The Power of

AND

Application of Machine Learning to Numerical Algebraic Geometry

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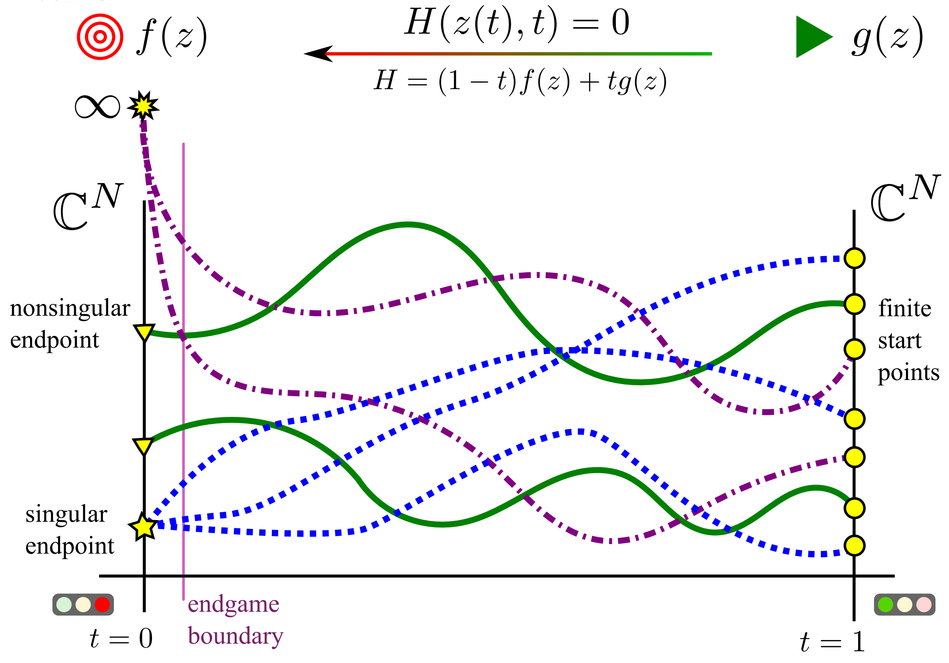
Bertini 2

- The solution of arbitrary polynomial systems is an area of active research, and has many applications in math, science and engineering.
- This program, Bertini 2, builds on the success of the first Bertini program, and seeks to eventually replace it entirely, as a powerful numerical engine.
- Pybertini is a package that lets you call Bertini2 functions from Python code.
- Python is a common programming language used for tasks such as data processing / computation.
- Both Bertini2 and Pybertini are written in C++.

Homotopy continuation

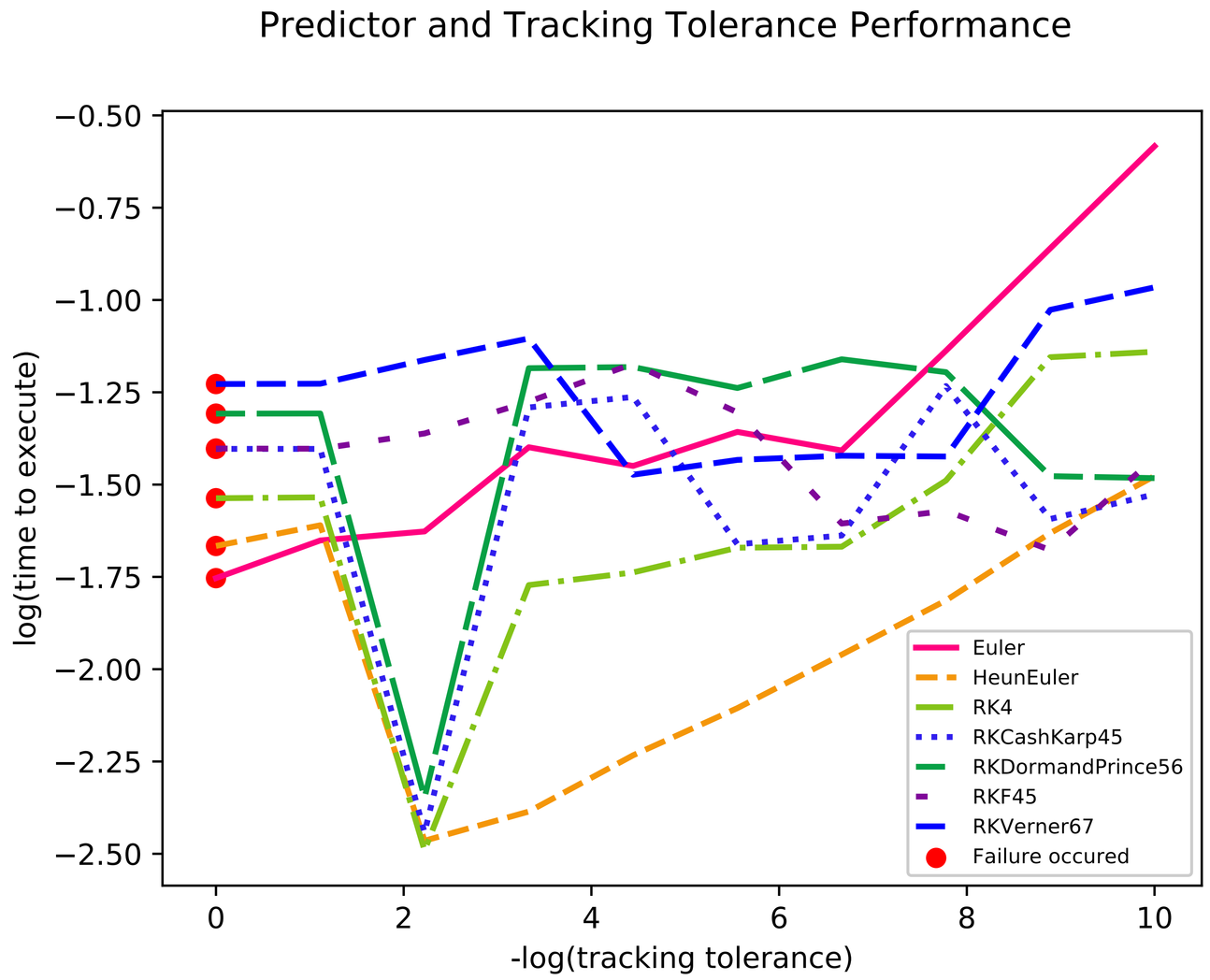
Bertini2 implements *homotopy continuation* to numerically solve $f(z) = 0$, for $\mathbb{C}$ (complex) polynomial systems.



To solve a particular polynomial system $f(z) = 0$,

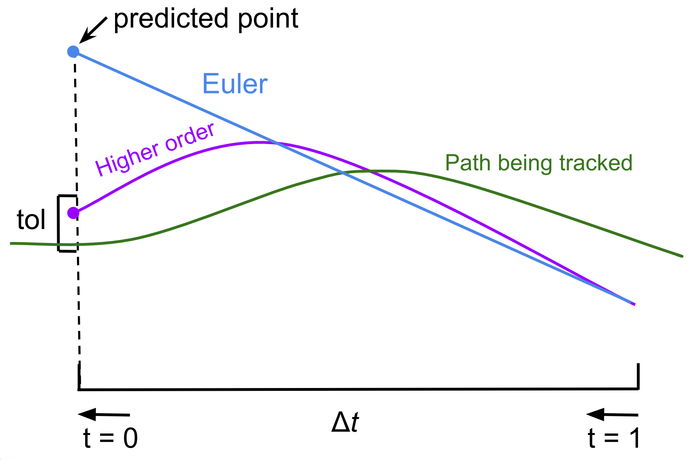
1. analyze the structure of $f(z)$, form a similar system $g(z)$,
2. numerically deform from $g(z)$ to $f(z)$ through time t using a continuous homotopy $H(z(t), t) = 0$.

Controlling Bertini2 with Tensorflow



In the plot, it is clear that there is no one best predictor. The predictor that is best depends on the tracking tolerance. Although, there are predictors that are deemed to never be appropriate to use by this plot.

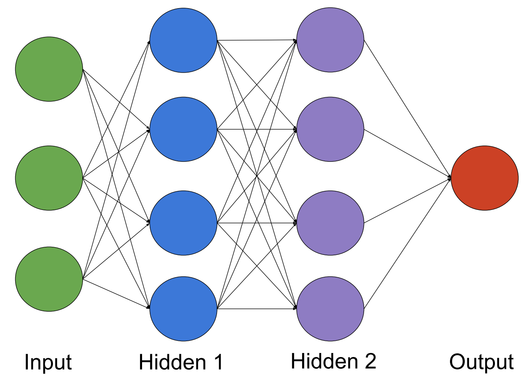
The goal of controlling Bertini2 with Tensorflow is to train Tensorflow in choosing the best path tracking settings in a given situation.



- **Predictor**: seven of the predictors from Bertini2 depicted in the graph above by the different lines.
- **Tolerance**: Bertini shrinks Δt if distance between path and predicted point is less than the tolerance. Higher order is more expensive to compute.
- **Failure occurred**: path crossings exist at this point.

Tensorflow

- Machine learning is one of the fastest growing fields in computing, with many applications.
- TensorFlow™ is an open source software library for high performance numerical computation and machine learning.
- Commonly used to make the creation of neural networks easier.
- Neural Networks consist of an input layer, multiple hidden layers, and an output layer. Every node in one layer is connected to every node in the next layer.
- Developed by Google. Free and Open-Source.



- Can also be used as a tool to optimize functions.
- The function we were trying to optimize is (to our knowledge) not differentiable, and because of this we are currently unable to use Tensorflow's optimization algorithms with Pybertini.

References and acknowledgements

[1] Bertiniteam: Brake, Collins, et al., "Bertini 2," 2014-2018. Available: `bertini2.org`

[2] M. Abadi, A. Agarwal, and et. al, "TensorFlow: Large-scale machine learning on heterogeneous systems," 2015. `tensorflow.org`

[3] D. Bates, J. Hauenstein, A. Sommese, and C. Wampler, *Numerically solving polynomial systems with Bertini.* SIAM, 2013, vol. 25.

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