

The Effects of Representational Math in a Montessori Classroom.

An Action Research Report
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Abstract:

This Action Research Study is designed to evaluate the effect of integrating representational math in the Montessori classroom during mathematical instruction and practice. In comparative studies, Montessori students have generally out-performed non-Montessori peers in the area of mathematics during the initial years of schooling. However, around the 2nd grade, Montessori students seem to underperform in comparison to non-Montessori peers in certain mathematical competencies. This research took place in a public school classroom with 28 students, ranging in age from 6 to 9 years old. This quantitative study looked at data from a pretest and posttest given to students. By introducing representational pictures alongside Montessori math work during a 25-day intervention, students showed an increase in ability to solve addition and subtraction problems.

Keywords: Montessori Mathematics, concrete-representational-abstract, number sense

Literature Review

In a traditional elementary classroom, there is importance in scaffolding math understanding along with the concepts and procedures that we are asking students to perform. Through the combination of mathematical understanding and computational accuracy, students develop fluency and flexibility with numbers. The *number sense* that is developed gives students the ability to perform mental math and solve problems in real world situations. My teaching career began in a traditional school where standardized test scores averaged around 80% proficiency for the 3rd -5th grades during my tenure. After moving to an urban public Montessori school for the 2014 - 2015 school year, math proficiency was 43% for the 3rd -5th grades. Both schools serve similar populations with similar socioeconomic groups. The traditional school relied heavily on the Every Day Math curriculum, while the Montessori school I currently teach uses the Montessori materials and lessons to teach state standards. This marked discrepancy intrigued me and I began to look for answers.

During the past year in the classroom, I found that students lacked the ability to apply their knowledge without the use of math manipulatives or without the use of paper and pencil. Students also experienced difficulty when they were asked to apply mathematical concepts to real world situations. The students attending my school were not developing the *number sense* that was being developed in traditional schools. Looking at the Minnesota State Standards for 3rd grade, nine of the thirteen standards require that students apply their understanding to real world situations. Students were not able to apply their own experiences and previous works in a Montessori classroom to a newly introduced task. The purpose of this research is to evaluate the effectiveness of using a Concrete–Representational–Abstract sequence (CRA) to help Montessori students succeed in mathematics. By use of this sequence, Montessori students

should be able to apply their wealth of knowledge to new applications and increase their understanding and enjoyment of mathematics.

Identifying the Needs

When looking at the comparative data of Montessori and traditional schools it is important to assess students' understanding and abilities in mathematics through standardized methods. Montessori education shows a significant benefit for students in the early years of Children's House and Lower Elementary (Peng & Md-Yunus, 2014). For those early years, the math work and understanding is greater than that of students of similar abilities in conventional schools; however, as the students move towards 3rd grade, students in Montessori schools lose this advantage. Data by Peng & Md-Yunus (2014) presented in their work *Do Children in Montessori Schools Perform Better in the Achievement Test?* indicate that as Montessori students progress through 1st, 2nd and 3rd grades, their peers in traditional schools catch up and then surpass them. The focus of the research was to compare scores on three standardized tests. The scores were measured with multiple sources: Elementary School Language Ability Achievement Test (ESLAAT), Elementary School Math Ability Achievement Test (ESMAAT), and Social Studies Ability Achievement Test (SSAAT). Peng and Md-Yunus (2014) found that Montessori students do hold many advantages, especially in the areas of social studies and language. However, in contrast to language and social skills, math gains by students in a traditional school were at a higher rate than that of students in a Montessori school.

There is a need to examine how Montessori mathematics is taught within the classroom. In the article *Preschool Children's Development in Classic Montessori, Supplemented Montessori, and Conventional Programs* Angeline Lillard (2012) highlighted the differences and

outcomes between Montessori programs and concluded that the lack of growth of Montessori students compared to their peers in traditional schools could be attributed to Montessori students not knowing the material, or having deep experience with word problems. “Math achievement was examined via the Applied Problems subtest. This task involves simple counting, addition, and subtraction, reading clock faces, and reporting and calculating coin values” (Lillard, 2012, p. 388). This work implies that it is important for us to examine the process of CRA mathematical teaching. Students in Montessori classes need to have experiences in solving math problems written into real world situations. The data from Lillard’s research show us that Montessori students again were underperforming on standardized tests no matter if they were in traditional or supplemental Montessori programs.

Current research suggests that one of the key factors for student success in mathematics is having *number sense* and the important role that teachers and classrooms have in building that into the curriculum. In his research, James R. Olsen (2015) focuses on the need for incorporating mental math in the classroom. The author argues, “Being able to do some math mentally leads to fluency and confidence. As with people who are fluent in foreign language, who can carry on a conversation without stopping to look up words in a dictionary or taking long pauses to compose sentences, those fluent with numbers can engage in the flow of the process of problem solving and move through many calculations” (Olsen, 2015, p. 544). This article focuses on the need to implement mental math strategies across all levels of schooling. By reinforcing the benefits of conceptual understanding, we are helping with procedural fluency and automaticity. Conceptual understanding is the comprehension of operations, concepts and number relations. Students in a Montessori classroom have heavy focus on the operational aspect of mathematics; however, Olsen’s research suggests a need for work in both understanding concepts and number relations.

Montessori Advantages

The use of concrete materials to help students has been proven to be an effective tool for teaching mathematics. For Montessori students in the early years of elementary education (such as Children's House), higher test scores for those children whose education includes the use of materials and manipulatives would suggest this might be the reason why Montessori students outperform their traditional school peers during the early years (Peng & Md-Yunus, 2014). Research completed by Donabella & Rule (2008) found that most students in traditional schools who struggle in mathematics lacked experience with concrete materials. The article *Four Seventh Grade Students Who Qualify For Academic Intervention Services in Mathematics Learning Multi-digit Multiplication With the Montessori Checkerboard*, Donabella & Rule (2008) found that students needed to have a foundation with concrete materials to be successful.

Further, data suggests that after students used the Montessori materials they were able to apply the knowledge and skill to new applications. These applications could be in the form of quizzes, standardized tests, or problem solving. This research focused on the deficiencies of middle school students in the area of math and suggests that the use of Montessori materials increased student achievement and confidence. The research showed that the students benefited in their math application because they were taught using the concrete materials. Montessori students have a lot of practice with these concrete materials, experience that helps them perform operational skills (Donabella & Rule, 2008). This research allows us to approach the dilemma of disparate test scores from a different angle. Montessori students have extensive work with concrete materials: Knowing that concrete materials are an important part of conceptual understanding, it allows us to look deeper into our work and develop our understanding of what elements Montessori students might be missing. Montessori students may be lacking the

scaffolding to complete problems where the operation is unknown, multi-stepped, or in real world situations.

When looking at the students that are not proficient on standardized tests, it is important to examine the scaffolding gaps that exist for all students. Pool, Carter, Johnson & Carter (2013) identified that the work of implementing and connecting interventions is more important than the interventions themselves. When thinking of the work that a classroom teacher presents to a student, it represents a Tier 1 service. When a student struggles with a mathematical concept, such as addition of two digit numbers, they would need a *second scoop* of instruction. This is called a Tier 2 service, whereby the student would be taught in small group or one on one the same mathematical concept. The connection between the Tier 1 and Tier 2 work was found to be vital to the growth and benefit of the students.

Montessori teachers need to examine the connection between the first best instruction and the support services that students need. Montessori educators need to connect the math work and interventions to the work that students are already doing in Montessori schools. Montessori students are receiving Tier 1 work in their classroom through proven Montessori curriculum. Montessori schools are not using Tier 2 level interventions based on Montessori principles and ideals for the students other than re-teaching. (Pool, et al., 2013) examined many of the techniques for interventions and their success. When looking at steps for increasing proficiency in math for Montessori students, teachers need to connect the interventions to the work they are doing in the classroom.

Researchers agree that building *number sense* into the classroom is important, and the work by Heirdsfield (2011) builds an argument for Montessori teachers to implement mental math strategies: In her work *Teaching Mental Computation Strategies in Early Mathematics*

Heirdsfield outlines a systematic approach to building *number sense* for students. The first part of the system is to know what the students' need and their current ability in math. When looking back at the research on Montessori teaching previously described, we know that Montessori students have an excellent understanding of concrete materials and operations. The second step is to map out the steps to achieve a particular goal set for each student to learn through Montessori teachings. The third step is to teach the materials and provide the resources to help the students achieve these goals. The fourth and final step is to maintain a place or area of study where students are able to complete the work. This fourth step fits perfectly within the environment established in a Montessori classroom. Evidence suggests that while students did benefit from the instruction, there was an additional benefit when students developed higher order reasoning, and making sense of numbers and operation (Heirdsfield, 2012). When looking at this particular process and how it should be implemented, only the second step is lacking for the Montessori student. Students in a Montessori classroom are moved from the Concrete to the Abstract too quickly. If students were taught techniques to bridge the gap between Concrete and Abstract, it would help them understand *number sense* at a much deeper level.

Representational Math

Students in a Montessori classroom that have an unparalleled experience with concrete materials still struggle with needing to apply understanding to new situations. Montessori students lack the *number sense* required for them to apply their knowledge to abstract concepts and real world situations. By sequencing instruction, and providing a processing bridge from concrete to abstract, I propose that Montessori students will improve their fluency and comprehension in *number sense*. In the article *Developing Representational Ability in*

Mathematics for Students With Learning Disabilities Van Garderen, Scheuermann, & Jackson (2012) suggest that more scaffolds and application with representational math are needed to support all learners. Research concluded that this area of mathematics is vital for teaching students. “One important content area that is critical for all students to develop mathematical understanding is representation. The importance of having a strong representational ability or fluency in mathematics, that is, knowledge about representations and facility to use that knowledge appropriately to solve problems, cannot be overlooked” (Van Garderen, Scheuermann & Jackson, 2012, p. 34).

When thinking about the work that is needed for Montessori students, it becomes apparent that we need to help students develop a deeper understanding of *number sense*. In the work *Conceptualizing Mathematics Using Narratives and Art*, Kurtz & Bartholomew (2013) relate that for many students, mathematics is simply something that they do. They work through it, but really do not view mathematics as a way of problem solving. Research found that the connections and interactions that students have with their work is one of the most important factors in student success and achievement. Additionally, students greatly benefited in the areas of confidence, understanding, and personal connection to their work after using art to understand the concepts. In describing their experience, students relate that the work became more exciting, and they found themselves becoming the teachers of the content through the use of their artistic medium (Kurtz & Bartholomew, 2013).

To support the necessity to link concrete to abstract and develop representational skills, Van Garderen, Scheuermann, & Jackson (2012), indicated that concepts within the sequence could not be taught in isolation, but rather, needed to be presented as part of an integrated system. Data presented in *Developing Representational Ability in Mathematics for Students With*

Learning Disabilities: A Content Analysis of Grades 6 and 7 Textbooks indicate that some schools and classrooms over emphasized representational work. This comparative study suggests there were problems with using representational mathematics, and that if not taught correctly, these techniques can actually become a problem for students. For example, one of the authors' concerns was how teachers and textbooks use representational math to teach content when they should be using concrete materials (Van Garderen, et al., 2012). Representation is used to help bridge the gap between concrete materials and abstract work. Based on findings:

Representational math was used to teach abstract math content. An example of this would be drawing three lemons and four cherries to solve a $3+4=7$ problem. What was not presented was how the student might mentally create these representations or using concrete models when using these concepts for problem solving (Van Garderen, et al., 2012). If students were provided with representations to use (as opposed to creating their own), there is concern that they are just using the representations as a tool to solve their work and not necessarily help them understand the concept. Research indicates that the use of representations in mathematics was key to helping children bridge the gap from concrete to abstract understanding (Kurtz & Bartholomew, 2013) (Van Garderen, et al., 2012).

Flores (2010) provided a model of representational work to help students who struggle in the area of subtraction when moving from concrete to abstract. Flores developed an intervention that helps students solve abstract work by teaching them how to use number lines and pictures. This work can be applied to a Montessori classroom due to the presence of concrete and abstract work and the missing element of representational mathematics. This was the first study that had shown the effectiveness of CRA, but also acknowledged the limitations of current Montessori techniques. Researchers found that using this model was effective for all the students as an

intervention. Data suggest that students internalized the lessons and techniques as evidenced by sustained performance levels throughout the school year. In contrast to other types of interventions, with CRA students maintained their understanding and skills (Flores, 2010).

The Concrete-Representational-Abstract model has been used in traditional classrooms to improve conceptual understanding and to build *number sense*. Data presented by Arroyo (2014) in *The Effects of Using a Systematic Approach During Mathematical Instruction* show an increase in all three levels of mathematics. Students were pleased and excited to show how they arrived at their mathematical answers: Further, student engagement showed a marked improvement (Arroyo, 2014). Arroyo's research opens the door for this work to be done in a Montessori classroom. Because Montessori students do not have much practice with representation, they cannot readily understand the abstract concepts we ask of them. Arroyo indicated that students did not need representational skills because they were being taught the abstract directly for the sake of her tests. However, when thinking about the ultimate goal of developing a deeper understanding by Montessori students, bringing representational strategies into the Montessori classroom allows students to begin to mentally apply math fluency and computations in the development of *number sense*. Having a systematic approach to mathematics instruction is important and by strengthening the Montessori method, our students can achieve at a high level.

Final Thoughts

The students attending the school where I conducted my research enjoy mathematics, and I have my students solve problems and complete works that surpass state grade level standards for mathematics. Once these Montessori students became inspired to do more and solve harder

and more complex problems, and they pushed themselves further than expected. Math education in the United States approaches the teaching and testing of mathematics as a way to find solutions. However, we never help them develop the conceptual understanding of why math works. Our approach to mathematics instruction in Montessori classrooms is similar in that we fail to teach the *number sense*. Through the use of representational mathematics in Montessori classrooms we allow the child to build the connections they need to be successful in mathematics, problem solving and real world solutions.

The purpose of my action research is to help Montessori students achieve at a higher level in mathematics. From my research it appears that the best way to help Montessori students is to emphasize representational math as a scaffold to help students solve problems abstractly without materials. My action research is focused on working with students in developing strategies from the concrete stage, through the representational stage, and ultimately to the abstract work and understanding of *number sense*. The CRA model is important to implement in traditional and Montessori classrooms to support all learners.

Research Design and Methodology

Purpose

The purpose of my action research topic is designed to evaluate the impact of integrating representational mathematics in the Montessori Learning Environment. In comparative studies, students attending Montessori schools have generally outperformed students attending non-Montessori schools in many areas of academic study. My goal is to create positive change for the participating students in their understanding of work with positive and negative integers by using number lines and other strategies that move students from a concrete stage to an abstract stage of

understanding through representational mathematics. The Concrete-Representational-Abstract (CRA) approach to mathematics will allow students to build number sense through their connected work in the classroom. Through the use of number lines as an effective representational strategy in math instruction, we will ideally see positive growth in the area of mathematics when students are working in number sense, subtraction, and addition problems.

Central Question

Would Montessori students benefit from direct instruction in representational mathematics?

Topical Questions

1. What lessons and materials can be used to help students move from the concrete stage to the abstract stage in mathematical understanding?
2. How can we produce a positive change in Montessori student's abilities at working with positive and negative integers?
3. How does the use representational mathematics impact student achievement?
4. What is the best way to introduce and practice representational mathematics in a Montessori classroom?

Participants

Participants in the study were 28 students from a lower elementary class in an urban public Montessori school. There were 8 students in first grade. There were 10 students in second grade. There were 10 students in third grade. There were two students that receive English Language Learner services and two students with Individualized Education Plans.

Setting

The study took place inside my classroom in an urban Montessori school featuring a magnet program that allows students from all areas of the city to attend. This school has enjoyed a rich tradition of Montessori for more than 20 years.

Materials

- Addition equations and sums box
- Subtraction equations and differences box
- Stamp Game
- Student Math Journals
- Daily math prompts featuring number lines problems on the board
- White board and markers
- Name that Number Work
- Number Line practice sheets
- 10 More-10 Less, 1 More- 1 Less Work

Data Collection

My research project is designed to look at both quantitative and qualitative information. The quantitative information will be from a test that students will use for their initial and final interview. Students will have an initial test to determine their mathematical understanding. The initial test consists of solving 5 math problems, each with two different tasks, orally without the aid of manipulative. The students will be invited to the table and asked to sit down. They will be given a visual picture of math problems and asked to solve them. After the student provides the

answer, the test administrator will ask, “How did you solve it?” Students will be asked to share their strategy. Test administrator will record their strategy. The initial tests will be completed when the student solves the 5 math problems or requests the test be stopped. After 25 days of work with representational mathematics in whole group instruction (Tier 1 support) and small group work (Tier 2 Support), I conducted the final interview using the same questions and materials from the initial test.

Results

The goal of this research was to examine the changes in math understanding in 28 students after a 25-day instructional period. This instructional period was selected so that it would help to isolate the growth to a typical instructional period in a traditional school. I collected my pretest data from all of the students on January 4th, 2016 and began introducing representational mathematics into the Montessori math lessons and daily math over the next 25 days. On February 10th, after the instructional practice period, I was able complete the posttest for all students. For the assessments, all 28 students were present for both days and the assessments were completed in a three-hour timeframe. The pretest and posttest took place during a normally scheduled morning work time. After having both the pre and posttests, I began to analyze the data of each individual student, group of students, and the class as a whole.

Test Content

The goal of the assessment was to assess fluency and flexibility of students when adding and subtracting numbers mentally. The tasks measured the effectiveness of their mental strategies without having to use manipulatives. All students that participated in the study had

already demonstrated an ability to break apart two digit numbers into 10's and 1's. This foundational starting point was an important part of the examining their mental fluency and flexibility during the assessment. Being able to decompose two digit numbers, demonstrated both place value and a beginning number sense. Being successful in decomposition demonstrated a readiness for a deepening *place value* understanding, as well as indicating that students were receptive to strategies that would lead to developing flexible mental strategies with two digit operations.

The assessment was created by the study researcher in based on an assessment with permission by Stephen Theomke, and designed to progressively increase in math knowledge and complexity – The first two tasks of the assessment measured the student's ability in working with a decade number (10, 20, 30, 40, etc.). In **Task 1** students were asked to jump forward from a decade number using a single digit number without counting. In **Task 2** students were asked to jump back to a decade number using a single digit number without counting. After students have shown an understanding in working to and from a decade number, I ask that students start with a number and move to the decade number through addition in **Task 3**. In **Task 4**, students are asked to jump back from a decade number without counting. Mastering both of these steps indicates students are now ready to start moving through the decade number in both addition and subtraction which are questions featured in **Task 5** and **Task 6**. The next two tasks again assessed the student's ability to use 10's and 1's, which had been established prior to the assessment's beginning. In **Task 7**, students were asked to add 10's and 1's from a decade number to derive at a new number. In **Task 8**, students were asked to subtract with 10's and 1's to a decade number. During **Task 9**, students needed to use flexible mental strategies for adding

two doubled-digit numbers. Within **Task 10**, students need to use flexible mental strategies for subtracting two double-digit numbers.

Data Analysis

After assessing pretest scores, it was important to first connect the best instruction for the whole group (Tier 1) with the work that students would be doing in small groups (Tier 2). This step leads us to answering the question, how can we produce a positive change in Montessori student's abilities at working with positive and negative integers? The instructional practice is centered around the Concrete-Representational-Abstract (CRA) model and how to build *number sense*. Data presented by Arroyo (2014) in *The Effects of Using a Systematic Approach During Mathematical Instruction* suggests improvement in all three levels of mathematics.

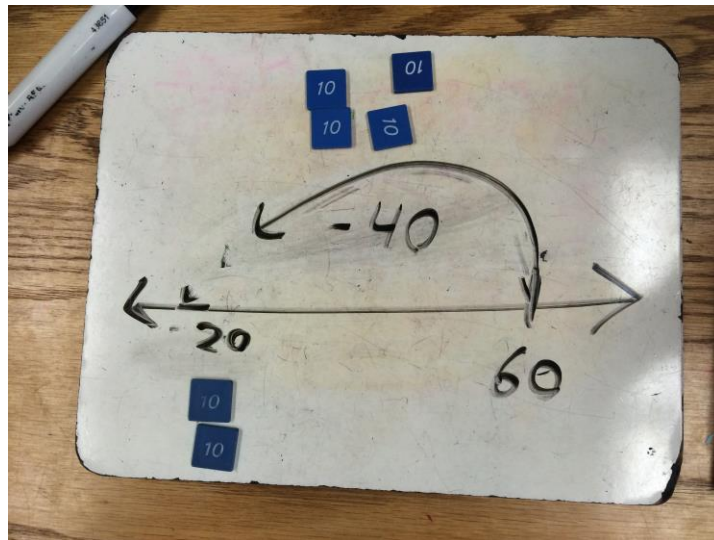


Figure 1: Example of Concrete Representational Work.

The instructional intervention I conducted for the students during whole class instruction revolved around the use of concrete to representational work (see Figure 1). I posed the question,

“We know how to use the materials to solve problems, but if we needed to draw a diagram to explain our work, how could we do that?” The students then began using number lines to show solutions to the concrete works that they were doing in class. Once students were able to draw problems from concrete materials, I started with problems in the representational stage: Starting with numbers and “adding on.”

Figure 2 shows examples of two problems that students in Group A solved during their small group instruction. For the problem, “ $347 + 298$ ” students were able to visualize the addition of 300, then counting back 2. Additionally, as shown in Figure 2, students were able to draw the addition of each place value. In the problem “ $168 + 257$ ”, students started with 168, then added two hundreds. Students then added 5 tens, which they needed to cross into another hundred with 11 tens. Then students added ones. The work that was done with the 100 more 10 more work, stamp game, and number line work helped students visualize the moves they would be doing to solve the problem.

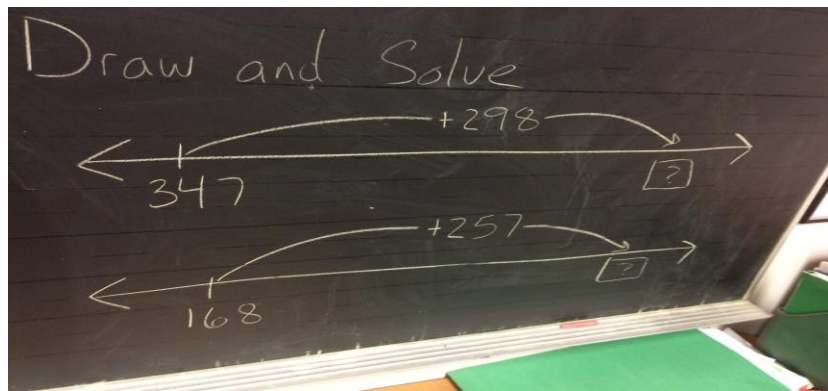


Figure 2: Example of problem posed to Group A.

The purpose of this research was to examine the results of the use of representational mathematics in a Montessori classroom and how it would impact student achievement through

the building of *number sense*. After collecting my initial data, I decided that the range of student's scores could be divided into three groups. All children would receive general instruction and practice together in whole group (Tier 1), but then based on their initial scores be given more specific classroom instruction. The student were sorted into one of three groups (A, B, and C) based on conceptual understanding, mathematics and *number sense*, as determined by pretest scores. Each of these groups was given connected instructional support to their level of learning (Tier 2 support).

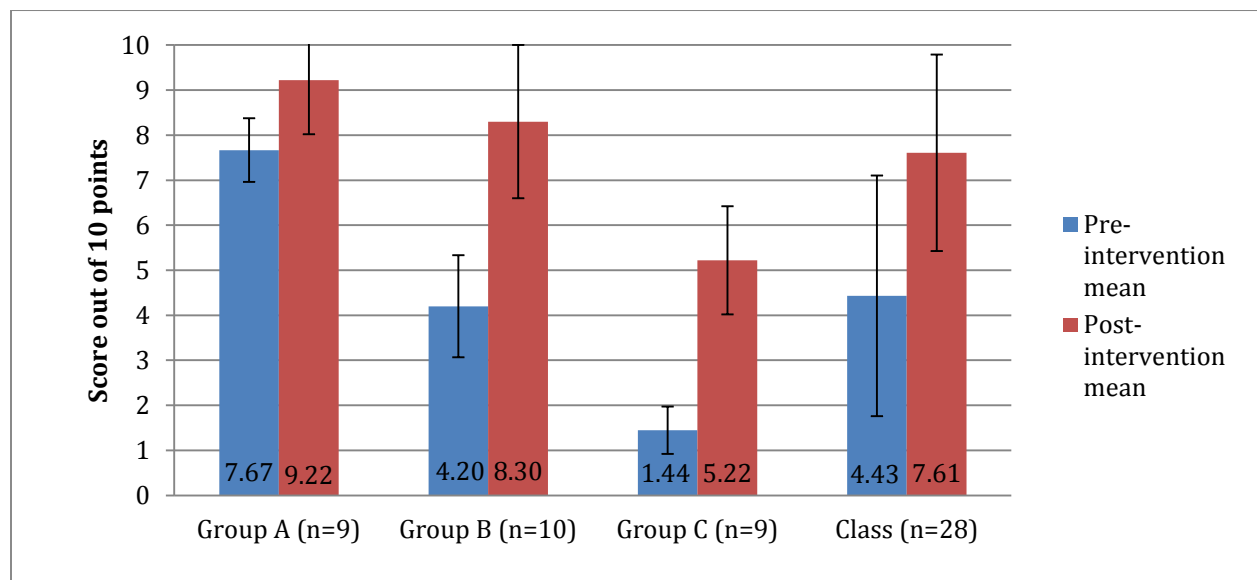


Figure 3. Mean test scores pre and post intervention.

Group A consisted of nine students who scored 7 or greater on the initial assessment. Group A students had knowledge and skills that allowed them to use 10's and 1's in their mental math. Group B consisted of 10 students, who scored 3-6 on the initial assessment. Group B students did not yet have the 10's and 1's work mastered mentally, but did have the concept of addition to a decade number. Group C consisted of nine students who scored 2 or less on the

initial assessment. These students were able jump from a decade number, but did not have the flexibility to jump from a number to a decade number. Figure 3 demonstrates the distribution of these scores for the whole class (n=28).

Throughout the 25-day intervention period, the purpose of assigning the class into groups was designed to achieve two outcomes. The first is that lessons could be tailored for the needs of the student during a small group lesson. The second outcome was to make sure that students were getting instruction closest to their level of achievement based on the pretest. This matched the work that was found in research conducted by Pool, Carter, Johnson & Carter (2013).

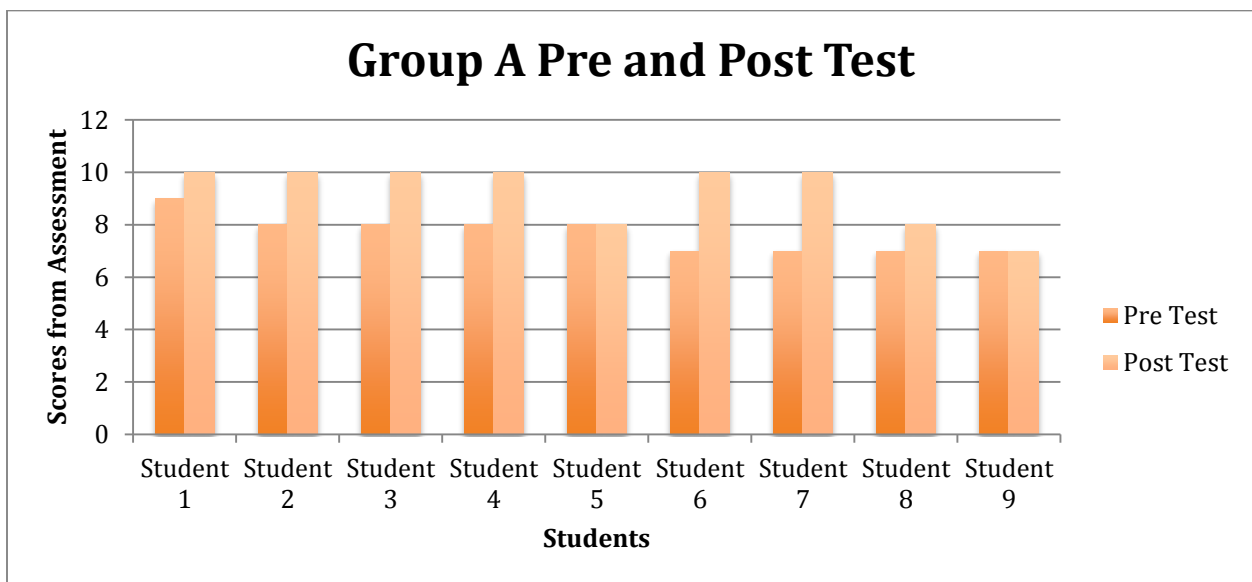


Figure 4. Group A Pre and Post test results.

Group A consisted of students that were able to move across a decade number in both addition and subtraction. Looking at Figure 5, the group had a mean pre-test intervention score of 7.67 points out of 10. This group practiced the same daily work as the rest of the class, but had 10 small group lessons that used both Montessori materials and representational tools. At the end

of the 25-day period, the average score was 9.22 ± 1.20 , a gain of 1.56 ± 1.13 points. This group had the lowest increase among all groups, but this is likely attributed to the group having scores near the predetermined ceiling of 10. Of the 9 students in this group, 6 of the students scored 10 on the posttest. The three students that did not score 10 points, were 2nd grade students who are not expected to achieve the second grade standard of successfully adding or subtracting two digit numbers until the end of the school year.

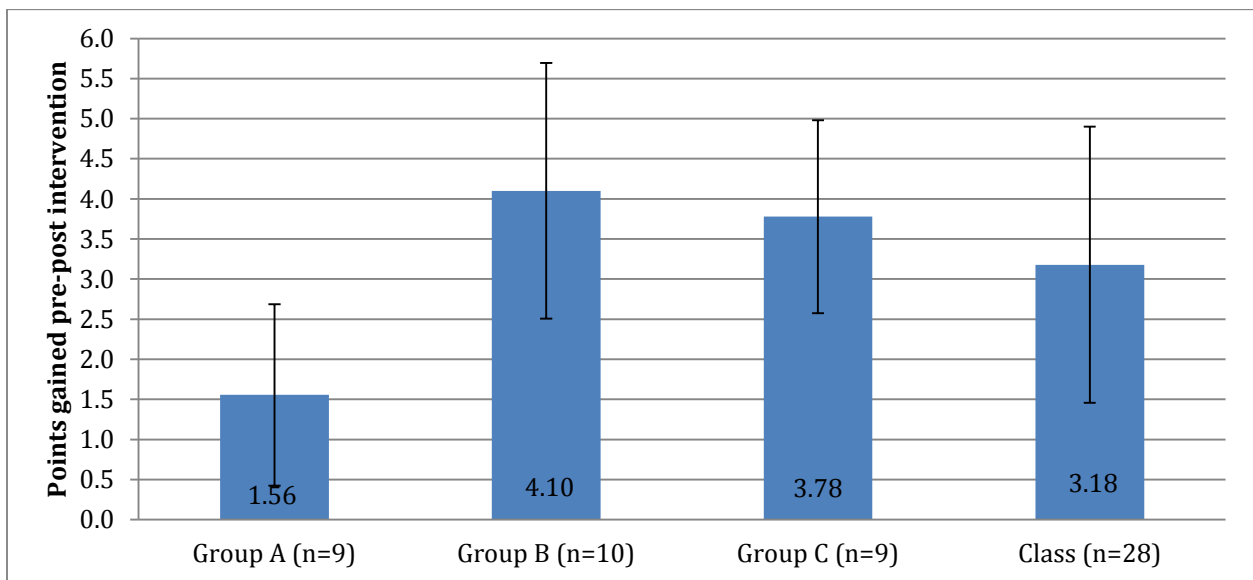


Figure 5. Gains Made by Group, comparing Standard Deviation.

Group B was comprised of students that did not yet have the 10's and 1's work mastered mentally, but did have the concept of addition to a decade number. The group needed further support in moving from a decade number in subtraction with 10's and 1's with both concrete materials and representational work. This group was comprised of students from all three grade levels. Looking at Figure 6, the average score for the group during the pre intervention work was 4.2 ± 1.14 . This group practiced the same daily work as the rest of the class, but had 10 small

group lessons that used both Montessori materials and representational tools. At the end of the 25-day period, the average score was 8.3 ± 1.70 , an increase of 4.1 ± 1.60 points out of 10. This group had the largest increase among all groups and also had the highest standard deviation.

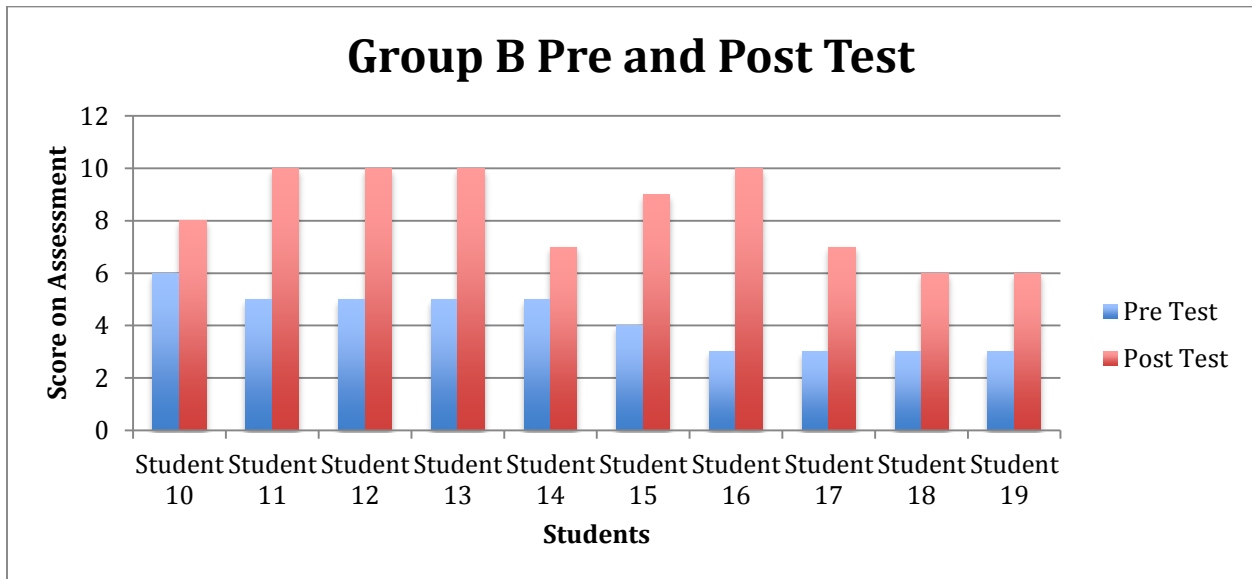


Figure 6. Group B Pre and Post test results.

Group C consisted of students that were able to move across a decade number in both addition and subtraction. This group was comprised of five-first grade students, and four-second grade students. There were two students that received ELL services and two students that have IEP's to support their learning. This may have been a factor, however each student increased his or her score from the initial assessment. Looking at Figure 7, the average score for the group was 1.44 ± 0.53 . This group practiced the same daily work as the rest of the class, but had 10 small group lessons that used both Montessori materials and representational tools. At the end of the 25-day period, the average score was 5.22 ± 1.20 (See Figure 8), an increase of 3.78 ± 1.20 . In this group, 80% of the students moved past the pre-test average.

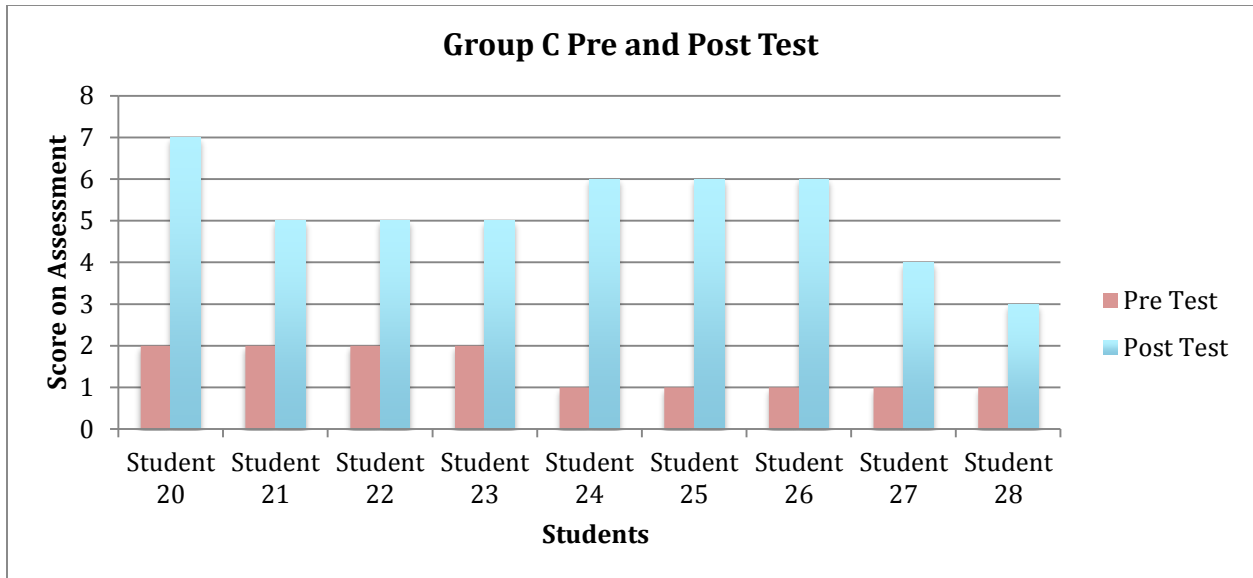


Figure 7. Group C Pretest and Post test results.

Statistically significant differences existed between pre and post scores when the whole class was analyzed together ($p < 0.001$) – see Figure 9. Data from subgroups are not statistically significant likely due to the small size of test group. A pronounced gain was made for subgroups B and C with non-overlapping SD, suggesting that the intervention was trending towards significance. However, Group A started at a mean of 7.67 points, and therefore experienced a ceiling effect with this test. There may have been more gains made via the intervention; however, due to the limitations of the pre and post testing as written, this data could not be captured. For further evaluation of the success of the intervention, in future research a different exam would be best used due to 10/28 students achieving a perfect score in post-testing.

	Pre-intervention mean	Post-intervention mean	p
Group A (n=9)	7.67±0.71	9.22±1.20	0.296
Group B (n=10)	4.20±1.14	8.30±1.70	0.220
Group C (n=9)	1.44±0.53	5.22±1.20	0.570
Class (n=28)	4.43±2.67	7.61±2.18	0.000

Figure 8. P chart for Pre and Post Test with Standard Deviation.

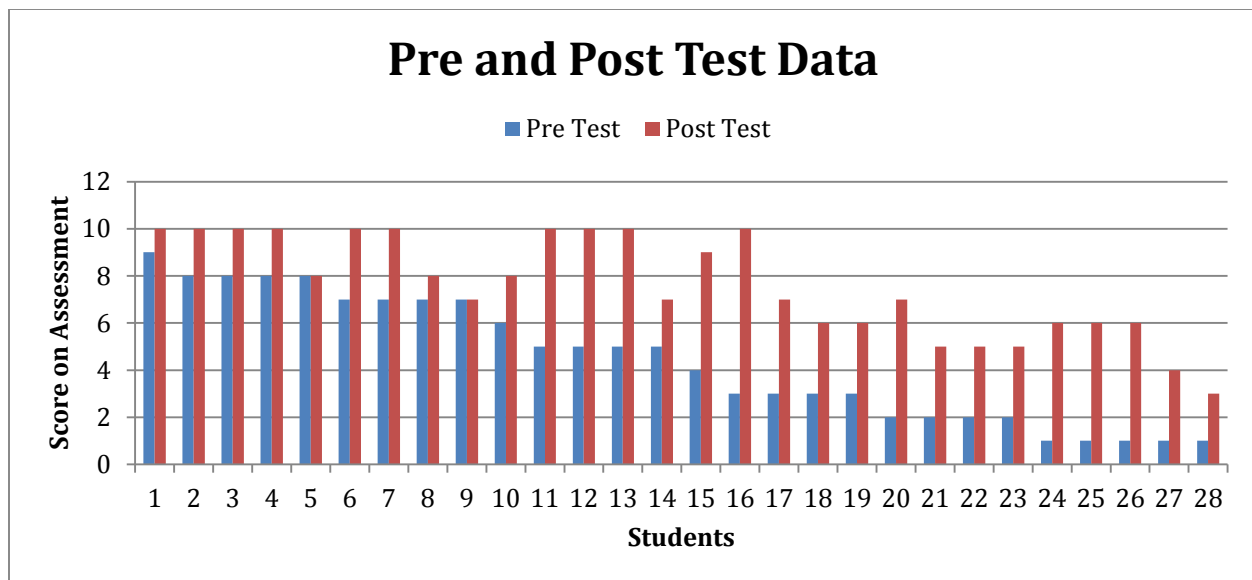


Figure 9. Pretest and Post-Test Results for Students

As shown in Figure 9, the pretest score is recorded in dark blue and the post-test score is recorded in red. The lowest score possible was a 0, the highest score possible was a 10. After the pretest, the class as a whole held an average of 4.42±2.67. The majority of the class (65%) began working in the score range from 2-7. This meant that most of the students were beginning to use decade numbers, move through decade numbers, and add and subtract 10 in order to solve mental math problems. Based on the data collected during the Pretest, I created three groups and sorted students into groups to receive representational mathematics instruction based on their

score level. After the 25-day intervention period, the average score of the class increased to 7.61 ± 2.18 . Upon further analysis, 26/28 students (92%) finished above the average score of the pretest. The data showed that 26 of the 28 students increased their ability in the area of mental math.

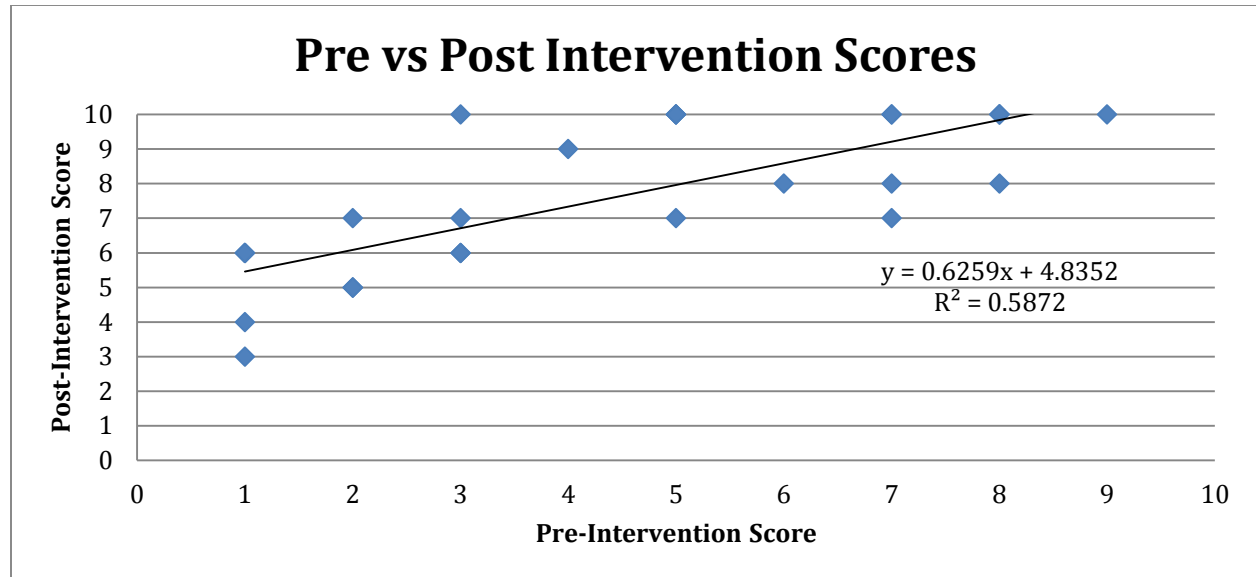


Figure 10. Correlation of Pre and Post intervention scores.

There is a moderate linear relationship between pre- and post-intervention scores ($R^2 = 0.587$), suggesting that all students would be expected to make an improvement in their test scores with this intervention.

Action Plan Moving Forward

The purpose of this action research was to study the impact of building *number sense* into daily math activities that supported learners in a Montessori environment. The goal was to have students build mental fluency and flexibility in their mathematical work to support their

understanding. Using the CRA methodology throughout the 25-day intervention cycle built a conceptual scaffold and supported the student's work with Montessori materials and showed a statistically significant gain (See Figure 10).

One component of my research that I would like to explore further is a replication of this study over an extended period of time. By conducting a study that focuses on a shorter period, I believe that I was able to study the effects of building *number sense* within the classroom and isolate outside influences and typical growth of a lower elementary aged child. However, after conducting my final assessment, the main observation made was the continued use of strategies by many of the students in their daily math following the intervention period. While each group made gains, Groups B and C students seemed to make additional gains after completion of the formal study. Additionally, observation proves that students who were not able to mentally process addition and subtraction problems during the formal study period are now successfully using those skills and strategies. The use of the strategies seemed to accelerate the growth of students throughout the room in the areas of addition and subtraction. Retesting at this point in the school year (2 months later) might show an increased understanding with more students scoring a 10. Also, replicating this work to a larger group of students may show statistically significant results, as I believe the sample sizes of the initial test group were too small to examine that element thoroughly.

Another area that I plan to pursue is to incorporate this data into the Montessori classroom by providing works and choices that allow the students to use these strategies when solving math. The absence of these works is reflected in the CRA model. Many students at the focus school moved from concrete materials to abstract work without having strategies for building representational models. When asked to apply knowledge, often abstractly in new

mediums, students at the focus school were not able to do so. I would like to create new materials that would support students and allow them to use the CRA model in their daily lessons. When thinking about one of my topical questions, “What is the best way to introduce and practice representational mathematics in a Montessori classroom?” the answer involves creating a visual for students as well as the Montessori materials. When teaching a lesson that involves adding and subtracting, supplementing that lesson with a number line as well as the abstract numbers would be beneficial for the students to conceptualize the concept.

Lastly, I believe that sharing this data with the Montessori community, families of students, and the city school district would be beneficial in developing strategies to support Montessori students. Current data show that Montessori students perform lower than their peers in the area of mathematics. This study may help to provide support for students and families. This information aligns with the work that the city school district uses in some of its assessments and interventions: In traditional schools, the city school district has been using methods closely tied to the CRA model for several years and is seeing the benefits for students. The students that worked with me in the area of mathematics outperformed their peers on the state standards test. The students in the subject school for third grade during the 2014-2015 were 50% proficient on the state math test. The students that worked with me and had representational math built into their work were 71% proficient over the same period of time. Currently, third graders at the test school are projected to score about 50% for the 2015-2016 school year; however, students that have worked with this model have scored 70% proficiency for the same period.

Dr. Maria Montessori knew that working with your hands and practicing mathematics concretely is the most essential scaffold that we could provide children. While Maria Montessori’s methods were revolutionary in that they moved the focus of education beyond a

particular discipline, she focused towards the development, efforts, and abilities of children.

Maria Montessori observed a natural desire in children toward Mathematics. However, her observations were not done with pencil and paper, but through applied science. She could only observe this if she thought of mathematics differently. It is with that same sense of discovery that I have undertaken this research for the development of children.

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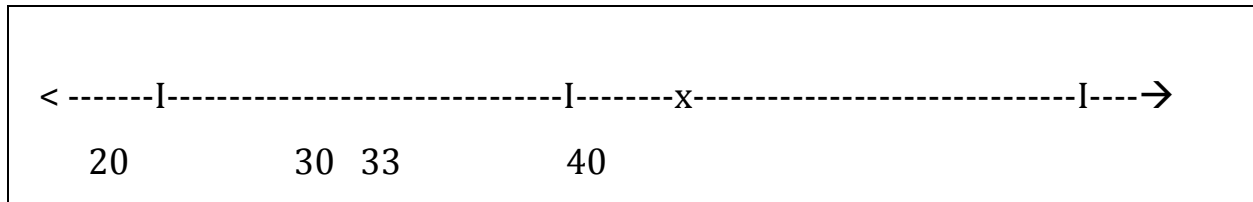
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Appendix A

Name: _____

Directions: Write the correct answer to each question. YOU NEED TO SHOW YOUR THINKING.

EXAMPLE: Is 33 Closer to 20 or to 40?**1. Is 78 closer to 30 or to 100?****2. Is 17 closer to -5 or 50?****3. Is 192 closer to 100 or 250?****4. Is 456 Closer to 300 or 700?**

Name: _____

10 Less	15	10 more
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10 Less	93	10 More
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	19	
--	----	--

	41	
--	----	--

	45	
--	----	--

	55	
--	----	--

	33	
--	----	--

	16	
--	----	--

$$44 + 10 =$$

$$78 = 10 + \underline{\quad}$$

$$38 + 10 =$$

$$99 + 10 =$$

$$13 + 9 =$$

$$14 + 7 =$$

$$11 + 7 =$$

$$15 + 8 =$$

Name: _____

Write 10 Names for each number in the boxes below.

1.

2.

3.

4.

5.

6.

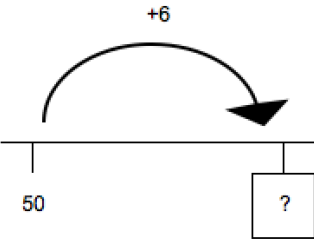
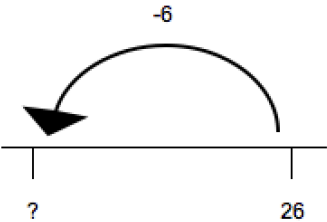

7.


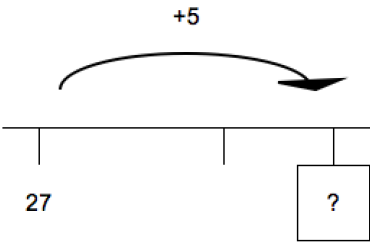
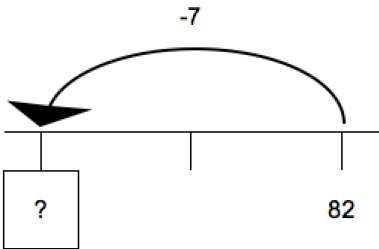
8.

9.

Student Number: _____	Date: _____
Score: _____	Pre _____ Post _____

Assessment adapted with permission from on the work *Conceptual Place Value* by Stephen Thoenke

Task 1 - Jumping forward from a decade number using a single digit number without counting	
Question for Student: Starting at 50 and jump forward 6 gets you to which number?	
	Correct Incorrect
Task 2 - Jumping back to a decade number using a single digit number without counting	
Question for Student: Starting at 26 and jumping back 6 gets you to which number?	
	Correct Incorrect
Task 3- Name the next decade number and jump to it without counting	
Question for Student: Starting at 77, and jumping to the next decade is how large a jump?	
	Correct Incorrect

<p>Task 4 - Jump back from a decade number without counting.</p>	
<p>Question for Student: Starting at 40 and jumping back 4 gets you to which number?</p>	
	<p>Correct</p> <p>Incorrect</p>
<p>Task 5 - Jump forward through a decade number to add a single digit number without counting.</p>	
<p>Question for Student: Starting at 27 and jump forward 5 gets you to which number?</p>	
	<p>Correct</p> <p>Incorrect</p>
<p>Task 6 – Jump through a decade number to subtract a single digit number without counting.</p>	
<p>Question for Student: Starting at 82 and jump back 7 gets you to which number?</p>	
	<p>Correct</p> <p>Incorrect</p>

Task 7 – Finding the difference.	
Question for Student: Please solve this problem.	
$20 + \square = 56$	<p>Correct</p> <p>Incorrect</p>
Task 8 – Finding the difference.	
Question for Student: Please solve this problem.	
$87 - \square = 70$	<p>Correct</p> <p>Incorrect</p>
Task 9 - Mental strategies for added two double-digit numbers	
Question for Student: Please solve this problem.	
$29 + 35$	<p>Correct</p> <p>Incorrect</p>
Task 10 - Mental strategies for subtracting two double-digit numbers.	
Question for Student: Please solve this problem.	
$84 - 37$	<p>Correct</p> <p>Incorrect</p>

Appendix C

SUMMARY OUTPUT - Whole Class								
<i>Regression Statistics</i>								
Multiple R	0.766280351							
R Square	0.587185576							
Adjusted R Square	0.571308098							
Standard Error	1.429367401							
Observations	28							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	75.55820106	75.5582	36.98229538	1.99696E-06			
Residual	26	53.12037037	2.043091					
Total	27	128.6785714						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	4.835185185	0.529845129	9.125658	1.375E-09	3.746072925	5.924297445	3.746072925	5.924297445
X Variable 1	0.625925926	0.102926217	6.081307	1.99696E-06	0.414358058	0.837493794	0.414358058	0.837493794

SUMMARY OUTPUT - Group A								
<i>Regression Statistics</i>								
Multiple R	0.39223227							
R Square	0.153846154							
Adjusted R Square	0.032967033							
Standard Error	1.181873681							
Observations	9							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	1.777778	1.777778	1.272727	0.296432925			
Residual	7	9.777778	1.396825					
Total	8	11.55556						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	4.111111111	4.547612	0.904015	0.396032	-6.6422828	14.86450502	-6.6422828	14.86450502

X Variable 1	0.66666667	0.590937	1.128152	0.296433	-0.730676917	2.06401025	-0.730676917	2.06401025
SUMMARY OUTPUT - Group B								
<i>Regression Statistics</i>								
Multiple R	0.425287356							
R Square	0.180869335							
Adjusted R Square	0.078478002							
Standard Error	1.634751906							
Observations	10							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	4.720689655	4.720689655	1.766451613	0.220475474			
Residual	8	21.37931034	2.672413793					
Total	9	26.1						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	5.620689655	2.081142328	2.700771389	0.027041714	0.821566841	10.41981247	0.821566841	10.41981247
X Variable 1	0.637931034	0.479979686	1.329079235	0.220475474	-0.468904107	1.744766176	-0.468904107	1.744766176

SUMMARY OUTPUT – Group C								
<i>Regression Statistics</i>								
Multiple R	0.219264505							
R Square	0.048076923							
Adjusted R Square	-0.087912088							
Standard Error	1.253566341							
Observations	9							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	0.555555556	0.555555556	0.353535354	0.570828987			
Residual	7	11	1.571428571					
Total	8	11.55555556						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	4.5	1.284523258	3.503245249	0.009949452	1.462585153	7.537414847	1.462585153	7.537414847
X Variable 1	0.5	0.840917866	0.59458839	0.570828987	-1.488454779	2.488454779	-1.488454779	2.488454779