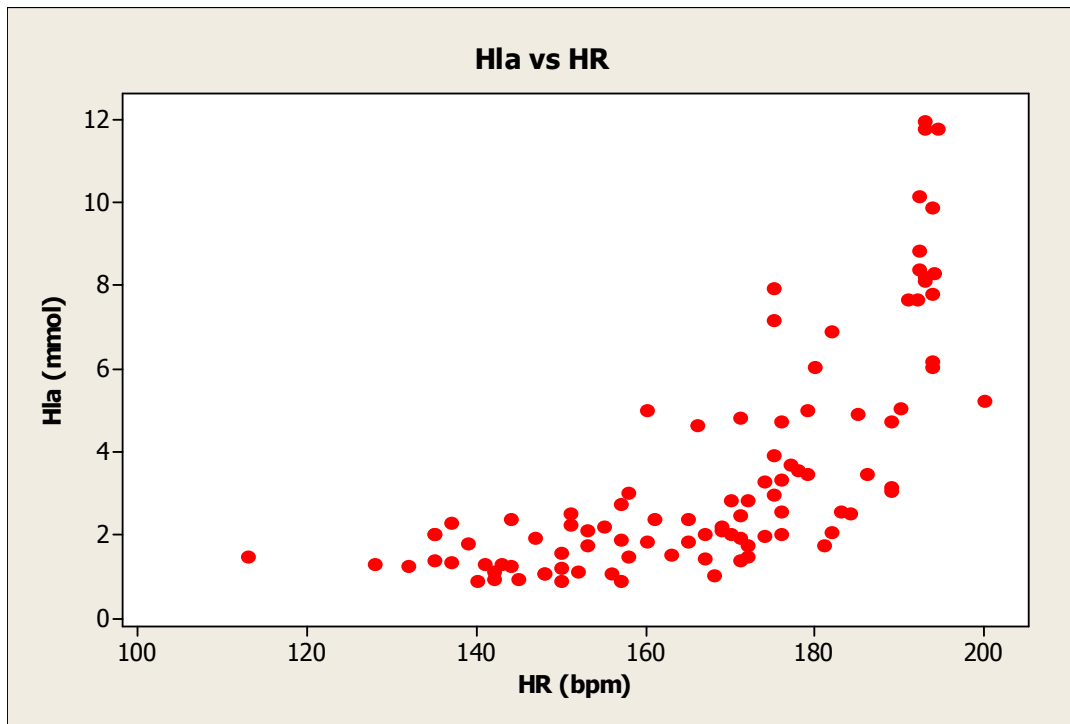


Figure A-1: Plot of Hla vs. HR.

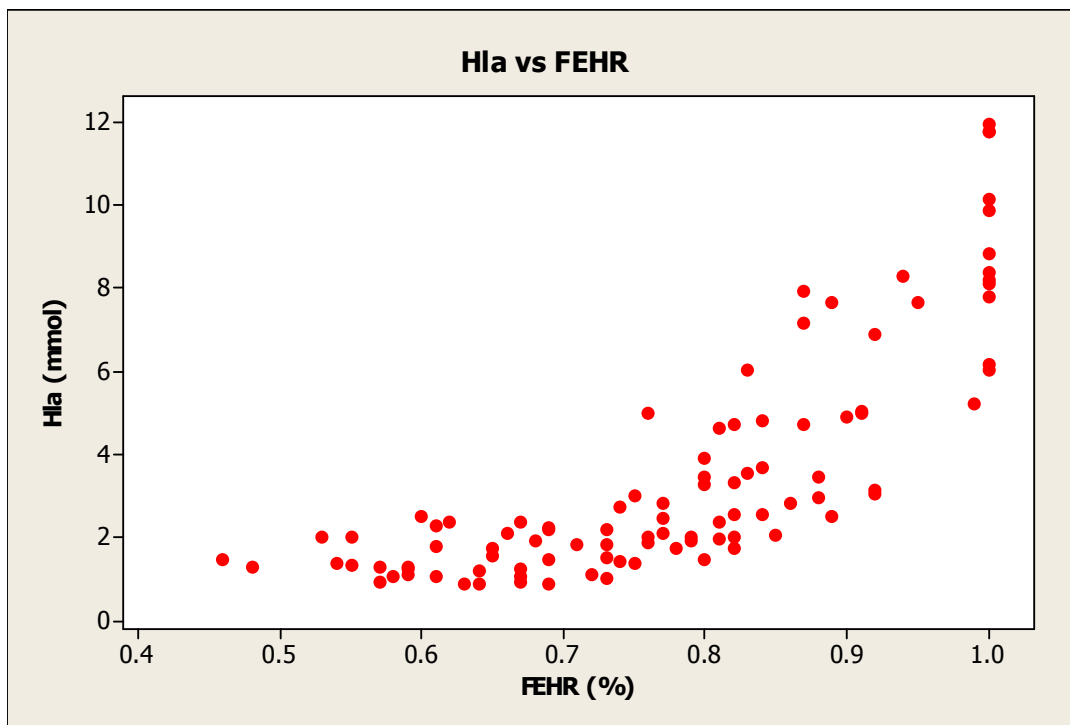


Figure A-2: Plot of Hla vs FEHR.

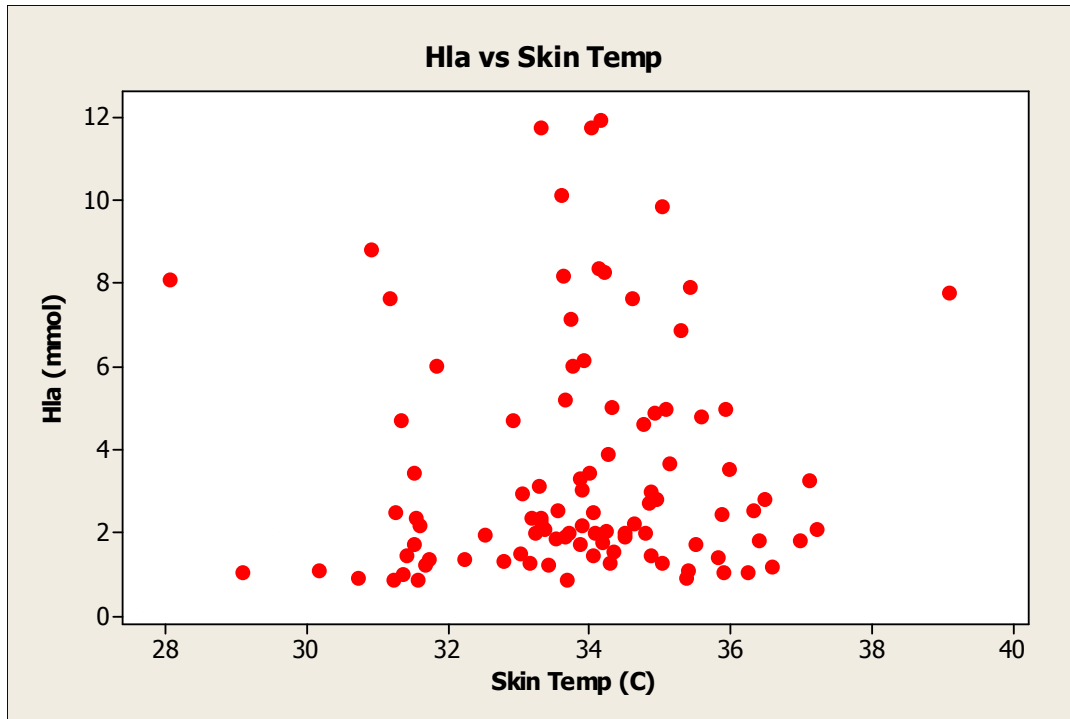


Figure A-3: Plot of Hla vs Skin Temp.

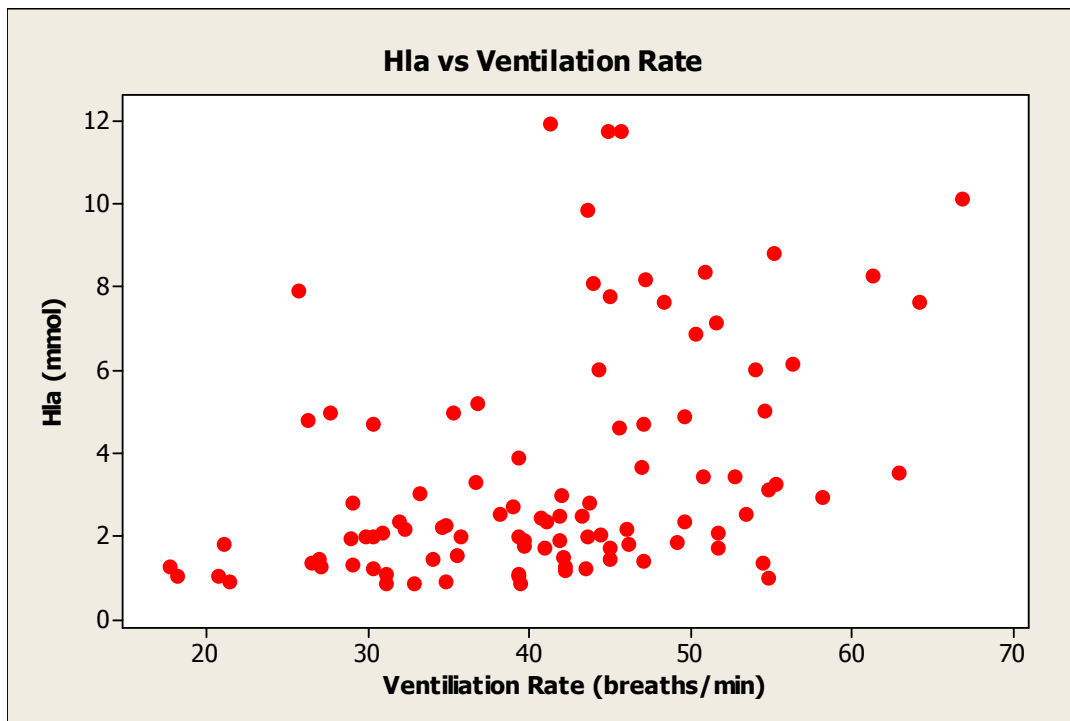


Figure A-4: Plot of Hla vs Ventilation Rate.

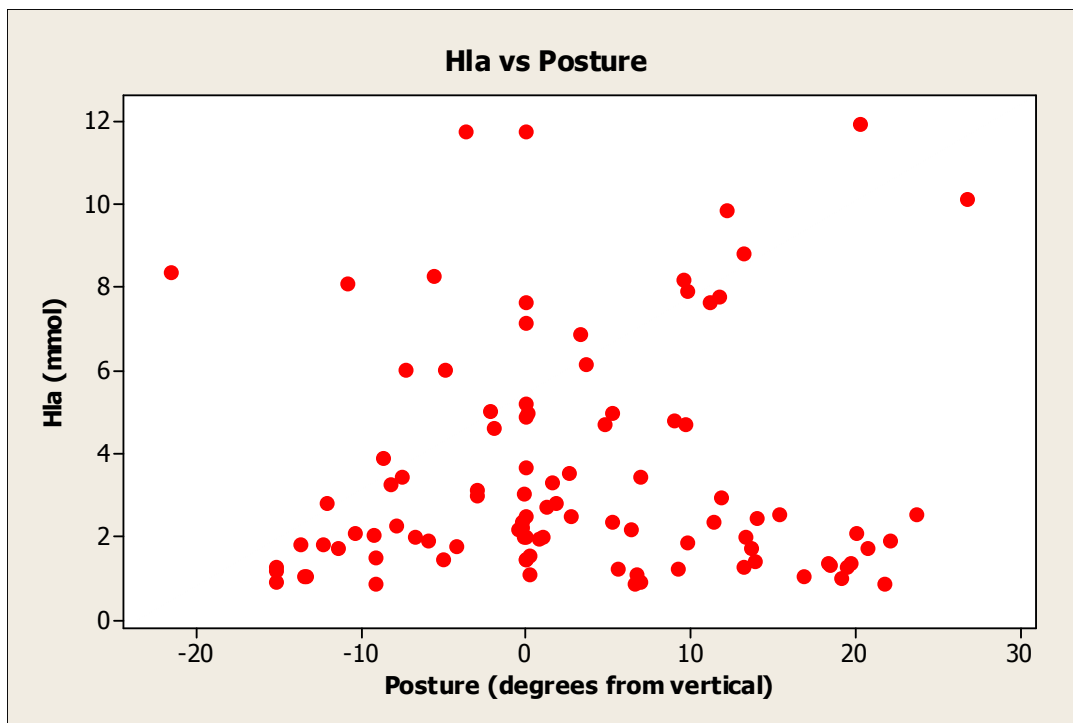


Figure A-5: Plot of Hla vs Posture.

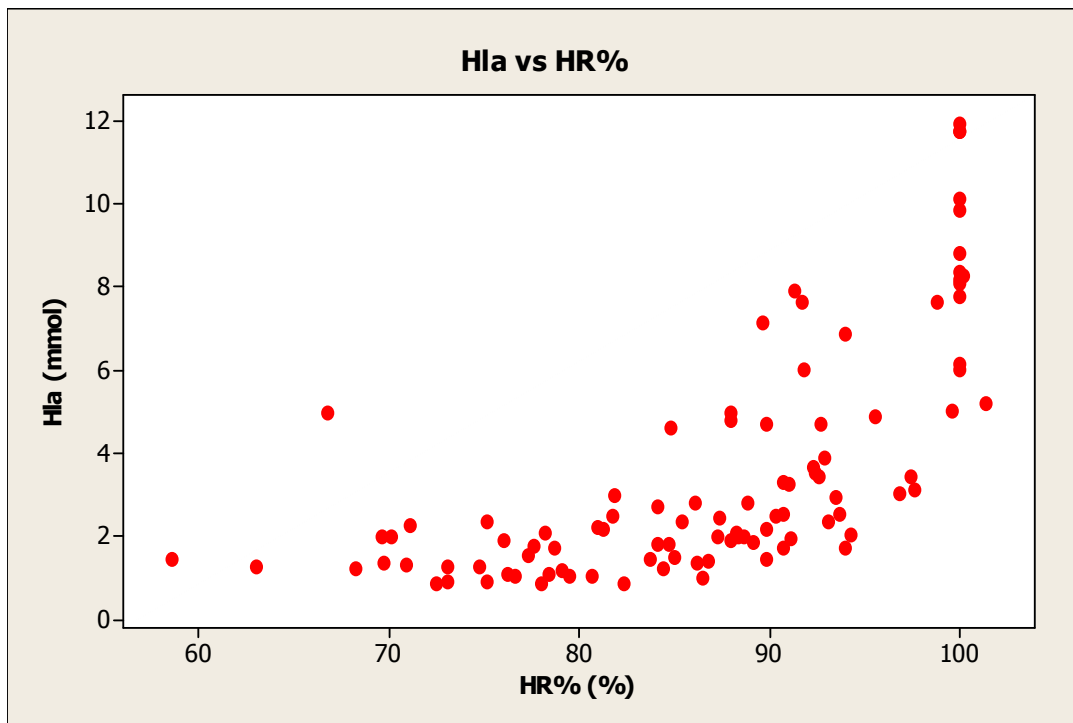


Figure A-6: Plot of Hla vs HR %.

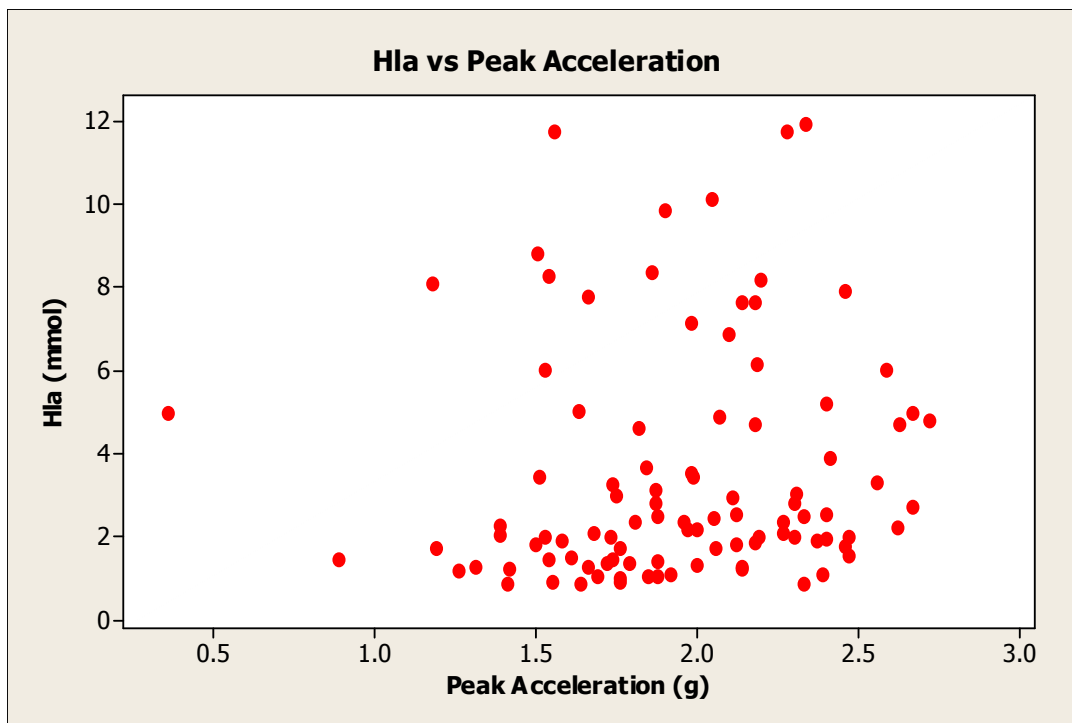


Figure A-7: Plot of Hla vs Peak Acceleration.

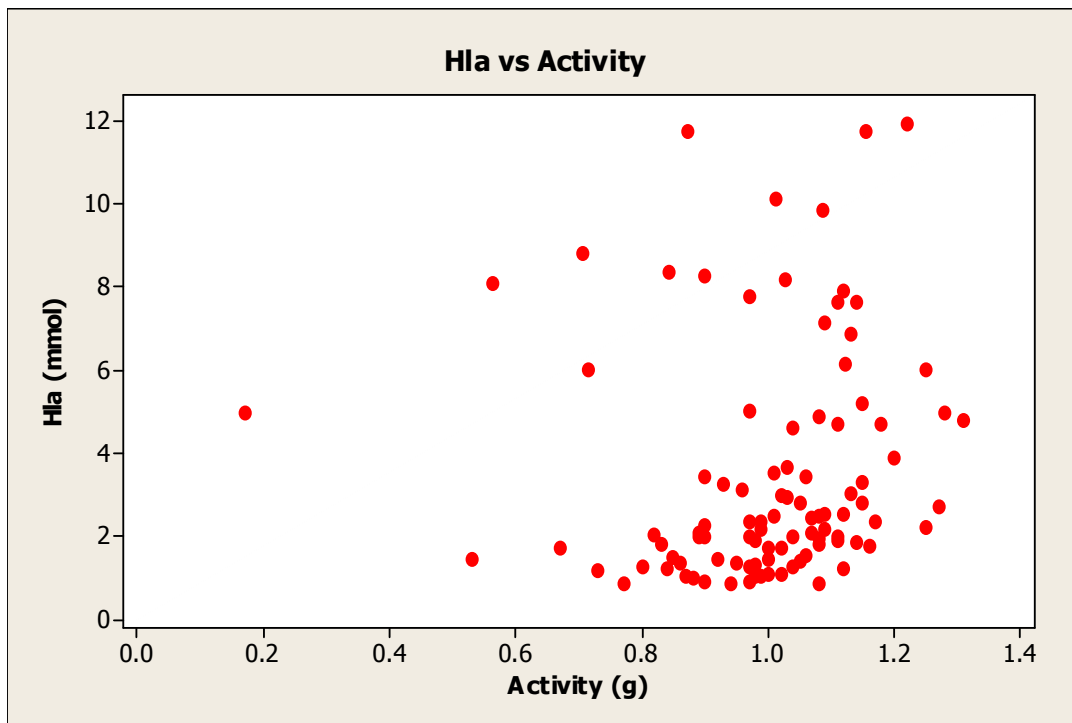


Figure A-8: Plot of Hla vs Activity.

## Appendix B

This appendix contains the single variable modeling for each the Heart Rate, FEHR and HR % for subjects 2, 15, 16, 22, and G.

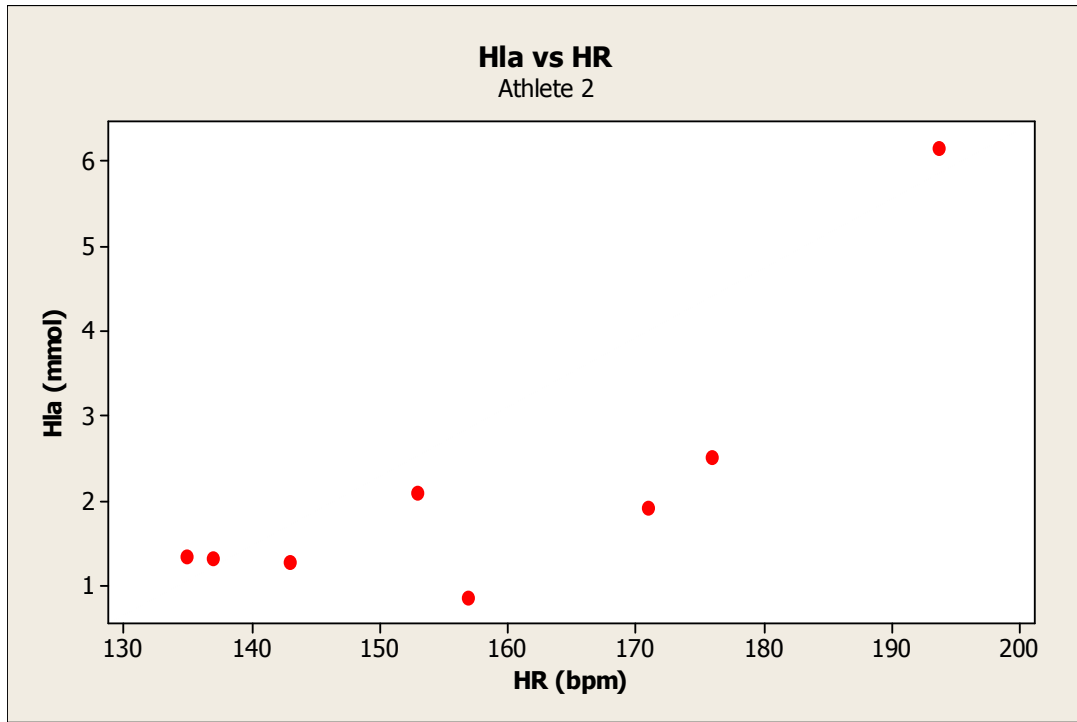


Figure B-1: Scatterplot of Hla vs HR for Athlete 2.

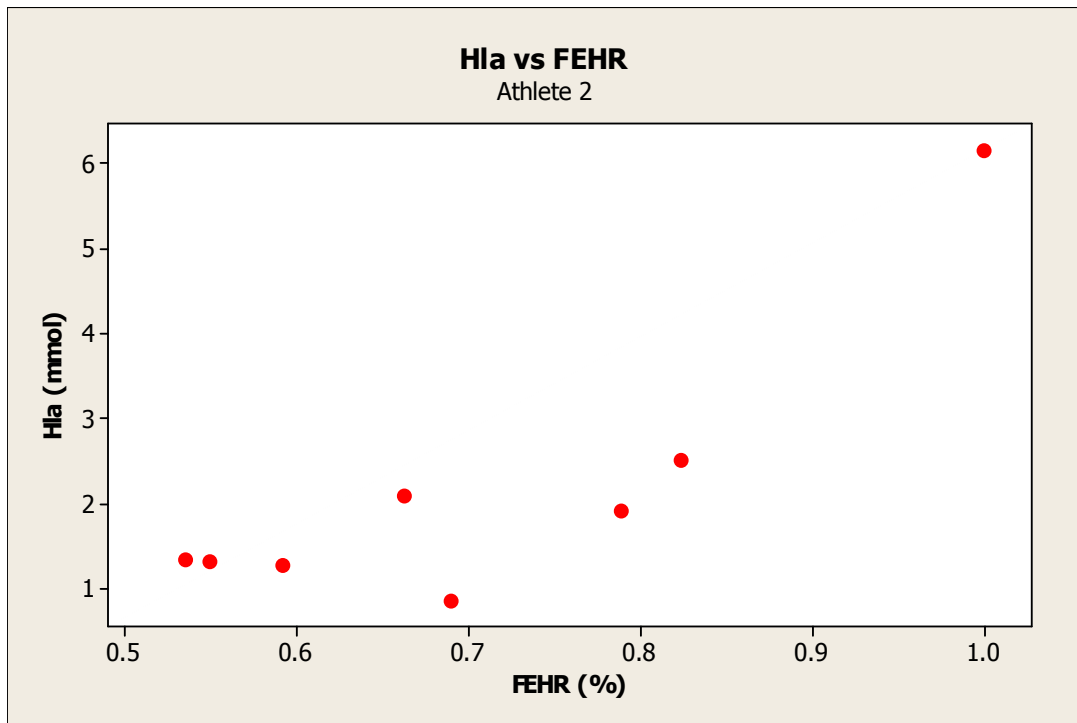
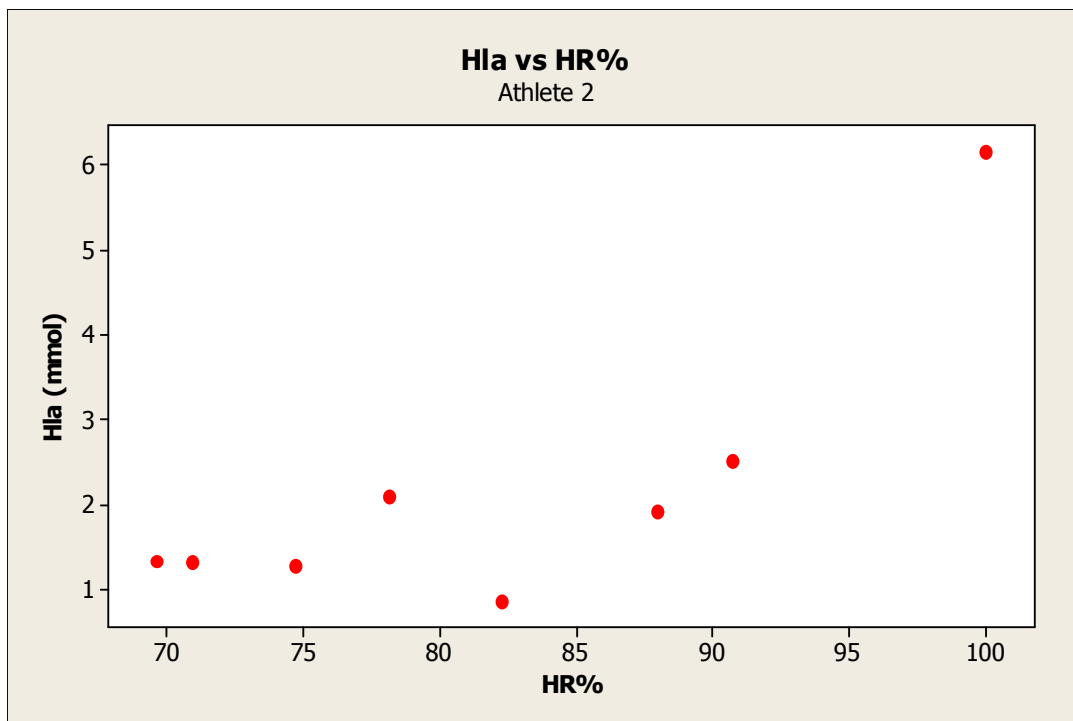
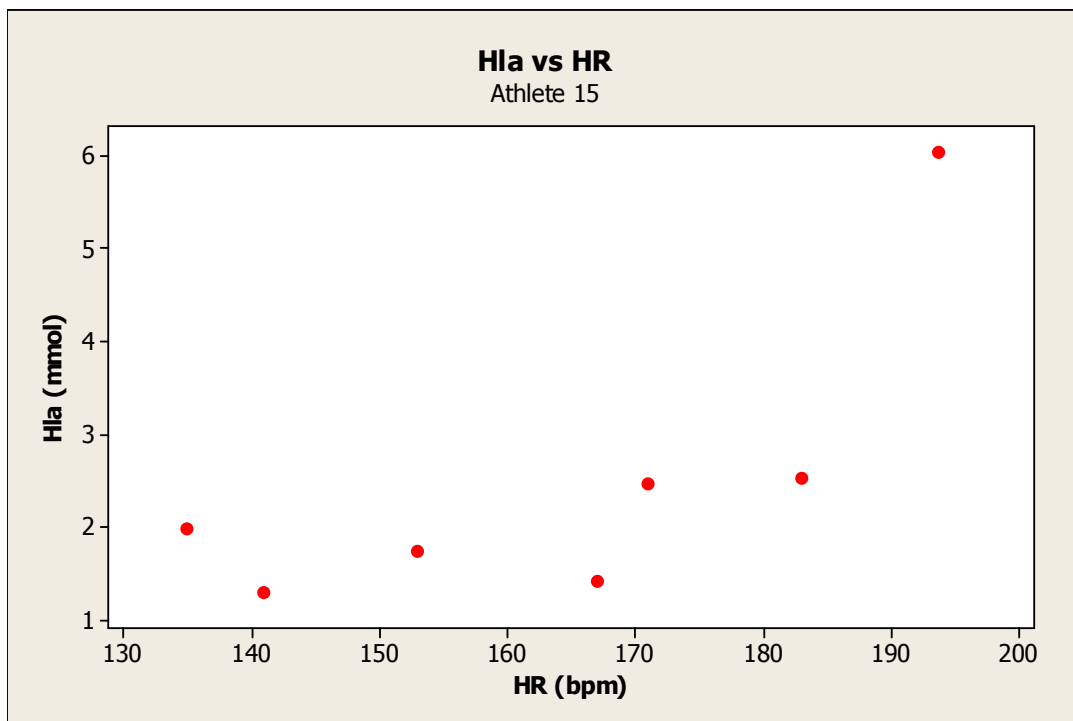


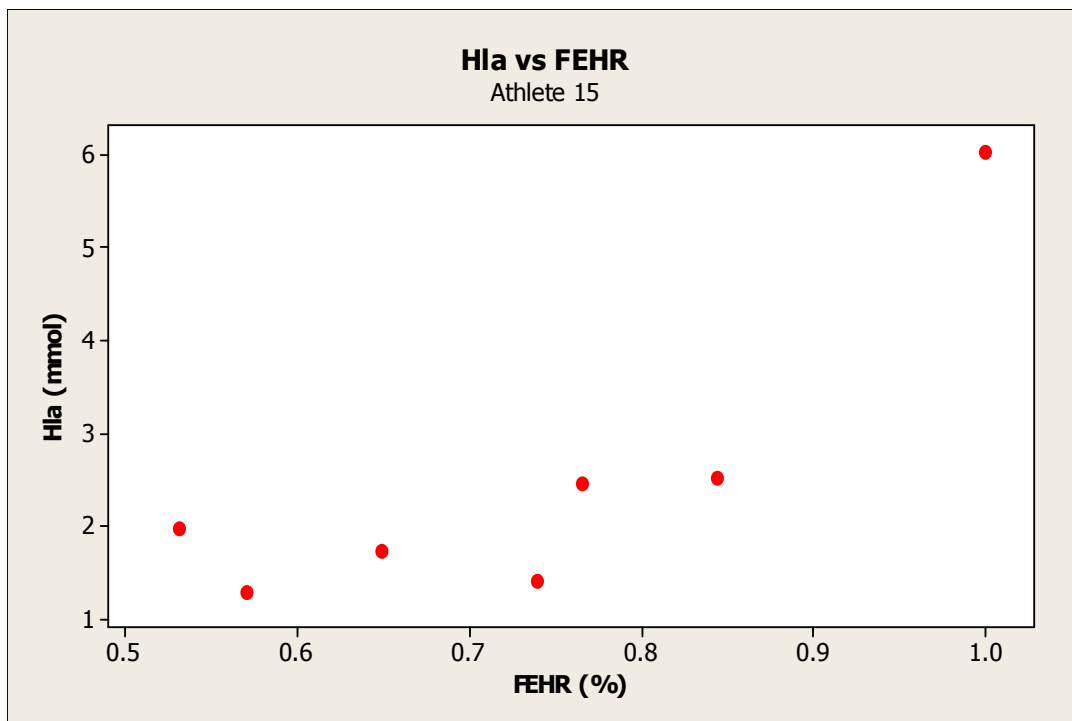
Figure B-2: Scatterplot of Hla vs FEHR for Athlete 2.



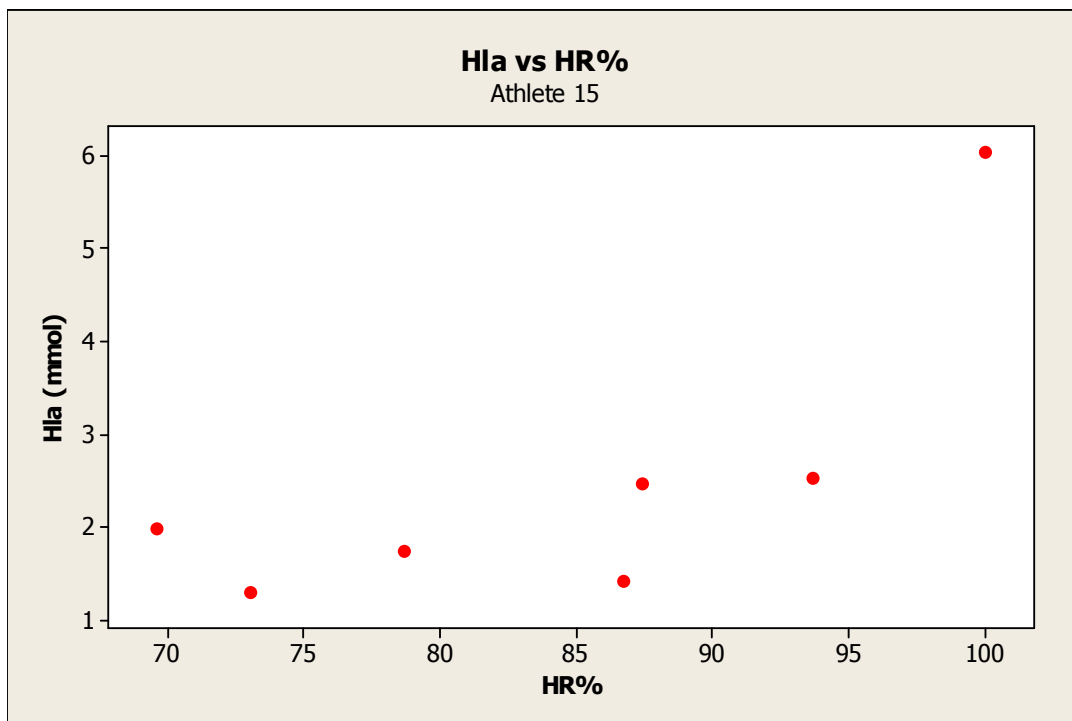
**Figure B-3:** Scatterplot of Hla vs HR% for Athlete 2.



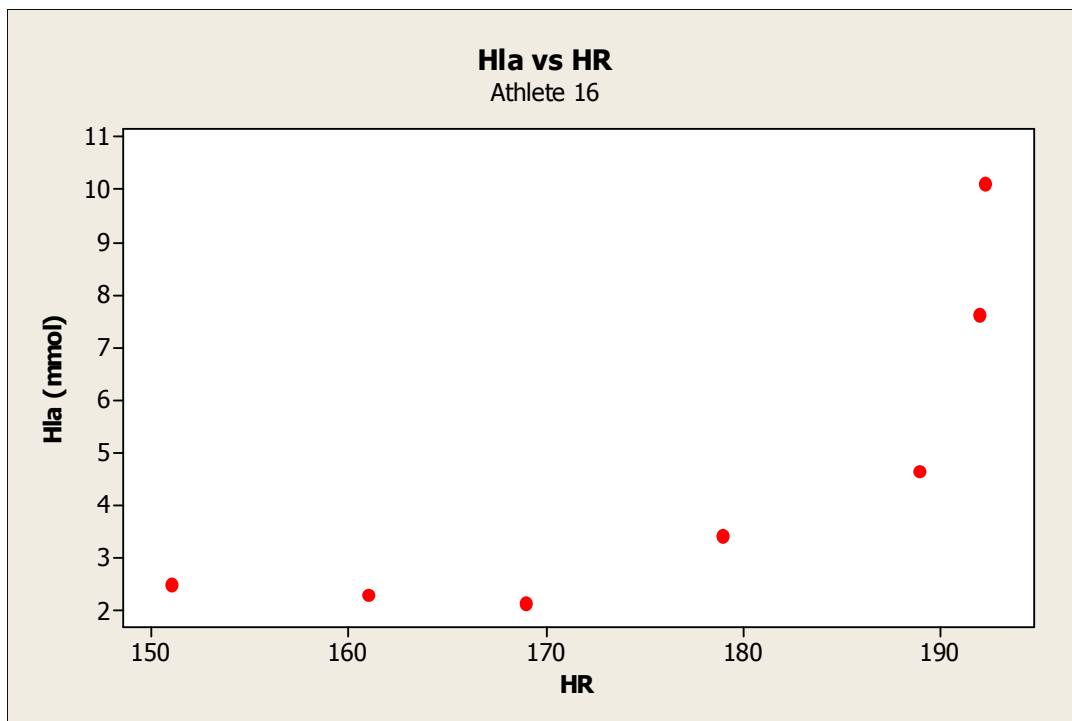
**Figure B-4:** Scatterplot of Hla vs HR for Athlete 15.



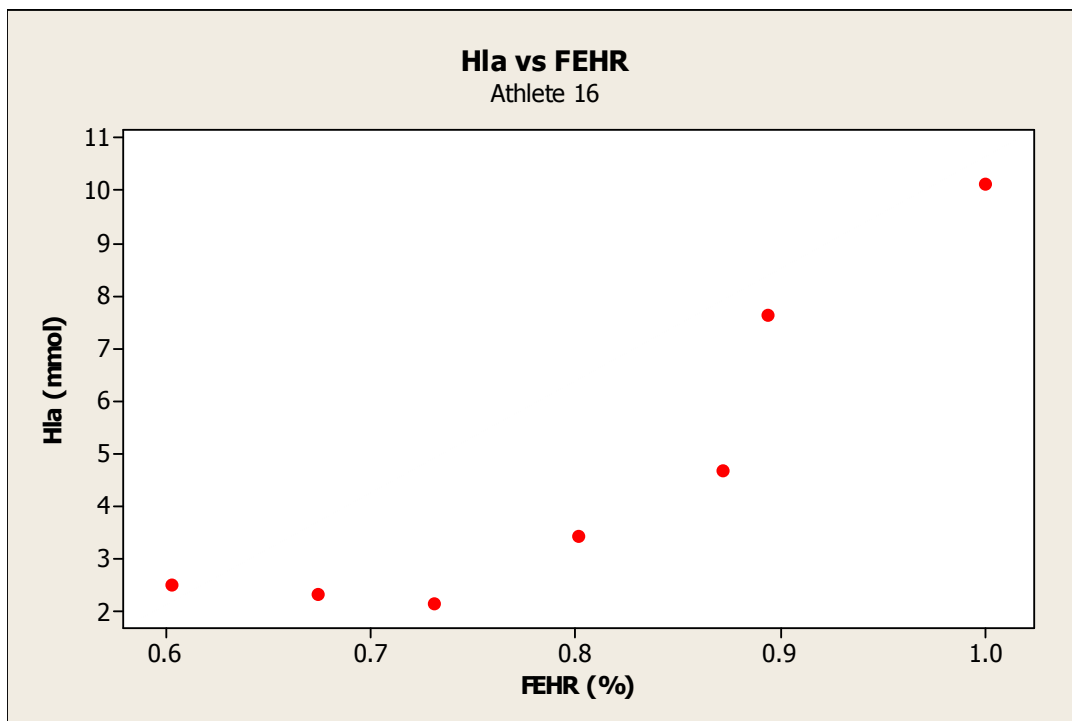
**Figure B-5:** Scatterplot of Hla vs FEHR for Athlete 15.



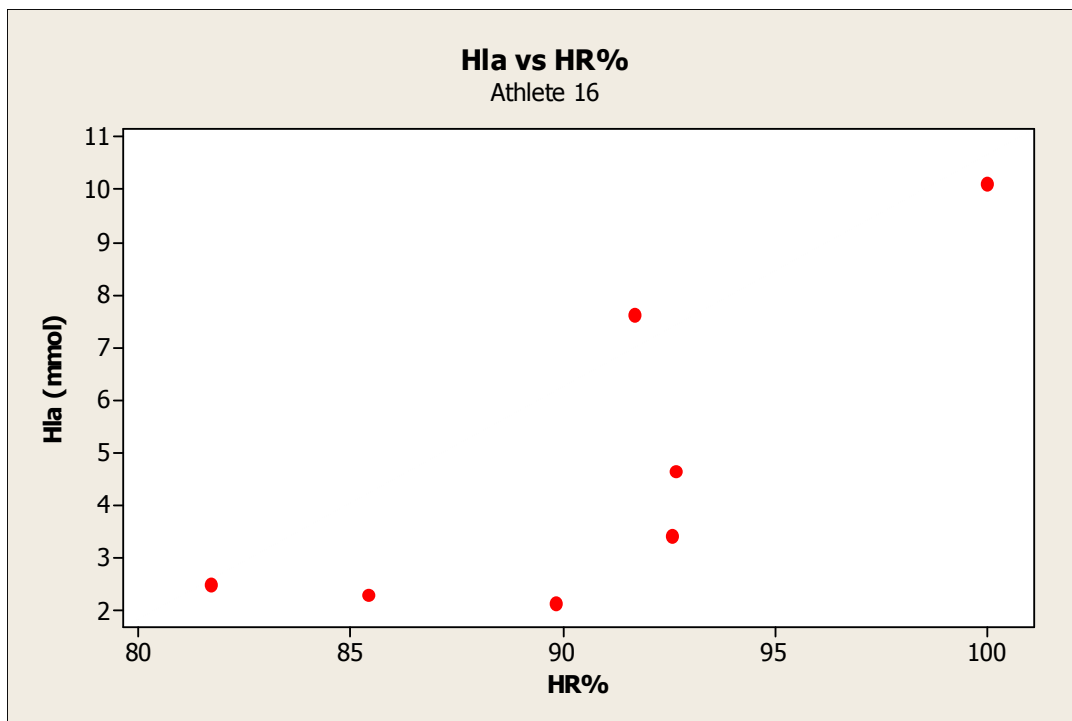
**Figure B-6:** Scatterplot of Hla vs HR % for Athlete 15.



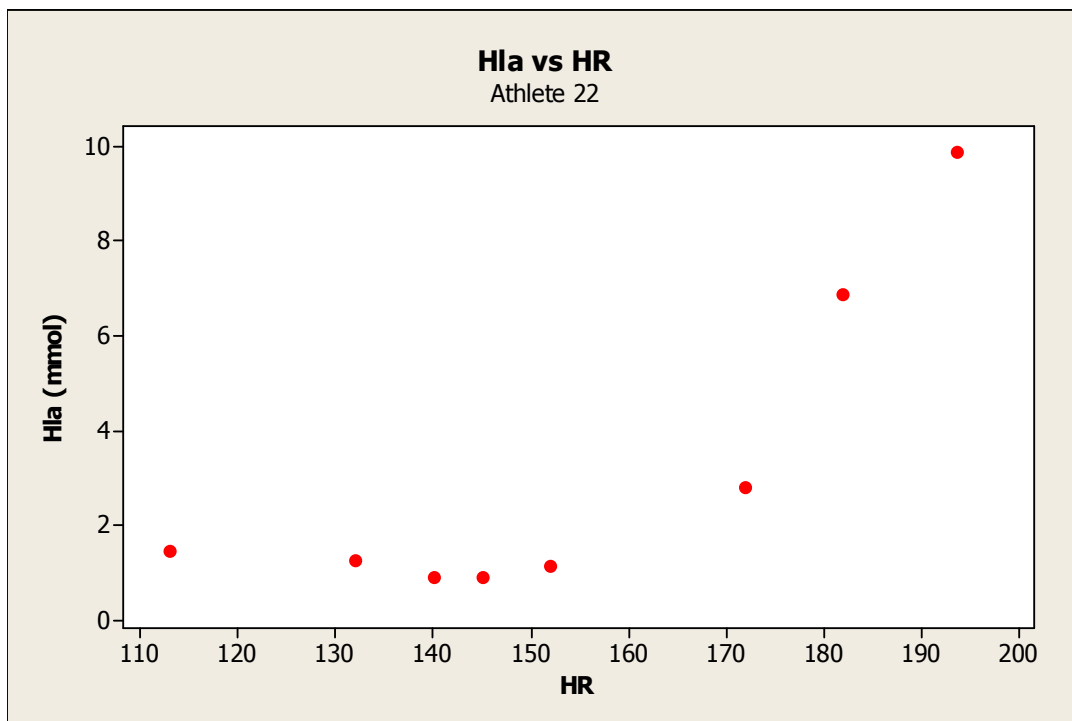
**Figure B-7:** Scatterplot of Hla vs HR for Athlete 16.



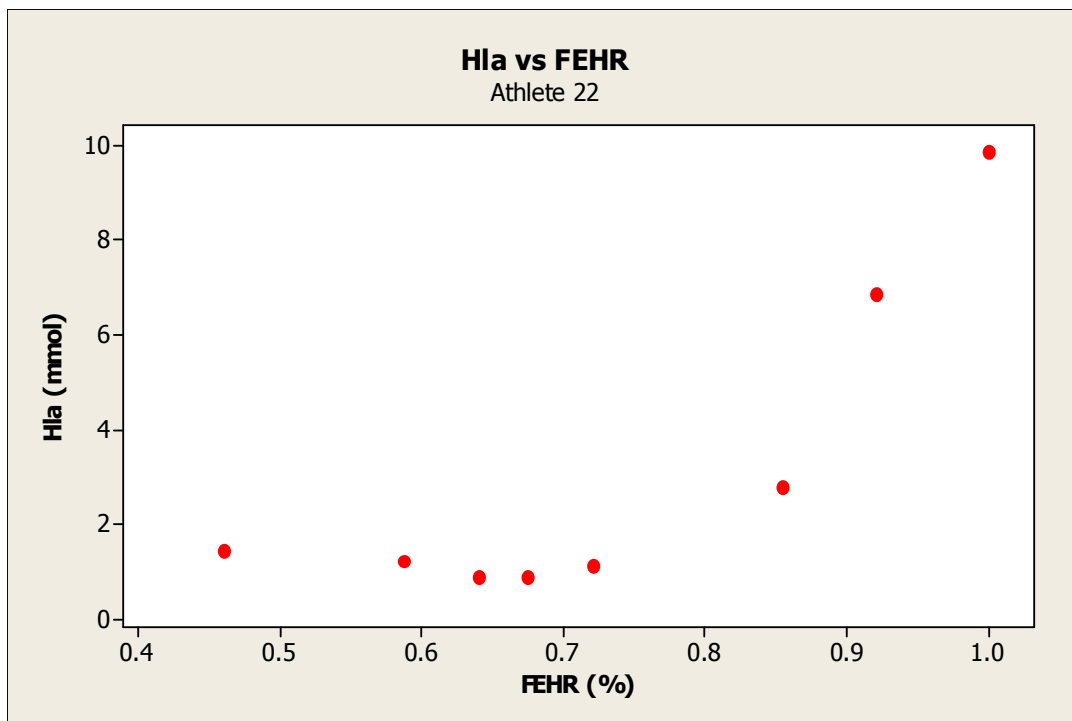
**Figure B-8:** Scatterplot of Hla vs FEHR for Athlete 16.



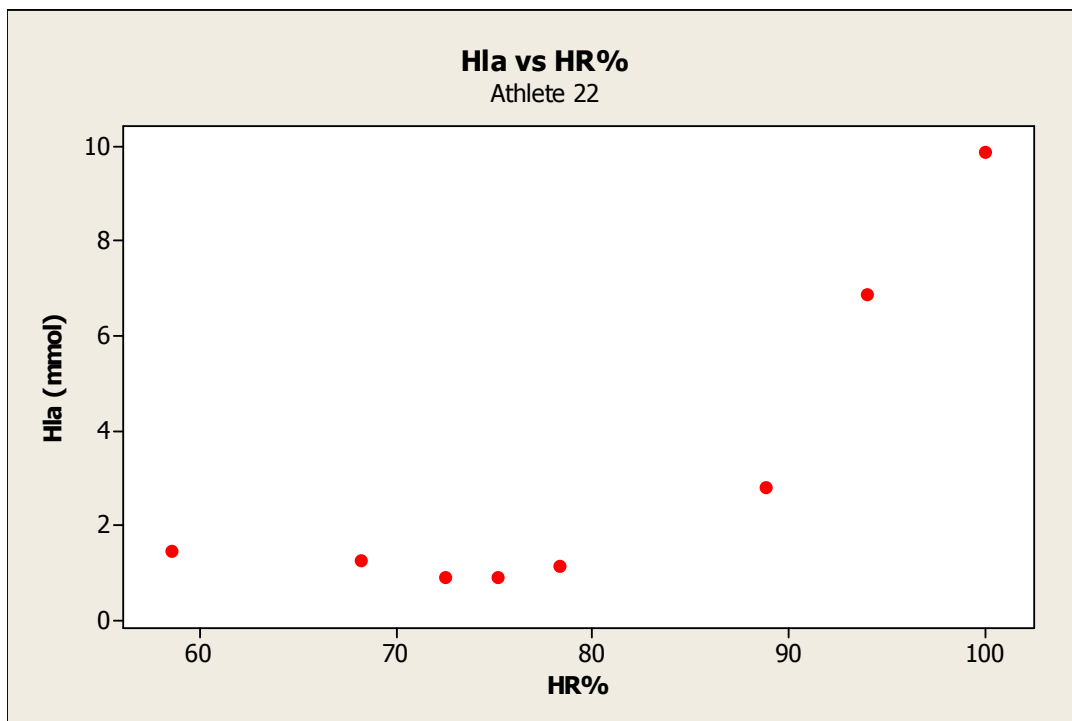
**Figure B-9:** Scatterplot of Hla vs HR% for Athlete 16.



**Figure B-10:** Scatterplot of Hla vs HR for Athlete 22.



**Figure B-11:** Scatterplot of Hla vs FEHR for Athlete 22.



**Figure B-12:** Scatterplot of Hla vs HR% for Athlete 22.

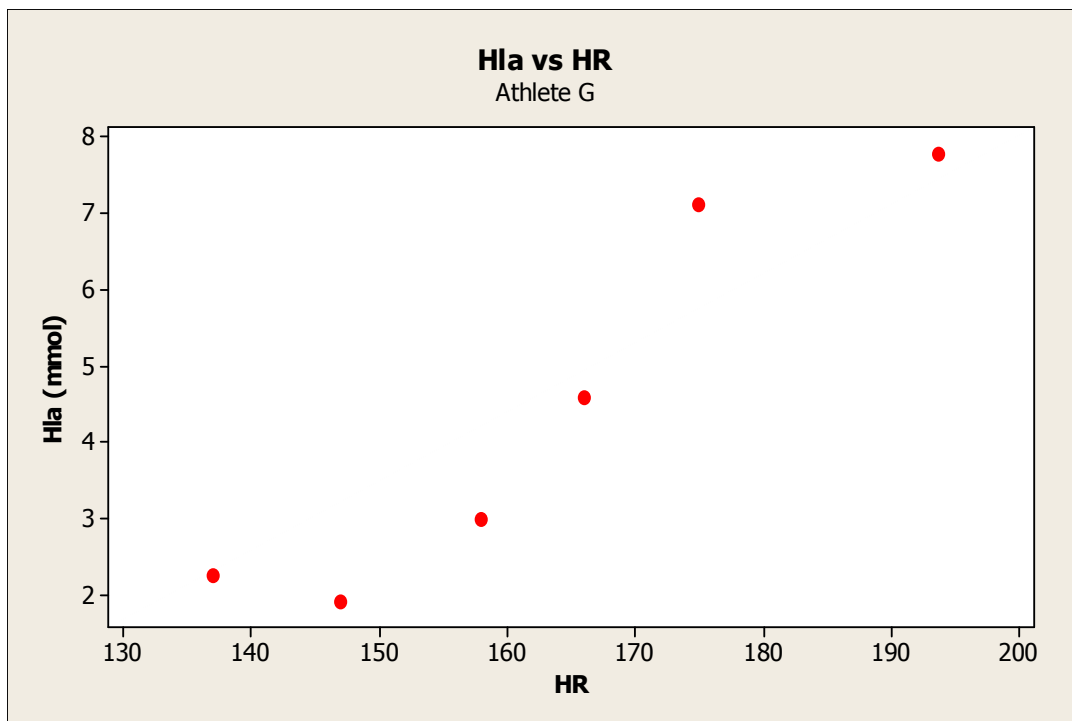


Figure B-13: Scatterplot of Hla vs HR for Athlete G.

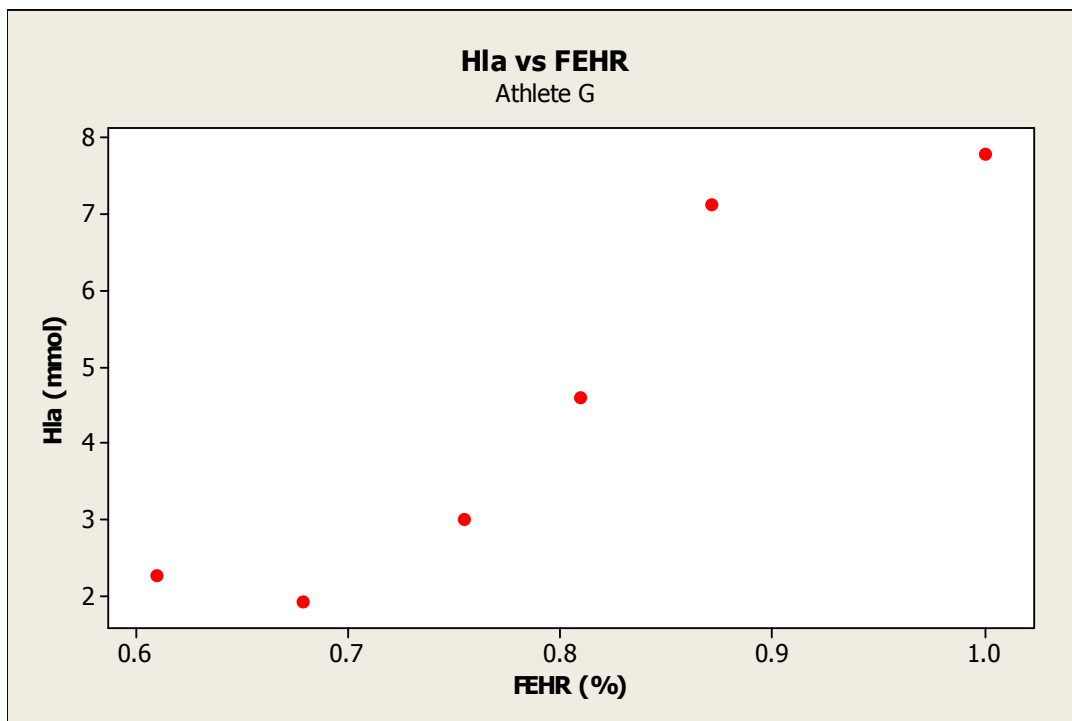
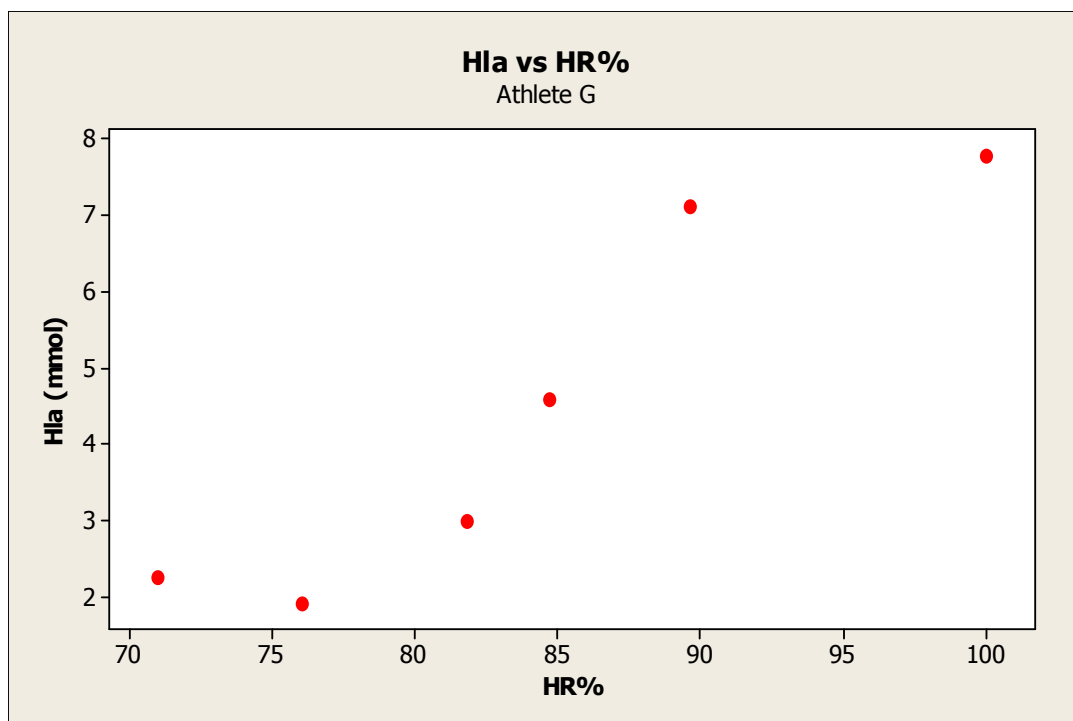


Figure B-14: Scatterplot of Hla vs FEHR for Athlete G.



**Figure B-15:** Scatterplot of Hla vs HR% for Athlete G.

## Appendix C

This appendix provides a breakdown of what the Minitab output statistics are when performing nonlinear regression (affine).

### Nonlinear Regression: Hla = Theta1 - Theta2 \* exp(-Theta3 \* 'Karvonen HR')

Method

Algorithm	Gauss-Newton
Max iterations	200
Tolerance	0.00001

Method to determine best fit model

3 cases with missing values were not used.

Starting Values for Parameters

Parameter	Value
Theta1	-1.06
Theta2	-1.436
Theta3	-1

Starting parameters determined by solving three equations in three unknowns.

Equation

Hla = 0.908886 + 0.00705673 \* exp(7.03434 \* 'Karvonen HR')

Fit equation

Parameter Estimates

Parameter	Estimate	SE Estimate
Theta1	0.90889	0.41993
Theta2	-0.00706	0.00797
Theta3	-7.03434	1.10071

Estimates of the parameters determined after the number of iterations.

Hla = Theta1 - Theta2 \* exp(-Theta3 \* 'Karvonen HR')

Lack of Fit

Source	DF	SS	MS	F	P
Error	95	165.800	1.74526		
Lack of Fit	41	59.580	1.45317	0.74	0.843
Pure Error	54	106.220	1.96703		

Lack of fit information provided if the data contains replicates. This explains the error in the model

Summary

Iterations	48
Final SSE	165.800
DFE	95
MSE	1.74526
S	1.32108

Warning messages associated with the data.

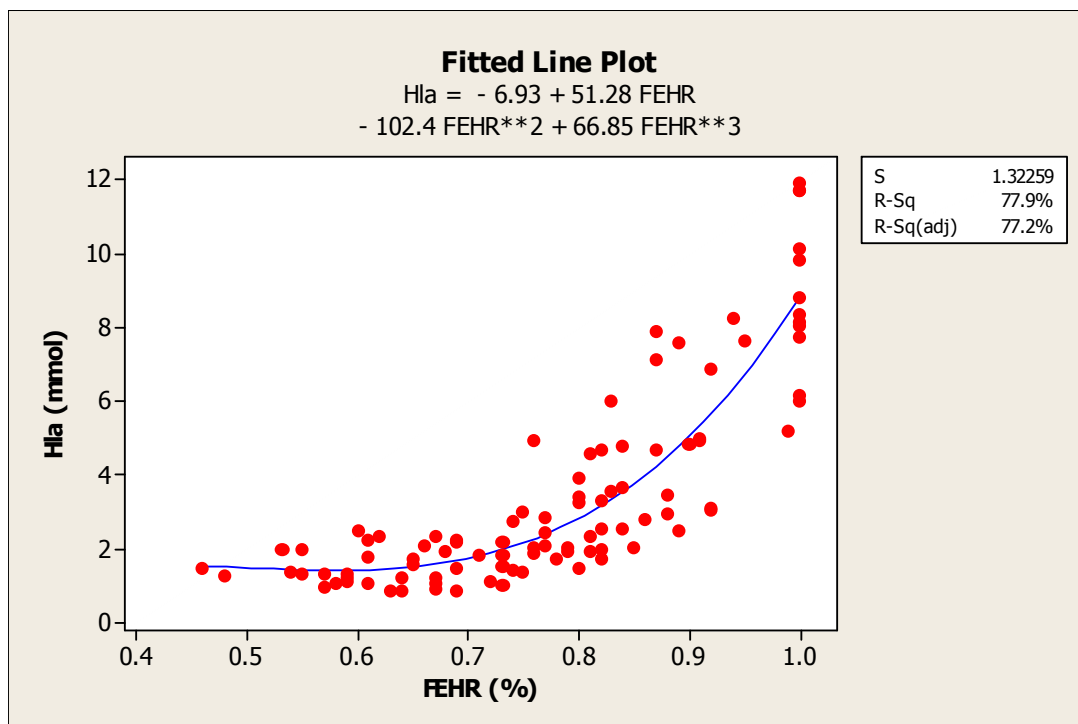
\* WARNING \* Some parameter estimates are highly correlated. Consider simplifying the expectation function or transforming predictors or parameters to reduce collinearities.

### Fitted Line: Hla versus Karvonen HR

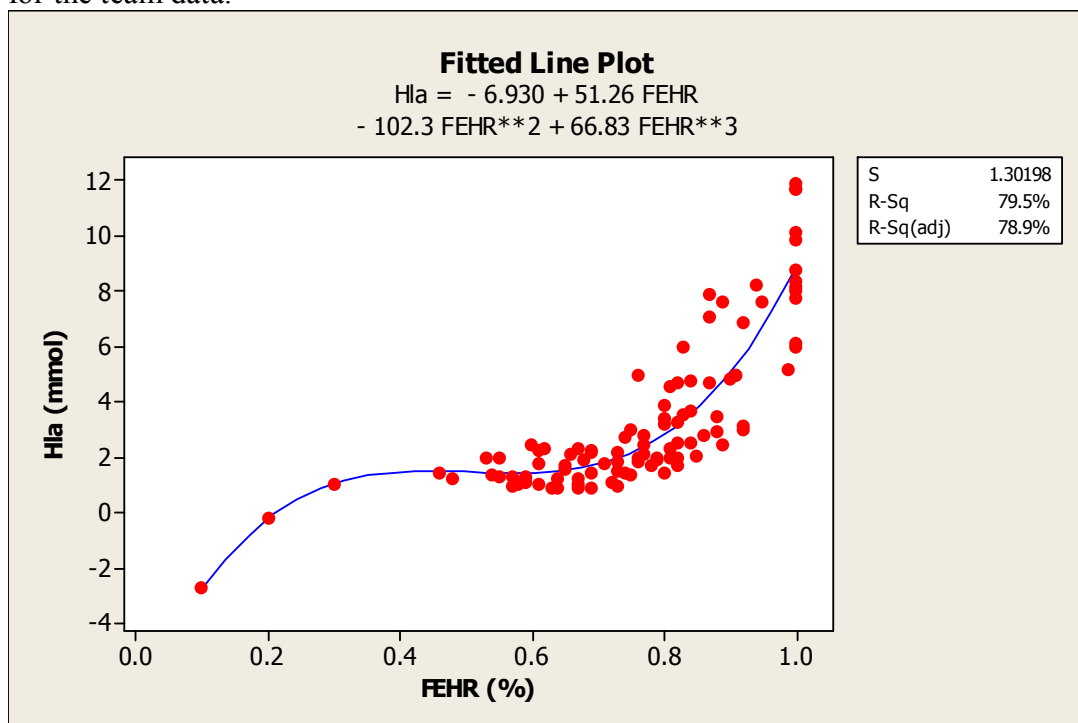
Figure C-1: Minitab statistical output for Hla vs. FEHR

## Appendix D

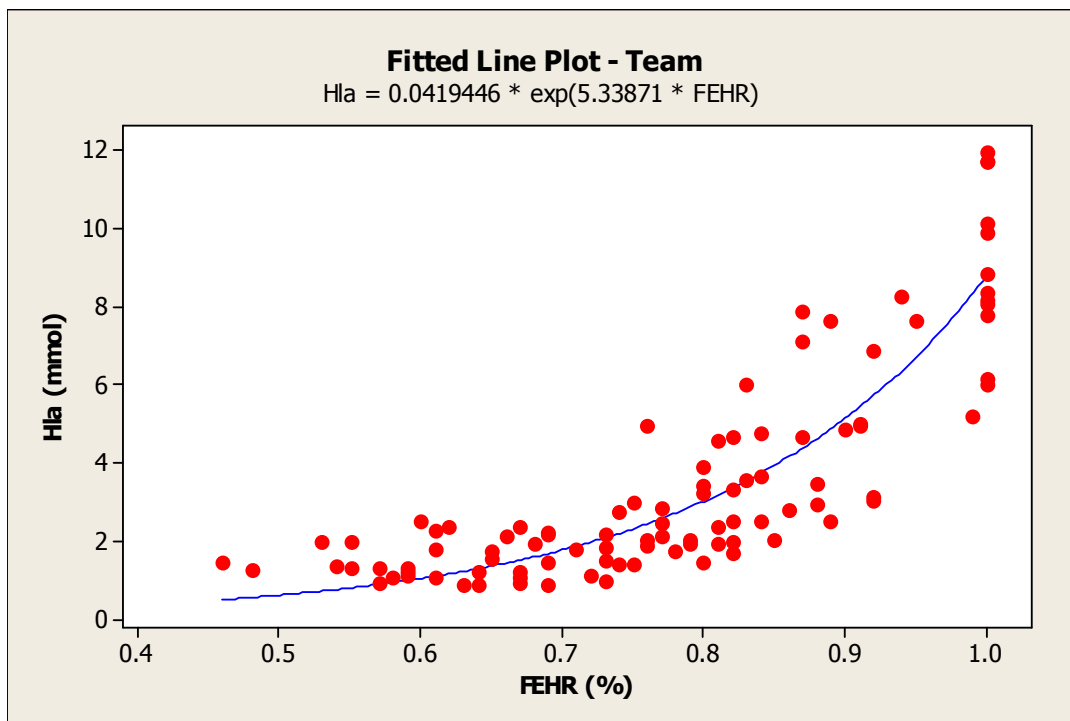
This Appendix contains the fitted models discussed throughout the paper.



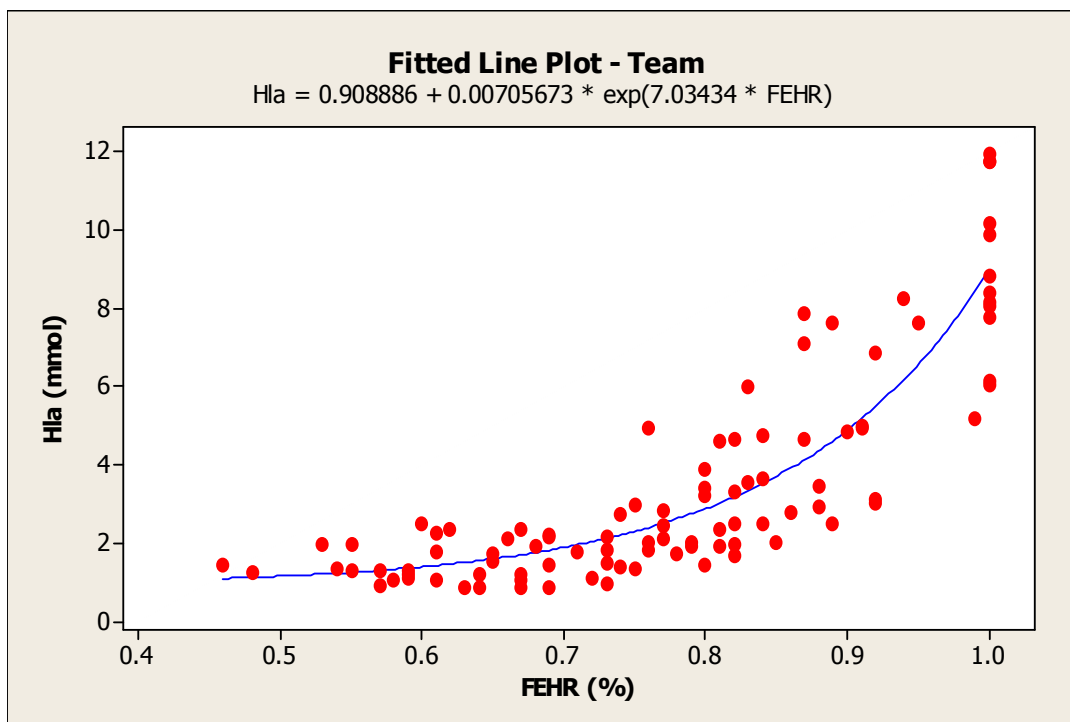
**Figure D-1:** Fitted line plot of Hla vs. FEHR using a 3<sup>rd</sup> degree polynomial regression for the team data.



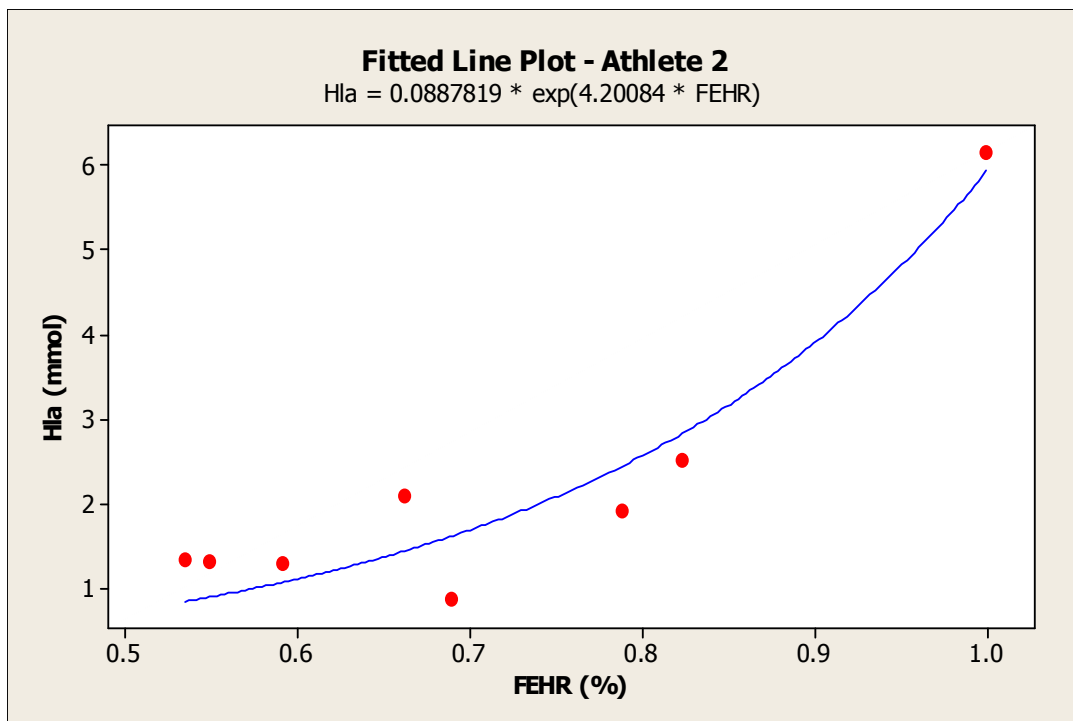
**Figure D-2:** Fitted line plot of the Hla vs. FEHR using a 3<sup>rd</sup> degree polynomial extended to smaller values of the FEHR for the team data.



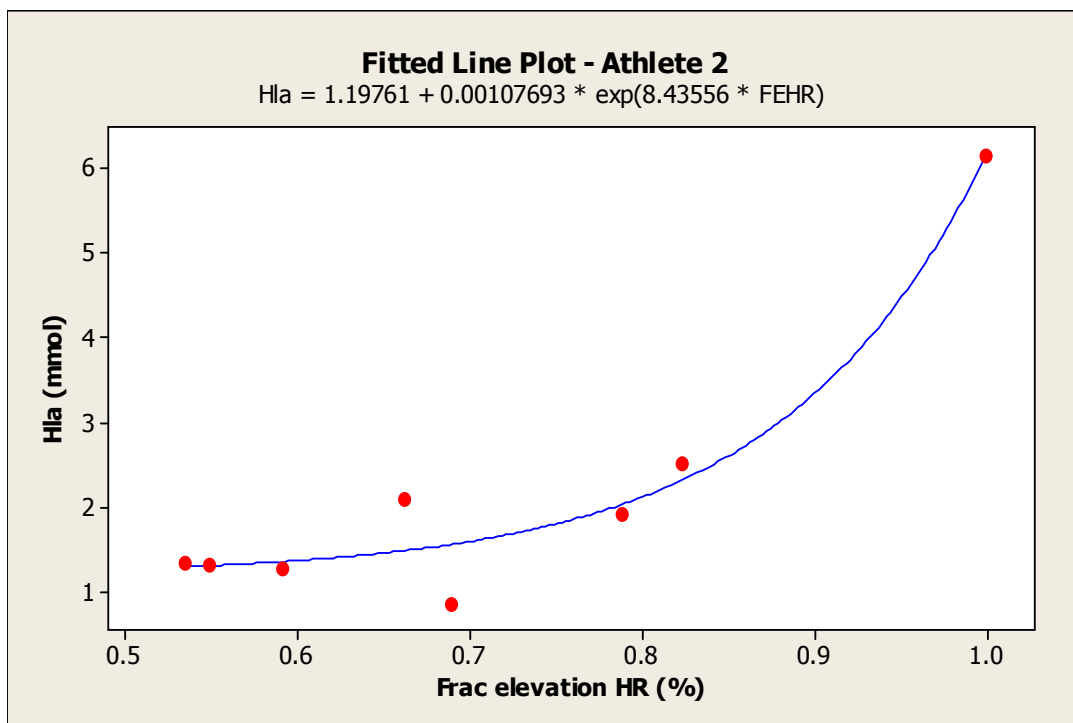
**Figure D-3:** Fitted line plot of Hla vs. FEHR using exponential regression for the team data.



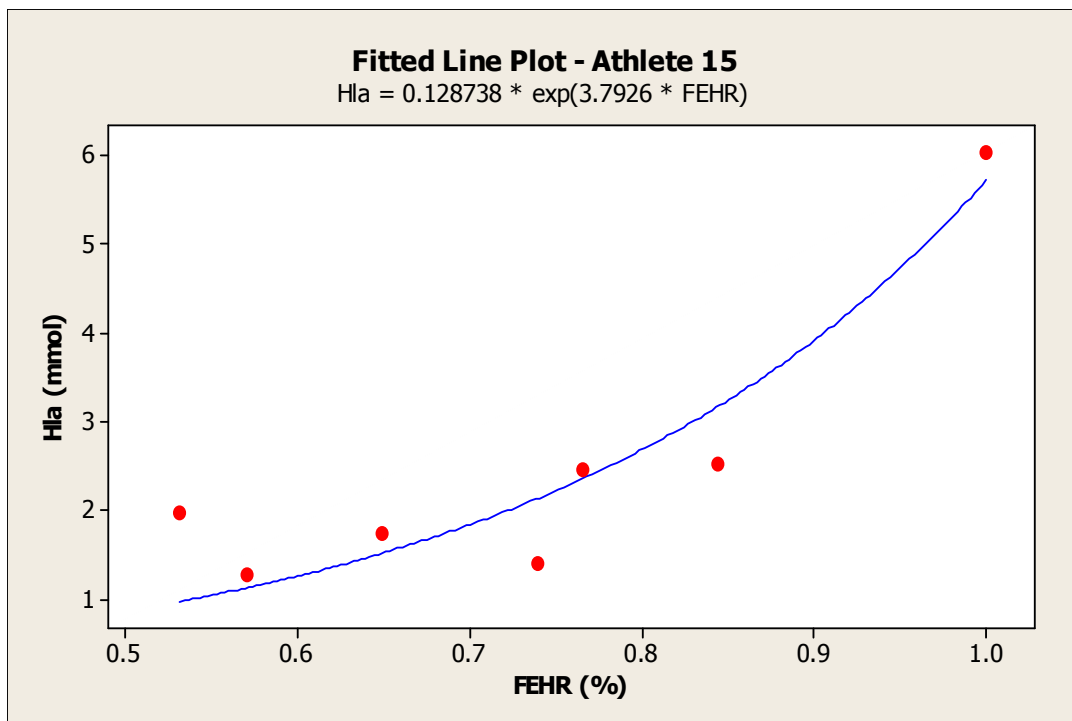
**Figure D-4:** Fitted line plot of Hla vs. FEHR using affine regression for the team data.



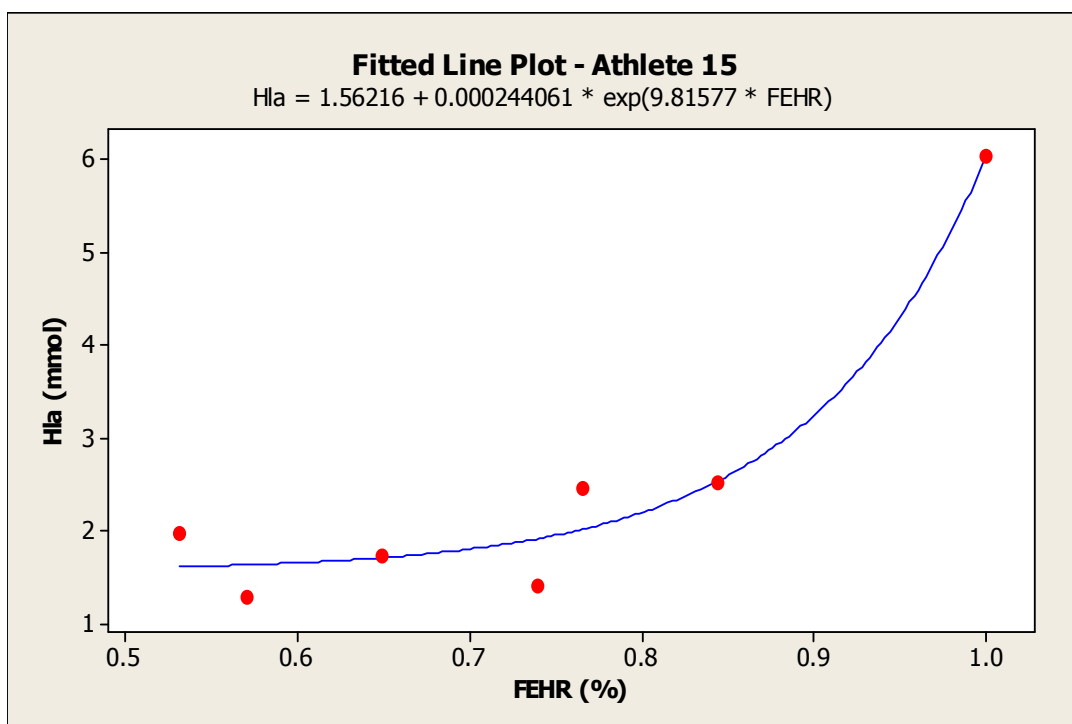
**Figure D-5:** Fitted line plot of Hla vs. FEHR using exponential regression for Athlete 2.



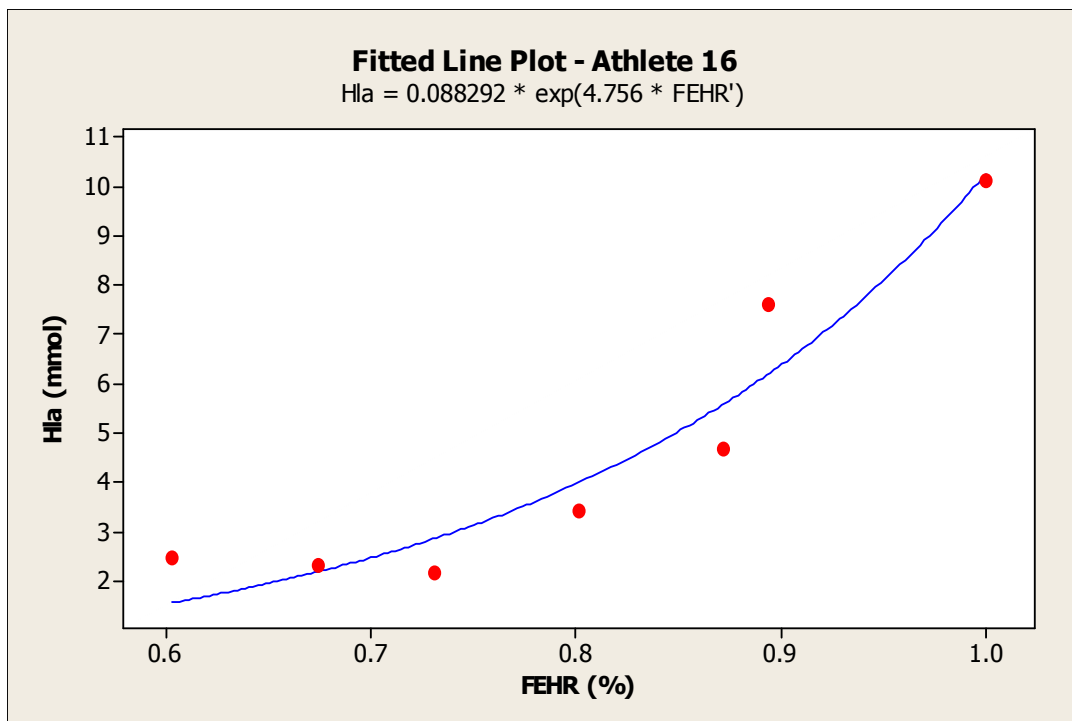
**Figure D-6:** Fitted line plot of Hla vs. FEHR using affine regression for Athlete 2.



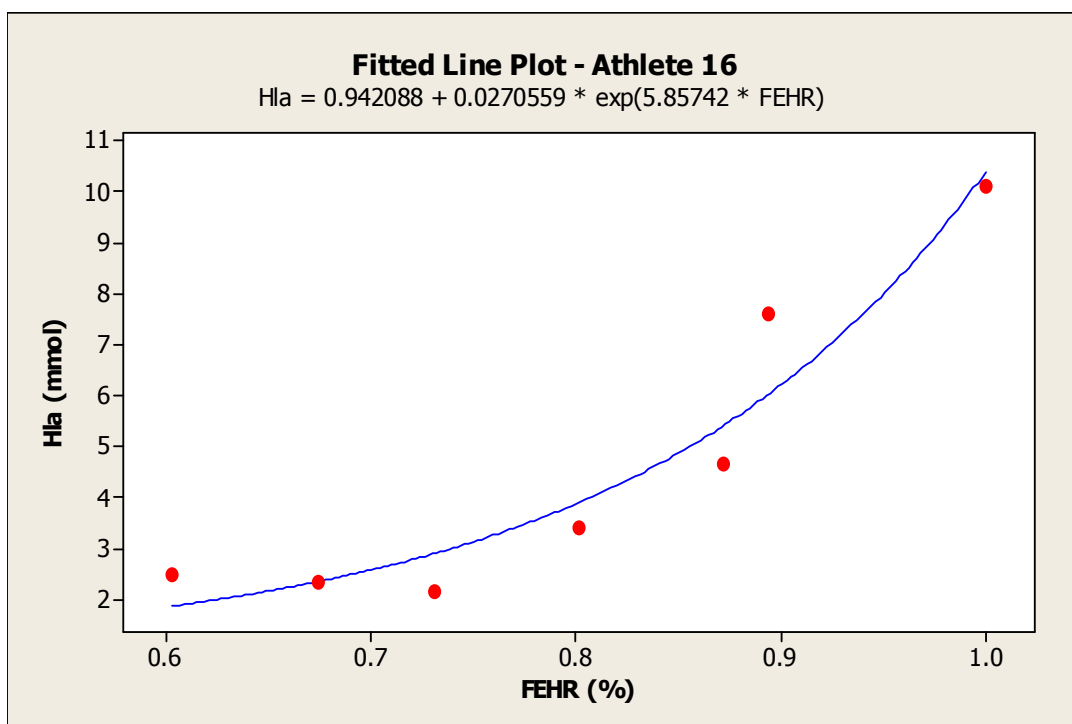
**Figure D-7:** Fitted line plot of Hla vs. FEHR using exponential regression for Athlete 15.



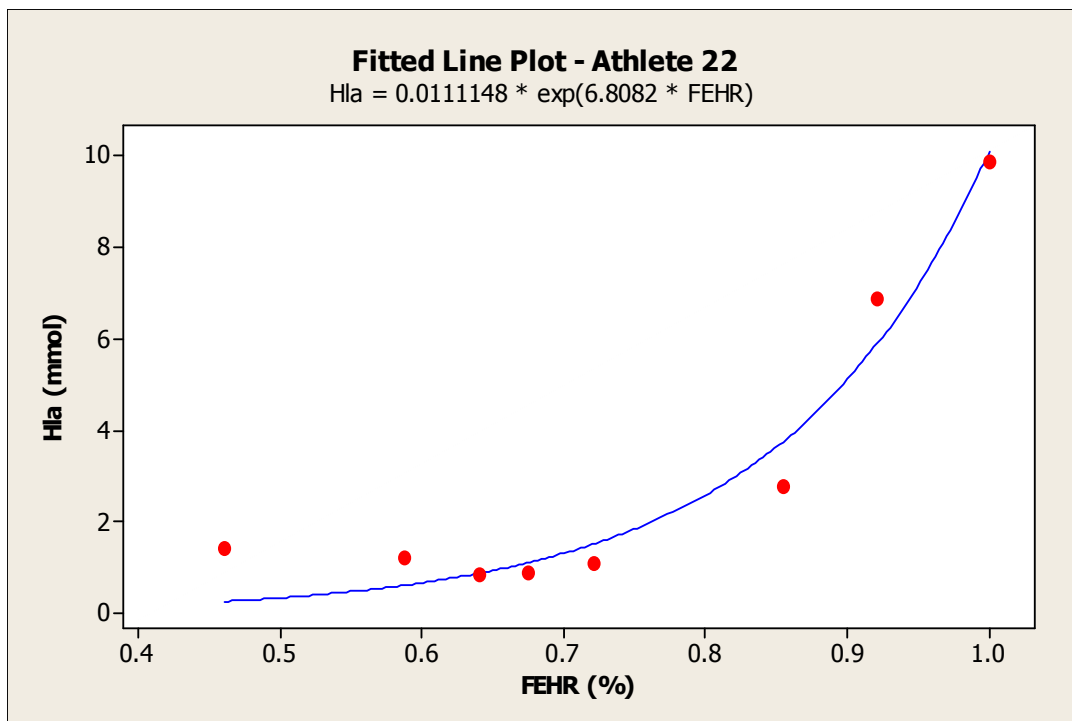
**Figure D-8:** Fitted line plot of Hla vs. FEHR using affine regression for Athlete 15.



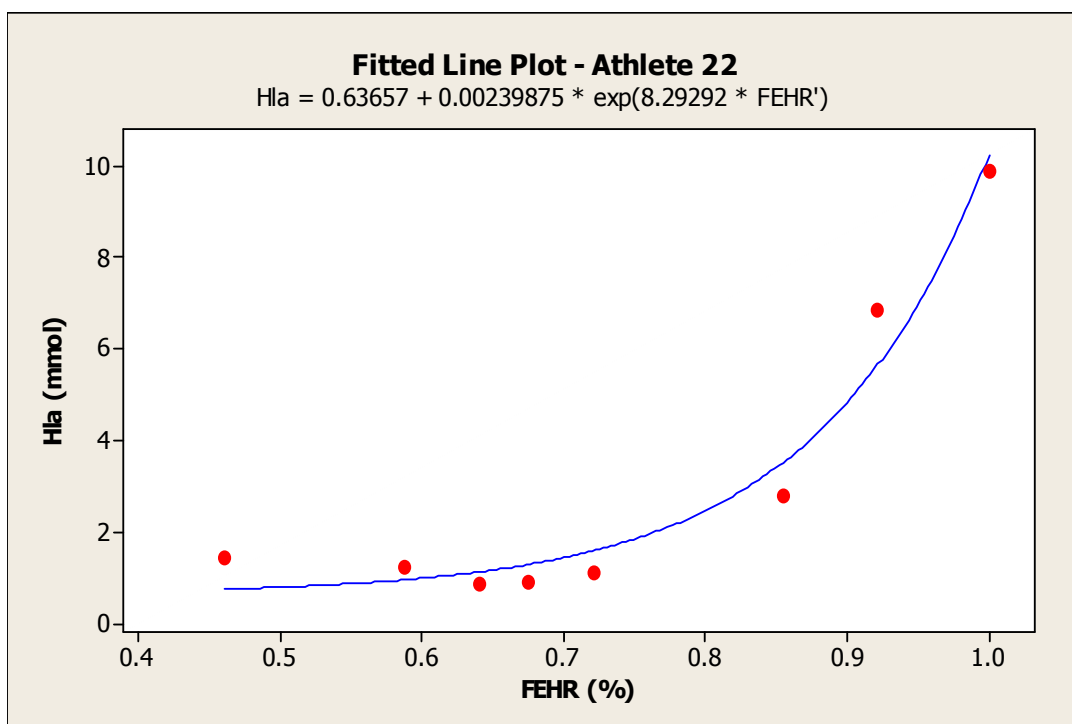
**Figure D-9:** Fitted line plot of Hla vs. FEHR using exponential regression for Athlete 16.



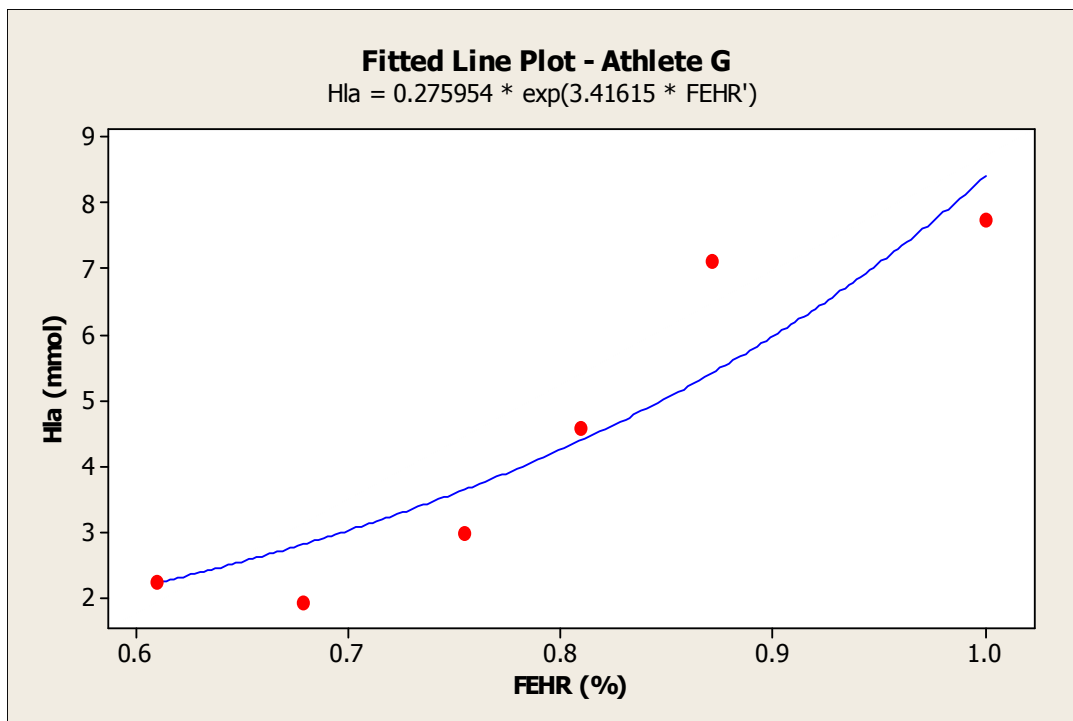
**Figure D-10:** Fitted line plot of Hla vs. FEHR using affine regression for Athlete 16.



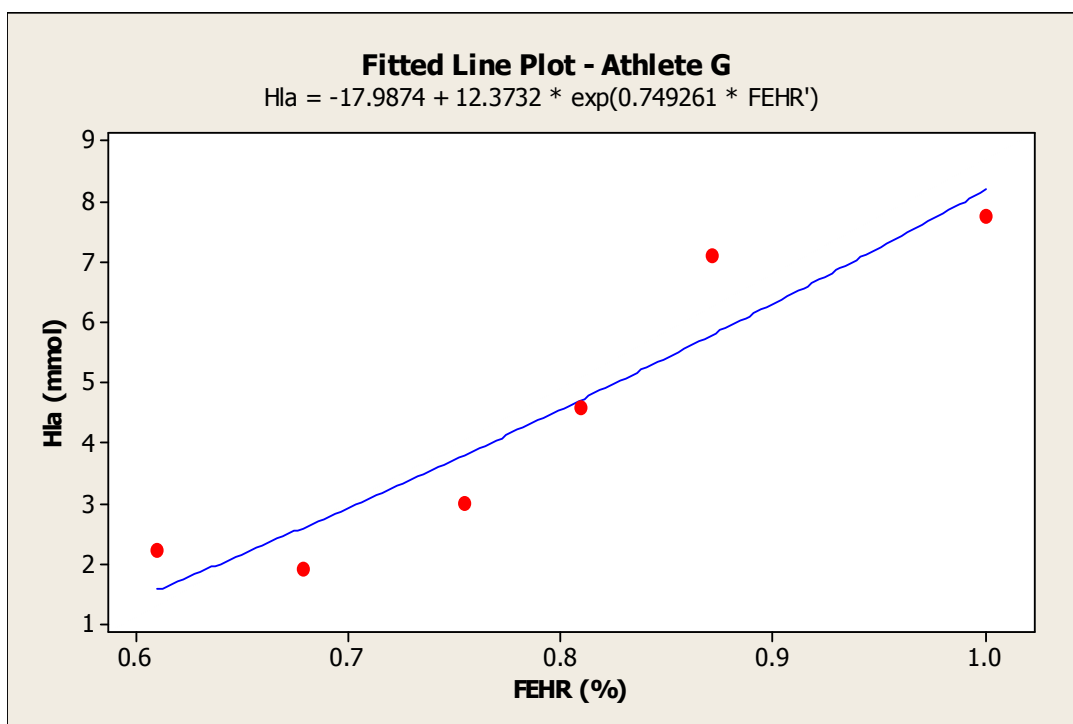
**Figure D-11:** Fitted line plot of Hla vs. FEHR using exponential regression for Athlete 22.



**Figure D-12:** Fitted line plot of Hla vs. FEHR using affine regression for Athlete 22.



**Figure D-13:** Fitted line plot of Hla vs. FEHR using exponential regression for Athlete G.



**Figure D-14:** Fitted line plot of Hla vs. FEHR using affine regression for Athlete G.

### Appendix E

This appendix provides the models and key statistics from the graphical models of Appendix D.

<u>Data Type</u>	<u>Regression Type</u>	<u>Equation</u>	<u>Q<sub>0</sub> SE Estimate</u>	<u>Q<sub>1</sub> SE Estimate</u>	<u>Q<sub>2</sub> SE Estimate</u>	<u>Final SSE</u>	<u>DF</u>	<u>MSE</u>	<u>S</u>
Team	Exponential	[HLa] = 0.0419446 * exp(5.33871 * FEHR)	0.0137	0.3473	N/A	171.733	96	1.789	1.337
Team	Affine	[HLa] = 0.908886 + 0.00705673 * exp(7.03434 * FEHR)	0.4199	0.00797	1.10071	165.800	95	1.745	1.321
Athlete 2	Exponential	[HLa] = 0.0887819 * exp(4.20084 * FEHR)	0.0774	0.662351	N/A	2.149	5	0.430	0.656
Athlete 2	Affine	[HLa] = 1.19761 + 0.00107693 * exp(8.43556 * FEHR)	0.3300	0.00281	2.5689	0.916	5	0.183	0.428
Athlete 15	Exponential	[HLa] = 0.128738 * exp(3.7926 * FEHR)	0.0774	0.662351	N/A	2.149	5	0.430	0.656
Athlete 15	Affine	[HLa] = 1.56216 + 0.000244061 * exp(9.81577 * FEHR)	0.3161	0.00084	3.40281	0.706	4	0.177	0.420
Athlete 16	Exponential	[HLa] = 0.088292 * exp(4.756 * FEHR)	0.0583	0.712202	N/A	4.606	5	0.921	0.960
Athlete 16	Affine	[HLa] = 0.942088 + 0.0270559 * exp(5.85742 * FEHR)	1.7845	0.0772	2.72533	4.349	4	1.087	1.043
Athlete 22	Exponential	[HLa] = 0.0111148 * exp(6.8082 * FEHR)	0.0100	0.932679	N/A	3.868	6	0.645	0.803
Athlete 22	Affine	[HLa] = 0.63657 + 0.00239875 * exp(8.29292 * FEHR)	0.5751	0.00492	2.03874	3.141	5	0.628	0.793
Athlete G	Exponential	[HLa] = 0.275954 * exp(3.41615 * FEHR)	0.1896	0.760371	N/A	4.533	4	1.133	1.065
Athlete G	Affine	[HLa] = -17.9874 + 12.3732 * exp(0.749261 * FEHR)	99.2576	87.1342	3.2732	3.528	3	1.176	1.084