Proceedings of the 33rd
Conference on Decision and Control
Lake Buena Vista, FL - December 1994

FA-13 9:30

Special Weighting and Objective Functions for Robust Synthesis with a Type of Coprime Factor Uncertainty Model

M. Abrishamchian and B. R. Barmish
Department of Electrical and Computer Engineering
University of Wisconsin
Madison, Wisconsin 53706

B. T. Polyak
Institute for Control Science
Moscow 117806
Russia

Abstract

The objective of this paper is to describe the construction of *special weighting and objective functions* which can be used to facilitate a robust synthesis which is motivated by the technique of Vidyasagar and McFarlane and Glover. To this end, in going from a physical description of the plant to a standard H^∞ problem to be solved, we include weights in a way which reduces the conservatism associated with the overbounding of uncertain blocks.

1. Introduction

The takeoff point for this paper is the following fact: Except for some highly restrictive uncertainty structures (for example, a single full complex block in the feedback loop leading to a classical H^∞ problem), robust synthesis cannot be carried out in a "one shot" manner. The avoidance of such iterations provides strong motivation for the work of McFarlane and Glover [1] and Vidyasagar [2]. By describing the plant via a coprime factor uncertainty model, they show that when finding a suitable robust controller, iterations can be avoided completely. This raises the following question: In massaging the given uncertainty description of the plant into the form required for a coprime factor style solution, can we reduce or eliminate conservatism which is introduced as a consequence of overbounding the uncertainty? In other words, there is a modeling step involved in going from the physical description of the problem to the coprime factor description of the plant. The success of the subsequent design process depends critically on the "tightness" of the resulting coprime factor model vis-a-vis the physically realizable uncertainties; for more detailed discussion; see [3].

2. The MIMO Case

In this paper, we consider a unity feedback MIMO system consisting of an uncertain plant $P(s,\Delta)$ connected in cascade with a proper controller $C(s)$.

2.1 Notation and Assumptions: More specifically, the plant family $\mathcal{P}$ which is described by the uncertain plant

$$P(s,\Delta) = (D_0(s)+\sum_{i=1}^{\ell}\Delta_i D_i(s))^{-1}(N_0(s)+\sum_{i=1}^{\ell}\Delta_i N_i(s))$$

with the polynomial matrices $N_i(s) \in \mathbf{R}^{r\times m}(s)$ and $D_i(s) \in \mathbf{R}^{r\times r}(s)$ for $i = 0,1,\ldots,\ell$ and the uncertainty $\Delta = [\Delta_1 \;\; \Delta_2 \;\; \cdots \;\; \Delta_\ell]$. Each complex block Δ_i is of size $r\times r$ and we take

$$\| \Delta \| \leq 1$$

where

$$\| \Delta \| \doteq \overline{\sigma}(\Delta)$$

and $\overline{\sigma}(\cdot)$ denotes the largest singular value. When $\Delta = 0$, we obtain the nominal plant

$$P_0(s) \doteq P(s,0) \doteq D_0^{-1}(s)N_0(s)$$

with $N_0(s)$ and $D_0(s)$ assumed to be coprime and $P_0(s)$ have a full rank over the field of rational functions. Furthermore, the nominal plant is proper and all the entries of $N_i(s)$ and $D_i(s)$ have degree less than the highest degree of entries of $N_0(s)$ and $D_0(s)$ respectively. Without loss of generality, we assume that $m \leq r$; i.e, the number of outputs is greater than or equal to the number of inputs.

2.2 The Set $\mathcal{C}$ and Robust Stabilization: The notation $\mathcal{C}$ is used to describe the set of all proper BIBO stabilizing controllers for the nominal plant. Then, given a compensator $C(s)$ connected in a feedback system, we write

$$C(s) \doteq N_c(s)D_c^{-1}(s)$$

with $N_c(s) \in \mathbf{R}^{r\times m}(s)$ and $D_c(s) \in \mathbf{R}^{r\times r}(s)$. Now, with the resulting closed loop polynomial matrix given by

$$\Psi(s,\Delta) = \Psi_0(s) + \sum_{i=1}^{\ell}\Delta_i\Psi_i(s)$$

where for $i = 0,1,\ldots,\ell$,

$$\Psi_i(s) = N_i(s)N_c(s) + D_i(s)D_c(s),$$

we call $C \in \mathcal{C}$ a ***robust stabilizer*** if the resulting family of closed loop polynomials described by

$$\psi(s,\Delta) \doteq det\; \Psi(s,\Delta)$$

and $\| \Delta \| \leq 1$ is robustly stable. That is, $\psi(s,\Delta)$ has all its roots in the open left half plane for all admissible Δ. In such a case, we say that $\mathcal{P}$ is ***robustly stabilizable***.

2.3 Special Weighting Function: We define the weighting matrix

$$W(s) \doteq [W_S(s) \quad W_T(s)]$$

Funding for this research was provided by the National Science Foundation under Grant No. ECS-9111570.

0-7803-1968-0/94$4.00©1994 IEEE

where

$$W_S(s) \doteq \begin{bmatrix} D_0^{-1}(s)D_1(s) \\ D_0^{-1}(s)D_2(s) \\ \vdots \\ D_0^{-1}(s)D_\ell(s) \end{bmatrix}$$

and

$$W_T(s) \doteq \begin{bmatrix} D_0^{-1}(s)N_1(s)(N_0^T(s)N_0(s))^{-1}N_0^T(s)D_0(s) \\ D_0^{-1}(s)N_2(s)(N_0^T(s)N_0(s))^{-1}N_0^T(s)D_0(s) \\ \vdots \\ D_0^{-1}(s)N_\ell(s)(N_0^T(s)N_0(s))^{-1}(s)N_0^T(s)D_0(s) \end{bmatrix}$$

2.4 Theorem (see [3] for proof): *The plant family $\mathcal{P}$ is robustly stabilizable if and only if*

$$\inf_{C \in \mathcal{C}} \|\mathcal{D}_0(W_S S_0 + W_T T_0)\mathcal{D}_0^{-1}\|_\infty < 1$$

where

$$\mathcal{D}_0(s) \doteq diag\ \{D_0(s), D_0(s), \ldots, D_0(s)\}.$$

2.5 Remarks: We note that our analysis simplifies considerably for the special case when the nominal plant $P_0(s)$ is square and open loop stable — but possibly destabilized by the uncertainty Δ. In this case, we first write

$$P_0(s) = \frac{1}{d(s)}N_0(s)$$

with $d(s)$ being a stable polynomial. Although this is not necessarily a coprime description of the nominal plant, the nominal plant is still detectable and stabilizable; therefore the result of Subsection 2.4 is still valid and it is easy to see that

$$D_0(W_{S,i}S_0 + W_{T,i}T_0)D_0^{-1} = W_{S,i}S_0 + W_{T,i}T_0$$

for $i = 1, 2, \ldots, \ell$. Hence, the relevant the H^∞ problem simplifies to

$$\inf_{C \in \mathcal{C}} \|W_S S_0 + W_T T_0\|_\infty < 1$$

with sensitivity weighting exactly as in Subsection 2.3 and complementary sensitivity weighting simplifying to

$$W_T(s) = \begin{bmatrix} N_1(s)N_0^{-1}(s) \\ N_2(s)N_0^{-1}(s) \\ \vdots \\ N_\ell(s)N_0^{-1}(s) \end{bmatrix}.$$

Notice that for the case of $r = m = 1$, that is, for the case of SISO systems, we obtain similar weighting and objective functions as given above.

3. Example for the SISO Case

To illustrate how the theory in this paper can address problems which are not readily transparent using the coprime factor uncertainty model, we consider an example involving three plants. Indeed, we begin with the nominal plant

$$P_0(s) = \frac{2s+1}{s^2+4s+3}$$

but also want to guarantee stability against the nonminimum phase operating condition

$$P_1(s) = \frac{s-1}{s^2+3s+4}$$

and the unstable operating condition

$$P_2(s) = \frac{s+2}{s^2-s+4}.$$

Now, we execute the recipe associated with the theory of Section 2 by considering the family of plants

$$P(s,\delta) = \frac{2s+1-\delta_1(s+2)-\delta_2(s-1)}{s^2+4s+3-\delta_1(s-1)-\delta_2(5s-1)}$$

where

$$|\delta_1|^2 + |\delta_2|^2 \le 1.$$

Notice that we recover the k-th plant $P_k(s)$ by setting $\delta_k = 1$ and $\delta_i = 0$ for $i \neq k$. Now, we carry out the required minimization in Theorem 2.4 and compute

$$\gamma^* = \inf_{C \in \mathcal{C}} \|\, W_S S_0 + W_T T_0 \,\|_\infty \approx 0.9727\ .$$

Since, $\gamma^* < 1$, we are guaranteed that $\mathcal{P}$ is robustly stabilizable. After carrying out appropriate order reduction, we obtain the compensator

$$C(s) = \frac{3s+2}{s+2}$$

which still guarantees

$$\|W_S S_0 + W_T T_0\|_\infty < 1.$$

Hence, $C(s)$ is a robust stabilizer.

4. Conclusion

In this paper, our main objective was to carry out a "one-shot" robust synthesis for a class of systems with structured uncertainty. This was accomplished by reducing the robust stabilization problem to a standard H^∞ problem with special weighting and objective functions. One direction for future research involves extending the results to systems with structured real parametric uncertainty.

References

[1] D. C. McFarlane and K. Glover, ***Robust Controller Design Using Normalized Coprime Factor Plant Descriptions***, Springer-Verlag, Berlin, 1990.

[2] M. Vidyasagar, *Control System Synthesis: A Factorization Approach*, MIT Press, Cambridge, 1985.

[3] M. Abrishamchian, B. R. Barmish and B. T. Polyak, "Special Weighting and Objective Functions for Robust Synthesis with a Type of Coprime Factor Uncertainty Model," Technical Report ECE-94-01, Department of Electrical and Computer Engineering, University of Wisconsin, Madison, 1994.