NEW INVESTIGATIONS OF VON NEUMANN TYPE FLUIDS

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Abstract

This paper presents some recent computer studies of a von Neumann type fluid. The particular model and the discrete dynamical equations utilized are described in detail and various results from fluid mechanics and statistical mechanics are discussed. Computer examples are presented and the model is shown to behave like a liquid as regards density, interparticle separation, and surface wave generation.

I. Introduction

During the late nineteen thirties, John von Neumann "became interested in questions of theoretical hydrodynamics, particularly in the great difficulties encountered in obtaining solutions to partial differential equations by known analytical methods." [1] From his studies, von Neumann published a paper [2] which described a new numerical method, discrete rather than continuous, for treating a class of hydrodynamic problems. In this approach, a quasi-molecular fluid model is developed in which the number of particles involved is reduced from that of a real fluid ($\sim 10^{23}$) to a much more manageable number (~10-100). In order that the smaller number of particles have the appropriate hydrodynamic behavior, the interparticle forces are adjusted and the physical parameters of each particle are modified. In a self-contained fashion, then, the present paper builds upon the results of an earlier paper [3] and extends the study of what has come to be called a von Neumann type fluid.

II. Basic Approach

The following physical situation provides a starting point for the computer model. A liquid is poured into a right-circular cylindrical container. Initially, on the macroscopic level, it's behavior will be quite violent and erratic. Splashing and various wave phenomena will occur. After the passage of time, the fluid will settle down and stabilize. A definite top surface will develop and motion, at least at the macroscopic level, will subside. Within the fluid itself, on the microscopic level, density and pressure gradients will arise and some particle motion will continue.

In order to model this situation numerically, we will proceed as follows: adopting the modern physical viewpoint, [4], [5], the liquid is assumed to be composed of interacting particles. Our original liquid contained 300 particles. This number was later reduced to 278, for reasons to be explained. The mass of each particle was taken to be 1.0 (CGS units are used throughout). The container itself was 10.0 units in diameter. A cylinder was chosen so that, by taking advantage of symmetry, only two-dimensional motion need be considered, i.e., the model represents a section through the vertical axis of the container. Particles were entered into the container with random positions and velocities and sufficient time was allowed for the liquid to settle. The average kinetic energy per particle was observed periodically and the liquid was deemed to have stabilized when further variations of this quantity was small. Liquid viscosity was taken into account at the container's boundaries by damping the velocity of any particle striking a wall to 10% of its original value. This damping factor is, of course, strictly a function of the material nature of the walls of the cylinder. Two different types of forces are allowed to act on and within the fluid. The forces between the particles are taken to be classical molecular in nature [5] and derivable from the Lennard-Jones potential. When any two particles are far away from each other, they experience an attractive force. As they approach each other, the attractive force is superceded by a much stronger repulsive force. Ultimately, these forces reach a relative state of equilibrium within the fluid. The entire fluid mass also experiences the force of gravity.

III. Dynamical Equations

In implementing the general approach described in Section II, let the computer generated liquid be composed of N particles, denoted $P_1, P_2, \ldots, P_N$, each of the same mass m. For positive time step Δt, let $t_k = k\Delta t$, $k = 0,1,2,3,\ldots$. The state of the liquid will be described completely by giving the position and velocity of each particle for every time t_k. At each time t_k, let particle P_i be located at $\vec{r}_{i,k} = (x_{i,k}, y_{i,k})$ and have velocity $\vec{v}_{i,k} = (v_{i,k,x}, v_{i,k,y})$. Furthermore, at t_k let P_i experience an acceleration $\vec{a}_{i,k} = (a_{i,k,x}, a_{i,k,y})$.

Let the position, velocity, and acceleration be related by the "leap-frog" formulas [6, p. 107]:

$$(3.1) \qquad \vec{v}_{i,\frac{1}{2}} = \vec{v}_{i,0} + \frac{\Delta t}{2}\vec{a}_{i,0}$$

$$(3.2) \qquad \vec{v}_{i,k+\frac{1}{2}} = \vec{v}_{i,k-\frac{1}{2}} + (\Delta t)\,\vec{a}_{i,k} \quad , \quad k = 1,2,3,\ldots .$$

$$(3.3) \qquad \vec{r}_{i,k+1} = \vec{r}_{i,k} + (\Delta t)\,\vec{v}_{i,k+\frac{1}{2}} \quad , \quad k = 0,1,2,\ldots .$$

One should note that since these formulas generate positions at time t_k and velocities at times $t_{k+\frac{1}{2}}$, position and velocity are never known simultaneously. This precludes computing the total energy at each step. At any time step, one can compute either kinetic energy or potential energy, but not both.

At t_k, let the force acting on P_i be $\vec{F}_{i,k} = (F_{i,k,x}, F_{i,k,y})$.

We assume that force and acceleration are related by the discrete Newton's law

$$(3.4)\qquad \vec{F}_{i,k} = m\,\vec{a}_{i,k}\,.$$

As soon as one specifies the form of $\vec{F}_{i,k}$, the motion of each particle, and hence the state of the liquid, will be determined recursively and explicitly by (3.1)-(3.4) from given initial data. The force $\vec{F}_{i,k}$ is defined as follows. Let $r_{ij,k} = |r_{i,k} - r_{j,k}|$ be the distance between P_i and P_j at time t_k and let G (coefficient of attraction), a (exponent of attraction), H (coefficient of repulsion), and b (exponent of repulsion) be given constants subject to the constraints $G \geq 0$, $H \geq 0$, $b \geq a \geq 2$ [5]. Then the force $\bar{F}_{i,k} = (\bar{F}_{i,k,x}, \bar{F}_{i,k,y})$ exerted by P_j on P_i is assumed to be

$$(3.5)\qquad \bar{F}_{i,k,x} = m^2 \left\{ -\frac{G}{(r_{ij,k})^a} + \frac{H}{(r_{ij,k})^b} \right\} \frac{x_{i,k} - x_{j,k}}{r_{ij,k}}$$

$$(3.6)\qquad \bar{F}_{i,k,y} = m^2 \left\{ -\frac{G}{(r_{ij,k})^a} + \frac{H}{(r_{ij,k})^b} \right\} \frac{y_{i,k} - y_{j,k}}{r_{ij,k}}\,.$$

The total force $(F^*_{i,k,x}, F^*_{i,k,y})$ on P_i due to all the other particles is given by summing over the expressions (3.5)-(3.6), excluding the ith term:

$$(3.7)\qquad F^*_{i,k,x} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{F}_{i,k,x}\,, \qquad F^*_{i,k,y} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{F}_{i,k,y}\,.$$

The force of gravity is given by (0,-980m). Adding this to (3.7) gives us the final expression for $\vec{F}_{i,k}$:

(3.8) $$F_{i,k,x} = F^*_{i,k,x};\ F_{i,k,y} = F^*_{i,k,y} - 980m$$

In liquids, all particles are always within the strong force fields of their neighbors [7, p. 46]. Furthermore, it is known "that the "intermolecular forces are shortranged, [with] interaction between molecules hardly extending beyond nearest neighbors." [8, p. 225] To emphasize the localness of particle interactions, only those particles within a fixed distance r_0 of P_i are allowed to contribute to the attractive and repulsive forces experienced by P_i. Besides being physically reasonable, allowing such local interactions makes for much faster computation times. With N particles there are N(N-1)/2 total interactions. Clearly, if this number can be appreciably reduced, the expressions (3.5)-(3.6) will be evaluated less often, and the computer code will be more efficient numerically.

Particle damping and rebounding from the container walls is implemented as follows. Let one end of the base of our rectangular section be at the origin (0,0) and let the other end be at (A,0). Then

(a) if $x_{i,k}<0$, reset P_i at $(-x_{i,k},y_{i,k})$

(b) if $x_{i,k}>A$, reset P_i at $(2A-x_{i,k},y_{i,k})$

(c) if $y_{i,k}<0$, reset P_i at $(x_{i,k},-y_{i,k})$.

Suppose that P_i has velocity $\vec{v}^*_{i,k}$ at t_k. After resetting the position, one resets the velocity to $\vec{v}_{i,k}$, where

(a) if $x_{i,k}<0$ or $x_{i,k}>A$, $\vec{v}_{i,k} \rightarrow (1-d)(-v^*_{i,k,x}, v^*_{i,k,y})$

(b) if $y_{i,k}<0$, $\vec{v}_{i,k} \rightarrow (1-d)(v^*_{i,k,x}, -v^*_{i,k,y})$,

where d is a fixed damping constant. After extensive computer experimentation, the following parameter selections were settled upon: N = 300 (later 278), A = 10.0, G = 25.0, a = 2, H = 50, b = 4, m = 1.0, d = 0.9, $r_0 = 0.25$, and $\Delta t = 10^{-4}$. The time step is chosen so as to satisfy the condition $\Delta t \ll \frac{r}{\|\vec{v}_{i,k}\|}$ for every i and every k [9, p. 123]. In general, r is taken to be a typical interaction distance. Here, $r = r_0$ was used. Time steps larger than 10^{-4} were tried with the result being, typically, the onset of instability. (The exact FORTRAN code which implements the model with the parameter choices given above is given in Appendix I of [10].)

Before any investigation of the liquid computer model can begin, a stable, settled configuration must be attained. Ideally, one could determine a stable initial state for every possible choice of parameters. This problem, however, is entirely intractible mathematically. Consequently, it is necessary to allow the liquid to find its own stable particle distribution. This is done by placing the particles at random positions in the container and giving them random velocities and then by observing the state of the fluid at successive time steps. This was done and, as time passed, the fluid was observed to stabilize from the bottom up with a new layer of 40 particles appearing approximately every .17 to .22 units. During this phase of the computations, it was felt that if this trend were to continue, 20 of the 300 particles would not fit into any level. Hence, the number of particles was reduced from N = 300 to N = 280. This, we believed, would result in a liquid of 7 levels with 40 particles in each level. After allowing about 21 seconds of physical (fluid) time to pass, the topmost row refused to conform and level out. A good deal of activity continued after the remainder of the liquid had settled. It was concluded that this surface activity was appropriate and reflected molecular activity, like Browman motion and evaporation, which occur in a real liquid at the surface/external medium interface. [4] Ultimately, two particularly incorrigible particles were removed. The result was a stable liquid of 278 particles. This particle arrangement can be seen in Fig. 1. The average kinetic energy per particle is 20.467. (The exact positions and velocities are given in Appendix II of [10].)

IV. Qualitative Nature of the Modeled Fluid

An interesting, but very difficult, question to consider is whether we have modeled a liquid or a gas. Let us now examine some of the various approaches to answering this question. One of them, for example, is statistical. It is known that "in assemblies where particles interact with short-range forces, of the order of 1,000 particles are sufficient to exhibit statistical behavior." [9, p. 122] With this number of particles, one can average over individual particle motions and speak meaningfully of the fluid's temperature, pressure, viscosity, thermal conductivity, and other thermodynamic properties. [11] One would then be able to compare values computed statistically against those computed experimentally for real liquids and gases. In this way, one would not only be able to distinguish between a liquid and a gas but would hopefully be able to specify the particular liquid or gas actually modeled.

An example of a specific difference between liquids and gases, arising out of statistical behavior, which would be testable readily, given a suitable number of particles, is the following. For homogeneous and incompressible fluids (liquids), pressure is proportional to depth while for gases, the natural logarithm of pressure is proportional to depth. [12, p. 31-32] One could graph these quantities to see which hypothesis, liquid or gas, best explains the data. A drawback to this approach is, of course, the large number of particles required. Limited

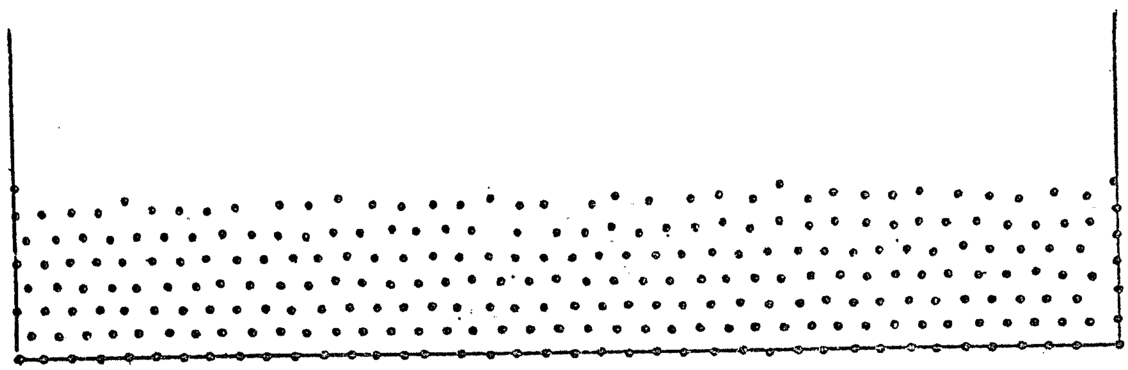


FIGURE 1 - Settled Fluid

funds and a lack of sufficiently powerful computing machinery have so far prevented our modeling of fluids of this size. However, other approaches to be described next, and which are based upon less formal properties, are available and do imply that our fluid is a liquid.

It is known [7, p. 45-46] that the average particle separation in a typical gas is of the order of 10d where d is the equilibrium distance at which attraction and repulsion balance. In a liquid, the average particle separation is of the order of d. For the particular choice of parameters used in this paper, d is 1.414. For the fluid presented in Fig. 1, the mean separation is 0.255. This is evidence in favor of the fluid generated being a liquid. Furthermore, "liquids can be regarded for most purposes as being incompressible." [7, p. 76] This means that further evidence for a liquid-like nature rather than a gaseous nature is the property of uniform particle density. Consideration of Table I shows a remarkably uniform distribution of particles. (The uppermost 39 particles at the surface have been excluded.)

TABLE I

Range in Height	Number of Particles
0.0 - 0.2	40
0.2 - 0.4	40
0.4 - 0.6	40
0.6 - 0.8	40
0.8 - 1.0	39
1.0 - 1.2	40

Another approach in the consideration of the type of fluid actually generated comes from the observation of the fluid's macroscopic properties.

Earlier studies of a von Neumann type fluid have demonstrated the properties of bouyancy, expansion due to heating, convection, free surface flow, and cavity flow. [3, 13] We next add to this list splashing and surface wave generation which result when a particle of greater relative mass is dropped into the fluid, from which will follow, again, the liquid nature of our fluid. Incidentally, the computer examples which follow are also of interest in themselves, since they describe the evolution of fundamental fluid phenomena.

V. Splashing and Surface Wave Generation

After a stable configuration had been obtained, a single additional particle was added to the other 278. This new particle, P_{279}, was placed above the surface at (5,1.7), released, and allowed to drop into the fluid. Three cases were considered: 1) particle has mass = 10.0 and initial velocity zero, 2) particle has mass = 10.0 and initial velocity of 30.0 downward, and 3) particle has mass = 250.0 and initial velocity zero.

Graphs of the resulting fluid motions are shown in Figures 2-4, and these will be discussed separately. Note first, however, that due to limitations of printer spacing, gaps appear in the automatic plots given in Figures 2-4 which do not actually occur in the fluid or in the computer output. This shortcoming is apparent immediately by comparing Figure 1 and Figure 2(a). The only difference that exists between these two graphs is the existence of particle P_{279} in Figure 2(a). Otherwise, the two graphs are of the identical fluid. Figure 1, however, was drawn by hand and is the more accurate of the two.

Figures 2(a)-(e) show the effect of dropping P_{279}, of mass 10.0, from a position of rest, into the fluid. The resulting motion is shown, respectively, at times t_0, t_{229}, t_{503}, t_{823}, and t_{1143}. This gentle drop into the fluid results in the splash effect which is seen clearly in Figure 2(c). The splash continues to increase in size, as is shown in Figure 2(d), while the rate of descent of P_{279} slows. Simultaneously, P_{279} is covered over by fluid particles. Figure 2(e) shows the fall of the two waves generated by the splash.

To accentuate the effects shown in Figure 2, we next reset the velocity of P_{279} to (0,-30). The results of this change are shown in Figures 3(a)-(d) at the respective times t_{137}, t_{229}, t_{824}, and t_{1464}. Figures 3(a) and (b) show a more rapid entry and descent than those shown in Figure 2. Figures 3(c) and (d) show the generation of outward moving waves at the later times t_{824} and t_{1464}.

In order, again, to accentuate the effects shown in Figure 2, P_{279} was once more dropped from a position of rest, but this time its mass was set at the extreme value $m = 250$. Figures 4(a) and (b), at the respective times t_{229} and t_{366}, show a more rapid rate of descent within the fluid and a much larger splash than that shown in Figure 2. However, it is likely that extensive calculation with such a large mass might require decreasing the time step Δt, since interparticle forces can now be exceptionally large.

In all three cases described above, note that the physical reactions are consistent with the contention that the fluid is a liquid.

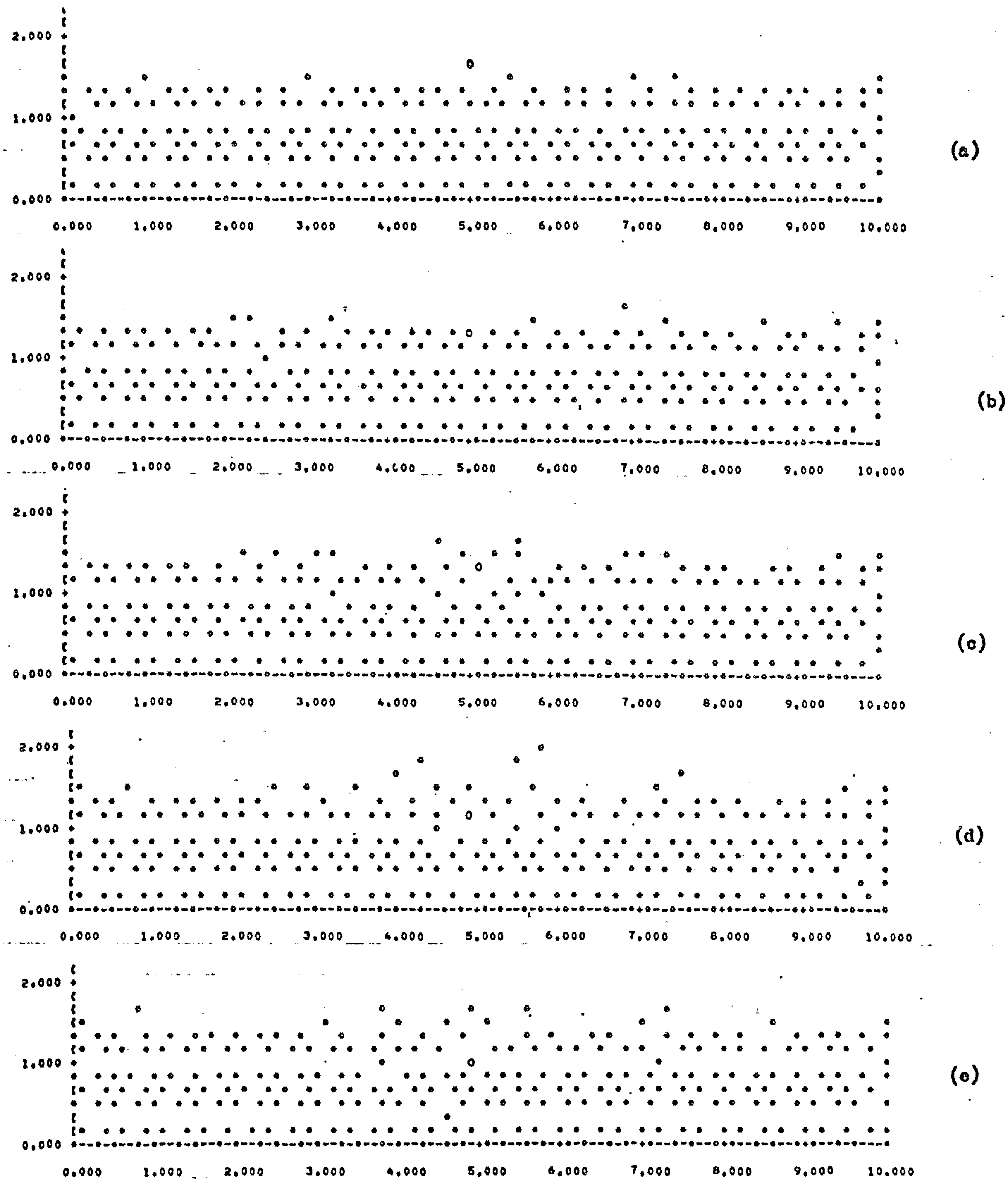

2,000
1,000
0,000
0,000 1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 9,000 10,000
(a)
(b)
(c)
(d)
(e)

FIGURE 2

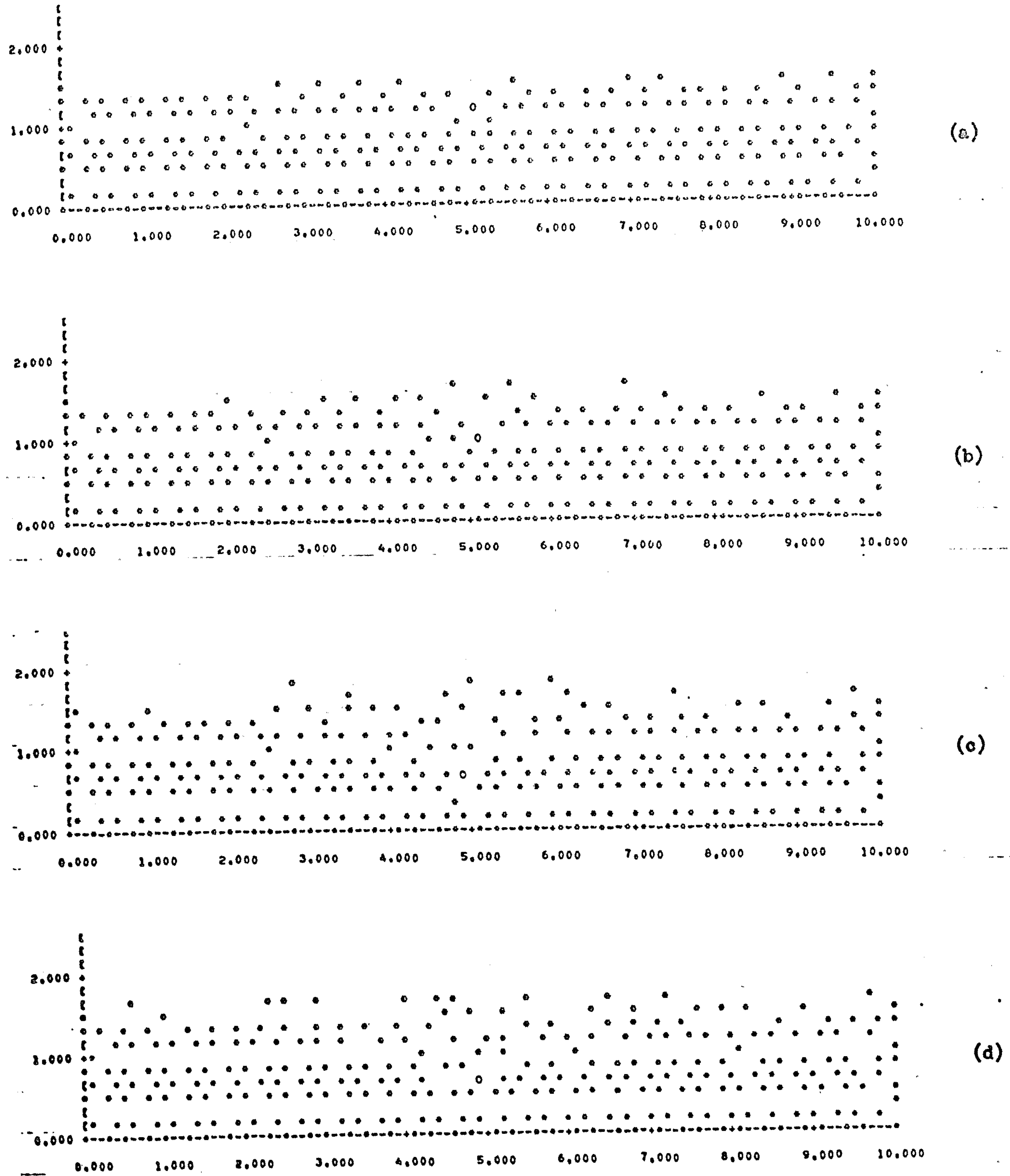
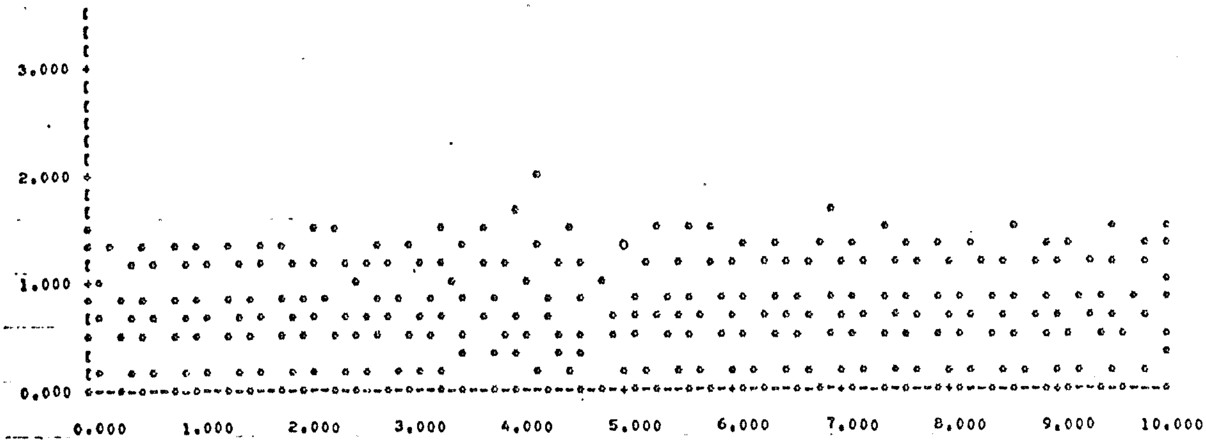
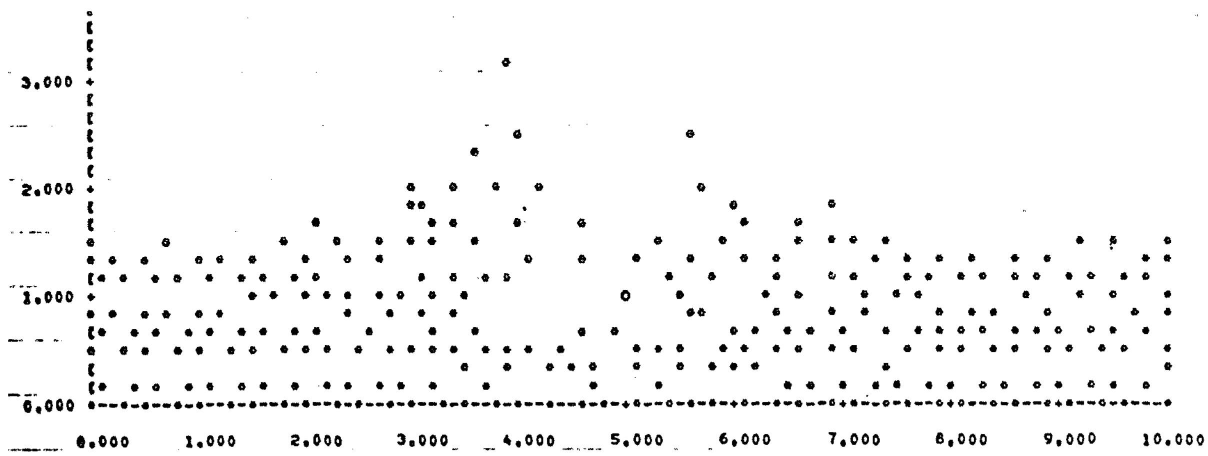

2,000
1,000
0,000
0,000 1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 9,000 10,000
(a)
(b)
(c)
(d)

FIGURE 3


3,000
2,000
1,000
0,000
0,000
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
10,000

(a)


3,000
2,000
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4,000
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8,000
9,000
10,000

(b)

FIGURE 4

V. Remarks

As was stated earlier, the liquid produced in this paper was the result of a great deal of experimentation in the selection of parameter values. Table II shows some of the parameter combinations and the nature of the density of the resulting fluid. N is the number of particles in the fluid, w is the width of the container, d is the depth of the fluid, r_0 is the radius of interaction, mean sep. is the mean separation of particles, and KE/p. is the mean kinetic energy per particle. All fluids have particle masses of 2.5.

While none of these earlier combinations generated a fluid with uniform density, a few of them, 4 and 10 for example, could be called heavy gases. (For exact positions and velocities, see Appendices III and IV of [10].) As such, they would prove useful in simulating the earth's atmosphere.

If the force expression contains only repulsion, it is reasonable to expect the resulting fluid to be gaseous in nature. This conclusion follows from the fact that "at the molecular level a perfect gas is characterized by the absense of intermolecular attraction." [7, p. 345] Fluids 2 through 5, which resulted solely from repulsive forces, do indeed have the compressibility which is characteristic of gases.

Not all of the fluids given in Table II were run to stability. In order to save time, some computations were stopped when it became clear that a particular set of parameters was not moving towards a liquid.

Table II

	N	H	b	G	a	w	d	r_o	mean sep.	KE/p.	nature of density
1	300	1	3	0	-	6	1.2	-	.13	.46	irregular
2	267	25	3	0	-	6	5	-	.36	4.62	increasing downward
3	267	50	3	0	-	6	8.5	-	.45	64.03	increasing downward thin at top
4	267	100	3	0	-	6	10	-	.50	.69	gradual increase downward
5	300	1	5	0	-	6	3.5	-	.25	.28	increasing downward
6	582	1	5	0	-	10	2.6	-	.25	.33	irregular
7	300	1	5	1	2	6	2.5	-	.24	.03	increasing downward irregular near top
8	267	100	3	.5	6	6	10	-	.50	.58	gradual increase downward
9	267	100	3	10	2	6	9	-	.48	.67	increasing downward some irregularity
10	300	100	3	75	2	6	5.5	2	.34	47.20	gradual increase downward
11	300	100	4	75	2	6	7	1.5	.40	34.54	irregular
12	300	50	5	25	2	6	5	1.5	.34	108.00	irregular
13	247	50	5	25	3	10	4	2	.43	3.45	irregular
14	300	50	5	25	3	6	8	2	.46	88.17	irregular
15	300	50	5	25	3	10	6	2	.46	86.87	irregular
16	300	50	6	25	2	6	5.7	1.5	.37	264.49	uniform in mid-section

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APPENDIX I

FORTRAN PROGRAM

```
C     ********************************************************************
C     *                                                                  *
C       THIS PROGRAM IS DESIGNED TO MODEL THE BEHAVIOR OF A FLUID IN A
C       CONTAINER. A LEAP-FROG FORMULATION IS BEING USED.
C     *                                                                  *
C     ********************************************************************

        INTEGER CF
        REAL MASS
        DIMENSION X(280),Y(280),VX(280),VY(280),FX(280),FY(280)
        COMMON X,Y,VX,VY,NPARTS,WIDTH,HEIGHT,TIME,K,KK
        COMMON /MNRTST/ CF
        IF(LINKME(0,7).LT.0) STOP NOLNK7
        IF(LINKME(1,6).LT.0) STOP NOLNK6

C ->    PARAMETER DEFINITION

        MASS=1.0
        DT=0.0001
        DTM=DT/MASS
        NTIME=1000000

C ->    RESTART THE PROGRAM USING THE APPROPRIATE 'STARTING FORMULA'. THIS
C ->    PORTION OF THE PROGRAM REPRESENTS STARTING THE COMPUTATION AT K=0.

        CALL RSTART
        CALL FORCE(FX,FY)
        DO 1 I=1,NPARTS
           VX(I)=VX(I)+0.5*DTM*FX(I)
           VY(I)=VY(I)+0.5*DTM*FY(I)
           X(I)=X(I)+DT*VX(I)
           Y(I)=Y(I)+DT*VY(I)

C ->    ADJUST POSITIONS AND VELOCITIES TO TAKE INTO ACCOUNT ANY
C ->    COLLISIONS WHICH HAVE OCCURRED WITH THE WALLS OF THE CONTAINER.

              IF((X(I).GT.WIDTH).OR.(X(I).LT.0.0).OR.(Y(I).LT.0.0))
     +     CALL CNTAIN(I)

      1 CONTINUE
        K=K+1

C ->    MOVE AHEAD FROM STEP K TO STEP K+1 AND COMPUTE NEW POSITIONS AND
C ->    VELOCITIES.

        DO 3 KK=K,NTIME
           TIME=TIME+DT
           CALL FORCE(FX,FY)

           DO 2 I=1,NPARTS
              VX(I)=VX(I)+DTM*FX(I)
              VY(I)=VY(I)+DTM*FY(I)
              X(I)=X(I)+DT*VX(I)
              Y(I)=Y(I)+DT*VY(I)

```

```
            IF((X(I).GT.WIDTH).OR.(X(I).LT.0.0).OR.(Y(I).LT.0.0))
     *        CALL CNTAIN(I)


    2 CONTINUE
      IF(MOD(KK,50).EQ.0) CALL STORE
    3 CONTINUE
      STOP OK
      END


      S U B R O U T I N E STORE

C   ****************************************************************************
C   *                                                                          *
C     STORE STORES THE MOST RECENT DATA AND PARAMETERS ON LOGICAL UNIT
C     NUMBER '35 AND '36.                                                      *
C   *                                                                          *
C   ****************************************************************************

      INTEGER CF,PAR(20)
      DIMENSION X(280),Y(280),VX(280),VY(280)
      DIMENSION LUNFLE(2),NF(2)
      COMMON X,Y,VX,VY,NPARTS,WIDTH,HEIGHT,TIME,K,KK
      COMMON/MNRTST/CF
      DATA LUNFLE/'35,'36/
      DATA NF/2,1/

      L=KK
                  WRITE('34,1) NPARTS,L,WIDTH,HEIGHT,TIME,CF
    1 FORMAT(2I7,3E15.7,I1)
      DO 6 I=1,NPARTS
      WRITE('34,2) X(I),Y(I),VX(I),VY(I)
    2 FORMAT(4E15.7)
    6 CONTINUE
      END FILE '34
      REWIND '34
      CF=NF(CF)
      LFLE=LUNFLE(CF)
      REWIND LFLE
      ICHECK=0
    3 READ('34,4,END=5) (PAR(J),J=1,20)
      ICHECK=ICHECK+1
      WRITE(LFLE,4) (PAR(J),J=1,20)
      GO TO 3
    5 CONTINUE
      IF(ICHECK.NE.NPARTS+1) STOP 1
      END FILE LFLE
      REWIND LFLE
      REWIND '34
      CLOSE 10
    4 FORMAT(20A3)
      RETURN
      END
```

```

      S U B R O U T I N E CNTAIN(I)

C    *****************************************************************
C    *                                                               *
C      DAMP = 1.0 IF DAMPING IS TOTAL
C      DAMP = 0.0 IF THERE IS NO DAMPING
C    *                                                               *
C    *****************************************************************

      DIMENSION X(280),Y(280),VX(280),VY(280)
      COMMON X,Y,VX,VY,NPARTS,WIDTH,HEIGHT,TIME,K,KK

      DAMP=0.9
      D1=0.1
C =>  D1 = 1.0 - DAMP

      IF(X(I).LE.WIDTH) GO TO 1
      X(I)=2.0*WIDTH-X(I)
      VX(I)=-D1*VX(I)
      VY(I)=D1*VY(I)
      GO TO 2
    1 IF(X(I).GE.0.0) GO TO 2
      X(I)=-X(I)
      VX(I)=-D1*VX(I)
      VY(I)=D1*VY(I)
C   2 IF(Y(I).GE.0.0) RETURN
    2 CONTINUE
      IF(ABS(VX(I)).LT.1.0E-10) VX(I)=0.0
      IF(Y(I).GE.0.0) RETURN
      Y(I)=-Y(I)
      VX(I)=D1*VX(I)
      VY(I)=-D1*VY(I)
      IF(ABS(VX(I)).LT.1.0E-10) VX(I)=0.0
      RETURN
      END


      S U B R O U T I N E FORCE(FKX,FKY)

C    *****************************************************************
C    *                                                               *
C      FKX = FORCE ON PARTICLE IN X-DIRECTION
C      FKY = FORCE ON PARTICLE IN Y-DIRECTION
C      H = CONSTANT OF REPULSION
C      A = EXPONENT OF REPULSION
C      G = CONSTANT OF ATTRACTION
C      B = EXPONENT OF ATTRACTION
C    *                                                               *
C    *****************************************************************

      DIMENSION X(280),Y(280),VX(280),VY(280),FKX(280),FKY(280)
      COMMON X,Y,VX,VY,NPARTS,WIDTH,HEIGHT,TIME,K,KK
      DO 1 M=1,NPARTS
      FKX(M)=0.0
```

```
      FKY(M)=0.0
    1 CONTINUE
      NPART1=NPARTS-1
      DO 2 I=1,NPART1
      IP1=I+1
      DO 2 J=IP1,NPARTS
      IF((ABS(X(I)-X(J)).GT.0.25).OR.(ABS(Y(I)-Y(J)).GT.0.25)) GO TO 2
C ->  PARTICLES A DISTANCE FARTHER AWAY THAN 0.25 ARE NOT ALLOWED TO
C ->  INTERACT.
      RIJ2=(X(I)-X(J))*(X(I)-X(J))+(Y(I)-Y(J))*(Y(I)-Y(J))

C ->  RIJ2 = SQUARED DISTANCE BETWEEN PARTICLE I AND PARTICLE J

      IF(RIJ2.GT.(0.0625)) GO TO 2
C ->  PARTICLES A DISTANCE FARTHER AWAY THAN 0.25 ARE NOT ALLOWED TO
C ->  INTERACT.

C ->  ONLY PARTICLES WITHIN A CERTAIN DISTANCE OF EACH OTHER ARE ALLOWED
C ->  TO INTERACT.

      RIJ4=RIJ2*RIJ2
      Z=(( 50.0/RIJ4)-( 25.0/RIJ2))*1.0
C ->  Z=((H/R**A)-(G/R**B))*MASS(I)*MASS(J)
      RIJ=SQRT(RIJ2)
      FX=Z*((X(I)-X(J))/RIJ)
      FY=Z*((Y(I)-Y(J))/RIJ)
      FKX(I)=FKX(I)+FX
      FKX(J)=FKX(J)-FX
      FKY(I)=FKY(I)+FY
      FKY(J)=FKY(J)-FY
    2 CONTINUE
      DO 3 I=1,NPARTS
      FKY(I)=FKY(I)-980.0
C ->  ACCELERATION OF GRAVITY = 980.0CM/S**2
C ->  CONSTANT TERM = MASS*980.0
    3 CONTINUE
      RETURN
      END


      S U B R O U T I N E RSTART

C  ********************************************************************
C  *                                                                  *
C     RSTART READS THE MOST RECENT DATA AND PARAMETERS FROM LOGICAL UNIT
C     NUMBER '33.
C  *                                                                  *
C  ********************************************************************

      INTEGER CF,PAR(20)
      DIMENSION X(280),Y(280),VX(280),VY(280)
      COMMON X,Y,VX,VY,NPARTS,WIDTH,HEIGHT,TIME,K,KK
      COMMON /MNRTST/ CF

      REWIND '33
```

```
      REWIND '35
      REWIND '36
      ICHECK=0
    1 READ('35,80,END=2) (PAR(I),I=1,20)
      ICHECK=ICHECK+1
      WRITE('33,80) (PAR(I),I=1,20)
      GO TO 1
    2 IF(ICHECK.EQ.280) GO TO 5
      ICHECK=0
      REWIND '33
    3 READ('36,80,END=4) (PAR(I),I=1,20)
      ICHECK=ICHECK+1
      WRITE('33,80) (PAR(I),I=1,20)
      GO TO 3
    4 IF(ICHECK.NE.280) STOP 4
    5 CONTINUE
      REWIND '33
      READ('33,81) NPARTS,K,WIDTH,HEIGHT,TIME,CF
      DO 6 N=1,NPARTS
      READ('33,9) X(N),Y(N),VX(N),VY(N)
    6 CONTINUE
      IF(CF.EQ.0) CF=1

    9 FORMAT(4E15.7)
   80 FORMAT(20A3)
   81 FORMAT(2I7,3E15.7,I1)

      RETURN
      END$
EOF..
```

APPENDIX II

EXACT POSITIONS AND VELOCITIES OF PARTICLES OF FIGURE 1.

X	Y	VX	VY
.5580E+01	.1035E-05	.7760E+00	-.9628E+00
.1010E+01	.1089E-05	.0000E+01	.8909E-02
.6848E+01	.1808E-05	.0000E+01	.8909E-02
.1264E+01	.3705E-05	.0000E+01	.8909E-02
.4311E+01	.3991E-05	-.6929E-06	.8909E-02
.8873E+01	.4074E-05	-.6649E-02	-.1488E+00
.3292E+01	.4636E-05	-.6834E-08	.8909E-02
.7863E+01	.5232E-05	.7404E-06	-.8019E-01
.1517E+01	.5390E-05	.0000E+01	.8909E-02
.6087E+01	.5643E-05	.4332E-01	-.1856E+01
.8115E+01	.5679E-05	.6268E-09	.8910E-02
.1000E+02	.6378E-05	.2528E-07	.9277E-02
.2533E+01	.6741E-05	.0000E+01	-.8018E-01
.4564E+01	.7103E-05	-.7046E-08	.8909E-02
.1772E+01	.7293E-05	-.6760E-07	.8911E-02
.7576E+00	.7557E-05	-.6962E-04	.8909E-02
.7354E+01	.8419E-05	.0000E+01	.8909E-02
.7610E+01	.8523E-05	.7481E-04	-.8087E-01
.3040E+01	.1534E-04	-.6103E-04	.1714E-01
.8621E+01	.1845E-04	-.6795E-07	-.7127E-01
.5045E+00	.3566E-04	-.6953E-02	.4811E-01
.3545E+01	.3679E-04	.6876E-01	.1120E+00
.3802E+01	.3836E-04	.7741E-01	-.2867E+00
.2785E+01	.4801E-04	-.7336E-01	-.2765E+00
.2528E+00	.4845E-04	.7543E+00	-.9583E+00
.2276E+01	.6236E-04	-.6873E-01	-.8937E-01
.8369E+01	.7939E-04	.2620E-02	.2056E+00
.9374E+01	.9540E-04	-.1417E+00	.2155E+00
.7103E+01	.9866E-04	.7071E-01	.1075E+00
.5323E+01	.1023E-03	.6140E-01	.1179E+00
.4819E+01	.1032E-03	.7054E-01	.1083E+00
.2025E+01	.1040E-03	.7049E-01	.1087E+00
.6339E+01	.1041E-03	.3056E-03	.7421E-02
.4603E-04	.1070E-03	.6281E-02	.1152E-01
.4053E+01	.1078E-03	.6846E-01	.1124E+00
.5831E+01	.1198E-03	-.4897E-02	.2061E+00
.9123E+01	.1287E-03	-.7536E+00	-.9652E+00
.6596E+01	.1513E-03	.6736E-01	-.2885E+00
.9624E+01	.1854E-03	-.3535E-02	.2063E+00
.5072E+01	.1923E-03	-.1101E+01	-.1770E+00
.9755E+01	.2114E+00	-.2804E+01	-.6026E-01
.5959E+01	.2136E+00	-.1293E+02	.3520E+01
.1644E+01	.2147E+00	.7979E+00	-.8681E+01
.4693E+01	.2148E+00	.1621E+01	-.3314E+01
.1900E+01	.2148E+00	-.6191E+01	-.4237E+01
.8244E+01	.2149E+00	.1826E+01	-.1290E+01
.9499E+01	.2151E+00	.2594E+01	.3085E+00
.6977E+01	.2154E+00	-.1016E+01	.4487E+01
.1262E+00	.2156E+00	-.8578E+00	-.4029E+00
.5453E+01	.2157E+00	-.3167E+01	.4342E+01
.1136E+01	.2160E+00	-.2693E+01	-.3435E+01
.2403E+01	.2161E+00	.8476E+01	-.2016E+01
.9247E+01	.2164E+00	.4353E+01	-.1215E+01
.8854E+00	.2169E+00	.4936E+01	-.1517E+00
.8493E+01	.2172E+00	-.1270E+01	.2033E+01
.6465E+01	.2175E+00	-.7811E+00	.7099E+01
.4440E+01	.2177E+00	.3445E+01	-.1536E+01

x	y	vx	vy
.3428E+01	.2177E+00	.1060E+02	.3785E+01
.6333E+00	.2177E+00	.3374E+01	-.6543E+00
.6212E+01	.2180E+00	.2114E+01	.1053E+02
.7988E+01	.2180E+00	-.3930E+01	-.2622E+01
.8997E+01	.2182E+00	-.6117E+00	-.6517E+00
.5203E+01	.2182E+00	.2260E+01	-.5672E+01
.3780E+00	.2182E+00	.2503E+01	.3725E+01
.3935E+01	.2184E+00	.3353E+01	.2612E+01
.5712E+01	.2189E+00	-.9197E+00	-.1931E+00
.7484E+01	.2189E+00	-.3995E+01	-.1031E+01
.2908E+01	.2192E+00	.1943E+01	.2967E+00
.6721E+01	.2193E+00	.3761E+00	.3233E+01
.4183E+01	.2194E+00	-.2525E+01	-.2385E+00
.2653E+01	.2196E+00	-.7430E+01	-.3101E+01
.3676E+01	.2197E+00	.3504E+01	.7239E+01
.4945E+01	.2201E+00	-.5614E+01	.7176E+01
.2150E+01	.2205E+00	.1531E+01	.2447E+01
.3159E+01	.2209E+00	-.5137E+01	.3167E+00
.7741E+01	.2213E+00	.3554E+01	-.4987E+01
.1389E+01	.2223E+00	-.5029E+01	-.3864E+01
.8745E+01	.2258E+00	.2411E+01	.1223E+00
.7229E+01	.2279E+00	.3490E+01	-.8590E+00
.1000E+02	.2503E+00	-.1154E+00	.1154E-01
.9638E+01	.4249E+00	.7596E+01	-.4983E+01
.4566E+01	.4310E+00	-.5307E+00	.2875E+01
.4293E-04	.4313E+00	.1134E+00	-.1273E-01
.2020E+01	.4316E+00	.2939E+01	-.2282E+01
.2513E+00	.4324E+00	.7893E+00	-.5008E+01
.4055E+01	.4327E+00	-.2715E+00	-.8475E+01
.1520E+01	.4328E+00	.1505E+01	-.3524E+01
.3289E+01	.4339E+00	-.2228E+01	-.6618E+01
.5577E+01	.4341E+00	-.2840E+01	-.7792E+01
.1002E+01	.4345E+00	-.3360E+01	-.8284E+01
.4818E+01	.4351E+00	.3080E+01	.3326E+01
.2278E+01	.4353E+00	.2907E+01	-.5644E+01
.3551E+01	.4355E+00	.7534E+01	-.8890E+01
.6596E+01	.4357E+00	-.1279E+00	.4241E-01
.5327E+01	.4360E+00	.6322E+01	-.8165E+00
.5839E+01	.4362E+00	-.1213E+01	-.3767E+01
.5021E+00	.4363E+00	.4611E+01	.1039E+01
.5071E+01	.4364E+00	.3345E+00	.2862E+01
.7538E+00	.4366E+00	.6286E+01	.1293E+01
.8370E+01	.4372E+00	-.2119E+01	-.2276E+01
.8875E+01	.4376E+00	.1970E+01	-.1221E+01
.6340E+01	.4377E+00	.4496E+01	.2139E+00
.9129E+01	.4378E+00	.3420E+01	-.6651E+00
.2533E+01	.4392E+00	.6284E+01	.4897E+01
.9385E+01	.4392E+00	-.6228E+01	.6604E+00
.6850E+01	.4393E+00	-.7286E+00	.5108E+01
.1261E+01	.4394E+00	.3239E+01	.1203E-01
.6092E+01	.4399E+00	-.2846E+01	.8716E+01
.3803E+01	.4399E+00	-.2460E+00	-.5323E-03
.7609E+01	.4412E+00	-.7220E+01	.8054E+01
.1770E+01	.4415E+00	.3802E+01	.5375E+01
.4309E+01	.4420E+00	.2690E+01	.9516E+00
.3045E+01	.4427E+00	-.1014E+02	-.7890E+01
.7860E+01	.4432E+00	.2802E+01	-.5128E+01

x	y	vx	vy
.2796E+01	.4435E+00	.4161E+01	.1609E+01
.7108E+01	.4449E+00	.9338E+01	-.8832E+01
.8625E+01	.4458E+00	.8396E+01	-.8907E+00
.8112E+01	.4470E+00	-.1993E+01	.7933E+01
.7360E+01	.4539E+00	-.1464E+01	-.1263E+01
.9997E+01	.5001E+00	-.6774E+00	-.4066E+01
.9774E+01	.6383E+00	-.4695E+01	.2357E+01
.3419E+01	.6476E+00	-.7239E+01	.8553E+00
.1241E+00	.6478E+00	.1463E+01	-.1224E+01
.9523E+01	.6494E+00	-.4834E+01	-.6574E+01
.3734E+00	.6503E+00	.6271E-01	-.2044E+01
.2147E+01	.6518E+00	-.6971E+01	.3954E+01
.4693E+01	.6524E+00	.2269E+01	.8792E+01
.1125E+01	.6531E+00	-.1020E+01	.1692E+01
.6723E+01	.6534E+00	-.7646E+01	-.1864E+01
.5204E+01	.6535E+00	-.2077E+01	-.8840E+01
.5710E+01	.6550E+00	-.1338E+01	.4234E+00
.4437E+01	.6554E+00	-.1096E+02	-.5132E+01
.8734E+00	.6558E+00	-.1180E+02	.3076E+00
.6243E+00	.6560E+00	.6732E+00	.2202E+01
.2408E+01	.6562E+00	-.1298E+01	.3177E+00
.2664E+01	.6569E+00	.3915E+01	.1055E+02
.6232E+01	.6573E+00	.5476E+00	-.8019E+01
.1642E+01	.6575E+00	-.4641E+00	.1706E+01
.6477E+01	.6577E+00	-.9123E+01	-.2776E-01
.4945E+01	.6577E+00	.1526E+01	-.1027E+02
.5454E+01	.6583E+00	-.4101E+01	.7464E+01
.8751E+01	.6593E+00	.8352E+01	-.3758E+01
.9010E+01	.6603E+00	-.2892E+01	.6644E+01
.3174E+01	.6616E+00	-.7208E+01	-.7296E+01
.5962E+01	.6616E+00	-.3039E+01	-.7491E+01
.6983E+01	.6631E+00	.7114E+01	.1020E+02
.3919E+01	.6633E+00	-.4973E+01	-.4570E+01
.8247E+01	.6639E+00	-.6844E+00	.4048E+01
.1380E+01	.6642E+00	-.1886E+01	-.4436E+01
.3666E+01	.6647E+00	.2063E+01	-.2052E+00
.7749E+01	.6649E+00	.5740E+01	.4830E+01
.1891E+01	.6661E+00	-.3587E+01	.2480E+01
.7498E+01	.6667E+00	-.3923E+01	.1417E+01
.7239E+01	.6693E+00	.1471E+02	-.1769E+01
.8499E+01	.6714E+00	.3116E+01	.6744E+01
.9268E+01	.6720E+00	-.1383E+01	.2611E+01
.8001E+01	.6771E+00	-.3148E+01	-.1925E+01
.2922E+01	.6783E+00	-.7270E+01	.5765E+00
.4172E+01	.6788E+00	-.1571E+02	-.2152E+01
.1000E+02	.7511E+00	-.4643E-01	.1743E+01
.1977E-03	.8638E+00	.2292E+00	-.1279E-01
.2478E+00	.8675E+00	.3088E+01	.2221E+01
.5077E+01	.8705E+00	-.1978E+01	.8680E+00
.4567E+01	.8723E+00	.9426E+00	.8749E+01
.7499E+00	.8724E+00	.4378E+01	.3740E+01
.2276E+01	.8726E+00	-.6290E+00	.5943E+01
.9981E+00	.8731E+00	-.2942E+01	.2638E+01
.5008E+00	.8739E+00	.1965E+01	-.3726E+00
.6601E+01	.8745E+00	-.1727E+01	-.4611E+01
.9683E+01	.8746E+00	-.2557E+00	-.4271E+01
.4817E+01	.8747E+00	.1202E+02	.2152E+00

x	y	vx	vy
.5582E+01	.8754E+00	.5282E+01	-.3232E+01
.5324E+01	.8762E+00	-.1078E+02	-.3021E+01
.2782E+01	.8800E+00	.2247E+01	-.2528E+01
.6086E+01	.8811E+00	.1308E+01	-.7149E+01
.2525E+01	.8820E+00	-.2855E+01	-.2607E+01
.1508E+01	.8822E+00	.1994E+01	.8953E+00
.3286E+01	.8823E+00	-.3710E+01	.1334E+01
.9140E+01	.8839E+00	.4264E+01	-.8254E+01
.7365E+01	.8840E+00	-.5459E+01	-.5534E+01
.8363E+01	.8856E+00	-.8493E+00	.2520E+01
.1258E+01	.8856E+00	.3427E+01	-.3919E+01
.4326E+01	.8859E+00	.3689E+01	.3573E+01
.2027E+01	.8861E+00	.5186E+01	-.3873E+01
.5835E+01	.8866E+00	.2303E+00	.4574E+01
.8891E+01	.8888E+00	-.1490E+01	.1169E+02
.1758E+01	.8893E+00	-.6995E+01	.3153E+01
.7614E+01	.8913E+00	-.3382E+01	.9474E+00
.3545E+01	.8926E+00	-.7152E+00	.5870E+01
.9401E+01	.8927E+00	-.6216E+01	-.3465E+01
.7866E+01	.8935E+00	-.5198E+01	.1398E+01
.7112E+01	.8954E+00	.2002E+01	.2081E+01
.8117E+01	.9003E+00	.4254E+01	.1587E+01
.4064E+01	.9040E+00	-.1179E+01	.1008E+02
.3035E+01	.9053E+00	.4181E+01	.9615E+01
.8619E+01	.9061E+00	.7801E+01	.6855E+01
.3808E+01	.9087E+00	-.1831E+01	-.1946E+01
.6346E+01	.9105E+00	-.7049E+00	.2873E+01
.6855E+01	.9121E+00	.5624E+00	.1238E+02
.1000E+02	.1001E+01	-.8855E-01	-.4587E+00
.1202E+00	.1083E+01	-.1068E+01	.3771E+01
.5202E+01	.1090E+01	-.1753E+01	-.8044E+01
.2658E+01	.1091E+01	.4015E+01	-.3096E+01
.8724E+00	.1091E+01	-.4509E+01	-.3843E+01
.5694E+01	.1093E+01	-.7343E+01	-.6982E+00
.3699E+00	.1093E+01	.2651E+01	-.5454E+01
.2405E+01	.1099E+01	-.5002E+01	-.4924E+01
.1116E+01	.1099E+01	-.1047E+01	-.1081E+01
.4687E+01	.1100E+01	.4771E-01	-.1019E+02
.4952E+01	.1101E+01	.3438E+01	.5935E+01
.1630E+01	.1102E+01	.8767E-01	-.7490E+01
.6232E+00	.1102E+01	.3486E+01	-.1304E+01
.1376E+01	.1105E+01	.3812E+01	.3136E+00
.9287E+01	.1105E+01	.4525E+01	-.8745E+01
.9017E+01	.1106E+01	.4265E+01	-.1086E+02
.6698E+01	.1106E+01	-.2197E-01	.6717E+01
.7991E+01	.1110E+01	.6768E+01	-.3679E+01
.9538E+01	.1111E+01	.4360E+00	-.3245E+01
.2155E+01	.1112E+01	.2480E+01	-.4892E+01
.6200E+01	.1112E+01	.4369E+01	-.8783E+00
.3164E+01	.1116E+01	-.9527E+01	-.3158E+01
.5946E+01	.1116E+01	.4165E+01	-.7175E+01
.7226E+01	.1116E+01	-.2917E+01	-.6427E+01
.1907E+01	.1116E+01	.1923E+01	-.8450E+00
.3674E+01	.1120E+01	.3990E+01	-.1182E+01
.8491E+01	.1122E+01	.2010E-01	-.1128E+01
.4433E+01	.1122E+01	-.6286E+01	.3509E+01
.2906E+01	.1124E+01	.3399E+01	-.2024E+01
.4179E+01	.1128E+01	-.1166E+02	-.1319E+01

x	y	vx	vy
.9785E+01	.1132E+01	-.4160E+01	-.3916E+01
.3931E+01	.1136E+01	-.2895E+01	.7365E+01
.7747E+01	.1138E+01	-.6630E+01	-.4946E+01
.3406E+01	.1138E+01	.1061E+01	-.3618E+00
.7483E+01	.1140E+01	-.2701E+01	.3468E+00
.8753E+01	.1142E+01	.4470E+01	.9234E+01
.5440E+01	.1148E+01	-.9296E+01	-.7206E+01
.8241E+01	.1154E+01	-.2885E+01	-.1293E+02
.6449E+01	.1156E+01	.2334E+01	-.3654E+01
.6979E+01	.1172E+01	.5354E+01	-.1394E+01
.1000E+02	.1252E+01	.9579E+00	.6707E+00
.2511E-04	.1299E+01	.6013E-01	-.9834E-02
.2509E+00	.1318E+01	-.4326E+01	-.1484E+01
.1773E+01	.1329E+01	.6521E+01	-.3808E+01
.1519E+01	.1333E+01	.1481E+01	-.9733E+01
.5263E+01	.1336E+01	.1607E+01	-.1608E+01
.9149E+01	.1341E+01	-.4420E+00	.2247E+01
.7835E+00	.1342E+01	.1490E+01	.2522E+01
.1259E+01	.1344E+01	.3413E+01	-.1026E+01
.5320E+00	.1345E+01	.2889E+00	-.1019E+00
.4885E+01	.1351E+01	.5436E+00	-.1307E+02
.3557E+01	.1353E+01	-.9324E+01	.1171E+01
.3267E+01	.1356E+01	.3056E+01	.4603E+01
.4614E+01	.1357E+01	-.3406E+01	-.8006E+01
.2438E+01	.1358E+01	.3944E+01	-.6204E+01
.3815E+01	.1358E+01	-.2846E+01	.4815E+01
.8885E+01	.1364E+01	-.3547E+01	.2977E+01
.7208E+01	.1368E+01	-.1506E+01	-.1061E+02
.2697E+01	.1370E+01	-.9083E+01	-.1227E+01
.2030E+01	.1371E+01	-.4534E+01	-.6666E+01
.6710E+01	.1376E+01	.5037E+01	-.1346E+02
.9757E+01	.1378E+01	-.8266E+00	.9666E+00
.4079E+01	.1379E+01	.4498E+01	.6053E+01
.6170E+01	.1385E+01	-.3666E+01	.6173E+01
.5786E+01	.1389E+01	-.3249E+01	-.2861E+01
.7989E+01	.1390E+01	-.3804E+01	-.2275E+01
.8598E+01	.1399E+01	-.4892E+01	.5438E+00
.7739E+01	.1403E+01	-.2093E+01	-.5695E+00
.4360E+01	.1405E+01	.3262E+01	-.1640E+01
.8250E+01	.1411E+01	-.8392E+01	-.1317E+02
.6423E+01	.1415E+01	.6396E+01	.1647E+01
.9472E+01	.1415E+01	.4491E+01	.9245E+01
.7460E+01	.1423E+01	-.8406E+01	.4747E+01
.1019E+01	.1424E+01	.4809E+01	-.1925E+01
.2970E+01	.1428E+01	.4182E+01	.5066E+01
.5493E+01	.1430E+01	.4813E+01	.8445E+01
.9999E+01	.1503E+01	-.1193E+00	.1065E+01
.6977E+01	.1514E+01	.8306E+01	-.5457E+01
.6765E-02	.1549E+01	.8169E+00	.1509E+01

APPENDIX III

EXACT POSITIONS AND VELOCITIES OF FLUID #4 IN TABLE II

X	Y	VX	VY
.0000E+01	.5797E+01	.1078E+00	.8583E-02
.2777E+01	.5116E+01	-.2990E+00	.7128E+00
.3336E+01	.5297E+01	-.1939E+00	.2060E+01
.4503E+01	.3587E+01	-.4623E+00	.1722E-01
.3293E+01	.0000E+01	.7153E-03	.2951E+00
.1243E+01	.6280E+01	.1819E+00	.6555E+00
.0000E+01	.1493E+01	.2729E+00	.2384E-02
.1905E+01	.3260E+01	-.5983E-01	.2715E+00
.3685E+01	.0000E+01	-.4292E-02	.2947E+00
.5155E+01	.8196E+01	-.8128E+00	.1620E+01
.1821E+00	.0000E+01	.1192E-03	.5636E+00
.6000E+01	.2510E+01	-.2189E+00	.4768E-02
.6000E+01	.3050E+01	-.1984E+00	.5960E-02
.3082E+01	.2188E+01	-.3898E-01	.2469E+00
.4662E+01	.0000E+01	.0000E+01	.3082E+00
.4322E+01	.4707E+00	.1887E+00	.9281E-01
.4535E+01	.3085E+01	-.6691E-01	.1354E+00
.5817E+01	.0000E+01	.4768E-03	.5705E+00
.0000E+01	.2506E+00	.5609E+00	.9239E-03
.0000E+01	.2280E+01	.2322E+00	.3576E-02
.2509E+01	.0000E+01	.0000E+01	.2971E+00
.3784E+01	.4287E+01	-.2607E+00	.1131E+01
.1630E+01	.4847E+00	.2387E-01	.3357E-01
.2901E+01	.0000E+01	-.2384E-03	.2960E+00
.9304E+00	.8689E+01	-.3379E+00	.1914E+01
.3557E+01	.6054E+01	.2987E+00	.6208E-01
.2734E+01	.8897E+00	.2476E+00	-.3001E-02
.1922E+01	.0000E+01	-.3576E-03	.2974E+00
.5628E+01	.0000E+01	.0000E+01	.3938E+00
.5166E+01	.1420E+01	.1545E-01	.2001E+00
.2221E+01	.1788E+01	.1619E+00	-.5859E-01
.2583E+01	.2105E+01	-.3888E-01	.2214E+00
.4701E+01	.1638E+01	-.7210E-01	.2498E+00
.2521E+01	.4872E+00	.2012E+00	.1201E-01
.6000E+01	.8578E+01	-.4005E-01	.1049E-01
.6000E+01	.1726E+01	-.2642E+00	.3099E-02
.6000E+01	.3605E+01	-.1764E+00	.6914E-02
.6000E+01	.1050E+02	-.9060E-02	.2861E-02
.1330E+01	.2001E+01	.2444E+00	.1867E+00
.3983E+01	.3224E+01	-.2535E+00	.1120E+01
.5392E+01	.5373E+01	-.9982E-01	.8247E+00
.3553E+01	.8963E+01	-.5288E+00	.2075E+01
.2108E+01	.4354E+01	-.4406E+00	.2704E+00
.5456E+01	.3684E+01	-.9003E-01	.2872E+00
.5075E+01	.2367E+01	-.4963E-01	.2482E+00
.3157E+01	.4152E+01	.3992E+00	.9712E+00
.6000E+01	.3889E+01	-.1636E+00	.7153E-02
.6456E+00	.5934E+01	-.6146E-01	.1218E+01
.4428E+01	.9731E+01	.4942E+00	.3190E+01
.2504E+01	.1327E+01	.2576E+00	-.3795E-02
.0000E+01	.3664E+01	.1795E+00	.5960E-02
.3501E+01	.6749E+01	.4603E+00	.5283E+00
.2053E+01	.1314E+01	.2047E+00	-.4431E-01
.5247E+01	.7424E+01	-.4783E+00	.2303E+01
.6000E+01	.1984E+01	-.2441E+00	.3695E-02
.1407E+01	.3042E+01	.3020E-01	.1610E+00
.5114E+01	.8948E+00	-.3745E-01	.5905E-01

.1825E+01	.2723E+01	.7638E-01	.2244E+00
.5051E+01	.0000E+01	.0000E+01	.3218E+00
.7547E+00	.0000E+01	.5364E-03	.3301E+00
.1880E+01	.6588E+01	-.2454E-01	.8165E+00
.0000E+01	.9691E+01	.2000E-01	.5722E-02
.0000E+01	.2014E+01	.2470E+00	.3099E-02
.4674E+01	.7012E+01	-.6573E-01	.1167E+01
.4089E+01	.2220E+01	-.2322E-02	.5702E+00
.4630E+01	.2568E+01	-.2295E+00	.1546E+00
.0000E+01	.4246E+01	.1579E+00	.6676E-02
.6000E+01	.9959E+01	-.1431E-01	.5722E-02
.7717E+00	.7948E+01	.3193E+00	.1189E+01
.6000E+01	.7256E+00	-.3214E+00	.1490E-02
.5623E+00	.0000E+01	.5364E-03	.3487E+00
.6000E+01	.8169E+01	-.4911E-01	.1049E-01
.3881E+01	.0000E+01	-.2384E-03	.3036E+00
.1470E+01	.3584E+01	-.2523E+00	.2028E+00
.3754E+01	.8019E+01	-.4994E+00	.1163E+01
.6000E+01	.6691E+01	-.8583E-01	.1049E-01
.4172E+01	.2708E+01	-.2576E+00	.7443E+00
.3583E+01	.2237E+01	.2572E-01	.4870E+00
.7289E+00	.7242E+01	.3425E+00	.1478E+01
.0000E+01	.1752E+01	.2618E+00	.2623E-02
.4948E+01	.3920E+01	-.1302E+00	.1381E+00
.0000E+01	.8341E+01	.4741E-01	.9537E-02
.4225E+01	.1744E+01	-.1532E+00	.4209E+00
.2974E+01	.6410E+01	.3214E+00	.1386E+01
.2785E+01	.1702E+01	.4907E-01	.1357E+00
.0000E+01	.3952E+01	.1634E+00	.6676E-02
.5517E+01	.2171E+01	-.2545E-01	.2247E+00
.6000E+01	.4846E+00	-.3772E+00	.1073E-02
.4076E+01	.0000E+01	.3815E-02	.2989E+00
.1599E+01	.1241E+01	.5048E-01	.1599E+00
.4911E+01	.4457E+01	.3357E-01	.3823E+00
.6000E+01	.1219E+01	-.2937E+00	.1788E-02
.4518E+00	.8363E+00	.9769E-01	.2386E-01
.5318E+01	.6712E+01	.3959E-01	.2001E+01
.8901E+00	.1311E+01	.5891E-01	.1651E+00
.5533E+00	.4182E+01	-.1067E+00	.6131E+00
.4396E+01	.4141E+01	.5286E+00	.4442E+00
.4594E+01	.2099E+01	-.4083E-01	.2057E+00
.9480E+00	.0000E+01	.2980E-03	.3159E+00
.1015E+01	.3891E+01	-.2658E+00	.1641E+00
.0000E+01	.1237E+01	.2915E+00	.1907E-02
.5417E+01	.4773E+01	.1369E+00	.4334E+00
.0000E+01	.3379E+01	.1835E+00	.5484E-02
.2696E+01	.4540E+01	-.2631E+00	.4703E+00
.4007E+01	.3755E+01	-.1247E+00	.1592E+01
.3752E+01	.1778E+01	-.9517E-01	.3680E+00
.9182E+00	.2304E+01	.6367E-01	.1270E+00
.3097E+01	.0000E+01	.2384E-03	.2941E+00
.5437E+01	.0000E+01	.4768E-03	.3460E+00
.1764E+01	.5909E+01	.2921E+00	.7587E+00
.2117E+01	.0000E+01	-.4768E-03	.3005E+00
.5184E+01	.4799E+00	-.3975E-01	.2413E-01
.8132E+00	.4726E+00	.1019E+00	.5680E-01
.1726E+01	.0000E+01	-.3576E-03	.3000E+00

.0000E+01	.1080E+02	.7311E-02	-.9537E-03
.1941E+01	.7307E+01	-.5661E+00	.1164E+01
.0000E+01	.6816E+01	.8196E-01	.9537E-02
.2209E+01	.2391E+01	.2083E+00	.5102E+00
.4549E+01	.7769E+01	-.1289E+01	.1478E+01
.0000E+01	.7365E+00	.3265E+00	.1371E-02
.3910E+01	.1319E+01	.7618E-01	.3039E+00
.4000E+01	.7289E+01	-.3216E+00	.9837E+00
.2305E+01	.5534E+01	-.2111E-01	.5833E+00
.6000E+01	.6019E+01	-.1044E+00	.1001E-01
.6000E+01	.0000E+01	-.2174E+00	.1043E+00
.3214E+01	.2683E+01	-.1087E+00	.2874E+00
.2437E+01	.3448E+01	-.8363E-01	.7102E+00
.3293E+01	.4688E+01	-.6489E+00	.1581E+01
.1066E+01	.4448E+01	-.3658E+00	.3953E+00
.3489E+01	.0000E+01	.0000E+01	.2991E+00
.0000E+01	.1149E+02	.4224E-02	-.8583E-02
.6793E+00	.6568E+01	.2371E+00	.1364E+01
.0000E+01	.6467E+01	.9141E-01	.9060E-02
.0000E+01	.6128E+01	.9906E-01	.9060E-02
.1701E+01	.5288E+01	.5096E-01	.7559E+00
.4468E+01	.0000E+01	-.9537E-03	.3054E+00
.2077E+01	.4815E+00	.7390E-01	.8792E-01
.0000E+01	.7176E+01	.7466E-01	.9537E-02
.1818E+01	.8630E+00	.7528E-01	.1099E+00
.9451E+00	.2813E+01	.8043E-01	.1525E-01
.3016E+01	.8255E+01	.2940E+00	.9810E+00
.9723E+00	.3346E+01	-.9985E-01	.6507E-02
.5816E+00	.4745E+01	.3847E-01	.9339E+00
.3334E+01	.9905E+01	-.3491E+00	.2017E+01
.1315E+01	.6969E+01	.5477E-01	.9216E+00
.5244E+01	.0000E+01	.4768E-03	.3234E+00
.6000E+01	.1112E+02	-.5722E-02	-.9537E-03
.5365E+01	.6020E+01	-.2975E-01	.1132E+01
.5297E+00	.3636E+01	-.3262E-01	.3414E+00
.0000E+01	.2549E+01	.2232E+00	.4053E-02
.4282E+01	.5955E+01	.1269E+00	.1202E+01
.6000E+01	.9467E+01	-.2289E-01	.7629E-02
.0000E+01	.1244E+02	.2350E-02	-.2289E-01
.0000E+01	.4919E+00	.3803E+00	.1252E-02
.0000E+01	.9212E+01	.2748E-01	.7629E-02
.2992E+01	.1286E+01	.1843E+00	.2755E-01
.0000E+01	.3099E+01	.2008E+00	.5007E-02
.5550E+01	.1247E+01	-.3680E-01	.9363E-01
.4860E+01	.5665E+01	-.7254E-01	.9112E+00
.0000E+01	.7549E+01	.6578E-01	.9537E-02
.0000E+01	.9849E+00	.3017E+00	.1431E-02
.4901E+01	.5026E+01	.3903E+00	.2597E+00
.3874E+01	.4820E+00	.2435E+00	.2265E+00
.0000E+01	.2822E+01	.2059E+00	.4530E-02
.3712E+00	.0000E+01	.4172E-03	.3960E+00
.6000E+01	.6351E+01	-.9155E-01	.1097E-01
.2567E+01	.3992E+01	-.1258E+00	.6990E+00
.4360E+01	.1277E+01	.6922E-01	.3205E+00
.5118E+00	.3107E+01	.1430E-01	.1699E+00
.5498E+01	.2657E+01	-.2572E-01	.2899E+00
.8836E+00	.8866E+00	.1564E+00	.1295E+00

.5583E+01	.4498E+00	-.2381E-01	.4782E-01
.1545E+01	.4133E+01	-.4627E+00	.3710E+00
.4954E+00	.2602E+01	.2037E-01	.1455E+00
.5021E+01	.9028E+01	-.7107E+00	.1159E+01
.1796E+01	.2171E+01	.3835E+00	.1017E+00
.4138E+00	.4466E+00	.9235E-01	.2144E-01
.1156E+01	.9608E+01	.8196E-01	.2250E+01
.6000E+01	.5696E+01	-.1087E+00	.1001E-01
.2197E+01	.4923E+01	-.3602E+00	.2901E+00
.6000E+01	.7779E+01	-.5674E-01	.1144E-01
.6000E+01	.3326E+01	-.1864E+00	.6437E-02
.5008E+01	.3387E+01	-.1776E+00	.2352E+00
.2980E+01	.3604E+01	-.1503E+00	.9015E+00
.5042E+01	.2879E+01	-.6970E-01	.2028E+00
.0000E+01	.4544E+01	.1438E+00	.7629E-02
.2931E+01	.5759E+01	-.2757E+00	.1313E+01
.4578E+00	.1239E+01	.3395E-01	.2408E-01
.4578E+01	.8340E+00	.1221E+00	.1851E+00
.3275E+01	.1748E+01	.2797E-01	.2572E+00
.3701E+01	.2732E+01	-.1404E+00	.6653E+00
.6090E+00	.5327E+01	.1214E+00	.9704E+00
.6000E+01	.2468E+00	-.5646E+00	.6706E-03
.4857E+01	.0000E+01	-.4768E-03	.3108E+00
.5092E+01	.1897E+01	-.1443E-01	.2080E+00
.2969E+01	.4848E+00	.2820E+00	.9068E-01
.1726E+01	.8842E+01	.4872E-01	.1756E+01
.4321E+01	.8591E+01	-.1219E+01	.2005E+01
.6000E+01	.9008E+01	-.3195E-01	.9537E-02
.6000E+01	.7042E+01	-.7439E-01	.1192E-01
.2005E+01	.3802E+01	-.3990E+00	.3246E+00
.3663E+01	.8866E+00	.2638E+00	.1039E+00
.4361E+01	.4674E+01	-.4256E+00	.4212E+00
.0000E+01	.4848E+01	.1369E+00	.7629E-02
.3841E+01	.4881E+01	-.4950E+00	.8764E+00
.3425E+01	.4816E+00	.2789E+00	.7708E-01
.2711E+01	.2589E+01	.6965E-02	.3860E+00
.0000E+01	.8765E+01	.3728E-01	.8583E-02
.1267E+01	.1517E+01	.7874E-01	.2078E+00
.6000E+01	.4767E+01	-.1397E+00	.8583E-02
.6000E+01	.1471E+01	-.2694E+00	.2623E-02
.0000E+01	.5158E+01	.1257E+00	.8106E-02
.1427E+01	.7791E+01	.6909E+00	.2417E+01
.1375E+01	.2504E+01	.1685E+00	.1040E+00
.6000E+01	.7404E+01	-.6723E-01	.1097E-01
.2608E+01	.7617E+01	-.4117E+00	.9067E+00
.5433E+01	.4216E+01	-.6118E-01	.2834E+00
.0000E+01	.5474E+01	.1177E+00	.8583E-02
.6000E+01	.4469E+01	-.1445E+00	.8106E-02
.3427E+01	.3199E+01	-.3060E+00	.4341E+00
.8856E+00	.1808E+01	.5896E-01	.2381E+00
.6000E+01	.5380E+01	-.1216E+00	.9060E-02
.2278E+01	.8924E+00	.1979E+00	-.3694E-02
.4272E+01	.0000E+01	-.4768E-03	.3090E+00
.2196E+01	.9848E+01	.7297E-01	.2641E+01
.3858E+01	.5521E+01	.1156E+01	.2248E-01
.2551E+01	.6904E+01	-.3894E+00	.1027E+01
.0000E+01	.1021E+02	.1276E-01	.2861E-02

.2705E+01	.0000E+01	.0000E+01	.2922E+00
.4664E+00	.1662E+01	.6013E-01	-.4943E-01
.4380E+01	.5273E+01	.8462E+00	.2600E+00
.2313E+01	.0000E+01	-.2384E-03	.2942E+00
.4792E+00	.2121E+01	-.2922E-01	.1223E+00
.3506E+01	.3726E+01	-.7785E-01	.3087E+00
.5535E+01	.1708E+01	-.5992E-02	.1467E+00
.6000E+01	.2779E+01	-.2089E+00	.5484E-02
.4116E+01	.8761E+00	.2178E+00	.1900E+00
.6000E+01	.2246E+01	-.2356E+00	.4053E-02
.5541E+01	.8425E+00	-.4405E-01	.1192E+00
.0000E+01	.7937E+01	.5910E-01	.9537E-02
.1142E+01	.0000E+01	.1192E-03	.3068E+00
.1220E+01	.4946E+00	.3128E-01	.8605E-01
.1312E+01	.9222E+00	.1554E+00	.1054E+00
.1175E+01	.5628E+01	-.6318E-01	.8407E+00
.3205E+01	.8749E+00	.2622E+00	.4922E-01
.5484E+01	.3162E+01	-.5815E-01	.3287E+00
.4807E+01	.1188E+01	.6562E-01	.2293E+00
.1742E+01	.1699E+01	.1886E+00	.3699E-01
.6000E+01	.4177E+01	-.1564E+00	.7629E-02
.6000E+01	.1192E+02	-.3338E-02	-.1049E-01
.3257E+01	.7377E+01	-.4513E+00	.5165E+00
.6000E+01	.5070E+01	-.1264E+00	.9060E-02
.6000E+01	.9702E+00	-.2995E+00	.1431E-02
.1336E+01	.0000E+01	-.2384E-03	.3021E+00
.3455E+01	.1311E+01	.1059E+00	.1087E+00
.2376E+01	.6167E+01	.3260E+00	.9599E+00
.1916E-05	.0000E+01	.2201E+00	.1008E+00
.1120E+01	.5027E+01	.1353E+00	.6940E+00
.4779E+01	.4762E+00	-.7042E-02	.5606E-01
.2092E+01	.8161E+01	.8859E+00	.1735E+01
.4799E+01	.6332E+01	.4439E+00	.1206E+01
.2323E+01	.2925E+01	.9646E-01	.6106E+00
.2869E+01	.3093E+01	-.5374E-01	.5990E+00
.4129E+01	.6567E+01	.8558E-01	.9549E+00
.2617E+01	.8982E+01	-.7750E+00	.1673E+01
.1531E+01	.0000E+01	-.3576E-03	.3023E+00
.1617E+01	.4698E+01	-.3685E+00	.4827E+00

APPENDIX IV

EXACT POSITIONS AND VELOCITIES

OF FLUID #10 IN TABLE II

X	Y	VX	VY
.7826E+00	.3530E+01	-.1224E+01	.2475E+01
.2138E+01	.1747E+01	-.1842E+01	-.8631E+01
.5027E+01	.1865E+01	-.8593E+01	.6766E+00
.5240E+01	.4947E+01	-.1017E+02	.5168E+01
.3286E+01	.0000E+01	-.4768E-02	.3867E-01
.1211E+01	.2662E+01	-.3027E+01	.7531E+00
.0000E+01	.1489E+01	.3967E-01	.2384E-02
.6578E+00	.1891E+01	-.3969E+01	.6695E+01
.3676E+01	.0000E+01	-.2384E-02	.3966E-01
.4789E+01	.4805E+01	.4357E+01	.6557E+01
.1868E+00	.0000E+01	.1043E-02	.4422E-01
.6000E+01	.2501E+01	-.2384E-01	.0000E+01
.6000E+01	.3039E+01	-.1907E-01	-.2384E-02
.1783E+01	.9135E+00	-.3059E+01	-.1105E+01
.4647E+01	.0000E+01	-.4768E-02	.4150E-01
.3908E+01	.6327E+00	.1817E+01	-.4131E+01
.5163E+01	.1194E+01	.1030E+01	.1504E+01
.5810E+01	.0000E+01	.0000E+01	.5209E-01
.0000E+01	.2485E+00	.4438E-01	-.2980E-03
.0000E+01	.2273E+01	.2812E-01	-.7153E-02
.2506E+01	.0000E+01	.0000E+01	.3960E-01
.4251E+01	.6299E+00	-.4442E-01	.1134E+01
.1040E+01	.3552E+00	-.5079E+01	.7716E+00
.2896E+01	.0000E+01	-.2384E-02	.3538E-01
.3514E+01	.2413E+01	.7775E+01	-.5090E+01
.5680E+01	.1554E+01	-.3078E+01	.3598E+01
.2451E+01	.3249E+00	-.3750E+01	-.5737E+00
.1922E+01	.0000E+01	.0000E+01	.3427E-01
.5616E+01	.0000E+01	-.4768E-02	.4384E-01
.5597E+01	.5878E+00	-.2885E+01	.8567E+00
.1407E+01	.3170E+00	-.1174E+01	.6465E+01
.1517E+01	.1200E+01	-.4106E+01	-.7364E+01
.3770E+01	.3259E+00	.1972E+01	-.6410E+01
.2534E+01	.1217E+01	-.3771E+01	-.4184E+01
.1460E+01	.2387E+01	-.6574E+01	-.8676E+00
.6000E+01	.1721E+01	-.3338E-01	.0000E+01
.6000E+01	.3595E+01	-.1907E-01	-.2384E-02
.2107E+01	.5499E+01	-.6795E+01	.1845E+01
.6023E+00	.9108E+00	.2505E+01	.3615E+00
.5249E+01	.6410E+00	.2225E+01	.3740E+01
.4293E+01	.3214E+01	-.3581E+01	-.2580E+01
.5599E+01	.2636E+01	-.9423E+01	-.2795E+01
.2333E+01	.2010E+01	-.3822E+01	.2854E+01
.5641E+01	.1889E+01	-.6479E+01	-.1147E-01
.5021E+01	.1551E+01	-.6594E+00	-.4438E+01
.1635E+01	.2668E+01	-.3243E+01	.3010E+01
.6000E+01	.3883E+01	-.1431E-01	.0000E+01
.1305E+01	.5004E+01	.1714E+01	-.6307E+01
.2239E+01	.4330E+01	-.1298E+01	.2550E+01
.2201E+01	.1210E+01	.3417E+00	-.2661E+01
.0000E+01	.3647E+01	.1484E-01	-.2384E-02
.3312E+01	.2753E+01	-.1282E+01	-.5342E+01
.1450E+01	.8970E+00	-.6718E+01	-.3571E+01
.2594E+01	.2998E+01	-.3186E+01	-.4163E+01
.6000E+01	.1977E+01	-.2861E-01	-.2384E-02
.8821E+00	.1051E+01	-.2973E+01	-.3881E+01
.5368E+01	.3292E+00	.1037E+01	.5568E+00

.9180E+00	.2041E+01	-.1059E+01	.5578E-01
.5034E+01	.0000E+01	-.4768E-02	.4034E-01
.7623E+00	.0000E+01	.2384E-02	.3746E-01
.1147E+01	.3507E+01	-.1885E+01	-.3867E+01
.3857E+01	.4900E+01	.7187E+01	.2308E+01
.0000E+01	.2007E+01	.2684E-01	.0000E+01
.2917E+01	.2840E+01	-.2279E+01	-.2710E+01
.2604E+01	.1807E+01	-.7353E+01	.3476E+01
.3436E+01	.9378E+00	.1854E+01	.1492E+01
.0000E+01	.4225E+01	.1127E-01	.0000E+01
.4873E+01	.2886E+01	.5722E+01	-.1135E+02
.5604E+01	.3343E+01	-.4795E+01	-.9104E+01
.6000E+01	.7255E+00	-.3815E-01	-.4172E-02
.5697E+00	.0000E+01	.0000E+01	.3508E-01
.4199E+01	.3707E+01	.6784E+00	.2036E+01
.3871E+01	.0000E+01	.0000E+01	.3834E-01
.9915E+00	.2427E+01	-.1052E+02	-.3906E+01
.4185E+01	.2749E+01	.6578E+00	-.3911E+01
.3000E+01	.4646E+01	-.1763E+01	-.1393E+02
.4076E+01	.9179E+00	-.5160E+01	.1919E+01
.3252E+01	.1860E+01	-.2698E+01	.2576E+01
.3893E+01	.3895E+01	-.2993E+01	.4964E+01
.0000E+01	.1747E+01	.3380E-01	.0000E+01
.3509E+01	.4625E+01	.2334E+01	.7943E+01
.3562E+01	.5499E+01	-.9074E+01	.7533E+01
.2963E+01	.6414E+00	-.6271E+00	-.1849E+01
.1993E+01	.2693E+01	.7586E+00	.4300E+01
.1952E+01	.6180E+00	-.2222E+01	-.3976E+01
.7195E-05	.3932E+01	-.2531E+00	.3071E-04
.5347E+01	.2084E+01	-.5026E+01	.5485E+01
.6000E+01	.4849E+00	-.3815E-01	.1490E-02
.4065E+01	.0000E+01	-.4768E-02	.1218E+00
.1748E+01	.3339E+00	.1616E+01	.1395E+01
.4682E+01	.1931E+01	.1436E+01	-.1915E+01
.6000E+01	.1217E+01	-.2337E+00	-.1192E-02
.6569E+00	.1557E+01	-.7347E+01	-.2864E+01
.3745E+01	.2753E+01	-.9598E+01	-.6806E+01
.6074E+00	.1236E+01	-.9127E+01	-.1446E+01
.2901E+01	.4179E+01	.2306E-01	.2060E+01
.4237E+01	.1193E+01	.6797E+01	-.3866E+01
.2743E+01	.9231E+00	-.2862E+01	.3147E+01
.9555E+00	.0000E+01	-.5960E-03	.3494E-01
.1581E+01	.1538E+01	-.7691E+01	.2351E+01
.0000E+01	.1235E+01	.3991E-01	-.3576E-02
.5376E+01	.1410E+01	.2346E+01	-.1820E+01
.0000E+01	.3365E+01	.1647E-01	-.2384E-02
.1551E+01	.3294E+01	-.6013E+01	.9090E+01
.3081E+01	.9408E+00	.8184E+00	-.1026E+01
.1963E+01	.1507E+01	-.3086E+01	-.3422E+01
.8745E+00	.6270E+00	.2375E+01	-.3142E+01
.3091E+01	.0000E+01	-.2384E-02	.3837E-01
.5422E+01	.0000E+01	.0000E+01	.1214E+00
.3791E+01	.1568E+01	-.1177E+02	-.7257E+00
.2116E+01	.0000E+01	.0000E+01	.3580E-01
.4909E+01	.5987E+00	.4147E+00	.1737E+01
.5684E+00	.6038E+00	.4145E+00	-.5575E+01
.1728E+01	.0000E+01	.1192E-02	.3845E-01

.4391E+00	.4889E+01	-.8535E+01	-.1211E+02
.8691E+00	.4938E+01	-.6849E+01	-.3219E+00
.5455E+01	.4597E+01	.6163E+00	-.1567E+02
.1865E+01	.2355E+01	-.2551E+01	-.1392E+01
.2752E+01	.3668E+01	-.1034E+01	-.5047E+01
.0000E+01	.7361E+00	.4806E-01	-.1192E-02
.2345E+01	.1525E+01	.5190E+01	.1658E+01
.4359E+01	.1972E+01	-.2149E+01	-.6530E+01
.2318E+01	.3618E+01	-.9034E+00	-.2596E+01
.5300E+01	.2436E+01	.1848E+01	.1693E+01
.6000E+01	.0000E+01	-.1907E-01	.7664E-02
.2943E+01	.1796E+01	-.5467E+01	.2587E+01
.1973E+01	.2060E+01	.1663E+01	-.1748E+01
.3590E+01	.1290E+01	-.3345E+01	-.3309E+01
.4118E+00	.4448E+01	-.1320E+01	-.6844E+01
.3481E+01	.8287E-05	.1238E-03	-.9105E+00
.3181E+01	.3913E+01	-.8705E+00	-.1697E+01
.2033E+01	.3344E+01	-.1120E+02	-.9535E+00
.8029E+00	.3908E+01	-.3372E+01	.1129E+01
.5579E+01	.2979E+01	.6705E+01	.2812E+01
.1206E+01	.3945E+01	.1716E+01	.6876E+00
.4453E+01	.0000E+01	-.4768E-02	.4062E-01
.2113E+01	.3495E+00	-.6682E+00	-.3848E+01
.1864E+01	.4420E+01	.8952E+01	.9155E+00
.2287E+01	.6198E+00	-.3119E+01	-.8433E+01
.1351E+01	.1818E+01	-.6449E+01	-.8989E+00
.1828E+01	.3692E+01	-.2743E+01	-.6424E+01
.3537E+00	.2811E+01	-.3089E+00	.9033E+01
.4458E+00	.3949E+01	-.5335E+01	-.5013E+00
.7566E+00	.3166E+01	.1846E+01	-.9687E+00
.7910E+00	.5500E+01	-.4167E+01	-.4329E-02
.5228E+01	.0000E+01	-.4768E-02	.4032E-01
.3903E+01	.1234E+01	.1533E+00	-.5116E+01
.3590E+01	.3710E+01	.1837E+01	-.4083E+00
.3320E+00	.2412E+01	-.6858E+01	-.4595E+01
.0000E+01	.2540E+01	.2688E-01	.0000E+01
.4416E+01	.1654E+01	-.2214E+01	-.3468E+01
.3402E+01	.1556E+01	.3736E+01	.3535E+01
.5160E+01	.5497E+01	-.1058E+02	.1012E+02
.0000E+01	.4906E+00	.3586E-01	-.2980E-02
.3150E+01	.3544E+01	-.2923E+01	-.7726E+01
.2091E+01	.9200E+00	-.6836E+00	-.3234E+00
.0000E+01	.3087E+01	.2055E-01	-.2384E-02
.5667E+01	.1208E+01	.4314E+00	.2387E+01
.1485E+01	.3664E+01	.2081E+01	.5322E+01
.2861E+01	.3256E+01	-.3133E+01	-.1243E+02
.0000E+01	.9843E+00	.3744E-01	-.2384E-02
.2176E+01	.3010E+01	.6676E+01	.2662E+01
.3095E+01	.3388E+00	-.2795E+01	.1486E+01
.0000E+01	.2811E+01	.2581E-01	-.2384E-02
.3783E+00	.0000E+01	-.8941E-03	.3522E-01
.5102E+01	.4011E+01	.2967E+01	-.6725E+01
.1823E+01	.1231E+01	-.7980E+01	-.7574E+01
.2875E+01	.1219E+01	-.1684E+01	-.1697E+01
.1014E+01	.1710E+01	-.5619E+01	.1552E+01
.4712E+01	.1560E+01	.6130E+00	-.1587E+01
.2996E+00	.1390E+01	-.7407E+01	.2492E+01

.5064E+01	.3405E+00	-.5202E+01	-.2655E+01
.3975E+00	.3155E+01	-.1279E+01	.3981E+01
.1231E+01	.2093E+01	-.4628E+01	-.2110E+01
.4424E+01	.4364E+01	.1122E+00	-.1598E+02
.1196E+01	.1154E+01	-.7495E+01	.8589E+01
.3250E+00	.1079E+01	-.4566E+01	-.8678E+00
.3317E+01	.2160E+01	-.1664E+01	-.9306E+01
.3891E+01	.4375E+01	-.1157E+02	.5004E+01
.2399E+01	.2656E+01	.6614E+01	-.7863E+00
.2234E+01	.4854E+01	.7512E+01	.8386E+01
.6000E+01	.3315E+01	-.1907E-01	-.2384E-02
.5043E+01	.9060E+00	-.5321E+01	-.3760E+01
.9346E+00	.1396E+01	-.5883E+01	-.2084E+01
.3594E+01	.6173E+00	-.2531E+01	-.8318E+01
.0000E+01	.4532E+01	.1333E-01	.4768E-02
.2747E+01	.2561E+01	.3881E+01	-.8888E+01
.3300E+00	.1733E+01	-.1377E+01	-.1577E+01
.4435E+01	.3313E+00	-.2763E+01	.3243E+01
.3274E+01	.1219E+01	.1332E+01	.3196E+01
.3701E+01	.2130E+01	-.1095E+02	.2559E+01
.6577E+00	.2644E+01	-.8430E+00	.4107E+01
.6000E+01	.2438E+00	-.5722E-01	.1043E-02
.4841E+01	.0000E+01	-.4768E-02	.4140E-01
.3430E+01	.3209E+00	-.4780E+01	.2274E+01
.2782E+01	.3617E+00	-.1751E+01	-.2918E+01
.9063E+00	.2820E+01	-.6684E+01	-.1284E+01
.4870E+01	.3254E+01	-.2680E+01	-.4642E+01
.1805E+01	.3014E+01	.2667E+01	.1478E+01
.2487E+01	.3309E+01	.1769E+01	.1396E+01
.3085E+01	.2433E+01	-.1887E+01	.2739E+01
.3290E+01	.6278E+00	-.3979E+01	.1582E+01
.3740E+01	.9421E+00	.1892E+01	.5033E+00
.4866E+01	.4384E+01	-.1775E+01	.7611E+01
.4529E+01	.1279E+01	-.5678E+00	.3173E+01
.3067E+01	.1489E+01	.9837E+00	.4427E+01
.2609E+01	.2240E+01	-.5494E+01	.1288E+01
.5226E+01	.3545E+01	.6747E+01	-.5206E+01
.2861E+00	.7197E+00	-.1554E+01	.2126E+01
.1221E+01	.5500E+01	-.4269E+01	-.1322E+01
.6000E+01	.1467E+01	-.3815E-01	-.3576E-02
.1560E+01	.4112E+01	-.8200E+00	-.4970E+01
.2696E+01	.4963E+01	.4392E+01	-.1351E+01
.1283E+01	.1475E+01	-.5623E+01	.3371E+01
.4584E+01	.2281E+01	-.2612E+01	-.7669E+01
.1386E+01	.2964E+01	-.9996E+00	-.3776E+01
.4017E+01	.1781E+01	-.2956E+00	-.8523E+01
.5245E+01	.3162E+01	-.5381E+01	-.1751E+01
.6000E+01	.4490E+01	-.9537E-02	.0000E+01
.5668E+01	.2264E+01	.3007E+01	-.6499E+00
.1124E+01	.8382E+00	-.6107E+00	.3791E+01
.4537E+01	.2991E+01	.7829E+01	.3013E+00
.1612E+01	.6184E+00	-.1672E+01	.2967E+01
.4259E+01	.0000E+01	-.4768E-02	.3677E-01
.3988E+01	.3001E+01	.2189E+01	-.8043E+01
.5348E+01	.1728E+01	-.8095E+01	-.6193E+01
.3951E+01	.3402E+01	-.7387E+01	-.2373E-01
.1472E+01	.4610E+01	.1191E+02	-.1304E+01

.2700E+01	.0000E+01	.0000E+01	.3521E-01
.6782E+00	.2281E+01	-.8597E+01	.1993E+01
.4844E+01	.1260E+01	-.3956E+00	-.5836E+01
.2311E+01	.0000E+01	-.2384E-02	.3711E-01
.3752E+00	.2072E+01	-.2039E+01	.3522E+01
.4979E+01	.2192E+01	.6319E+00	-.4391E+01
.5694E+01	.3220E+00	-.3365E+01	-.5576E+01
.6000E+01	.2768E+01	-.2384E-01	-.2384E-02
.4586E+01	.6488E+00	-.4709E+00	-.1832E+01
.6000E+01	.2237E+01	-.3338E-01	-.2384E-02
.5390E+01	.9465E+00	-.5408E+00	-.7687E+00
.4059E+01	.2124E+01	-.5046E+01	-.6982E+01
.1148E+01	.0000E+01	-.1192E-02	.3467E-01
.7021E+00	.3351E+00	-.4044E+01	-.1926E+00
.3389E+00	.3588E+00	.3166E+01	.2746E+00
.1080E+01	.3167E+01	-.5432E+01	-.4398E+01
.2405E+01	.9210E+00	-.1693E+01	-.6932E+01
.4740E+01	.9496E+00	-.8038E+01	-.1144E+01
.4110E+01	.3482E+00	.5631E+01	.2651E+01
.1263E+01	.5750E+00	-.7625E+01	.3411E+01
.6000E+01	.4179E+01	-.1431E-01	.0000E+01
.4161E+00	.3548E+01	-.4516E+01	.1210E+01
.5697E+01	.8698E+00	-.5741E+01	-.6872E+01
.2946E+01	.2125E+01	.9699E+01	.2469E+01
.6000E+01	.9705E+00	-.4292E-01	.5960E-03
.1341E+01	.0000E+01	.0000E+01	.4048E-01
.2620E+01	.6290E+00	.3849E+01	-.2373E+01
.3482E+01	.3338E+01	-.2829E+01	-.4567E+01
.1097E-06	.0000E+01	.1630E-01	.7590E-02
.2233E+01	.2365E+01	.7947E+00	.3971E+01
.4746E+01	.3228E+00	-.2642E+01	.2877E+01
.2980E+01	.5500E+01	-.3608E+01	-.1029E-01
.3923E+01	.2445E+01	.8824E+01	-.5993E+01
.2689E+01	.1501E+01	.4622E+01	-.2807E+01
.1604E+01	.2079E+01	-.4546E+01	.4514E+00
.4432E+01	.9217E+00	-.7656E+01	-.4668E+01
.7535E+00	.4388E+01	.6787E-01	-.2812E+01
.1535E+01	.0000E+01	.1192E-02	.3918E-01
.1775E+01	.1807E+01	-.1333E+01	.3436E+01
.3374E+01	.4227E+01	.7781E+01	.1895E+00
.4165E+01	.5354E+01	.5739E+01	-.1218E+02
.4524E+01	.3460E+01	-.4128E+01	.2071E+01
.4356E+01	.4856E+01	.2572E+01	-.1583E+01
.0000E+01	.4872E+01	.1137E-01	.4768E-02
.1743E+01	.4870E+01	.6932E+01	.3959E+01
.2070E+01	.3918E+01	-.3740E+01	-.5886E+01
.3605E+01	.1842E+01	-.5774E+01	.5226E+01
.3307E+01	.5158E+01	.8193E+01	.1742E+01
.3577E+01	.3050E+01	-.8270E+01	.1635E+01
.2483E+01	.5408E+01	-.1963E+02	-.5453E+01
.4537E+01	.3944E+01	.1411E+01	.3897E+01
.2513E+01	.3951E+01	.1825E+01	-.4627E+00
.5570E+01	.3730E+01	-.1275E+02	-.1350E+01
.2648E+01	.4475E+01	.3544E+01	.2445E+01
.4883E+01	.3722E+01	-.5939E+01	.1074E+02
.4945E+01	.2534E+01	-.1032E+01	-.3553E+01
.4551E+01	.2608E+01	.5643E+00	-.5094E+01

.6000E+01	.4815E+01	-.9537E-02	-.4768E-02
.4155E+01	.1498E+01	-.6243E+00	-.8133E+01
.3195E+01	.3124E+01	-.3099E+01	-.1217E+02
.1170E+01	.4344E+01	-.4107E+00	-.7111E+01
.1784E+01	.5296E+01	.8742E+01	-.8255E+01
.4156E+01	.4154E+01	.4279E+01	.6341E+01
.4242E+01	.2382E+01	.1993E+01	-.1026E+01
.6000E+01	.5142E+01	-.9537E-02	.0000E+01
.0000E+01	.5294E+01	.1376E-01	.0000E+01
.5514E+01	.5273E+01	-.7565E+00	.1294E+01
.3419E+00	.5472E+01	.1116E+02	-.1295E+01
.5236E+01	.2775E+01	.4241E+01	-.4880E+01
.5572E+01	.4176E+01	.6726E-01	.5108E+00
.4626E+01	.5322E+01	.8750E+01	-.9769E+00
.6000E+01	.5500E+01	-.9537E-02	-.9537E-02

COMPUTER SCIENCES DEPARTMENT

University of Wisconsin - Madison

NEW INVESTIGATIONS OF VON NEUMANN TYPE FLUIDS

by

Craig L. Hougum

and

Donald Greenspan

Computer Sciences Technical Report #323

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