

ALGEBRA FOR EVERY EIGHTH GRADE STUDENT:

ONE SIZE FITS ALL?

by

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“Algebra For Every Eighth Grade Student: One Size Fits All?”

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The purpose of this paper is to investigate the reasons why Minnesota enacted legislation in 2007 requiring the enrollment in and successful completion of algebra 1 by every eighth grade student, and how, specifically, one school district implemented changes to meet this requirement. From the “new math” of the 1960’s to standards-based reform of the 21st century, United States public school students have been subjected to various trends in education; yet when compared to other countries, standardized test scores have fallen. How can this trend be reversed? The U.S. federal government has imposed legislation to require all students to demonstrate proficiency as measured on annual high-stakes testing. Each state has been challenged to increase the rigor in math as public schools across the country are working to meet the ever-increasing proficiency targets laid out in No Child Left Behind. Algebra 1 content, in both the Minnesota academic standards and the Common Core math standards, is expected to be mastered by students at an earlier age than ever before. How can students achieve this goal?

In this paper you will learn how the administration and math department at the middle school in Hastings, Minnesota, began increasing the rigor for students with new curriculum and teacher staff development. They recognized the success of a program dedicated to the lowest scoring 8th grade math students. This required a change in the math scheduling model and allocated fifty percent more time for students in math class. Teachers used the time for more instruction, one-on-one guidance, hands-on activities, technology exploration, and discovery lessons. The results, as measured by the state’s annual assessment, showed a noticeable increase in proficiency for Hastings eighth graders while the overall state average decreased from the previous year. These results and scheduling model may be helpful to districts interested in increasing student proficiency rates.

Introduction

Algebra 1 is considered to be a pivotal course in a student's mathematical study. In the 1960's and 1970's, Algebra 1 was typically taken in the 9th grade. Those who were successful moved on to study geometry and higher levels of math, continuing on a college preparatory track. For those who were unsuccessful, they either repeated the course, took lower levels of mathematics (i.e. "consumer math"), or simply ended their study of mathematics in high school. It was considered to be an "effective filter to separate the college-bound students from their work-bound classmates". (Steen, 1999) Beginning in the last part of the 20th century, Algebra 1 became a course available to higher-level students in 8th grade, allowing them to reach the study of calculus in high school. As expectations and rigor have increased, state standards have followed this trend embedding more algebra 1 topics at the 8th grade level. In fact, a number of states (such as California and Minnesota) have mandated legislatively that the successful completion of Algebra 1 be required for all students in 8th grade. Why has this "pushing down" of algebra become so important? Is this mandate what is best for all students? Is this level of study appropriate for all students? Is the successful completion of algebra 1 in eighth grade possible for all students, or is this a goal which is statistically impossible to attain? The purpose of this investigation is to examine the reasons behind requiring algebra 1 at the eighth grade level, how this drive for rigor at earlier ages has developed throughout the 20th century to today, the role of policy set at the national and state levels, and how the Hastings, Minnesota district has devoted extra time and resources to meet this challenge.

Why is the eighth grade deemed so important? Although much focus and attention is given to high school data, test scores, graduation rates, etc., more and more attention has recently been given to education and proficiency in the middle grades. While it may seem obvious that a

student's success in later years (high school and beyond) is built upon prior success, a report released in 2008 by the ACT organization shows the correlation between student performance in the middle grades and future college and career readiness. "Our research shows that, under current conditions, the level of academic achievement that students attain by eighth grade has a larger impact on their college and career readiness by the time they graduate from high school than anything that happens academically in high school" (ACT, 2008). The report further reinforces the idea that trying to increase the rigor at the high school level will not be successful unless the rigor at earlier grades is increased beforehand. This scaffolding approach must start in the primary grades and build from there to ensure students have a solid foundation upon which to build successfully (ACT, 2008). This is true in all states, and especially in Minnesota, where this paper focuses special attention.

History

During the post-World War II era, the United States experienced an unprecedented growth in its standard of living. The baby boom era had begun, and the cold war helped fuel a competitive spirit to be the best in the world in math, science, and technology for the preservation and defense of the "free world". The National Science Foundation was created in 1950. Its purpose was "to promote the progress of science; to advance the national health, prosperity, and welfare; to secure the national defense..." (NSF, 2011); however, the launch of the Soviet Union's Sputnik satellite in 1957 prompted America to quickly realize that it must continue to work hard to compete internationally. Also in 1957, the NSF expanded to examine and promote change in secondary math, biology, chemistry, and social sciences and to address the growing need for more science and math educators (NSF, 1956). The space race coupled with the cold war further fueled more competition, especially with the Soviet Union. The United

Stated educational system came under intense scrutiny. NASA was created by Congress in July of 1958. The National Defense Education Act was signed into law in September of 1958, providing funding for national defense fellowships and to provide financial assistance to the ever-growing population of college students (Kosar, 2011). Prior to the U.S. involvement in World War II, approximately 30.7% of high school graduates enrolled in college. By 1960, approximately 45.1% of the nation's high school graduates enrolled in college. (Young, 1982). [To give some perspective, in the fall of 2010 that percentage had soared to 68.1%. (US Dept. of Labor, Bureau of Labor & Statistics, 2011).] As more students enrolled in college and as more emphasis and need for technical careers grew, more attention was placed on K-12 education.

Trends and “new” theories and methodologies began to influence educational practices as the U.S. sought to remain an international leader. The National Defense Education Act of 1958 also provided funding to states to improve the teaching of math, science, and modern languages (i.e. Russian). During the 1960's the focus of education was on the content being taught, not the skill or performance of the students. “New math” was thrust into the curriculum, with more attention being devoted to set theory, number theory, modular arithmetic, and more abstract concepts. Schools built in the 1970's often had open classrooms, and the trends in education became more student-centered: choice, inclusiveness, innovation, active learning, holistic learning, and self-motivated/self-paced learning were common threads in instruction. By the mid- to late 1970's, standardized test scores began to decline and math textbooks returned to traditional content and methods. The back-to-basics movement took hold and more attention was given to not just content but student learning.

In April 1983, the National Commission on Excellence in Education released the report, “A Nation at Risk”. It noted that the United States was, “nation at risk ... [whose] educational

foundations ... are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people” (The National Commission on Excellence in Education, 1983). The report suggested that America’s young people were not learning enough nor demonstrating significant academic achievement (The National Commission on Excellence in Education, 1983) . Significant findings in the study were quite disturbing and sparked the need for further change in education. At that time, the achievement levels by high school students on standardized tests were lower than 26 years prior (around the time that Sputnik was launched). Scores on the SAT declined from 1963 to 1983, with verbal scores dropping 50 points and math scores dropping 40 points. From 1975 to 1980, colleges had to increase the amount of remedial math courses by 72%. These deficiencies in education were occurring at a time when the need for more highly skilled workers and workers in new fields were rapidly increasing (The National Commission on Excellence in Education, 1983). All of these factors led to a shift in how to measure academic quality.

Instead of focusing on the inputs of education (resources, curriculum, spending), the outputs (goals, products and results) became more important. Outcome-based learning and emphasis on learner objectives became popular in the 1980’s. One of the voices for such reform in the 1980’s was William Spady. While speaking against the confines of time, nine months at a time, he proposed the concept of outcome-based education. (Spady, 1988). This model of education reform, focusing on learner outcomes, is a student-centered learning style with more progressive types of instruction, such as block scheduling, reform math, project based learning, etc. Students must demonstrate mastery of an objective in order to move ahead, and if such mastery was not passed, the student would not be promoted. Outcome based education was popular in the 1980’s and 1990’s and was also referred to as mastery education or performance-

based education (Spady, 1988), and as a result many states began to implement outcome-based types of programs.

In Minnesota, for instance, the effort to establish an outcome-based approach to student learning began in 1972. The state Department of Education began developing Some Essential Learner Outcomes (SELOS) which specified the content students would be taught. In the aftermath of the “Nation at Risk” report, the Minnesota Legislature requested a report on education. The Commissioner of Education, Ruth Randall, introduced proposals considered to be quite radical at the time. (Manno, 1995) First, she proposed changing the graduation rule from traditional “seat time” graduation standards with measurable learner outcomes. She also asked for the creation of state achievement tests to measure individual student learning of these outcomes. The legislature was not satisfied with Commissioner Randall’s report, and as a result little action was taken. In 1984 the Minnesota Business Partnership produced a report which called for a major reorganization of K-12 education. The recommendations included that all students master certain common competencies and that the state develop uniform achievement tests to measure whether students had attained them, very similar to Commissioner Randall’s recommendations. After finally agreeing that the outcome-based approach would be required, people from various roles (the governor, the legislature, the Department of Education, and many others) began work on determining such outcomes. In 1991, the Minnesota State Board of Education released its Outcome-Based Graduation Rule, including the following proposed outcomes:

“In order to lead productive, fulfilling lives in a complex and challenging society and to continue learning, the graduate demonstrates the knowledge, skills, and attitudes essential to:

- Communicate with words, numbers, visuals, symbols and sounds;

- Think and solve problems to meet personal, social and academic needs;
- Contribute as a citizen in local, state, national and global communities;
- Understand diversity and the interdependence of people;
- Work cooperatively in groups and independently;
- Develop physical and emotional well-being;
- Contribute to the economic well-being of society.” (Manno, 1995)

There was strong opposition to these outcomes by several organizations, including parent groups, because some of the outcomes described values and attitudes. It was not clear and specific as to what some of these objectives meant, how they would be taught in a public school setting, nor how they would be evaluated and measured by teachers. More discussion and work followed. The opposition and lack of clarity in the objectives prompted then Governor Arne Carlson in 1993 to recommend delaying the implementation. By late 1993, the Minnesota Department of Education (then known as the Department of Children, Families, and Learning) brought forth a plan known as the Profile of Learning. It included thirteen competencies to be met for high school graduation:

- Comprehends, interprets, and evaluates information received through reading, listening, and viewing;
- Uses strategies to understand and apply information from technical reading, such as manuals and research documents;
- Writes and speaks clearly for academic, technical, and personal purposes with a variety of audiences;
- Analyzes patterns and functional relationships in order to solve problems and determine cause/effect relationships;
- Applies data handling and measurement techniques to solve problems and justify conclusions;
- Applies methods of inquiry needed to conduct research, draw conclusions, and communicate and apply findings;
- Understands the past and continuous development of societies and cultures in human history;
- Understands how principles of interaction and interdependence affect physical and social situations;
- Applies informed decision-making processes to promote healthy lifestyles, social well-being, and stewardship of the environment;
- Understands the processes and meaning of artistic expression;
- Understands application of technological systems;

- Understands the effective management of resources in a household, business, community, and government; and
- Communicates using a language other than English.

(MN House of Representatives, 1998)

Again, more debate ensued and more revisions followed, until 1998 when a two-fold plan for graduation requirements was finally adopted. The first part was the Basic Standards Tests: one in reading (testing the ability to read at a degree of difficulty equal to popular adult nonfiction; students tested in 8th grade), one in writing (testing the ability to write a composition in response to a request for information; students tested in 10th grade), and in math (testing for understanding through pre-algebra; students tested in 8th grade). The second part was the Profile of Learning, which had been modified to include ten broad learning areas:

- | | |
|------------------------------|---|
| 1. Read, view, and listen | 6. Scientific applications |
| 2. Write and speak | 7. People and cultures |
| 3. Literature and the arts | 8. Decision making |
| 4. Mathematical applications | 9. Resource management |
| 5. Inquiry | 10. World languages (optional for students) |
- (Manno, 1995)

Within these areas, certain content standards are given (56 standards in grades K-8, 48 standards in grades 9-12). To graduate from high school, students were required to complete:

- complete one content standard within each group or cluster;
- complete two standards within learning area 6;
- complete at least 21 of 47 standards in learning areas 1 to 9;
- complete at least three other content standards as electives; and
- use computer technology in completing a required content standard in learning areas 2, 4, 5, and 6. (Larson, 1998)

In theory, the concept of setting high, rigorous expectations that are assessed and measured in order to demonstrate learning is a good, positive policy that should benefit students, teachers, and families alike. So why has this approach been changed and/or completely abandoned?

Other countries, such as Australia, South Africa, and Hong Kong, implemented OBE models, but have since dropped these methods. What led to turning away from this reform? Minnesota's objectives, like others, were worded in such a broad, vague manner, which were difficult for consistent implementation across districts. Assessments were difficult to measure and these outcomes did not definitively seem to raise student achievement (Manno, 1995).

In the mid 1980's, the National Governor's Association devoted 12 months to study the issue of education. The driving force was that better schools will produce more educated workers in order to compete with workers around the world (National Governor's Association, 1986). Later, in 1989, President Bush and the nation's governors met at an Education Summit held in Charlottesville, Virginia. The goal was to develop ambitious, yet realistic goals for the performance of all students. To do this the country must have a common mission. The following six goals for the year 2000 were set:

- All children will start school ready to learn
- at least 90 percent of all students will graduate from high school
- all students will demonstrate competence in challenging subject matter
- U.S. students will be first in the world in math and science
- every adult will be literate
- every school will be safe and drug-free. (NY Times, 1989)

Also in 1989, the National Council of Teachers of Mathematics published Curriculum and Evaluation Standards for School Mathematics. This laid the groundwork or foundation for states to model their own standards. The United States, unlike many countries, did not (and still does not) have a national curriculum for education. Therefore, education is a function of each of the states. This first publication was published with the intent that states would use it as a guide in the development of their own standards. (NCTM, 1989).

During the 1990's, developing standards and the standards-based reform movement took hold. In 1990, only 1 in 6 eighth grade students were enrolled in algebra 1 (Loveless, 2008). When President Clinton took office, his administration made it a national goal to get 8th graders enrolled in algebra. The feeling was that the U.S. was falling behind other countries, and setting this goal would begin to close that gap (Loveless 2008). In 1996, Washington D.C. schools led the nation with 53% of its 8th graders enrolled in algebra. From 1990 to 2000, enrollment in algebra by 8th graders increased from 16% to 24% nationally. By 2007, the national rate had grown to 31%. Some of this push for algebra seemed to be motivated by equity issues. (Loveless, 2008). Robert Moses, civil rights leader, teacher, and founder of The Algebra Project (a foundation dedicated to improving minority education in math), suggested that algebra should be considered “the new civil right”. The study of algebra meant access; it represented open doors to productive careers and access to big ideas (Moses, 1995). This prompted more attention and action to improve student achievement in mathematics.

In November of 1998, Richard Riley, then U.S. Secretary of Education said, “Today’s students must master high-level mathematical concepts and complex approaches to solving problems to be prepared for college, careers of the 21st century and the demands of everyday life.” Shortly after this, Secretary Riley appointed a 25 member commission on the teaching of math and science for the 21st century. This commission, chaired by former astronaut and senator John Glenn, was charged with the task “to investigate and report on the quality of mathematics and science teaching in the nation, directing us to consider ways of improving recruitment, preparation, retention, and professional growth for mathematics and science teachers in K–12 classrooms nationwide. You reminded us that, three decades after a historic achievement, ‘we need to set the stage for advancement in mathematics and science for the next thirty years.’ ”

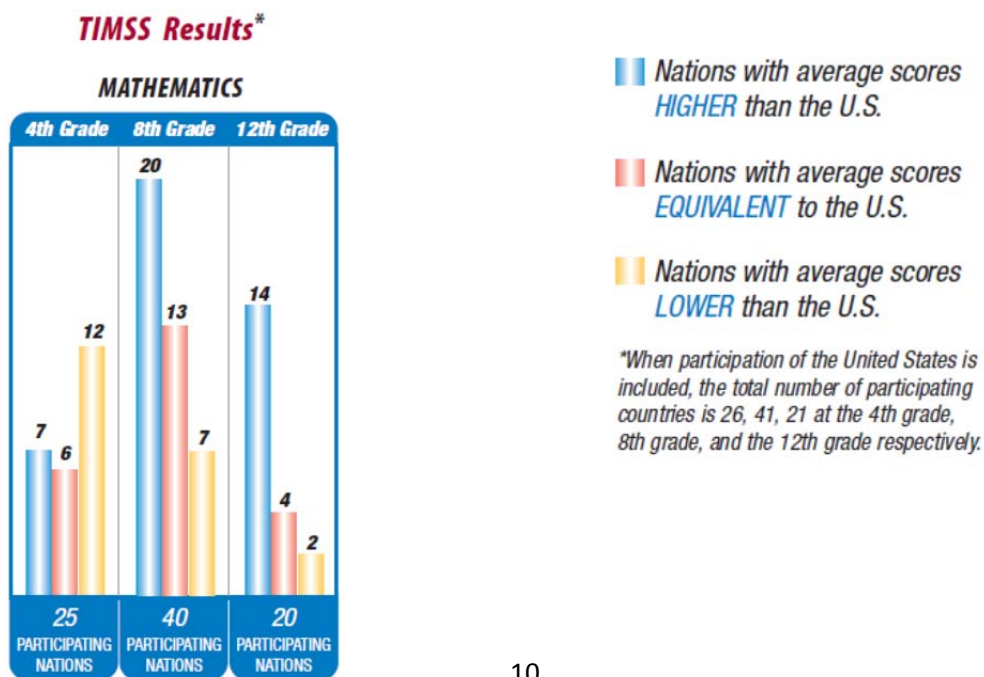
(U.S. Dept. of Education, 2000). The commission identified four reasons that children in the U.S. should achieve competency in math and science:

- (1) the rapid pace of change in both the increasingly interdependent global economy and in the American workplace demands widespread mathematics- and science-related knowledge and abilities;
 - (2) our citizens need both mathematics and science for their everyday decision-making;
 - (3) mathematics and science are inextricably linked to the nation's security interests; and
 - (4) the deeper, intrinsic value of mathematical and scientific knowledge shapes and defines our common life, history, and culture.
- (U.S. Dept. of Education, 2000)

It further identified five factors which made it the perfect time to enact change:

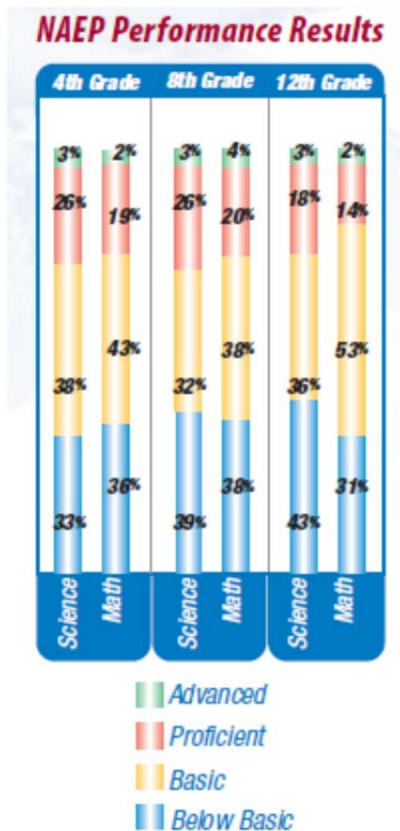
- (1) reform efforts have sharply focused the attention of the American people on education as a public issue;
 - (2) the nation now has a surplus of resources to invest in education;
 - (3) a coming demographic shift in the teaching force—two thirds of which will be retiring in the next decade—offers an unparalleled chance to plan for and make changes at the core of education itself;
 - (4) our schools can now put to work what educators have learned in the past generation about curriculum, high standards, effective teaching, assessment, and how children learn; and
 - (5) the rising generation of college graduates is once again showing an interest in teaching as a profession.
- (U.S. Dept. of Education, 2000)

The main message from the report was that American students must perform better in order to compete in a global economy. Analyzing the data from TIMSS (the Third International Math and Science Study, 1995), students from other countries were still outperforming their American peers.



(U.S. Dept. of Education, 2000)

The NAEP (National Assessment of Educational Progress) results from 1996 corroborated the lack of improvement in American student performance. Periodically, students are assessed in math and science by NAEP and rated in one of four categories: Below Basic, Basic, Proficient, or Advanced. Less than one third of the students tested were proficient, and more than one third of the students were below the basic level.



Science Results Reported in "What Do Students Know?": Results for 4th, 8th & 12th Grade Students, National Center for Educational Statistics, www.nagb.org/pubs/students/nation.html

NAEP 1996 Mathematics: Report Card for The Nation and the States, Washington D.C.: National Center for Educational Statistics, February 1997, p. 59

The commission's solution to improve student performance is to improve math and science teaching. Three goals were formulated:

Goal 1: Establish an ongoing system to improve the quality of mathematics and science teaching in grades K–12.

Goal 2: Increase significantly the number of mathematics and science teachers and improve the quality of their preparation.

Goal 3: Improve the working environment and make the teaching profession more attractive for K–12 mathematics and science teachers.

(U.S. Dept. of Education, 2000)

In the same year that this report was released, the NCTM released the Principles and Standards for School

Mathematics. This publication reflected the efforts of the

NCTM as an ongoing process. As the new millennium

began, the NCTM recognized the need for mathematics in a

changing world. Everyone needs to be able to understand and use mathematics in everyday life, in the workplace, and in the scientific and technical communities. "In this changing world, those

who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures.” (NCTM, 2000).

The standards-based reform movement of the 1990’s took a giant step when Congress re-authorized the Elementary and Secondary Education Act in 2001. Known now as the No Child Left Behind Act (NCLB), this act now required that all states test students annually in reading and math, and that school must demonstrate that students are making adequate yearly progress (AYP) in order to avoid sanctions. Each state is responsible for setting and implementing its own standards and assessments. By the year 2014, all students are expected to be proficient in reading and math, as measured on each state’s version of its reading and math assessments.

One of the unfortunate outcomes from NCLB is that some states have enacted more rigorous standards than others. It is difficult to compare one state’s proficiency rates to another because of this. Some states may show a higher proficiency rate in passing their own proficiency tests—are those students truly more knowledgeable or is this because their states’ standards are lower, making it easier for students to pass test? There exist some large discrepancies in student proficiency between the states; one of the ways to illustrate this is by using the Northwest Evaluation Association’s Scale Alignment studies for the states that use NWEA’s Measure of Academic Progress testing products in their schools.

“NWEA has conducted Scale Alignment studies to link the RIT scale to the proficiency levels from many state assessments. Each alignment study identifies the specific Rasch Unit (RIT) scale scores from MAP assessments that correspond to the various proficiency levels for each subject (reading, mathematics, etc.) and for each student grade.

Alignment studies also estimate the probability that a student with a specific RIT score will achieve a status of proficient or better on her or his state test. Because all states set their own standards for proficiency and may use different tests for measuring student achievement, alignment studies are usually necessary for each state.

When states use this information with NWEA assessments, member schools can monitor their students’ progress toward the proficiency standards for their states. This has been an important step toward helping our schools and districts to meet the

No Child Left Behind legislation. School districts can get copies of their state alignment studies from the [research section](#) of the NWEA web site and use the data for planning of curricular and instructional improvements.” (NWEA, 2007-2011)

The recommended fall MAP test scores to be proficient on state assessments in math in grade 8 vary greatly, from lows of 211 in Georgia and 213 in Maryland to 237 in Massachusetts and 235 in California (see table below).

State	RIT score	State	RIT score	State	RIT score
AZ	227	KY	226	NC	225
AR	225	ME	231	ND	225
CA	235	MD	213	OH	222
CO	233	MA	237	OR	224
DE	224	MI	222	PA	223
FL	222	MN	231	RI	231
GA	211	MT	234	SC	227
ID	229	NV	225	UT	226
IL	215	NH	231	VT	231
IN	224	NJ	224	WA	233
IA	225	NM	231	WI	217
KS	225	NY	225	WY	231

(NWEA, 2007-2011)

From the 2011 normative data compiled by NWEA, the mean RIT score for an eighth grade student in the fall of the year is 230.2. Clearly, a number of states are setting proficiency targets far below this mean score.

2011 MATHEMATICS STATUS NORMS (RIT VALUES)			
Grade	Beginning-of-Year Mean	Middle-of-Year Mean	End-of-Year Mean
K	143.7	150.5	156.1
1	162.8	172.4	179.0
2	178.2	185.5	191.3
3	192.1	198.5	203.1
4	203.8	208.7	212.5
5	212.9	217.8	221.0
6	219.6	222.8	225.6
7	225.6	228.2	230.5
8	230.2	232.8	234.5
9	233.8	234.9	236.0
10	234.2	235.5	236.6
11	236.0	237.2	238.3

(NWEA, 2011)

The expectation of the states, as put forth in the NCLB legislation, was that states would set high academic standards. With such variation on the interpretation of where the level of proficiency should be set amongst the states, there became questions of why the U.S. does not have national academic standards.

Eighth Grade Focus

Adding to the need for common standards across the country was the importance of success in the middle grades. It is critical for students to be successful in the middle grades, especially by eighth grade, because it is an accurate predictor of future success— “...the level of academic achievement that students attain by eighth grade has a larger impact on their college and career readiness by the time they graduate from high school than anything that happens academically in high school.” (ACT, 2008). The findings emphasize that “...students’ academic readiness for college and career can be improved when students develop behaviors in the upper elementary grades and in middle school that are known to contribute to successful academic performance.” (ACT, 2008)

California’s legislature was one of the first to mandate algebra 1 testing for all eighth graders in July of 2008 (Minnesota’s legislature passed the requirement in 2007 that eighth

graders successfully complete algebra 1 beginning in 2010-2011). The California mandate was blocked when it came before the appeals court in December of that year, because the process used by the State Board of Education did not adequately inform the public. During this time, much discussion took place in education circles whether “algebra for all” at the eighth grade level is appropriate. While a push for higher standards and equity among all students is noble and good, care must be taken to determine if every student is ready. Many students are still unprepared as they enter eighth grade—they must first have mastered basic skills—in order to be successful in algebra (McKibben, 2009). Steps must be taken to ensure that teachers are adequately trained to teach algebra 1—a study from California in 2008 found that of the 3,790 teachers in middle school teaching algebra, one third are either underprepared to teach the subject or are teaching out of their licensure area/field (Guha, 2008). (At Hastings Middle School, Hastings, Minnesota, all 7th and 8th grade math teachers are considered “highly qualified”, holding licensure in either grade 7-12 mathematics (six of eight teachers) or grades 1-6 with a grade 5-8 mathematics specialty (two of eight teachers)). With the continued idea of pushing algebra down to the 8th grade level for more and more students (if not all), this study highlighted the need to recruit, prepare, retain and support middle school teachers as well as teachers in earlier grades so that students are well prepared for algebra 1 and subsequent higher math classes (Guha, 2008).

The public schools in Norfolk, VA, have been dedicated since 2000-2001 to increase the number of students in algebra 1 in middle school through a project called “Algebra for All”. Though not mandated by law, their goal is to close the achievement gap, provide higher quality mathematics learning at earlier ages, and to have all students master algebra 1 prior to high school, as measured by proficiency on the statewide end-of-course algebra 1 exam. To achieve

this, they integrated algebra content in all grades (pre K-7) prior to grade 8, increasing (doubling) class time in grades 6 and 7, and focusing more professional development and training for teachers. (Paek, 2008, Algebra for All). Enrollment in the algebra 1 course has increased and at the same time, the percentage of students passing the course and the state's end-of-course exam has increased from 41% to 69% (Paek, 2008, Raising Student Achievement).

Minnesota

The Minnesota legislature passed an algebra requirement in 2007 for all eighth graders. It states that all eighth graders must complete an algebra 1 credit by the end of eighth grade, to be implemented by the 2010-2011 school year (Larson, 2010). This requirement was timed concurrently with the adoption of the 2007 mathematics standards, which would first be assessed on the Minnesota Comprehensive Assessment III (MCA III) in the spring of 2011. The 2007 standards for eighth grade focus heavily on the algebra strand and primarily on linear equations, slope/rate of change, and systems of linear equations with the ability to represent them graphically, algebraically, and numerically (see appendix 1). Quadratic functions and other traditional non-linear topics typically included in an algebra 1 textbook are not included in the eighth grade standards; they are included later, at the high school level. Therefore, in order to comply legally, a number of middle schools in Minnesota have created an algebra course which satisfies the legislative requirement, incorporates the standards for grade 8, and covers the linear topics/chapters often found in the first half of a traditional algebra 1 textbook.

Shortly after the passage of the Minnesota and California laws requiring algebra 1 for eighth graders, the National Council of Teachers of Mathematics (NCTM) published a position paper on this same issue. Released in the fall of 2008, the paper explains that algebraic concepts and skills should be a focus in the entire pre K-12 curriculum and that all students must have

access to learning algebra, but the timing of learning algebra is critical. “Only when students exhibit demonstrable success with prerequisite skills—not at a prescribed grade level—should they focus explicitly and extensively on algebra, whether in a course titled Algebra 1 or whether any integrated mathematics curriculum. Exposing students to such coursework before they are ready often leads to frustration, failure, and negative attitudes towards mathematics and learning.” (NCTM, 2008). Despite this position, the states must comply with the grade specific standards and tests of No Child Left Behind.

When the Elementary and Secondary Education Act of 1965 was originally passed, it prohibited the establishment of a national curriculum. Since the act was reauthorized in 2001 as NCLB, and as states have each been setting their own standards and assessments, there has been a push for some type of national academic standards in math and reading. In 2009, the National Governor’s Association and the Council of Chief State School Officers, in partnership with Achieve, Inc., ACT, and the College Board, worked to lead the Common Core State Standards Initiative. The goal was to develop a set of standards in English/Language Arts and Mathematics that the states can voluntarily adopt. Each state may also add to these standards, as long as the core standards comprise at least 85% of the state’s standards (National Governor’s Association, 2009). In June 2010, the Common Core State Academic Standards in English-Language Arts and Mathematics were released for adoption by the states (see appendix 2). To date, 45 of 50 states and 3 of 6 territories/districts have adopted these standards. Minnesota has chosen to adopt the English-Language Arts standards but not the Mathematics standards. Initially, it was reported that then Governor Tim Pawlenty and Education Commissioner Alice Seagren felt the Common Core standards in mathematics were not as rigorous as the 2007 Minnesota standards. (Stacey, 2010). The Minnesota legislature has also enacted specific timetables for the revision

of academic standards. The mathematics standards are not set to be revised again until 2015 by the Commissioner of the Department of Education. The commissioner does not have the authority to revise the math standards earlier unless further legislative action is taken. (MN Dept. of Education, November 2011).

District

At Hastings Middle School in Hastings, Minnesota, the process of ramping up the rigor began in earnest in school year 2004-2005. New statewide math standards had been adopted in 2003, and a concerted effort was made to target the lowest performing students in grade 8. At this time, the students in 8th grade took the Minnesota Basic Skills Test in Mathematics, which was the test required for high school graduation. Several budget cuts had been made in order to improve the district's financial situation; certain electives and other programming had been eliminated (The district was working its way out of statutory operating debt.). Many class sizes were quite large (eighth grade pre algebra sections numbered in the upper 30's per class and some sections of algebra 1 above 40 students per class). Despite this, class sizes for the lowest performing math students were intentionally held below 20 students, and an increase of student contact time given to improve math performance. The school operates on a day 1, day 2 schedule (a modified block schedule with certain classes meeting every other day). Math classes for all grades meet for one class period every day; however, the lowest performing students in 8th grade were scheduled with an extra class period every other day in order to give teachers more instructional time for discovery, exploration, lab activities, and more in depth study of topics. The course was named (MS)², "ms squared" meaning *Middle School Math Success*. Teachers were given extra staff development time to create and implement a variety of additional hands-on activities, technology labs, and real world activities, explorations, and projects to enrich students'

learning experiences. The results for all students, as measured in 2005 by the Minnesota BST, were noticeable.

8 th Grade Data	Hastings Percent Proficient	Hastings Average Score	State Percent Proficient	State Average Score
2003 MN Basic Skills Test (BST)	74.6%	629.1	71.7%	629.7
2004 MN Basic Skills Test (BST)	69.5%	623.8	70.7%	630.8
2005 MN Basic Skills Test (BST)	76.1%	631.3	74.3%	632.0

(MN Dept. of Education)

In 2006, the first MCA-II (Minnesota Comprehensive Assessments) tests were administered. This was the first year the new academic standards passed in 2003 were assessed. Because of the success students demonstrated, particularly the lowest achieving students, and the expectations set forth in NCLB for ever-increasing levels of proficiency amongst students, the school restructured its daily schedule to allow for the extra class time (referred to as *block-and-a-half scheduling*) for all 7th and 8th grade math classes and students. (Unfortunately, the schedule didn't allow for such restructuring at the 6th grade level.) In the fall of 2007, new curriculum was adopted and implemented from kindergarten through 12th grade. A significant amount of time was dedicated to plan and structure the progression of curriculum vertically K-12 for all learners. Although the data in the table below shows a dip in MCA-II scores in 2008 for both Hastings and statewide, the proficiency levels have made an overall increase in the past five years. The MCA-III tests were first administered in the spring of 2011 and assessed student performance based on the academic standards adopted in 2007.

8 th Grade Data	Hastings Percent Proficient	Hastings Average Score	State Percent Proficient	State Average Score
2006 MCA-II	53.86%	849.87	56.68%	850.81
2007 MCA-II	60.5%	851.4	58.7%	851.2
2008 MCA-II	56.0%	851.1	58.2%	851.3
2009 MCA-II	60.8%	853.3	59.6%	852.0
2010 MCA-II	60.5%	852.9	58.5%	851.6
2011 MCA-III	64.7%	854.0	53.1%	849.8

(MN Dept. of Education)

While not able to truly compare the 2010 results to the 2011 results (the standards changed and an entirely different test, MCA-II versus the MCA-III, was administered), Hastings eighth graders managed to increase the level of proficiency by 4.2 percentage points, while the state overall decreased by 5.4 percentage points. Because of such significant improvement in scores, the administration and teaching staff strongly advocated for the increased time schedule for all grades; as a result, every student in Hastings Middle School now receives this modified block schedule of time for math: 48 minutes one day, 96 minutes the next. The extra class time is intended to improve student learning and to enrich and deepen understanding of the math content. Teachers utilize the extra time in a variety of ways: providing additional practice, giving additional instruction, remediating as needed, enhancing and enriching learning through hands-on activities and technology-rich applications. A few examples of these lessons are provided in appendix 3. Because of the increased rigor in the new curriculum, each year's lessons and timeframes are modified to adjust for higher levels of prior learning. More professional development opportunities are available for teachers—the Minnesota Department of Education has initiated nine regional teacher academies for math and science, providing professional development and STEM initiatives in order to impact classroom instruction.

In order to comply with state law, Minnesota schools also had to ensure each 8th grade student was enrolled in an Algebra 1 course by the 2010-2011 school year. In order to move the

lowest achieving students up to this level, each school year from 2007-2008 required students in the lowest track of math to be advancing in rigor and curriculum. In the first years our lowest performing students were placed in a course 1 middle school textbook, the text used for our at-grade-level fifth grade students. Each year the lowest track's students progressed to the next highest level (course 2, course 3) until attaining Algebra 1. Today at Hastings Middle School, there are four levels/tracks of math in the 8th grade: Linear Algebra Essentials (students complete the first half of the textbook through lines and systems of linear equations with modifications and accommodations necessary for struggling learners and students with special education requirements), Linear Algebra (students complete the first half of the textbook through lines and systems of linear equations), Algebra 1 (students complete the entire textbook, linear and quadratics), and Honors Geometry (which qualifies for high school credit). These paths/tracks will ensure that during high school, students will complete an Algebra II credit (mandated by the state legislature by the year 2014-2015). It is important to note that while much emphasis and resources are placed on struggling learners, the gifted and talented learners are being provided with additional options too. For those students on the highest accelerated path, AP Calculus AB could be taken as a junior, with the option of taking AP Calculus BC as a senior. Students could also take AP Statistics anytime after completing Algebra II. The students with these options are currently sophomores (the class of 2014). The high school is also considering offering a College-in-the-Schools course in College Algebra, for those college-bound students who complete algebra II but may not be ready for pre-calculus or may not wish to take a pre-calculus or calculus course.

Conclusion

Since the launch of Sputnik, the educational system and teaching practices in the United States have been scrutinized. Over the decades, the emphasis has shifted from the inputs and resources to the outputs and student learning. This shift has led to the development and continued revision of state standards in mathematics. Federal legislation (No Child Left Behind) requires that proficiency rates continue to increase each year until 2014 when it is expected that all students are proficient. While most states have continued to update and increase the rigor of such standards, there remains great inconsistency from one state to another in determining what “proficient” means at each grade level. With the release of the Common Core State Standards in June of 2010 and the overwhelming response by the states to adopt them (45 of 50 at the time of this writing), it appears that the majority of the country’s students will soon be held to more consistent levels of proficiency. The debate continues as to whether all eighth grade students are ready for algebra 1. While many would agree that most students, given the proper preparation and strong work habits, can rise to meet the challenge of more rigor, we must remain flexible enough to accommodate those students who have not developed the skills to succeed. Teachers may not agree with the laws imposed upon the educational system, nor do they condone the judgment of student/teacher/school quality based on only one high stakes test; but they do want students to be challenged with rigor and high quality instruction. Not every eighth grade student may be ready for the abstraction of a traditional algebra 1 course. In Hastings, this was acknowledged, but the enactment of legislation requiring algebra 1 concepts for every eighth grader forced teachers to plan and prepare students for this level. By preparing students well in the K-7 curriculum and modifying and differentiating instruction, more students than ever can learn algebraic concepts laid forth in the standards. Whether every eighth grade student can

demonstrate mastery and score at a proficient on one high stakes test remains to be seen—and this will continue to be the challenge.

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Appendix 1: Minnesota K-12 Grade 8 Mathematic Standards

Minnesota K-12 Academic Standards in Mathematics

Strand	Standard	No.	Benchmark
8	Number & Operation	Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.	<p>8.1.1.1 Classify real numbers as rational or irrational. Know that when a square root of a positive integer is not an integer, then it is irrational. Know that the sum of a rational number and an irrational number is irrational, and the product of a non-zero rational number and an irrational number is irrational.</p> <p><i>For example:</i> Classify the following numbers as whole numbers, integers, rational numbers, irrational numbers, recognizing that some numbers belong in more than one category: $\frac{6}{3}$, $\frac{3}{6}$, $3.\overline{6}$, $\frac{\pi}{2}$, $-\sqrt{4}$, $\sqrt{10}$, -6.7.</p>
			<p>8.1.1.2 Compare real numbers; locate real numbers on a number line. Identify the square root of a positive integer as an integer, or if it is not an integer, locate it as a real number between two consecutive positive integers.</p> <p><i>For example:</i> Put the following numbers in order from smallest to largest: 2, $\sqrt{3}$, -4, -6.8, $-\sqrt{37}$.</p> <p><i>Another example:</i> $\sqrt{68}$ is an irrational number between 8 and 9.</p>
			<p>8.1.1.3 Determine rational approximations for solutions to problems involving real numbers.</p> <p><i>For example:</i> A calculator can be used to determine that $\sqrt{7}$ is approximately 2.65.</p> <p><i>Another example:</i> To check that $1\frac{5}{12}$ is slightly bigger than $\sqrt{2}$, do the calculation $(1\frac{5}{12})^2 = (\frac{17}{12})^2 = \frac{289}{144} = 2\frac{1}{144}$.</p> <p><i>Another example:</i> Knowing that $\sqrt{10}$ is between 3 and 4, try squaring numbers like 3.5, 3.3, 3.1 to determine that 3.1 is a reasonable rational approximation of $\sqrt{10}$.</p>
			<p>8.1.1.4 Know and apply the properties of positive and negative integer exponents to generate equivalent numerical expressions.</p> <p><i>For example:</i> $3^2 \times 3^{(-5)} = 3^{(-3)} = (\frac{1}{3})^3 = \frac{1}{27}$.</p>
			<p>8.1.1.5 Express approximations of very large and very small numbers using scientific notation; understand how calculators display numbers in scientific notation. Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation, using the correct number of significant digits when physical measurements are involved.</p> <p><i>For example:</i> $(4.2 \times 10^4) \times (8.25 \times 10^3) = 3.465 \times 10^8$, but if these numbers represent physical measurements, the answer should be expressed as 3.5×10^8 because the first factor, 4.2×10^4, only has two significant digits.</p>

Minnesota K-12 Academic Standards in Mathematics

Strand	Standard	No.	Benchmark
8	Algebra	Understand the concept of function in real-world and mathematical situations, and distinguish between linear and nonlinear functions.	<p>8.2.1.1 Understand that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. Use functional notation, such as $f(x)$, to represent such relationships.</p> <p><i>For example:</i> The relationship between the area of a square and the side length can be expressed as $f(x)=x^2$. In this case, $f(5)=25$, which represents the fact that a square of side length 5 units has area 25 units squared.</p>
			<p>8.2.1.2 Use linear functions to represent relationships in which changing the input variable by some amount leads to a change in the output variable that is a constant times that amount.</p> <p><i>For example:</i> Uncle Jim gave Emily \$50 on the day she was born and \$25 on each birthday after that. The function $f(x)=50+25x$ represents the amount of money Jim has given after x years. The rate of change is \$25 per year.</p>
			<p>8.2.1.3 Understand that a function is linear if it can be expressed in the form $f(x)=mx+b$ or if its graph is a straight line.</p> <p><i>For example:</i> The function $f(x)=x^2$ is not a linear function because its graph contains the points (1,1), (-1,1) and (0,0), which are not on a straight line.</p>
			<p>8.2.1.4 Understand that an arithmetic sequence is a linear function that can be expressed in the form $f(x)=mx+b$, where $x=0, 1, 2, 3, \dots$</p> <p><i>For example:</i> The arithmetic sequence 3, 7, 11, 15, ..., can be expressed as $f(x)=4x+3$.</p>
			<p>8.2.1.5 Understand that a geometric sequence is a non-linear function that can be expressed in the form $f(x)=ab^x$, where $x=0, 1, 2, 3, \dots$</p> <p><i>For example:</i> The geometric sequence 6, 12, 24, 48, ..., can be expressed in the form $f(x)=6(2^x)$.</p>

Minnesota K-12 Academic Standards in Mathematics

Strand	Standard	No.	Benchmark	
8	Algebra		Represent linear functions with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another.	
		Recognize linear functions in real-world and mathematical situations; represent linear functions and other functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions and explain results in the original context.	8.2.2.1	Identify graphical properties of linear functions including slopes and intercepts. Know that the slope equals the rate of change, and that the y-intercept is zero when the function represents a proportional relationship.
		8.2.2.2	Identify how coefficient changes in the equation $f(x) = mx + b$ affect the graphs of linear functions. Know how to use graphing technology to examine these effects.	
		8.2.2.3	Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems. <i>For example:</i> If a girl starts with \$100 in savings and adds \$10 at the end of each month, she will have $100 + 10x$ dollars after x months.	
		8.2.2.4	Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems. <i>For example:</i> If a girl invests \$100 at 10% annual interest, she will have $100(1.1^x)$ dollars after x years.	
		Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.	8.2.3.1	Evaluate algebraic expressions, including expressions containing radicals and absolute values, at specified values of their variables. <i>For example:</i> Evaluate $\pi^2 h$ when $r = 3$ and $h = 0.5$, and then use an approximation of π to obtain an approximate answer.
	8.2.3.2	Justify steps in generating equivalent expressions by identifying the properties used, including the properties of algebra. Properties include the associative, commutative and distributive laws, and the order of operations, including grouping symbols.		

Minnesota K-12 Academic Standards in Mathematics

	Strand	Standard	No.	Benchmark
8	Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear expressions and inequalities symbolically and graphically. Interpret solutions in the original context.	8.2.4.1	Use linear equations to represent situations involving a constant rate of change, including proportional and non-proportional relationships. <i>For example:</i> For a cylinder with fixed radius of length 5, the surface area $A = 2\pi(5)h + 2\pi(5)^2 = 10\pi h + 50\pi$, is a linear function of the height h , but the surface area is not proportional to the height.
			8.2.4.2	Solve multi-step equations in one variable. Solve for one variable in a multi-variable equation in terms of the other variables. Justify the steps by identifying the properties of equalities used. <i>For example:</i> The equation $10x + 17 = 3x$ can be changed to $7x + 17 = 0$, and then to $7x = -17$ by adding/subtracting the same quantities to both sides. These changes do not change the solution of the equation. <i>Another example:</i> Using the formula for the perimeter of a rectangle, solve for the base in terms of the height and perimeter.
			8.2.4.3	Express linear equations in slope-intercept, point-slope and standard forms, and convert between these forms. Given sufficient information, find an equation of a line. <i>For example:</i> Determine an equation of the line through the points $(-1,6)$ and $(2/3, -3/4)$.
			8.2.4.4	Use linear inequalities to represent relationships in various contexts. <i>For example:</i> A gas station charges \$0.10 less per gallon of gasoline if a customer also gets a car wash. Without the car wash, gas costs \$2.79 per gallon. The car wash is \$8.95. What are the possible amounts (in gallons) of gasoline that you can buy if you also get a car wash and can spend at most \$35?
			8.2.4.5	Solve linear inequalities using properties of inequalities. Graph the solutions on a number line. <i>For example:</i> The inequality $-3x < 6$ is equivalent to $x > -2$, which can be represented on the number line by shading in the interval to the right of -2 .
			8.2.4.6	Represent relationships in various contexts with equations and inequalities involving the absolute value of a linear expression. Solve such equations and inequalities and graph the solutions on a number line. <i>For example:</i> A cylindrical machine part is manufactured with a radius of 2.1 cm, with a tolerance of 1/100 cm. The radius r satisfies the inequality $ r - 2.1 \leq .01$.
			8.2.4.7	Represent relationships in various contexts using systems of linear equations. Solve systems of linear equations in two variables symbolically, graphically and numerically. <i>For example:</i> Marty's cell phone company charges \$15 per month plus \$0.04 per minute for each call. Jeannine's company charges \$0.25 per minute. Use a system of equations to determine the advantages of each plan based on the number of minutes used.

Minnesota K-12 Academic Standards in Mathematics

Strand	Standard	No.	Benchmark
Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear expressions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	8.2.4.8	Understand that a system of linear equations may have no solution, one solution, or an infinite number of solutions. Relate the number of solutions to pairs of lines that are intersecting, parallel or identical. Check whether a pair of numbers satisfies a system of two linear equations in two unknowns by substituting the numbers into both equations.
		8.2.4.9	Use the relationship between square roots and squares of a number to solve problems. <i>For example:</i> If $\pi x^2 = 5$, then $ x = \sqrt{\frac{5}{\pi}}$, or equivalently, $x = \sqrt{\frac{5}{\pi}}$ or $x = -\sqrt{\frac{5}{\pi}}$. If x is understood as the radius of a circle in this example, then the negative solution should be discarded and $x = \sqrt{\frac{5}{\pi}}$.
8 Geometry & Measurement	Solve problems involving right triangles using the Pythagorean Theorem and its converse.	8.3.1.1	Use the Pythagorean Theorem to solve problems involving right triangles. <i>For example:</i> Determine the perimeter of a right triangle, given the lengths of two of its sides. <i>Another example:</i> Show that a triangle with side lengths 4, 5 and 6 is not a right triangle.
		8.3.1.2	Determine the distance between two points on a horizontal or vertical line in a coordinate system. Use the Pythagorean Theorem to find the distance between any two points in a coordinate system.
		8.3.1.3	Informally justify the Pythagorean Theorem by using measurements, diagrams and computer software.
	Solve problems involving parallel and perpendicular lines on a coordinate system.	8.3.2.1	Understand and apply the relationships between the slopes of parallel lines and between the slopes of perpendicular lines. Dynamic graphing software may be used to examine these relationships.
		8.3.2.2	Analyze polygons on a coordinate system by determining the slopes of their sides. <i>For example:</i> Given the coordinates of four points, determine whether the corresponding quadrilateral is a parallelogram.
		8.3.2.3	Given a line on a coordinate system and the coordinates of a point not on the line, find lines through that point that are parallel and perpendicular to the given line, symbolically and graphically.

Minnesota K-12 Academic Standards in Mathematics

	Strand	Standard	No.	Benchmark
8	Data Analysis & Probability	Interpret data using scatterplots and approximate lines of best fit. Use lines of best fit to draw conclusions about data.	8.4.1.1	Collect, display and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit and determine an equation for the line. Use appropriate titles, labels and units. Know how to use graphing technology to display scatterplots and corresponding lines of best fit.
			8.4.1.2	Use a line of best fit to make statements about approximate rate of change and to make predictions about values not in the original data set. <i>For example:</i> Given a scatterplot relating student heights to shoe sizes, predict the shoe size of a 5'4" student, even if the data does not contain information for a student of that height.
			8.4.1.3	Assess the reasonableness of predictions using scatterplots by interpreting them in the original context. <i>For example:</i> A set of data may show that the number of women in the U.S. Senate is growing at a certain rate each election cycle. Is it reasonable to use this trend to predict the year in which the Senate will eventually include 1000 female Senators?

Appendix 2: Common Core State Standards Grade 8 Mathematic Standards

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
 - a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
 - b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
 - Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
 - Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions

8.F

Define, evaluate, and compare functions.

- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.¹
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

- Verify experimentally the properties of rotations, reflections, and translations:
 - Lines are taken to lines, and line segments to line segments of the same length.
 - Angles are taken to angles of the same measure.
 - Parallel lines are taken to parallel lines.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

¹Function notation is not required in Grade 8.

- Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

- Explain a proof of the Pythagorean Theorem and its converse.
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

- Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability

8.SP

Investigate patterns of association in bivariate data.

- Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
- Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

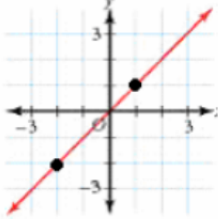
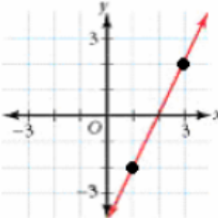
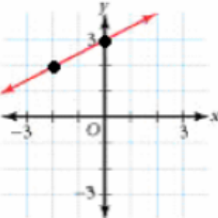
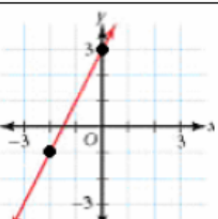
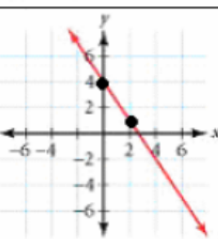
Appendix 3: Curricular Materials
 From www.ilovemath.org

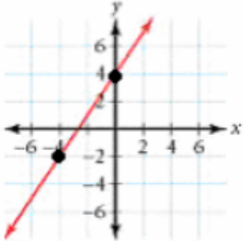
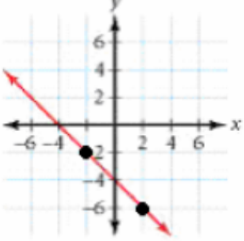
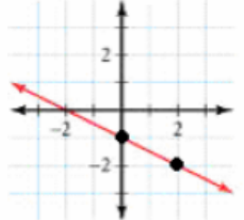
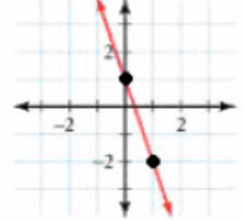
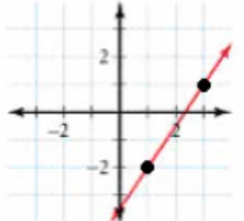
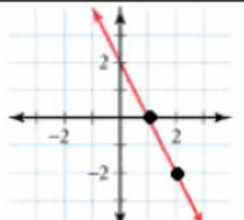
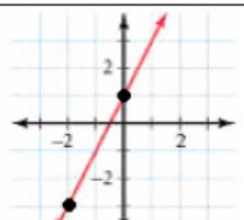
Algebra Lab

Name _____

Matching Graphs, Points, and Slopes _____

For each graph in the left hand column, find the points plotted and the slope of the line. Then, glue the matching points and slope in the blank columns on the right.

Graph	Points	Slope
		
		
		
		
		

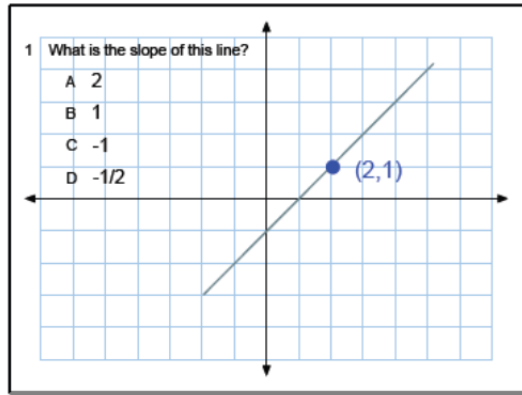
		
		
		
		
		
		
		

$(1, -2)$ $(3, 1)$	$slope = -1$	$slope = 1$	$(2, -2)$ $(0, -1)$
$slope = -\frac{3}{2}$	$(-4, -2)$ $(0, 4)$	$(-2, -1)$ $(0, 3)$	$slope = -2$
$(-2, 2)$ $(0, 3)$	$slope = 2$	$slope = \frac{3}{2}$	$(1, -2)$ $(3, 2)$
$slope = -\frac{1}{2}$	$(-2, -2)$ $(2, -6)$	$(1, -2)$ $(0, 1)$	$slope = \frac{1}{2}$
$slope = -3$	$slope = 2$	$(0, 4)$ $(2, 1)$	$(-2, -3)$ $(0, 1)$
$(-2, -2)$ $(1, 1)$	$(2, -2)$ $(1, 0)$	$slope = 2$	$slope = \frac{3}{2}$

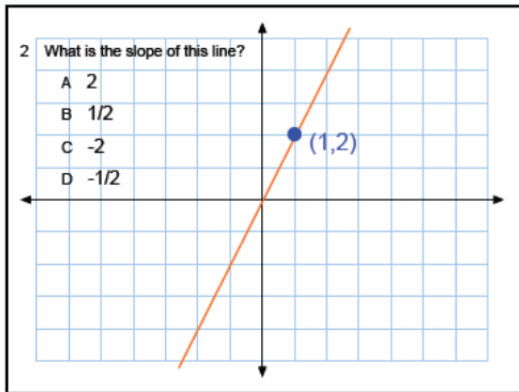
Slope

Grade: 7th - 8th
Subject: Algebra 1
Date: today

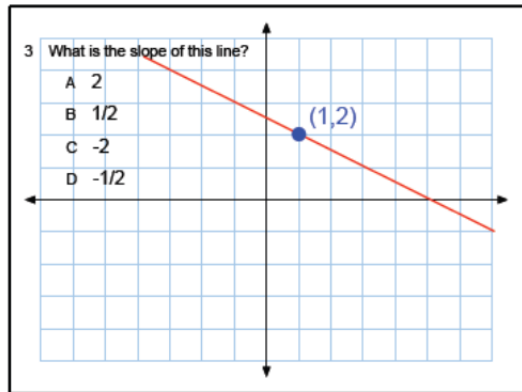
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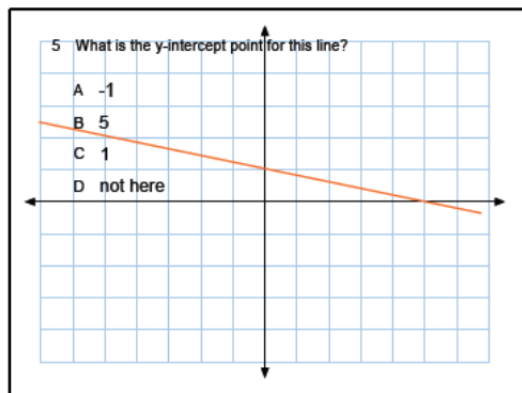
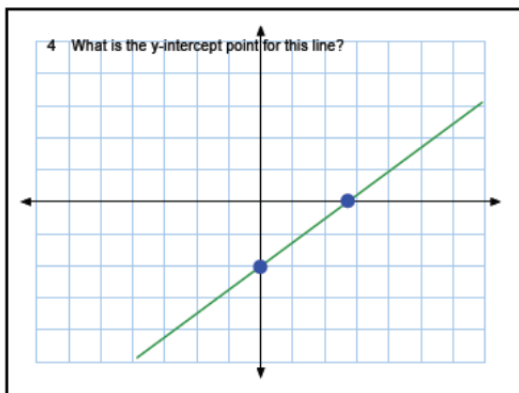
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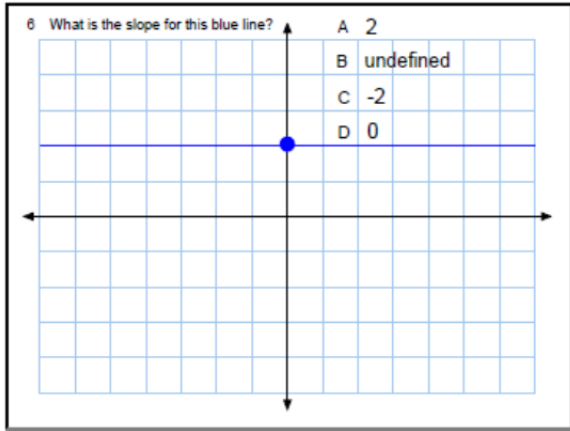


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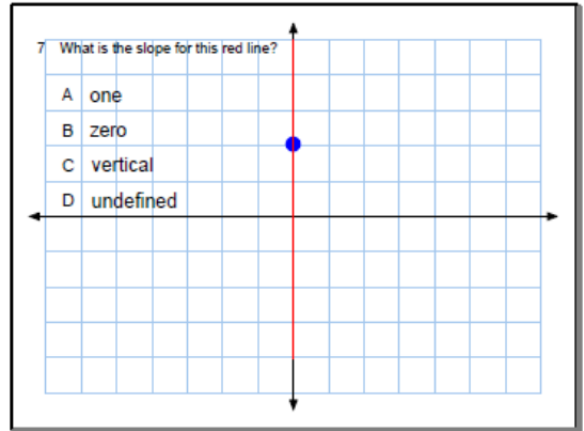


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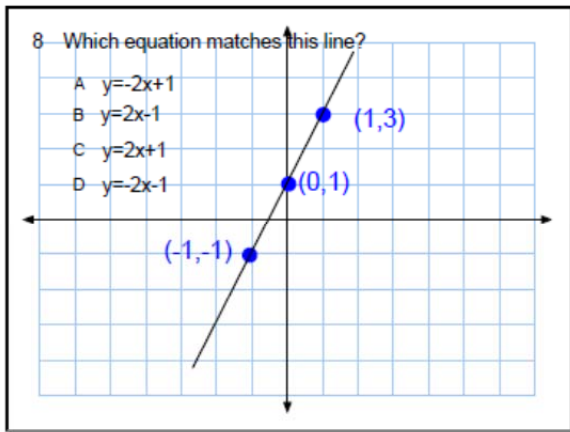




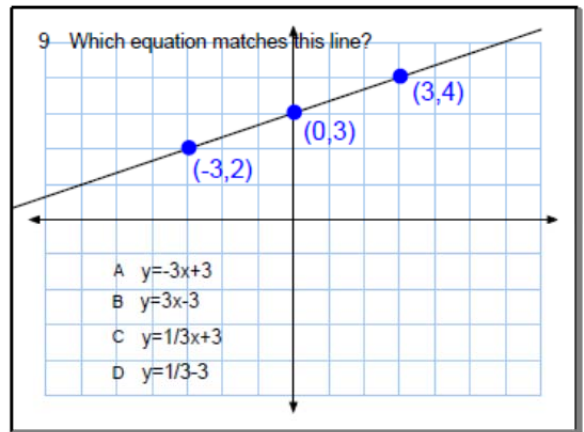
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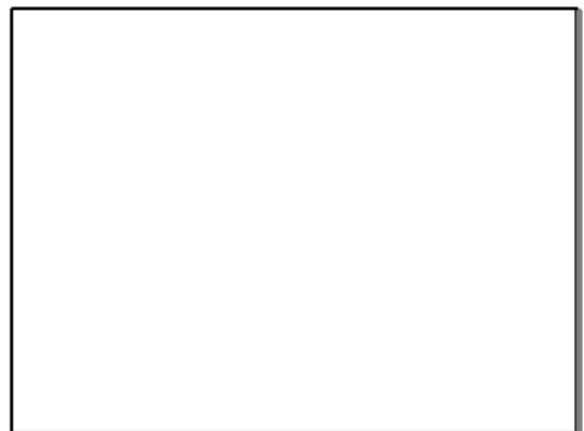
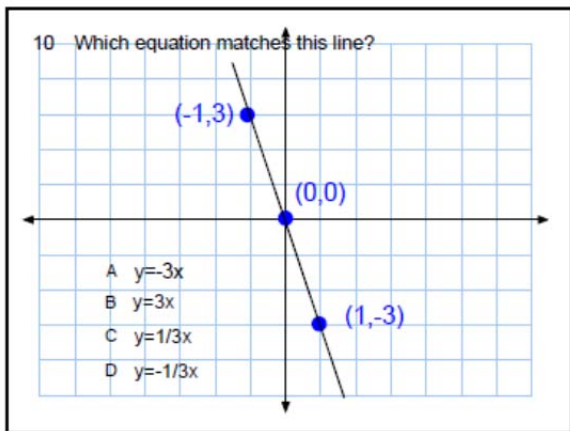
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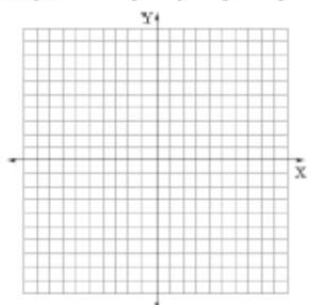
Algebra/Algebra 1
5-6A Slope Intercept Form

Name _____
Hour _____

Given the slope and the y-intercept of each line, graph the line, write the equation of the line in slope intercept form ($y = mx + b$) and provide 4 other points on the line (2 points to the left of the y-axis, 2 points to the right of the y-axis).

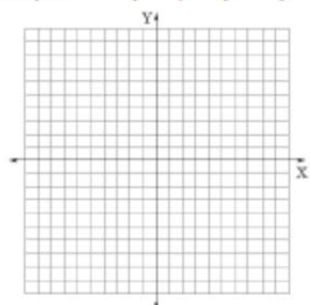
1) slope = $\frac{3}{4}$ y-intercept = -2
Equation _____

to the Left (,) (,)
to the Right (,) (,)



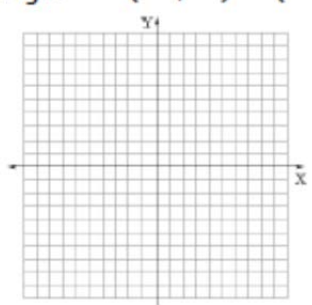
2) slope = -2 y-intercept = 3
Equation _____

to the Left (,) (,)
to the Right (,) (,)



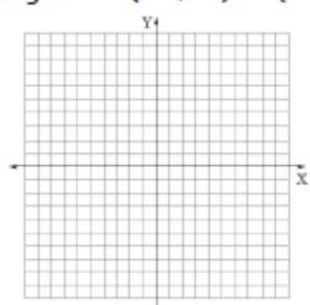
3) slope = 0.5 y-intercept = -6
Equation _____

to the Left (,) (,)
to the Right (,) (,)



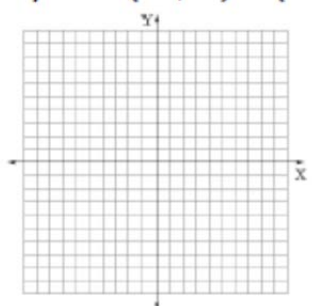
4) slope = $\frac{1}{3}$ y-intercept = 0
Equation _____

to the Left (,) (,)
to the Right (,) (,)



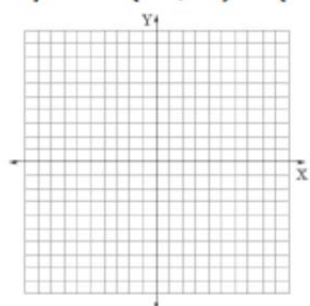
5) slope = $-\frac{2}{3}$ y-intercept = 5
Equation _____

to the Left (,) (,)
to the Right (,) (,)



6) slope = 0 y-intercept = -6
Equation _____

to the Left (,) (,)
to the Right (,) (,)



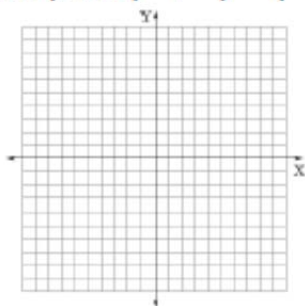
Given the slope and a point on each line, graph the line, find the y-intercept, write the equation line in slope-intercept form ($y = mx + b$) and provide 2 other points on the line.

7) slope = -3 point = $(-3, 4)$

y-intercept = _____

Equation _____

Two other points (,) (,)

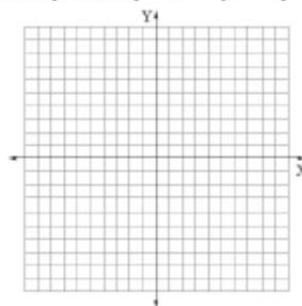


8) slope = $-1/2$ point = $(2, -4)$

y-intercept = _____

Equation _____

Two other points (,) (,)

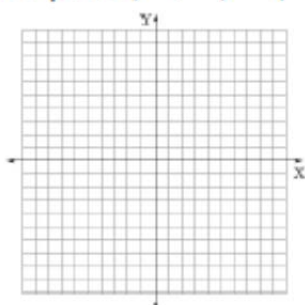


9) slope = $3/2$ point = $(-6, -8)$

y-intercept = _____

Equation _____

Two other points (,) (,)

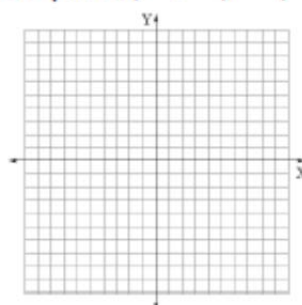


10) slope = $-4/3$ point = $(-9, 9)$

y-intercept = _____

Equation _____

Two other points (,) (,)

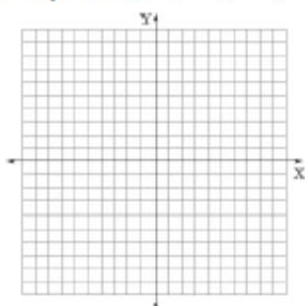


11) slope = $2/7$ point = $(-7, 0)$

y-intercept = _____

Equation _____

Two other points (,) (,)

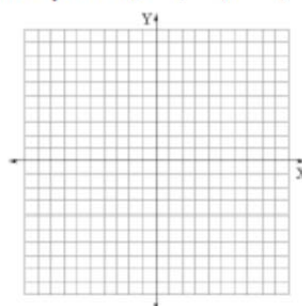


12) slope = $1/6$ point = $(-3, 1)$

y-intercept = _____

Equation _____

Two other points (,) (,)



Stained Glass Window

Graph each of the following lines as indicated by your teacher. Draw the lines to the edge of your graph paper. When you are finished, neatly color the sections to make a stained glass window design.

1. $y = -\frac{1}{2}x - 4$

7. $y = -\frac{1}{2}x + 4$

2. $y = \frac{3}{2}x + 12$

8. $y = -\frac{3}{2}x + 12$

3. $y = 12$

9. $y = -12$

4. $x = 0$

10. $y = -\frac{3}{2}x - 12$

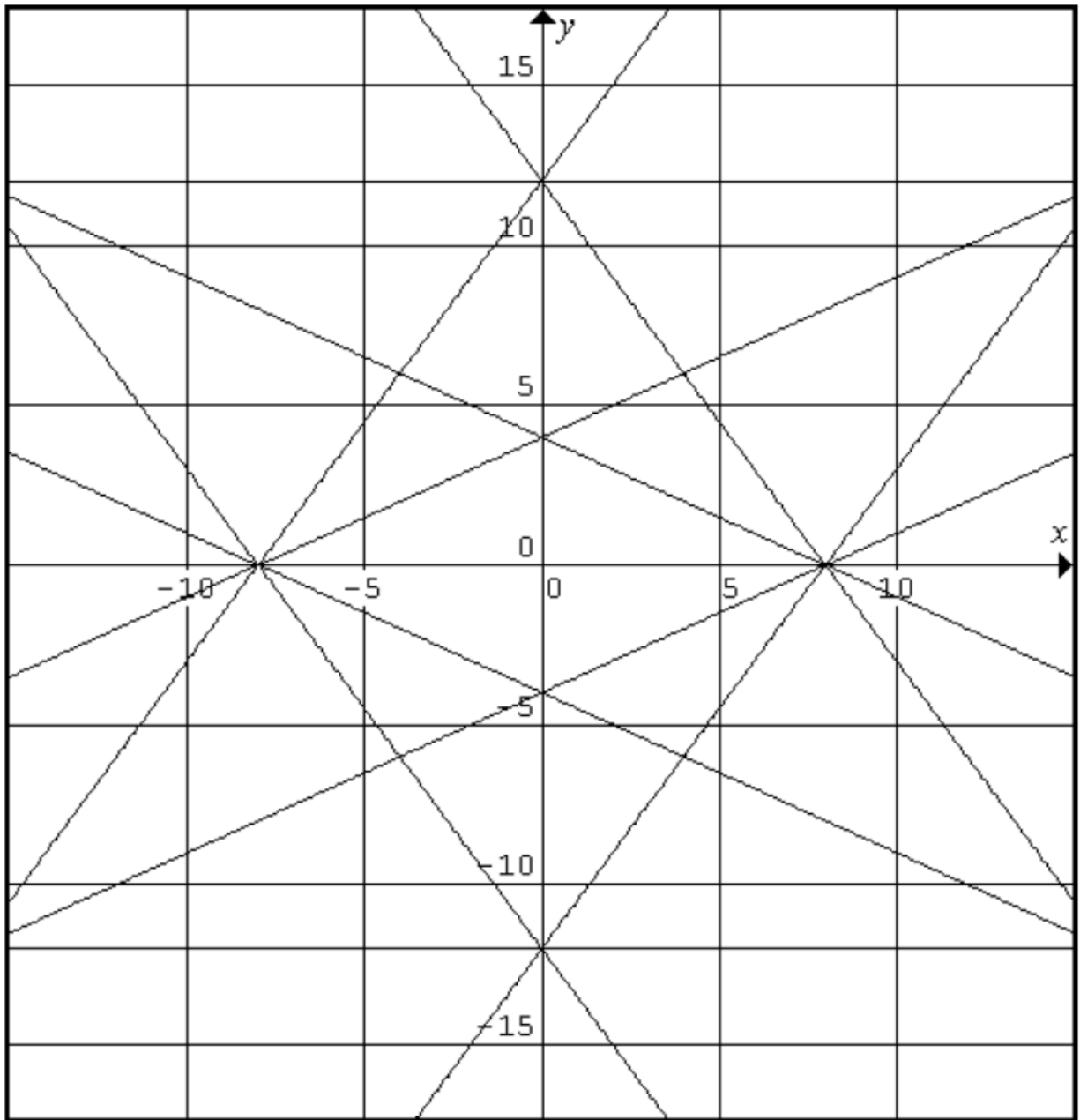
5. $y = \frac{1}{2}x + 4$

11. $y = \frac{3}{2}x - 12$

6. $y = \frac{1}{2}x - 4$

Tape/Glue your stained glass window to a sheet of colored construction paper. You will be graded on the accuracy of graphing the lines, and the neatness & creativity of your coloring.





Slope-Intercept Form: $y = mx + b$

Point-Slope Form: $y - y_1 = m(x - x_1)$

1) Write the equation of the following line in slope-intercept form: $-2x + 2y = 10$

2) Write the equation of the following line in slope-intercept form: $4y = 4x - 8$

3) Write the equation of the following line in slope-intercept form: $3y + 4 = -3x + 4$

4) A) Find the slope of the line between (4,3) and (-4,1) _____
 B) Write the equation of the line between these points in slope-intercept form

5) Write the equation of the line in slope-intercept form that contains the point (-2,5) and has a slope of -4.

6) Write the equation of the following line in slope-intercept form: $8x + 2y = 0$

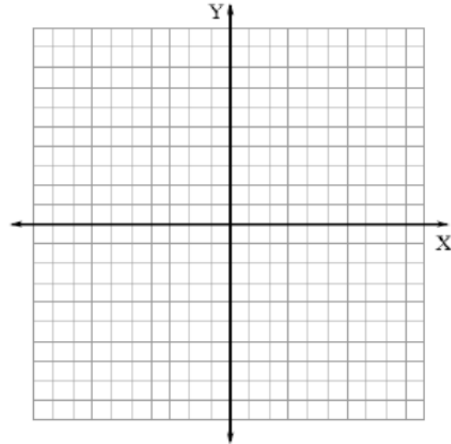
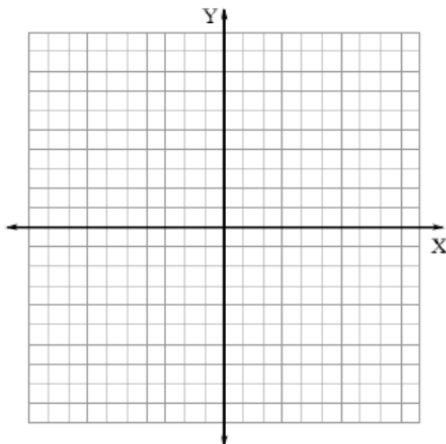
7) Complete the table:

Line	1	2	3	4	5	6
Slope						

Graph lines 1 & 2 on the graph below:

Graph lines 5 & 6 on the graph below:

(Use the graphing calculator to check your graphs—be sure to set the "Zoom" setting on Z Square



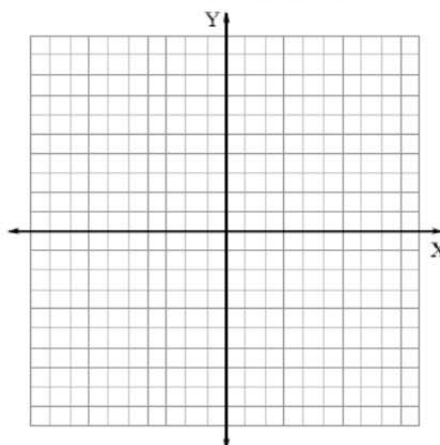
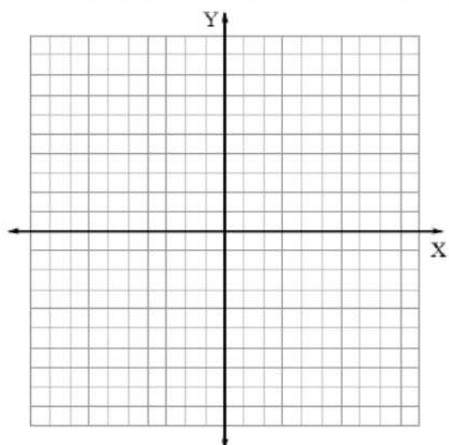
8) Compare the slopes of lines 1 & 2. _____

9) Describe the relationship between the graphs of lines 1 & 2 _____

10) Graph lines 1 & 3 on the graph below:

Graph lines 4 & 5 on the graph below:

(Use the graphing calculator to check your graphs—be sure to set the "Zoom" setting on Z Square)



11) Compare the slopes of lines 5 & 6. _____

12) Describe the relationship between the graphs of lines 5 & 6 _____

13) Multiply the slope of line 4 times the slope of line 5 _____

14) Describe the relationship between the graphs of lines 4 & 5 _____

15) Multiply the slope of line 1 times the slope of line 3 _____

Drawing Conclusions

16) What is the relationship between lines with the same slope? _____

17) What is the relationship between lines if the product of the slopes is equal to -1? _____

18) Write the equation of a line parallel to $y = \frac{2}{3}x + 7$ _____

19) Write the equation of a line perpendicular to $y = \frac{2}{3}x + 7$ _____

How Strong is Spaghetti?

Objective: To find a linear function that fits a set of real world data.

Procedure:

- ❖ Puncture two holes in the top of the cup and thread a string through the holes. Tie the ends of the string together so that the string acts like a handle.
- ❖ Place one piece of spaghetti under the string so that the cup hangs from the middle of the piece of spaghetti. One person should hold both ends of the spaghetti.
- ❖ Another person should begin to add pennies to the cup. When the spaghetti breaks, record the number of pennies needed to break the spaghetti.
- ❖ After you have broken one piece of spaghetti, use two new pieces and again place pennies in the cup until the spaghetti breaks. Repeat the experiment until the table below is completed.

Data Table and Exploration:

1. Complete the table below based on your experiments.

Pieces of Spaghetti	Number of Pennies Needed
1	
2	
3	
4	
5	

2. Graph your results on graph paper. Use the pieces of spaghetti on the x-axis and the number of pennies on the y-axis. Remember to label your axes and scale on the graph.
3. Describe what you see on your graph. What kind of model do you think is appropriate for this data?
4. Pick any two points from your line. Write a function in slope-intercept form.
5. Using your equation above, estimate the amount of pennies needed to break the pieces of spaghetti.

Pieces of Spaghetti	Number of Pennies Needed
20	
50	
150	

6. Using your calculator, calculate a line of best fit. Write the function.
7. How does the line of best fit compare with your line?
8. Use the calculator function to predict how many pennies are needed to break 20, 50, and 150 pieces of spaghetti. How do these values compare to the predictions you made from your equation?

Created/adapted from a similar worksheet from www.ilovemath.org

Algebra/Linear Algebra
05-08 Slopes, Parallel Lines, Perpendicular Lines

Name _____
Hour _____

In the center of the bull's eye, there is a coordinate pair (point). In the first ring, there is another coordinate pair. In the following ring, calculate the slope of the line between the two points. In the next ring, write the equation of the line in point-slope form. Then write the equation of the line in slope-intercept form in the next ring. Finally, make up an equation for a line that is parallel to this line and an equation that is perpendicular to this line.

