THE EFFECT OF DEMAND ELASTICITY ON SECURITY PRICES FOR THE POOLCO AND MULTI-LATERAL CONTRACT MODELS

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Abstract. Optimum power flows and security constrained power flows assume that customer demand is a fixed quantity. In the new competitive environment, it is necessary to assume that demand is elastic and will vary as a function of price. A critical element in any competitive model, whether it be a PoolCo or a multi-lateral contract model, is for a system operator to ensure reliability and feasibility of the power system operating point. Security pricing for feasibility was first advanced by Schweppe et al for the case of line flow constraints for a PoolCo model. Using geometry, this paper generalizes the approach to include any security constraint for a general competitive model. The paper discusses interpretations of security pricing and its possible implementation in Energy Management Systems.
Keywords. Spot prices, customer demand response, elasticity of demand, system security, optimization, security pricing.

1 Introduction

As electric utilities restructure and move towards deregulation, there is an increasing need to analyze the new pricing structures that have been proposed. Various models like PoolCo [1] and multi-lateral contract models (a more general version of the bilateral contract model) [2] have been advocated. Each of these models requires a system operator to ensure reliable coordinated power system operation. The responsibilities of the system operator range from many (PoolCo) to a few (multi-lateral contract model). However, at a minimum, the system operator would be responsible for maintaining system security.

Whether the system operates as a PoolCo or using a multi-lateral contract model, many consumers (especially large consumers) are likely to see the marginal short run spot pricing of electricity advanced by Schweppe et al [3, 4]. Prices will be set by the laws of supply and demand. The price will vary spatially (across customers) and temporally (by time). The price at any time will be determined by (a) available generation, (b) available transmission and distribution capacity, and (c) demand. The price will be made up of different components, among them:

- Generation fuel costs: These include fuel costs for meeting the consumer demand along with the relevant component of system losses.
- Ancillary services costs: Include costs for supporting such services as reactive power, voltage control, frequency control, etc.
- System security costs. These refer to the costs associated with the need to maintain feasibility for many credible contingencies. Alternatively, these costs can be quantified as the expected value of the cost of a partial outage. In a rationally designed system, both these costs should be equal.

The fundamental spot pricing principles have been established in [3] and in references therein. However, the pricing of system security for reliability has not received much attention. According to a recent study [5], "Too many of the current discussions of industry restructuring pass over this complexity [of the transmission grid operation] with a cursory reference to the control function or the need to maintain operational reliability".

The basic idea of pricing security for the case of line flow constraints was first advanced in [3]. Kaye et al [6] extended this approach to consider pricing in the presence of contingencies and determine, via a feedback mechanism, the optimal societal consumption and optimal prices. Alvarado et al [7] considered the formal quantification of system security by computing the outage cost associated with specific operating points, as well as the influence of actions on this cost. The present paper offers a different perspective of security prices, but retains the spirit of [3]. It uses a geometric approach to generalize the method in [3], to consider pricing near any security constraint (e.g., voltage collapse, Hopf bifurcation instabilities, line flow constraints, etc.).

The main assumptions in this paper are:

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- The customer demand of electricity depends on its elasticity [8], i.e., it is controlled by the price of electricity. Higher price implies lower demand and vice-versa.
- There is a practical way of measuring consumer demand-price relationships (or elasticity of demand); e.g., EPRI's C-VALU software [9].
- The system security limits, characterized by the notion of an Operation Limit Boundary (OLB), must not be violated.
- The system operates under ideal market conditions. That is, suppliers sell (electricity) at the marginal cost and the customer consumption (of electricity) is determined by the price and the elasticity of demand, without a time lag.
- Security prices are used for maintaining security (or feasibility of contracts) in the power system.

The objectives of this paper are:

- Derive prices when both customer demand price relationships and security limits are included in a market driven system. The methods are general and are valid under either PoolCo or multi-lateral contract models.
- Provide an intuitive understanding of the effect of prices when the system is at or near security constraint and differentiate security costs from congestion costs [10].

The paper is organized as follows. Section 2 introduces the concept of the Operation Limit Boundary (OLB), and the notion of security in power systems. Section 3 briefly describes the problem setting. Section 4 obtains prices taking full account of system constraints and customer price-demand response. Section 5 interprets spot prices. Section 6 discusses various computational issues and the implementation of prices in energy management systems.

2 System Security

Any physical system is subject to operational constraints. An operating point will be called feasible provided it violates no system constraint. The collection of all feasible operating points constitutes the *Feasibility Region* (FR), and can be viewed as bounded by the intersection of the various constraint sets. The *Operation Limit Boundary* (OLB) is the boundary of the feasibility region. For convenience it is assumed that each individual system constraint is smooth. Thus, the OLB is a smooth surface except at the points where multiple constraints become binding.

For a power system, operational constraints arise because of a number of reasons:

- Generation limits: There is a maximum total amount of generation that the system can produce.
- Transmission and Distribution limits: The system can produce the power, but cannot transport this power to one or more demand locations as a result of either:
  - Overloaded lines.
  - Voltage collapse limits.
  - Transient stability limits.

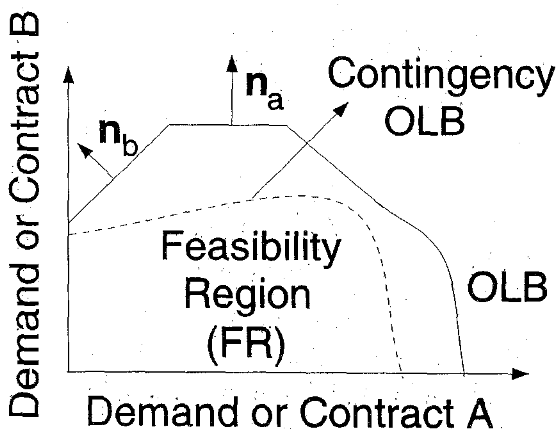

Fig. 1: *Feasibility Region and the OLB.*

Fig. 1 illustrates the concept of the OLB in a power system setting. For a PoolCo model, the horizontal and vertical axis represent the demand of two customers A and B. For a multi-lateral contract model, the axes would represent two contracts A and B. The OLB represents a hard limit to the demand (or contracts) that can be met by the system. Any demand (or contracts) outside the boundary is infeasible, and hence cannot be satisfied by the system. Depending on the nature of the OLB, an attempt to satisfy a combination of demands or contracts outside the OLB can result in a blackout. The horizontal and vertical distances from a current operating point to the OLB are the security margins of A and B respectively. Each location can have a different security margin. The margin at any location depends on the demands or contracts at other locations. For instance, even if A is held constant, the security margin at A may change according to changes in B.

Contingencies affect the OLB. Specifically, outages of lines and/or generators result in a new OLB. Contingencies typically shrink the feasibility region FR and hence decrease the security margins (the OLB for a contingency is shown by dotted lines in Fig. 1). The OLB could also vary with time, depending on which system resources are available (generation, transmission, etc.).

In practice, it will be necessary to use a conservative FR and OLB which takes into account a safety margin of error. For instance, it may be required that the system survive one or more credible contingencies at any given time. Whatever the criterion for its determination, an OLB exists and the power system must operate within its confines at all times.

3 A Restructured Electric Industry

This section briefly describes a possible restructured energy marketplace. Producers of electric power are called suppliers. Customers are the consumers of the electric power. A system operator acts as a middleman between the suppliers and customers. The system operator manages the transmission and distribution of electrical power. The system operator is also responsible for maintaining the system security.

In the PoolCo model, the system operator coordinates the sale of electricity and sets spot prices. The system operator buys generation and ancillary services (voltage support, load following, operating reserve, etc.) and dispatches these in an economically efficient way.

In the multi-lateral contracts model, the prices are set by mutual contracts between suppliers or groups of suppliers and customers or groups of customers, with some participation by the system operator. The system operator checks for feasibility of various contracts and also imposes any applicable additional costs on the participants. For instance, the system operator could provide (or contract with other parties to provide) reactive power support, losses, operating reserve and other ancillary services.

Let $\mathbf{x} = [x_1, \ldots, x_m]^T$ be a vector of customer demands (in a PoolCo model) or a vector of contract transactions for power transfer (in a multi-lateral contract). The corresponding prices (per unit quantity consumed) are denoted by $\boldsymbol{\rho}^* = [\rho_1, \ldots, \rho_m]^T$. In an efficient market system, these prices are the costs incurred in satisfying $\mathbf{x}$ and are composed of several components: the cost of producing $\mathbf{x}$, the cost of ancillary services and the cost of security.

Let $C^*(\mathbf{x})$ be the *optimal cost* for satisfying $\mathbf{x}$. In a PoolCo model, $C^*(\mathbf{x})$ is the (minimum) cost of optimally dispatching resources (generation, reactive power support, etc.) to satisfy $\mathbf{x}$ taking full account of system constraints like load flow equations (equality constraints), and voltage, generation and flow limits (inequality constraints). The mathematical optimization of a constrained problem generally calls for the construction of a Lagrangian function that incorporates the cost function as well as the constraints [3, 11, 12, 13]. The optimal cost $C^*(\mathbf{x})$ will typically vary with time of day and across seasons, depending on which units are supplying power on the margin.

In a multi-lateral contract model, it is convenient to unbundle $C^*(\mathbf{x})$ as $C^*_{prod}(\mathbf{x}) + C^*_{anc}(\mathbf{x})$. Then $C^*_{prod}(\mathbf{x})$ is the cost that the suppliers charge the customers for producing $\mathbf{x}$; this cost depends on the contractual agreement between suppliers and customers. The system operator will impose a cost $C^*_{anc}(\mathbf{x})$ for ancillary services[1] (e.g., voltage support, reactive power, losses, etc.). The system operator would commit resources to minimize $C^*_{anc}(\mathbf{x})$.

To introduce the notion of customer response [3], assume that customers consume electric power because they derive a benefit from the use of electric power, and this benefit has a value to them. Let $B_k(x_k)$ denote the benefit derived by customer k (or contract k) as a result of x_k units of power being consumed. Assume that $B_k(x_k)$ is a smooth function of x_k and represents the total benefits of consuming x_k. Let $B(\mathbf{x}) = \sum_i^n B_i(x_i)$ be the total societal benefit function, which is the sum of individual benefits $B_k(x_k)$.

The consumption of electric power will be affected by price. Let ρ_k be the price quoted for consumer k (or for contract k). The *net benefit* seen by a customer (or for a contract) is the difference between the benefit derived by the customer and the cost of paying for that benefit. A rational customer will determine the level x_k of consumption of electric power to maximize net benefits, i.e., $\max(B_k(x_k) - \rho_k x_k)$. This occurs when $\rho_k = \partial B_k(x_k)/\partial x_k$, that is, when the marginal benefit to a customer (or for a contract) equals the price.

Define $g_k(x_k) = \partial B_k(x_k)/\partial x_k$ for $k = 1, ..., m$ and $\mathbf{g}(\mathbf{x}) = [g_1(x_1), ..., g_m(x_m)]^T$. The vector $\mathbf{g}(\mathbf{x})$ is the customer demand-price relationship and represents the customer response. That is, the demand price relationship is given by:

$$\boldsymbol{\rho} = \mathbf{g}(\mathbf{x}) = [g_1(x_1), \ldots, g_m(x_m)]^T \tag{1}$$

$g_k(x_k)$ is assumed to be a decreasing function of x_k, i.e., higher the price, lower will the level of consumption be, and vice-versa. The relationship $\boldsymbol{\rho} = \mathbf{g}(\mathbf{x})$ need not be fixed. On the contrary, it is quite likely that it will depend on exogenous factors like weather, time and season. Therefore, prices may have to be set periodically, e.g., hourly or daily.

4 Price-Sensitive Security Constrained Optimum Power Flow

The classical optimal power flow problem does not consider customer demand price relationships. Instead a projected customer demand $\tilde{\mathbf{x}}$ is used. If $\tilde{\mathbf{x}}$ lies in the interior of the *Feasible Region FR*, the marginal cost of supplying x_k to customer k is the price ρ_k for customer k: $\rho_k = \partial C(\tilde{\mathbf{x}})/\partial x_k$. However when customer demand response and system security constraints are considered, a different approach must be used.

Problem Statement. *Determine the optimum consumer demand* $\mathbf{x}^*$ *and prices* $\boldsymbol{\rho}^*$ *such that (a)* $\mathbf{x}^*$ *is feasible, and the resources for producing* $\mathbf{x}^*$ *are dispatched*

[1] Some of these services may be provided by the suppliers themselves

at optimal cost, and (b) all customers maximize their net benefits and respond optimally to the spot prices $\boldsymbol{\rho}^*$.

This is a non-standard optimization problem, and as such cannot be cast as a single minimization problem. A two-tiered approach [3] is needed: First the necessary conditions for the most efficient spot pricing must be determined by constraining the customer demand (or contracts) to lie in the feasibility region FR. (goal (a) above). Second, customer response must be taken into account to close the feedback loop (goal (b) above). The two-tiered approach maximizes the benefits of the suppliers, system operator and customers and results in optimal pricing and consumption of demand, taking full account of security constraints.

The system operator minimizes net societal costs to derive prices[2] $\boldsymbol{\rho}^*$, assuming that $\mathbf{x}^*$ is consumed. The system operator minimizes $C^*(\mathbf{x}) - B(\mathbf{x})$ (the first level of optimization) subject to the feasibility constraint, $\mathbf{x} \in FR$, *and* assuming optimal customer response when a price is seen, i.e., $\boldsymbol{\rho} = \mathbf{g}(\mathbf{x})$ (this is the second level of optimization). The solution to the two layered problem is summarized in the Proposition below.

Proposition. Let $\mathbf{x}^*$ be the optimal demand vector (or contract vector), and a regular point of the feasibility constraint [14]. Let $\boldsymbol{\rho}^*$ be the corresponding optimal prices seen by the customers. Then $\mathbf{x}^*$ and $\boldsymbol{\rho}^*$ satisfy:

$$\begin{array}{l} \mathbf{x}^* \in \mathrm{int}(FR): \ \boldsymbol{\rho}^* = \mathbf{g}(\mathbf{x}^*) = \frac{\partial C^*}{\partial x^*}(\mathbf{x}^*) \\ \mathbf{x}^* \in \mathrm{bdy}(FR): \boldsymbol{\rho}^* = \mathbf{g}(\mathbf{x}^*) = \frac{\partial C^*}{\partial x^*}(\mathbf{x}^*) + \lambda^* \mathbf{n}(\mathbf{x}^*) \end{array} \tag{2}$$

where $\mathbf{n} = [n_1(\mathbf{x}^*), ..., n_k(\mathbf{x}^*), ...]$ is the unit *outward normal* (in, say, the 2-norm) at the OLB and $\lambda^* \geq 0$. Here $\frac{\partial C^*}{\partial x^*}(\mathbf{x}^*)$ is the marginal cost incurred by the system operator and suppliers when producing and delivering $\mathbf{x}^*$ and $\lambda^* \mathbf{n}(\mathbf{x}^*)$ is the marginal security price.

To prove (2), note that if $\mathbf{x}^*$ is in the interior of the *Feasible Region*, then the feasibility constraint is not binding and the first line of equations is obtained. If $\mathbf{x}^*$ is on the OLB, then the feasibility constraint is binding, and using the theory of Lagrange multipliers, the second line of equations is obtained. λ^* is the Lagrange multiplier associated with the security constraint, which the system operator will use to set prices so that the security constraints are not violated.

One may use Figs. 2, 3, to visualize the two-tiered approach. This is especially helpful for a multi-lateral contract model. The system operator would be given a set of all contracts for power transfer. If this contract set lies within the OLB, the system operator would compute the ancillary costs $C^*_{anc}(\mathbf{x})$ and allocate these costs to the contracts. If the contract set is infeasible, the system operator would impose a security price on the contracts to force the customers to ration demand till feasibility is achieved. In either case, the participants would respond by maximizing their benefits, and

[2]For a multi-lateral contract model, $C^*_{prod}(\mathbf{x})$ depends only on contractual agreements between customers and suppliers.

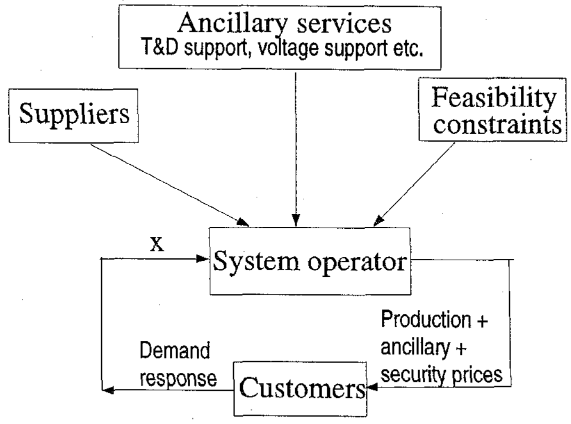

Fig. 2: *Feedback process for the PoolCo model.*

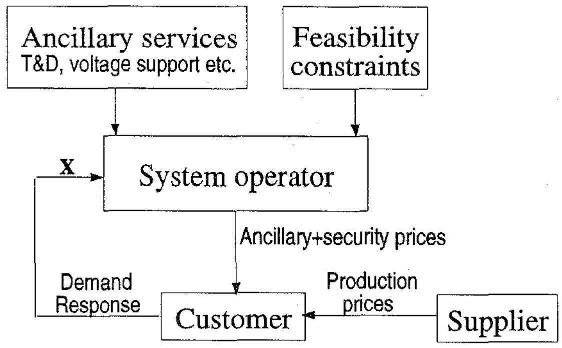

Fig. 3: *Feedback process for multi-lateral model.*

the feedback process would ensure that the system operates at an optimal point $\mathbf{x}^*$ satisfying (2).

Remarks.

1. The Proposition assumes that only one feasibility constraint is binding at the optimal solution. Should multiple feasibility constraints be binding, the expressions above must be modified to include Lagrange multipliers and outward normals for each of the binding constraints.

2. Optimal spot pricing near security constraints assumes rational customer response and that this response can be predicted. Security pricing alone cannot preserve system security. Security pricing would not be applicable in emergencies, or when unanticipated contingencies occur. In such cases, the system operator should have the power to curtail load and/or generation to maintain security.

3. If the various participants (system operator, suppliers, customers) "cooperate" and ration demand or contracts voluntarily (rather than by rationing in response to prices), security prices can be avoided. However, customers with low demand elasticities would not be agreeable to ration their

demand or contracts as much as those with relatively higher elasticities. A volunteer program like an *Interruptible Service Program* (ISP) managed by the system operator may be a possible solution.

5 Interpretation of Security Prices

1. *Geometric Interpretation.*

Fig. 4 shows the marginal costs associated with the single customer-single supply model. It is assumed that because of transmission constraints, the system cannot transfer more than d units of power (shown by dashed lines). Solving (2): the consumption level $x^* = [d]$. $\mathbf{n} = [1]$, $\lambda^* = g_1(d) - a$, the price $\boldsymbol{\rho}^* = [g_1(d)]$. Thus the security cost corresponds to a vertical portion added to the supply curve. When the system is not at feasibility limits, the price is effectively set by the suppliers. At system feasibility limits, the price is effectively set by the customers.

Demand elasticities of customers range from highly elastic to highly inelastic. Customers with high elasticities will be highly sensitive to security prices; even a low security price will induce them to ration their demand significantly. Customers who are more inelastic will be less sensitive to security prices and a relatively high price is needed for rationing their demand.

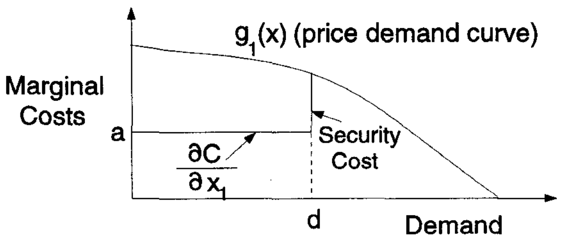

Fig. 4: *Geometric interpretation of security pricing*

Physically, n_k represents the optimal first order sensitivity of system security at the OLB with respect to customer k (or contract k). Alternatively, $-\mathbf{n} = -[n_1, ..., n_m]^T$ represents the optimal direction along which the customers (or contracts) should increase their consumption to get away from the OLB (see e.g., [16]).

Since n_k typically varies across customers (or contracts), they see spatially different security prices in general. For example, in Fig. 1, if the system is currently operating at the top where the normal is $\mathbf{n}_a = [1, 0]^T$, an increase in A's demand (or contract transaction) will violate the security constraint, while B's behavior near the operating point locally has no impact on system security. It is also theoretically possible for some customers to see a *negative* security price (Fig. 1) (when $\mathbf{n}_b$ has a negative component along the x-axis). In this case, system security is improved by having A *increase* demand (or contract transaction). Negative security costs may be seen for the case of security constraints caused by line flow limits. The combination of engineering tools (sensitivity with respect to security constraints) and economics (law of supply and demand) send the right pricing signals to customers (or to contracts) near security constraints.

2. *Security costs and Congestion costs.*

There is an important distinction between the security costs arising at a *feasibility* constraint and the costs arising as a result of congestion problems [10]. Both security costs and congestion costs [10] are obtained as the difference between the customer payments and the costs incurred by the supplier (power generation) and system operator (ancillary services). Informally, congestion costs are incurred when the vector of demands (or contracts) $\mathbf{x}^*$ lies in the interior of the feasibility region, and when $\mathbf{x}^*$ can be satisfied by using more expensive power, i.e., by "out of merit dispatch" (because the use of cheaper power is blocked by congested lines). *Hence, in the case of congestion, all customers will see prices that will be equal to the marginal cost of delivering power to them; these marginal costs, however, are higher (than usual) for one or more customers because the power from units with lower marginal costs cannot be used without violating system constraints.* Security costs are incurred when $\mathbf{x}^*$ lies on the OLB and when $\lambda^* > 0$; if these costs are not imposed, the system will become infeasible and operational reliability will be lost. *When the system is operating at the boundary of the feasibility region, prices in general will not equal the marginal cost of delivery; typically, they will be higher.* The difference between congestion and security costs can be explained another way. Congestion costs arise when suppliers are forced to modify their behavior (change the dispatch order) to conform to system constraints to satisfy $\mathbf{x}$. Security costs arise when $\mathbf{x}$ cannot be satisfied by the system, and the system operator uses pricing signals to make customers modify their behavior (rationing $\mathbf{x}$) to maintain feasibility.

Hogan [10] proposed the idea of transmission capacity rights (TCR) as a method to hedge congestion. Under his proposal, a TCR holder (who is entitled to a delivery of P MW to a specific point of the system) could either perform the act of delivering P or could have it done by the system operator (e.g., if congestion problems exist) and receive an equivalent rental payment.

Security prices arise out of feasibility constraints. While there is no hedge for feasibility, security prices can be avoided by participating in an Interruptible Service Program (ISP).

3. *Interruptible Service Program.*

In an ISP, participants enter into a contract with the system operator and agree to have part (or all) of their service interrupted at certain times of the day in return for (monetary) compensation. One possible approach to design an ISP follows.

Assume a participant in an ISP agrees to have some of his service interrupted at the system operator's convenience. This gives the system operator a powerful control tool to manage the system security. A reasonable compensation for the participant would be[3]:

- The participant should not see the security component of the price at any time (since the participant is not a threat to system security).

- In addition, the system operator agrees to pay a participant a compensation that depends on how much service was interrupted. Given the number of participants in the ISP, this compensation should be designed such that the system operator is revenue neutral. If the ISP has many participants, the compensation per participant would be a small percentage of the expected security prices revenue; if the ISP has few participants, the compensation would be a relatively higher percentage.

Under this approach, it makes sense for the system operator to interrupt service only when the system is near the OLB. Likewise, if the pricing scheme has been properly done, a participant should be indifferent to being interrupted. Customers with high elasticity of demand would participate in the ISP to hedge security costs; customers with high inelasticity would find the ISP less useful. The ISP gives a way for the participants to have a claim on security costs collected by the system operator and gives the system operator flexibility in managing system security.

4. *Transmission and distribution system expansion*

The use of security costs can (only) be advocated as a short run measure, since in the short run, the system operator cannot expand capacity. In the short run, security prices send efficient pricing signals to manage the system better. This is especially true for models like the multi-lateral contract model, in which the system operator plays a minimal role, but could use the security costs as a tool to avoid infeasibility of contracts. Note however that the transmission and distribution services (and the system operator) are a natural monopoly and hence will be regulated. Regulators will not allow security prices to be used as a frequent tool to manage security. The transmission company would be required to expand the system.

6 Computational Issues

This section discusses computational issues involved when the system operator solves (2). The computation of the optimal spot prices taking full account of customer demand response and system security limits can be integrated into Energy Management Systems.

The computation of $\partial C^*(\mathbf{x})/\partial x_k$ can be done by an optimal power flow solver. The computation of $\partial C^*(\mathbf{x})/\partial x_k$ is subject to load flow constraints and depends on system conditions. (For the multi-lateral contract model, the system operator has to solve for $C^*_{anc}(\mathbf{x})$ only.) The total cost of generation is assumed to be nearly quadratic, so the marginal cost of generation is nearly linear. The transmission system losses are approximately quadratic in generation produced, so the losses have an approximately linear marginal cost; this cost is typically less than 10% of the marginal generating costs. The marginal costs of the remaining ancillary services (voltage support, distribution losses, load following, etc.) could vary depending on system conditions, but they are typically also a small fraction of the marginal generating costs (though in some cases, these costs could be significantly higher [15]). Thus, it may be helpful to think of $\frac{\partial C^*(\mathbf{x})}{\partial x_k}$ as a linear function in $\mathbf{x}$ multiplied by a penalty factor which is close to 1. (This multiplicative factor would vary spatially among customers.)

The computation of $\mathbf{n} = [n_1, ..., n_m]$ is more complex. For security constraints caused by voltage collapse problems, $\mathbf{n}$ can be shown to be a left eigenvector of the load flow Jacobian corresponding to an eigenvalue of 0[16]. The sensitivity of $\mathbf{n}$ with respect to $\mathbf{x}^*$ has been derived in [17]. The computation of $\mathbf{n}$ at voltage collapse limits has been a subject of much research [16]. The computation of $\mathbf{n}$ for the case of Hopf instabilities is given in [16]. If the security problems are caused by line overloads, one may use sensitivity analysis to compute $\mathbf{n}$ (see, e.g., [11]).

One may solve (2) by using Newton type methods [18, 19]. First order derivatives of the terms in (2) with respect to each x_k make the implementation of a Newton type method simpler. A possible solution method that does not depend on explicit derivatives of the terms in (2) is described next.

Let $\mathbf{x}^i$, $\boldsymbol{\rho}^i$, λ^i be the vectors of demands, contracts and prices respectively after i iterations.

Start with an initial estimate $\mathbf{x}^0$.

- 1a. If $\mathbf{x}^i \in FR$, set optimal spot prices $\boldsymbol{\rho}^i = \boldsymbol{\rho}^*$ by solving (2) with $\mathbf{x}^i = \mathbf{x}^*$ on the right hand side.

- 1b. If $\mathbf{x}^i \notin FR$, ration the demands or contracts and select some $\mathbf{y}$ (heuristically) on the OLB with a corresponding outward normal $\mathbf{n}(\mathbf{y})$ and set optimal spot prices $\boldsymbol{\rho}^i = \boldsymbol{\rho}^*$ by solving (2) with $\mathbf{x}^i = \mathbf{x}^*$ on the right hand side; λ^i is chosen large enough so that $\mathbf{x}^{i+1} \in Bdy(FR)$ (see step 2).

- 2. Solve (2) for $\mathbf{x}^*$, with $\boldsymbol{\rho}^*$ set to $\boldsymbol{\rho}^i$ on the left hand side, to get $\mathbf{x}^{i+1} = \mathbf{x}^*$.

- 3a. If $\mathbf{x}^i$ is not close to $\mathbf{x}^{i+1}$, set $i \rightarrow i+1$ and go to step 1

- 3b. If $\mathbf{x}^i$ is close to $\mathbf{x}^{i+1}$, $\mathbf{x}^* = \mathbf{x}^i$, $\boldsymbol{\rho}^* = \boldsymbol{\rho}^i$. Stop.

If this iterative method converges, the optimal spot prices $\boldsymbol{\rho}^*$ and optimal consumption $\mathbf{x}^*$ are obtained.

[3]Assuming that security prices will typically be positive.

7 Conclusions

The need to maintain operational reliability is very important in power systems. The current trend in the electric power industry is towards deregulation. Two different methods for operating the electrical power systems have been proposed — the PoolCo and the multilateral contract models. The new structure will lead to increased competition and but will also lead to less coordinated actions among the various competitors. A system operator is seen as an essential component of the restructured system. The operator will be in charge of system security. Since some demands are, in fact, elastic, one of the tools of the operator will be the use of supplementary price signals to manage system security. This paper has discussed in detail how to compute optimal security prices near security constraints, taking full account of customer demand response as a method to help in preserving system security.

The method has been presented as a two level approach that maximizes the benefits of all the participants in the restructured energy marketplace — the suppliers, the customers and the system operator. The implications and interpretations of the method have been discussed in detail. The net result is a procedure that permits an operator to set security prices in an optimal manner.

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Biography

Rajesh Rajaraman has completed his Ph. D. requirements in the ECE Dept. at the Univ. of Wisconsin-Madison. He is working at Christensen Associates as a Senior Engineer in the area of power system economics. His main interests are in the areas of pricing of electricity services, FACTS devices and subsynchronous resonance.

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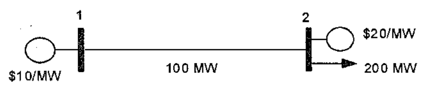
Discussion

Harry Singh, (*Pacific Gas and Electric Company, San Francisco, CA*) The authors have presented interesting and useful observations on the distinction between congestion costs and security costs. This discusser has the following comments for the authors consideration:

Significant attention has been given to how best the externalities associated with the transmission system can be represented in setting prices in a competitive power pool. It is generally accepted that a failure to represent transmission constraints can eliminate desirable locational price signals (e.g. for proper siting of new power plants). This issue is addressed by proposals that advocate using spatially varying spot prices that consider transmission losses and congestion. Unfortunately (in terms of preserving the locational signals), the congestion component of spatial variations in spot prices is subject to arbitrage opportunities. For example, in a lossless two node network shown in the figure below, one might expect the spot price at node 1 to be 10 $/MWh and that at node 2 to be 20 $/MWh when the line between the two nodes is congested. However, the generator at node 1 may raise its price bid to equal or approach that of the more expensive generator at node 2. Alternatively, it could submit a bid that restricts its available capacity to just less than the capacity of the transmission line and eliminate congestion congestion altogether. Similar arguments apply to the load at node 2 who might enter into a bilateral contract with the supplier at node 2 for an amount slightly larger than the capacity of the transmission line. In either instance, the price difference across the line is reduced to zero. This paper discusses a security component in spot prices that is independent of the price bids of generators. An obvious question is whether spatial price differences that result from security costs are less susceptible to arbitrage resulting from strategic behavior by bidders.



The second comment concerns the treatment of demand elasticity. Recent proposals to restructure electricity markets, allow for explicit demand side bidding where a load submits its demand curve in the daily power auction. The Optimal Power Flow problem in such cases is easily solved without the additional complexity of the two-level approach discussed in the paper. Have the authors considered how their proposal for security pricing might be implemented in a competitive power pool that allows for demand side bidding? The information available through demand side bids might allow for alternatives to the use of an estimated "value of lost load" needed to compute security prices.

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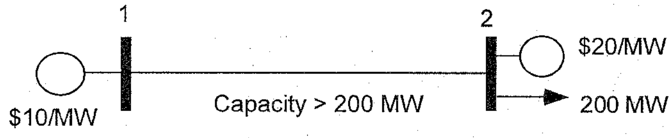
Rajesh Rajaraman (Christensen Associates), Jayant Sarlashkar and Fernando Alvarado (University of Wisconsin-Madison) :

The authors thank Dr. Singh for his insightful discussion. We address his questions below.

Response to Comment 1:

We agree with Dr. Singh's overall comment and would like to add to his observation. Undesirable game playing opportunities arise as a result of market power of participants; in Dr. Singh's well constructed example, the game playing opportunities are due to the monopoly position of the generator at bus 1 and/or the monopsony position of the load at bus 2. We further illustrate this in the figure below, where the line connecting nodes 1 and 2 have a transmission capacity well in excess of 200 MW.



There is no transmission congestion problem in the system. However, Dr. Singh's game playing opportunities --- that reduce the nodal price difference to zero --- also exist for this system: (a) The generator at bus 1 could raise its price bid to equal or approach that of the more expensive generator at bus 2, resulting in a $20/MWh price; (b) the generator at bus 1 could restrict its capacity to just less than the load at bus 2, resulting in a $20/MWh price; (c) if the generator at bus 1 bids its capacity to be x < 200 MW [1], then the load at bus 2 could sign a bilateral contract with generator 2 for an amount just slightly larger than 200-x MW, resulting in a $10/MWh price.

We recognize, however, that both congestion and system security issues could have a significant effect on the degree of market power of participants. Therefore congestion/security could have an important, albeit indirect, effect on anti-competitive game playing opportunities.

Response to Comment 2:

We agree with Dr. Singh that an OPF would solve for the optimal nodal prices in the pool model if the demand curves are known. However, we note that while the use of demand side bids is useful in ascertaining the value of load, there is a distinction to be made between the value of a "bid" load curve (where a bidder has presumably had an opportunity to make adjustments and be selective about which portions of a load might be interruptible) and the value of a no-notice, no contract load outage. While the former knowledge may put a bound on the latter, this bound is by no means tight.

The two-level approach suggested in the paper is also needed to account for demand elasticity in the multilateral contract model (or minimum ISO model), where the ISO would not have information about demand curves. The paper [1] illustrates how the two-level approach can be used by an ISO to adapt to the market, and anticipate the behavior of market participants.

The security component of the nodal prices is currently being developed as part of EPRI/Laurits R. Christensen Associates' Marginal Costing Within Transmission Network (MCTN) software. We have used this software to estimate bus marginal costs for the Southern Electric utility system, and are currently applying this methodology to estimate transmission marginal costs for an international utility system. Appropriate results from these projects will be published shortly as EPRI reports.

REFERENCE

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[1] If x > 200 MW, both nodes will have a price of $10/MWh.

Manuscript received October 30, 1996.