A Generalized Hamiltonian Model for Power System Dynamics with Relay Action

by

Raja Timihiri

Project Advisor: Christopher DeMarco

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ABSTRACT

The electrical system is a complex network of buses, loads, transmission lines in addition to other voltage and current regulating components. This complexity makes dealing with power system stability an ongoing challenge that has become more significant in recent years as renewables are integrated into the power grid. Solving these instabilities in the power grid requires a suitable model of the power system in order to analyze steady state and transient stability of such systems. In this paper, we consider an n-bus system model that consists of lossless transmission lines connecting generation and load buses. We develop a Hamiltonian function to represent the dynamics of this system and study the transient stability for selected system disturbances. These initiating disturbances may include transmission line failure, addition or removal of loads, short circuit or generator outage. In this report we focus is on simulating subsequent transmission line relay action following such disturbances, within a 14-bus example power system. In particular we examine the impact of overcurrent relay action, which disconnects of one or more network branches, on the system's transient and dynamic stability.

Index Terms — Bus System, Complex Voltage, Dispatch, Equilibrium, Gradient, Hessian, Network Branch, Stable Trajectory, Transient, Transmission

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1 INTRODUCTION

1.1 Objectives

Adequate analysis of our electrical system and its overall stability requires a model that takes into account the dynamic impact of sudden disturbances that occur during the operation, and the subsequent response of relay actions in system protection, that can change the network topology as the system response evolves in time. Many models have been developed along with analytical methods for control and analysis of such systems. However, relatively few analysis methods explicitly treat the (nearly) discontinuous change in structure associated with relay action, and in this context, analysis of the stability of the power network remains a challenge. This challenge stems from the nature of the power flow equations which are non-linear for a period of time of several seconds to minutes. Solving such non-linear equations can be achieved using several numerical methods, such as Newton-Raphson. However, when sudden changes occur, the power system may not re-gain its steady state operation, in which case severe system failures could impact large portions of the power network and create blackouts. Therefore, it is important to continue with the efforts to develop a suitable and simple model to represent the power network, in order to study the system trajectories after a disturbance, and determine the initial conditions that can cause a deviation from equilibrium operating points. To this end, a new “Hamiltonian-like” potential function is introduced. The close relationship of the gradient of this potential to the dynamics of an n-bus system is examined, allowing us to analyze its transient stability, ascertaining its ability to return to steady state operation a short time after a disturbance occurs. The dynamic and transient behavior of the system is simulated with various initial conditions and the transient responses are analyzed. This new potential function is derived to allow the control of transmission line connections within the n-bus system, thus we are able to study the stability of this system following predetermined transmission line failures. The important analysis issue that is addressed here is: Once the fault is cleared, does this disturbance take the system's operating trajectory far away from a stable equilibrium, or does the system's trajectory remain within the domain of attraction of the operating equilibria?

Starting from an equilibrium operating point, we analyze the stability of the system as a function of time as we control the connectivity of various branches in the system. One goal is to determine initial conditions that maximize the probability of the system returning to the desired equilibrium, examining the hypothesis that this objective may be approximately achieved when one equalizes the height of saddle exit points from the stable well of the potential function.

1.2 Power Flow Equations

For an n-bus system, starting from the apparent power at each bus, we get [2]:

$$S_i = V_i I_i^* = V_i \sum_{k=1}^{n} (y_{ik} V_k)^*, \qquad i = 1,2,\dots n \tag{1.1}$$

Where:

$$y_{ik} = g_{ik} - jb_{ik}$$

Our system constraints are described by the admittance matrix y_{ik}, representing the nodal admittance of the system. For a lossless system, we take the line conductances to be zero, i.e. $g_{ik} = 0$. This simplifies the admittance matrix:

$$y_{ik} = -jb_{ik} \tag{1.2}$$

Taking the real and imaginary parts of the above complex form of the power flow equation, we obtain:

$$P_i = \sum_{k=1}^{n} |v_i||v_k|[b_{ik} \sin(\delta_i - \delta_k)] \tag{1.3}$$

$$Q_i = -\sum_{k=1}^{n} |v_i||v_k|[b_{ik}\cos(\delta_i - \delta_k)]$$

Combining terms in the left, and scaling by $|v_i|$:

$$-P_i + \sum_{k=1}^{n} |v_i||v_k|[b_{ik}\sin(\delta_i - \delta_k)] = 0$$
$$\frac{Q_i}{|v_i|} + \sum_{k=1}^{n} |v_k|[b_{ik}\cos(\delta_i - \delta_k)] = 0 \tag{1.4}$$

1.3 Lagrange Formulation

Potential and kinetic energies of any system can be written as a function of system variables such as position angle and rotor speed of a generator. The difference between the total kinetic energy and the total potential energy of a system is referred to as the Lagrangian. Therefore, the Lagrangian formulation is:

$$L = Kinetic\ Energy - Potential\ Energy \tag{1.5}$$

Hamilton's principle provides the following relationship in autonomous systems, with no external forces applied.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \qquad i = 1,2,\dots N \tag{1.6}$$

q_i: System Variables (or coordinates)

Taking friction into account, we include the Rayleigh dissipation function D:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = 0, \qquad i = 1,2,\dots N \tag{1.7}$$

Thus the n-bus power system can be represented as a dynamical system using LaGrange's equation, with δ, ω as variables:

$$\frac{d}{dt}\frac{\partial L}{\partial \omega}(\delta, \omega) - \frac{\partial L}{\partial \delta}(\delta, \omega) + \frac{\partial D}{\partial \omega}(\delta, \omega) = 0 \tag{1.8}$$

The generators have inertia M and rotor speed ω, with rotor position angles δ_i and linear damping with respect to frequency. The bus voltage magnitudes and angles are expressed with respect to a reference bus (slack bus). The loads attached to each bus are constant impedances. Several mathematical steps (Appendix) lead to the following Lagrange formulation:

$$L(\tilde{\delta}_i, \tilde{\omega}_i) = \frac{1}{2}\sum_{i=1}^{n-1} M_i {\tilde{\omega}_i}^2 + \frac{1}{2} M_T \omega_o^2 + \sum_{i=1}^{n-1} [\, P_{mi}\tilde{\delta}_i - V_i \sum_{k=1}^{n} V_k\, B_{ik} \cos(\tilde{\delta}_i - \tilde{\delta}_k)$$

$$- \frac{M_i}{M_T} \sum_{i=1}^{n} [\, P_{mi}\tilde{\delta}_i - V_i \sum_{k=1}^{n} V_k\, B_{ik} \cos(\tilde{\delta}_i - \tilde{\delta}_k)]\,] + \sum_{i=1}^{n} [\, P_{mi}\tilde{\delta}_0$$

$$- V_i \sum_{k=1}^{n} V_k\, B_{ik} \cos(\tilde{\delta}_0 - \tilde{\delta}_k) \qquad (1.9)$$

This result is consistent with the work of Michel and Vittal [1].

We can express the motion of the generators:

$$M_i \dot{\omega}_i = P_i - P_{ei} - D_i \omega_i, \; i = 1, \dots, n$$

$$\dot{\delta}_i = \omega_i \qquad (1.10)$$

Where D_i represent the damping coefficients of each generator.

1.4 New Function Phi

A. Hypothesis

The power flow equations (1.4) organized in vector form, can be obtained as the gradient of a scalar function of δ and $|v|$, where b_{ik}, P_i, and Q_i are treated as fixed parameters.

Let function Φ: $\mathbb{R}^{2n-m} \rightarrow \mathbb{R}^{2n-m}$ be this scalar function. Then we assume that Φ exists such that:

$$\nabla\Phi\ (\delta_i, |v_i|) = \begin{bmatrix} -P_i + \sum_{k=1}^{n} |v_i||v_k|[b_{ik} \sin(\delta_i - \delta_k)] \\ \frac{Q_i}{|v_i|} + \sum_{k=1}^{n} |v_k|[b_{ik} \cos(\delta_i - \delta_k)] \end{bmatrix} \tag{1.11}$$

B. Verification

Focusing on the structure of the power equations for a lossless transmission network case, consistent with a Hamiltonian system, we assume the existence of function Φ that satisfies equation (1.11).

Following work by Bergen [2] and DeMarco [3], we successfully derive an expression for Φ as a function of state variables $\omega, \alpha, |v|$ in the form,

$$\Phi(\omega, \alpha, |\vec{v}|) = Kinetic\ Energy + Potential\ Energy \tag{1.12}$$

This Hamiltonian formulation can be obtained from previously derived Lagrangian formulation using a Legendre transformation (appendix). Thus we define Φ as:

$$\Phi(\omega, \alpha, |\vec{v}|) = \frac{1}{2}\omega' M\ \omega\ - \frac{1}{2}\sum_{k=1}^{n} Im[\vec{v}_{bus}\,(Y_{bus}\,\vec{v}_{bus})^{*}] \\ -{P_d}'\ \angle(\vec{v}_{bus}) + {Q_d}'\ \log|\vec{v}_{bus}| \tag{1.13}$$

Arguments

$\vec{v}_{bus} = complex\ vector\ of\ bus\ voltage\ phasors, such\ that:$

$\vec{v}_{bus} = |\vec{v}| * e^{j\delta}$

$where\ \ \alpha_k = \delta_k - \delta_1\ \ and\ \ \alpha_1 = 0$

$\omega = generator\ frequency\ \left(= \frac{d\delta}{dt}\right)$

$Y_{bus} = lossless\ bus\ admittance\ matrix, such\ that:$

$Y_{bus} = AA\ diag(-j\ bb)\ AA'$

$where\ \ AA = bus - branch\ incidence\ matrix$

$bb = column\ vector\ of\ non-negative\ line\ (branch)\ susceptances$

$Q_d = demand\ reactive\ power$

$P_d = demand\ real\ power$

It can be easily verified that:

$$\frac{\partial \Phi}{\partial \delta_i} = \sum_{k=1}^{n} |v_i||v_k|[b_{ik}\sin(\delta_i - \delta_k)] - P_i$$
$$\frac{\partial \Phi}{\partial |v_i|} = \sum_{k=1}^{n} |v_k|[b_{ik}\cos(\delta_i - \delta_k)] + \frac{Q_i}{|v_i|} \quad (1.14)$$

Thus, our calculations confirm that the power equations of this n-bus system can in fact be written in terms of the gradient of function Phi.

1.5 Effect of Relay Action on Transmission Line Connection

Faults in the power system can be due to different kinds of disturbances, one being temporary removal via relay action of a transmission line or branch within the power network. Such failures are common types of disturbances in the power grid however, deriving an appropriate model is difficult. Following the bistable branch threshold model [3], we chose to model the removal of transmission line from service, by action of relay using control parameter. The state space of our n-bus system model is augmented by variable γ, which corresponds to the connectivity of the transmission line:

$$\gamma_i = \begin{cases} 1, & tranmission\ line\ i\ is\ connected \\ 0, & transmission\ line\ i\ removed \end{cases} \quad (1.15)$$

Thus γ is assigned to each transmission line, this is reflected in the admittance matrix

$$Y_{bus} = AA\ diag(-j\ \gamma\ bb)\ AA' \quad (1.16)$$

Therefore Φ is now a function of γ.

The behavior of γ is represented by function θ [3]; we implement the following modified equation:

$$\theta(\gamma) \coloneqq 2 * thresh * \left(\frac{1}{20}e^{-20\gamma} - \frac{1}{200}e^{-200\gamma}\right) + 2\left(\frac{1}{20}e^{20(\gamma-1)} - \frac{1}{200}e^{200(\gamma-1)}\right) - 0.395\gamma$$

$thresh$: Column vector of non − negative branch failure threshold

(1.17)

2 FUNCTION Φ: GRADIENT AND HESSIAN

2.1 System Equilibrium

The system equilibrium will change given a change of parameters or variables. Allowable changes will maintain the power system within the region of attraction of a stable equilibrium point. We determine such equilibrium point we use the power flow equations represented by the gradient of function Φ.

We use the Newton-Raphson routine [4] to find an equilibrium point starting from an initial guess $\underline{X}_0$. This method is based on local information, thus it can work well when provided with a proper initial guess. The Newton-Raphson method is a powerful technique for solving equations numerically it is based on the simple idea of linear approximation. For an initial guess we choose vector $\underline{X}_0 = \begin{bmatrix} \underline{\omega_0} \\ \underline{\alpha_0} \\ \underline{|v|_0} \\ \underline{\gamma_0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. This is a 62x62 vector, with variables corresponding to 14 buses and 20 transmission lines.

The iteration equation for the Newton-Raphson is as follows:

$$\underline{X}_{n+1} = \underline{X}_n - \frac{f(\underline{X}_n)}{f'(\underline{X}_n)} \quad \rightarrow \quad \Delta\underline{X} = -\frac{f(\underline{X}_n)}{f'(\underline{X}_n)} \tag{2.1}$$

The vector $\underline{X}_n$ represents all the state variables $\omega, \delta_i, |v_i|$, and γ such that:

$$f(\underline{X}_n) = \nabla\, \Phi(\underline{X}_n) \tag{2.2}$$

$$f'(\underline{X}_n) = \nabla^2\, \Phi(\underline{X}_n)$$

Multiple iterations are necessary before the convergence of the Newton Raphson method. In addition, we chose a saturation function with slope of 1 at origin, and a decreasing slope away from the origin:

$$sat(z) = \text{atan}(\beta\, z) \tag{2.3}$$

$$\beta = \text{Real valued positive scaling coefficient}$$

First a candidate step size $\Delta\underline{X}$ is calculated:

$$\Delta\underline{\hat{X}}_{n+1} = -\left[\nabla^2\, \Phi\left(\underline{X}_n\right)\right]^{-1} \nabla\, \Phi\left(\underline{X}_n\right) \tag{2.4}$$

Then actual step size is used:

$$\Delta\underline{X}_{n+1} = \frac{sat\left(\left\|\Delta\underline{\hat{X}}_{n+1}\right\|\right)}{\left\|\Delta\underline{\hat{X}}_{n+1}\right\|}\, \Delta\underline{\hat{X}}_{n+1} \tag{2.5}$$

However, one issue arises due to the nature of function Φ: the invertibility of $\nabla^2\, \Phi\left(\underline{X}_n\right)$. The Hessian function is not always invertible; some choices of parameter values lead to a singular matrix. One solution is to add the quantity $\varepsilon\, {V_{eig}}' V_{eig}$ to the hessian, with ε on the order of $10^{-3} \sim 10^{-5}$, and V_{eig} is the Eigen vector corresponding to the zero Eigen value of the hessian matrix. Thus, the candidate step becomes,

$$\Delta\underline{\hat{X}}_{n+1} = -\left[\nabla^2\, \Phi\left(\underline{X}_n\right) + \varepsilon\, {V_{eig}}' V_{eig}\right]^{-1} \nabla\, \Phi\left(\underline{X}_n\right) \tag{2.6}$$

2.2 Gradient Vector and Hessian Matrix

First, we derive expressions for the gradient of Φ in terms of the state variables $(\omega, \alpha, |\vec{v}|, \gamma)$

$$\nabla\Phi\,(\omega, \delta_i, |v_i|, \gamma) = \begin{matrix} \frac{\partial \Phi}{\partial \omega} \\ \frac{\partial \Phi}{\partial \delta_i} \\ \frac{\partial \Phi}{\partial |v_i|} \\ \frac{\partial \Phi}{\partial \gamma} \end{matrix} = \begin{matrix} MM\,\omega \\ Re[\vec{v}_{bus}\,(Y_{bus}\,\vec{v}_{bus})^*] + P_d \\ Im\left[\frac{\vec{v}_{bus}}{|\vec{v}_{bus}|}(Y_{bus}\,\vec{v}_{bus})^*\right] + \frac{Q_d}{|\vec{v}_{bus}|} \\ \frac{1}{2}(AA'\vec{v}_{bus})^*\,(AA'\vec{v}_{bus})\,bb - \nabla\theta(thresh, \gamma) \end{matrix} \tag{2.7}$$

Then an equivalent expression for hessian of Φ is

$$hess(\Phi) = \nabla^2\Phi = \begin{bmatrix} MM & 0_{nxn} & 0_{nxn} & 0_{nxl} \\ 0_{nxn} & B_{22} & B_{23} & B_{24} \\ 0_{nxn} & {B_{23}}' & B_{33} & B_{34} \\ 0_{lxn} & {B_{24}}' & {B_{34}}' & -B_{44} \end{bmatrix} \quad (2.8)$$

Where

$$l = \#\ of\ transmission\ lines$$
$$n = \#\ of\ buses$$

And

$$B_{22} = Re[diag(\vec{\imath}_{bus}{}^{*}\ \vec{v}_{bus}) - j\ diag(\vec{v}_{bus}Y_{bus}{}^{*}\ diag(\vec{v}_{bus}{}^{*})]$$

$$B_{23} = Re\left[diag\left(\vec{\imath}_{bus}{}^{*}\frac{\vec{v}_{bus}}{|\vec{v}_{bus}|}\right) + diag\left(\vec{v}_{bus}Y_{bus}{}^{*}\ diag\left(\frac{\vec{v}_{bus}{}^{*}}{|\vec{v}_{bus}|}\right)\right)\right]$$

$$B_{24} = Re\left[diag(\vec{v}_{bus})\big(AA\ diag(-j\ bb\ AA'\vec{v}_{bus})\big)^{*}\right]$$

$$B_{33} = Im\left[diag\left(\frac{\vec{v}_{bus}}{|\vec{v}_{bus}|}Y_{bus}{}^{*}\right)\ diag\left(\frac{\vec{v}_{bus}{}^{*}}{|\vec{v}_{bus}|}\right) - diag\left(\frac{\frac{Q_d}{|\vec{v}_{bus}|}}{|\vec{v}_{bus}|}\right)\right]$$

$$B_{34} = Im\left[diag\left(\frac{\vec{v}_{bus}}{|\vec{v}_{bus}|}\right)\ \big(AA\ diag(-j\ bb\ AA'\vec{v}_{bus})\big)^{*}\right]$$

$$B_{44} = hessian\_theta\_eval(thresh, \gamma) = \nabla^2\theta(thresh, \gamma)$$

2.3 Test of Consistency

A routine to test consistency and correctness of ∇Phi versus Phi functions confirms that ∇Phi is the correct matrix of partial derivatives for the vector valued potential function Phi. A similar routine is used to test that Hessian Phi is consistent with the function developed for ∇Phi. This routine consists of examining the following expression:

$$\left\|f(x_0 + h*\Delta x) - f(x_0) - \frac{\partial f}{\partial x}\bigg|_{x_0} * (h*\Delta x)\right\| = error(h) \quad (2.9)$$

Where: $h = \frac{1}{2^k}$

x Represents all the state variables of the system

x_0 Represents the initial value of x

The quadratic convergence of the error defined above is investigated as k increases. For a sufficiently small h the error is found to decrease with a factor of $\frac{1}{4}$ at each step.

3 STATE SPACE MODEL

3.1 14 Bus System

The disturbance considered in this project are transmission lines that are out of operation for several seconds up to several minutes. This disturbance is modeled into the derived function Phi. For our purposes we use a 14 node network consisting of 20 lossless transmission lines.

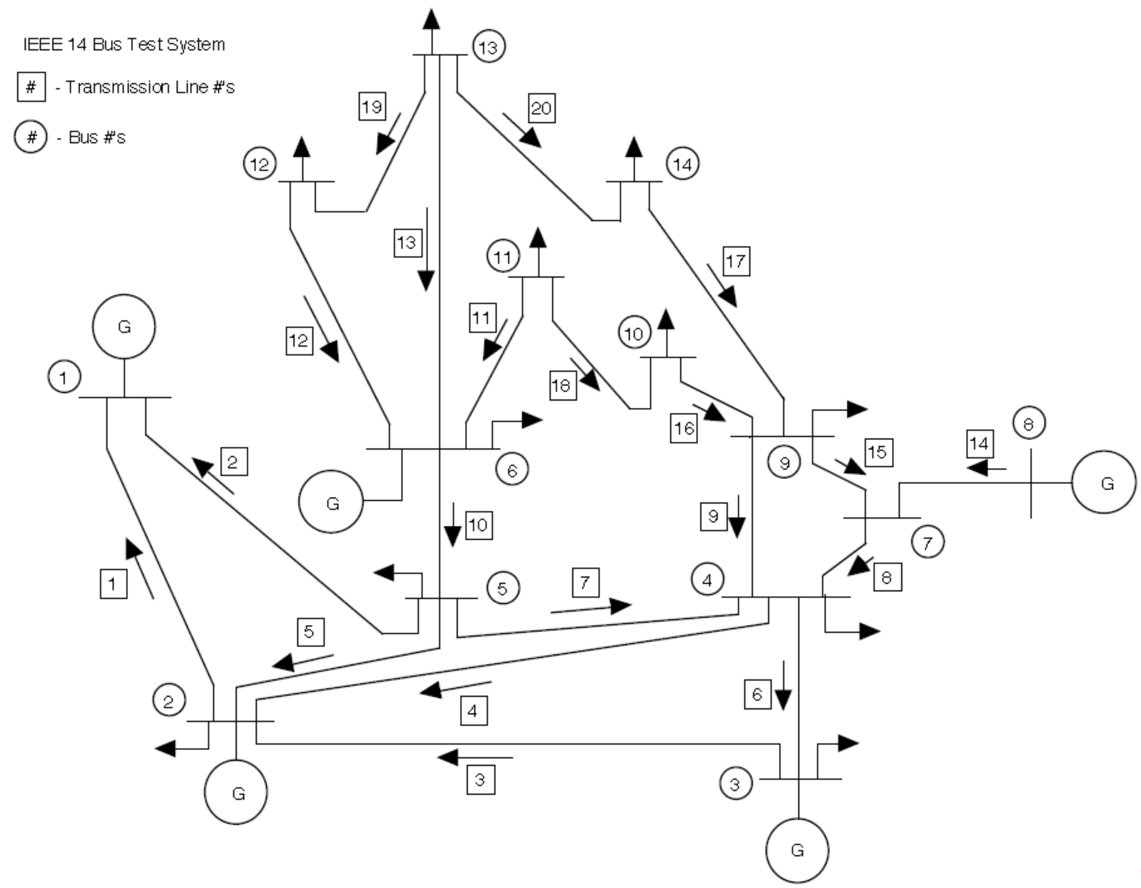

Figure 3.1: Example 14-bus System (IEEE)

To model a removal of a transmission line we consider an indicator variable, gamma, associated with each transmission line. For normal operation gamma is near a value of 1, then transitions to zero indicate the removal of that transmission line, i.e. the removal of the line from service:

$$\gamma = \begin{cases} 1, & \text{Transmission line operational} \\ 0, & \text{Removal of transmission line} \end{cases} \tag{3.1}$$

The overall state space system to be treated here can take the form:

$$\begin{bmatrix} \omega \\ \vec{v} \\ \gamma \end{bmatrix} = A_{weight} * \nabla\Phi(\omega, \vec{v}, \gamma) \qquad (3.2)$$

$$\vec{v} = \vec{v}_{bus} = |\vec{v}| * e^{-j\delta} \qquad (3.3)$$

Where, we've previously shown that the power systems equations relating power and voltage at each bus can be obtained as the gradient of function Φ.

3.2 Weighting Matrix

The matrix A_{weight} consists of small positive parameter ε_i (epsilon) representative of the system damping terms. Here, we let $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$ and $MM = I$ of dimension $nbus$

$A_{weight} =$ (3.4)

		nbus	nbus	nbus	nline
	nbus	O	$-MM^{-1}$	O	O
$A_{weight} =$	nbus	MM^{-1}	$-\varepsilon_1 * I$	O	O
	nbus	O	O	$-\varepsilon_2 * I$	O
	nline	O	O	O	$-\varepsilon_3 * I$

The state equations are numerically integrated in the MATLAB environment, using the Ode15s routine due to the stiffness of the system. Initial conditions are chosen along with the A_{weight} matrix. The solutions for each state variable are shown on separate plots as a function of time _ the time range of interest is several seconds, up to tens of minutes.

Several configuration of the weighting matrix A_{weight} (as described in Appendix B) are used in initial simulations to insure that the system converges and does so rapidly.

3.3 Generator Voltage Magnitudes

The 14-node bus system consists of a number of generator buses, loads and one slack (or reference) bus. The generator buses have constant voltage magnitude which is maintained using voltage regulator, while the slack bus balances the active and reactive power in the system. The load bus is generally referred to as P-Q bus and the generator bus as P-V bus.

Because the voltage magnitudes for the generator buses are constant we can reduce the dimension of the power system by getting rid of the lines and columns corresponding to voltage magnitudes of these buses.

3.4 Voltage Angle Reference

The model we consider for our 14-bus system uses angles differences at each bus. These angles differences are taken with respect to a reference angle, that of bus 1. Using angle differences is advantageous because the power flow equations are functions of angle differences, and this allows us to downsize the system further by eliminating the row and column corresponding the angle of bus 1. Getting rid of the reference angle also eliminates an inherently zero eigenvalue of the Hessian matrix. With the modification, the differential equations that govern this system can be written as follows:

Given

$$\vec{v}_{bus} = |\vec{v}| * e^{-j\delta}, \tag{3.5}$$

$$\delta = \text{Angle at each bus}$$

Define $\alpha_k = \delta_k - \delta_1 \;\; with \;\; \alpha_1 = 0$

Then for any $i, j \in \{1, \dots n\}$

$$\delta_i - \delta_j = \alpha_i - \alpha_j \tag{3.6}$$

The differential equations for α are:

$$\dot{\alpha}_1 = 0$$
$$\dot{\alpha}_k = \dot{\delta}_k - \dot{\delta}_1 \tag{3.7}$$

In matrix form:

$$\underline{\dot{\alpha}} = \left[\begin{array}{c|c} 0 & \cdots \quad 0 \\ \hline -1 & \\ \vdots & \mathrm{I}_{(n-1)x(n-1)} \\ -1 & \end{array}\right] \tag{3.8}$$

The state space system now takes the form:

$$\begin{bmatrix} \dot{\omega} \\ \dot{\alpha} \\ |\dot{v}| \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & T_{mat} & & \\ & & 1 & \\ & & & 1 \end{bmatrix} A_{weight} * \nabla\Phi(\omega, \vec{v}, \gamma) \tag{3.9}$$

Given that:

$$\mathrm{T_{mat}} = \left[\begin{array}{c|c} 0 & \cdots \quad 0 \\ \hline -1 & \\ \vdots & \mathrm{I}_{(n-1)x(n-1)} \\ -1 & \end{array}\right] \tag{3.10}$$

4 SIMULATIONS AND RESULTS

4.1 Initial Conditions

The previously calculated system equilibrium serves as a starting point for the initial conditions. This equilibrium point is perturbed by a different amount for each state variable according to the magnitude and order of the state variable. We generate a random value d_i then set initial condition:

$$\underline{x}_{(t=0)} = x_{equilibirum} + \varepsilon_i \frac{d_i}{\|d_i\|} \tag{4.1}$$

$d_i =$ Vector of normally distributed random values between 0 and 1

$$\varepsilon_i = \text{Scaling factor} = \begin{cases} 10^{-3}: 10 : 1.0 & for\ \ \omega, \alpha, |v| \\ 10^{-7}: 10 : 10^{-4} & for\ \ \gamma \end{cases} \tag{4.2}$$

The matrix whose columns are these initial conditions is saved to the Matlab workspace as **vecIC**; the **ode15s** solver is executed for each of these initial condition vectors.

4.2 Interpretation of Plots

We run simulations with several of the initial conditions previously calculated. The solutions to the ode15s solver converge to a stable value for all of the state variables. To illustrate figures are shown with initial conditions corresponding the most perturbed vector of **vecIC** matrix, i.e. the vector with the largest deviation from equilibrium within the initial condition matrix. This corresponds to perturbation of ± 1 for $\omega, \alpha, |v|$ and $\pm 10^{-4}$ for γ (gamma).

We let the ode solver run for t=1500 secs. We note if the system returns to a stable trajectory, and for all but one initial condition vector, a steady state value is in fact reached for all of state variables. To make the figures more readable, we show $\omega, \alpha, |v|$ for the first 5 buses:

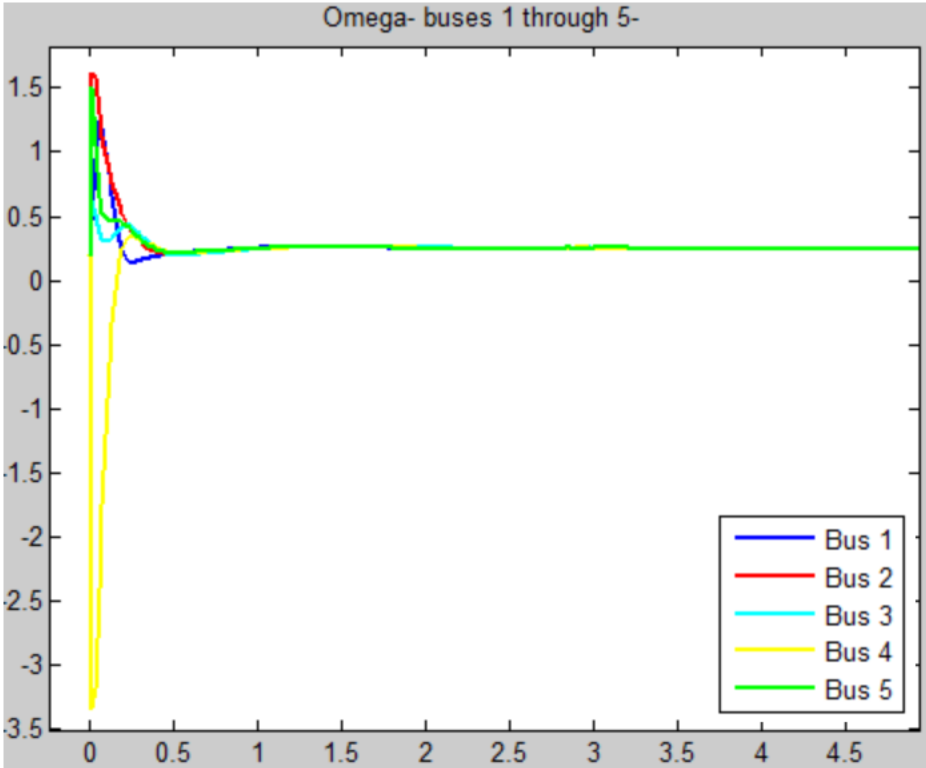

Figure 4.2.1: Bus Frequency ω vs. time (secs)

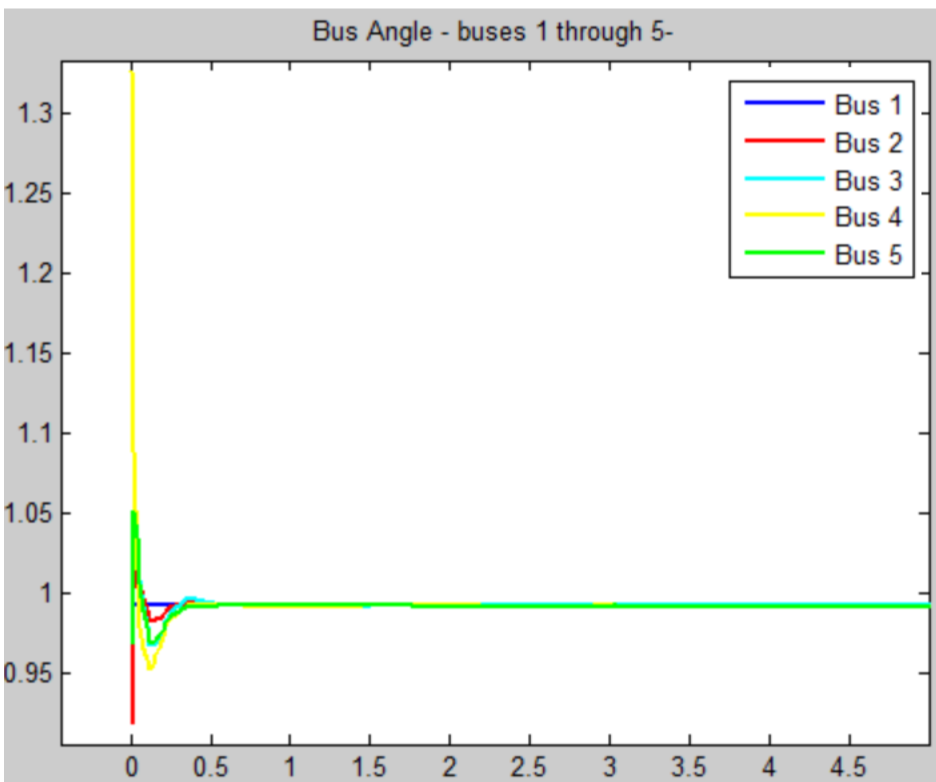

Figure 4.2.2: Bus Angle α vs. time (secs)

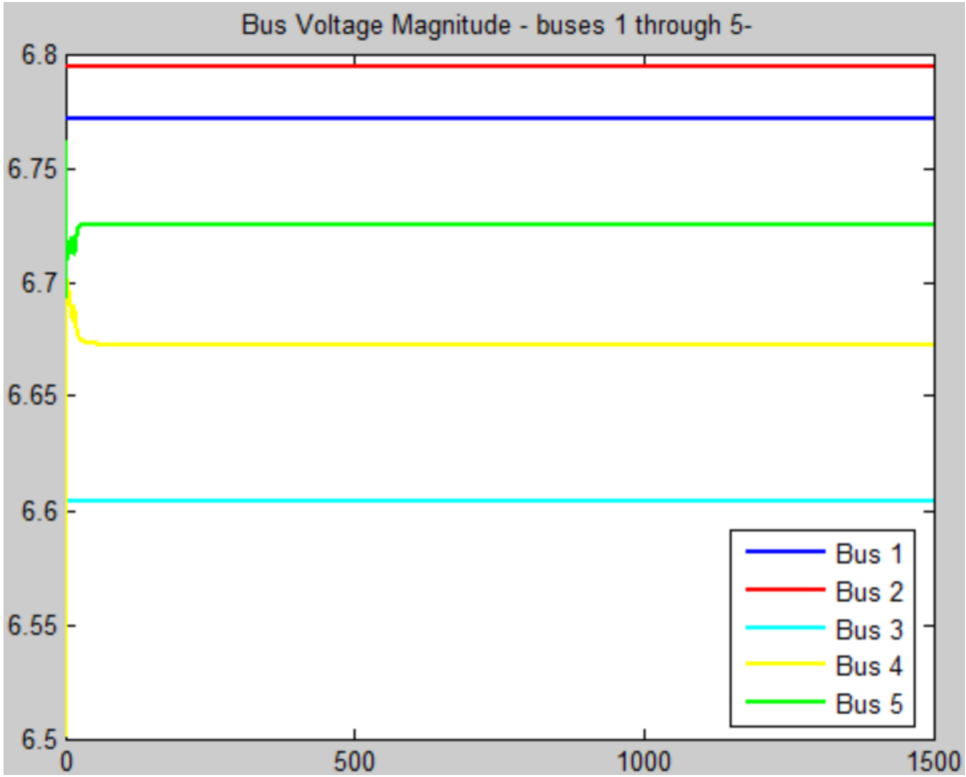

Figure 4.2.3: Bus Voltage Magnitude $|v|$ vs. time (secs)

The solutions to the set of differential equations with respect to transmission line control variable γ are approximately 1 and remain so for the duration of the simulation. This means that none of the transmission lines fail. Values of γ are shown for 5 transmission lines or branches.

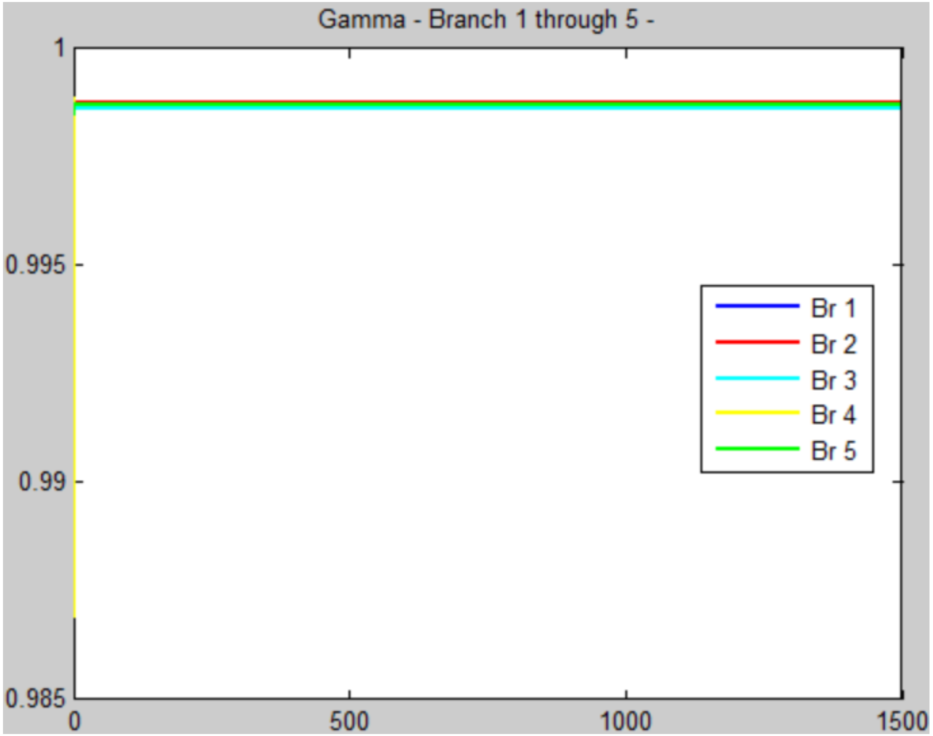

Figure 4.2.4: Line Failure Indicator γ vs. time (secs)

The variable gamma (γ) associated with the failure of a branch or transmission line, is not perturbed by a large epsilon due the nature of the function $\theta(\gamma)$. Any variation of γ larger than $\pm 10^{-4}$ lead to unstable results. In fact to guarantee that all the transmission lines will remain connected in our 14-bus system, γ cannot be perturbed by more than $\pm 10^{-4}$. Evidence of this can be seen when we run the simulation with initial conditions corresponding to a perturbation of 10^{-3} of state variable γ; this leads to failure of transmission line 1:

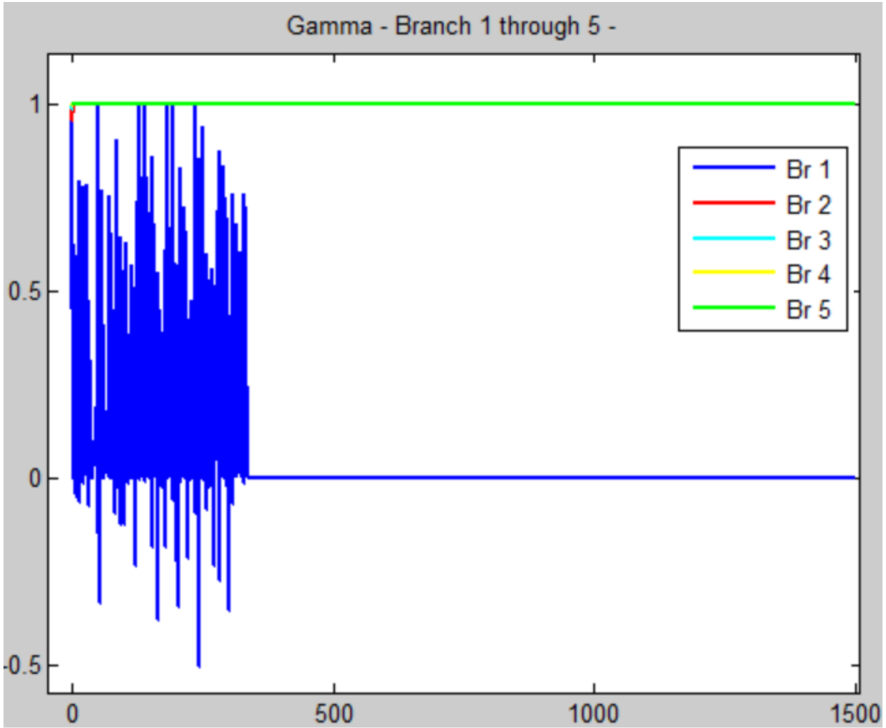

Figure 4.2.5: Line Failure Indicator γ vs. time (secs)

4.3 Induced Transmission Line Failure

Starting from a vector of initial conditions for the state variables, we can specify γ , the indicator associated with every line, to disconnect one or more branches within the 14-bus system. First we verify that our model is successful in disconnecting the specified line(s) by running the simulation with 1 transmission line (or branch) set to fail, then with 2 transmission lines set to fail.

Then, two important analysis questions arise:

- Are we able to maintain a stable system with these predetermined transmission line failures?
- Does the predetermined disconnection of one or more transmission line lead to a cascading failure of other transmission lines?

A. Disconnecting One Branch

We check that we can control the connectivity of every transmission line within our power system by setting the value of $\gamma_k = 0$ in the initial conditions, forcing the k^{th} branch to be disconnected. First, we use the equilibrium vector as initial conditions, and our results successfully show that the specified branch disconnects, while the rest of the transmission lines remain connected.

However, when we perturb the equilibrium vector and use that as initial values for the ode solver, we obtain different results:

Case 1: The k^{th} branch successfully disconnects, and the system converges to a stable equilibrium.

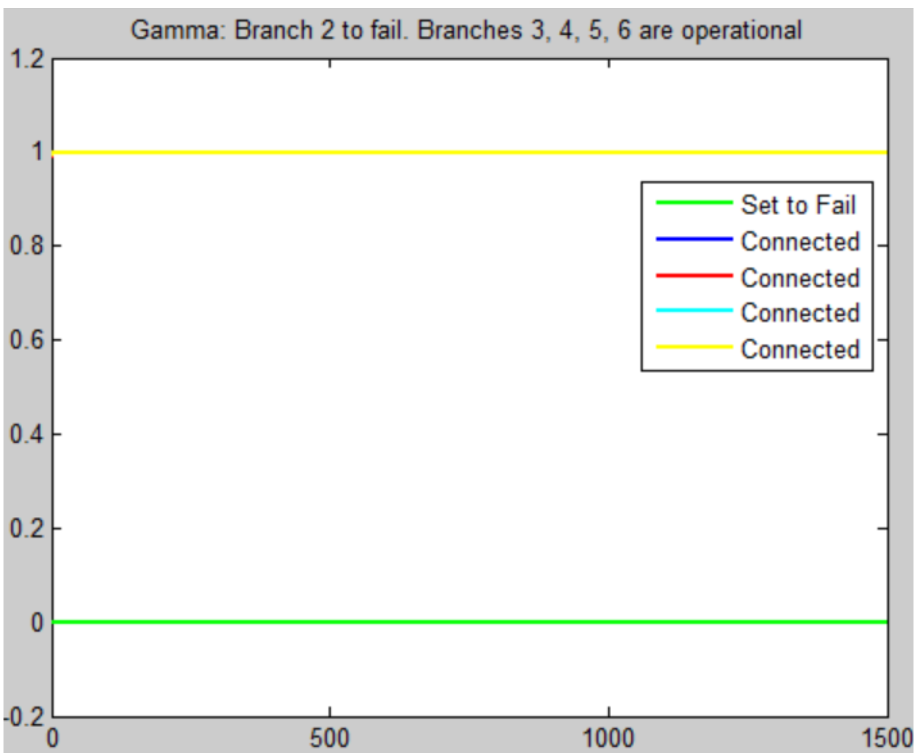

Figure 4.3.1: Line Failure Indicator γ vs. time (secs)

We verify that the connected branches are in fact all connected:

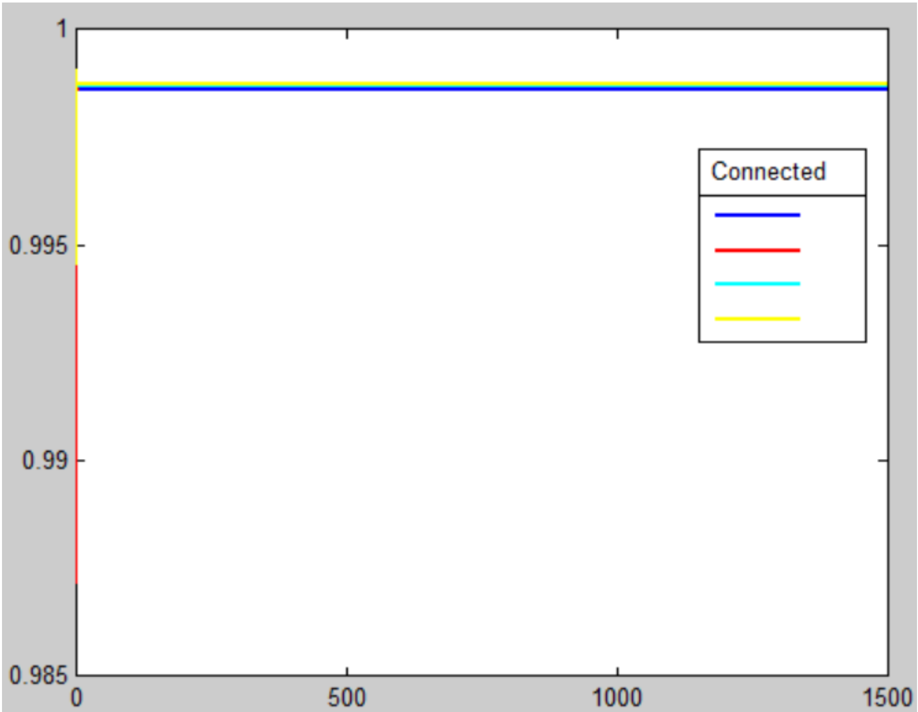

Figure 4.3.2: Line Failure Indicator γ vs. time (secs)

In the following figure, setting a branch to fail leads to the temporary removal of another transmission line during the course of the simulation interval:

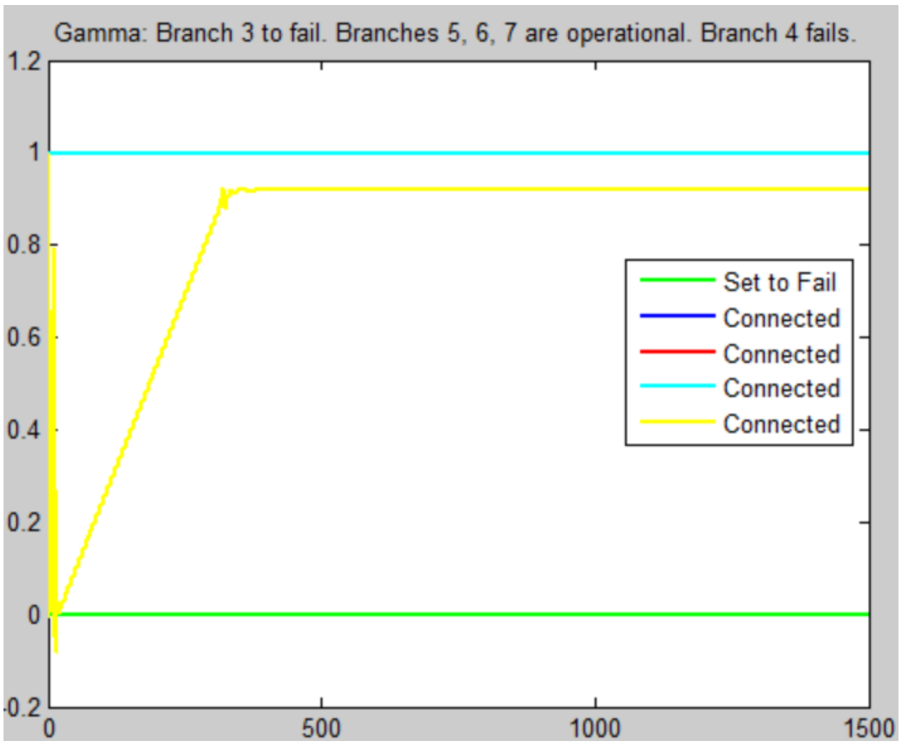

Figure 4.3.3: Line Failure Indicator γ vs. time (secs)

Case 2: The k^{th} branch successfully disconnect, but leads to the failure of other transmission lines. The other state variables still reach an equilibrium.

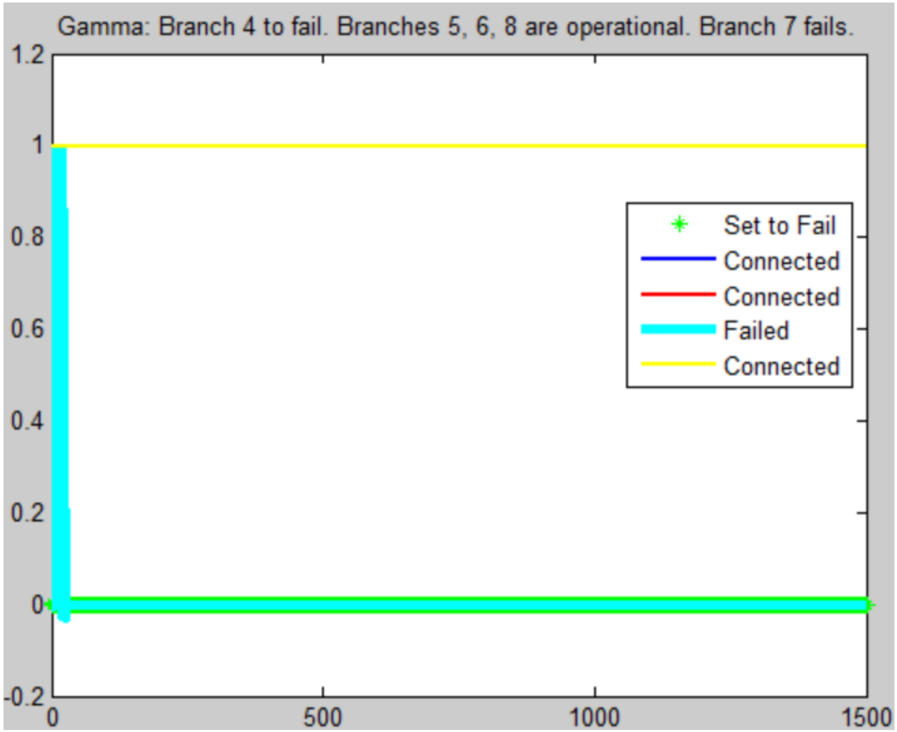

Figure 4.3.4: Line Failure Indicator γ vs. time (secs)

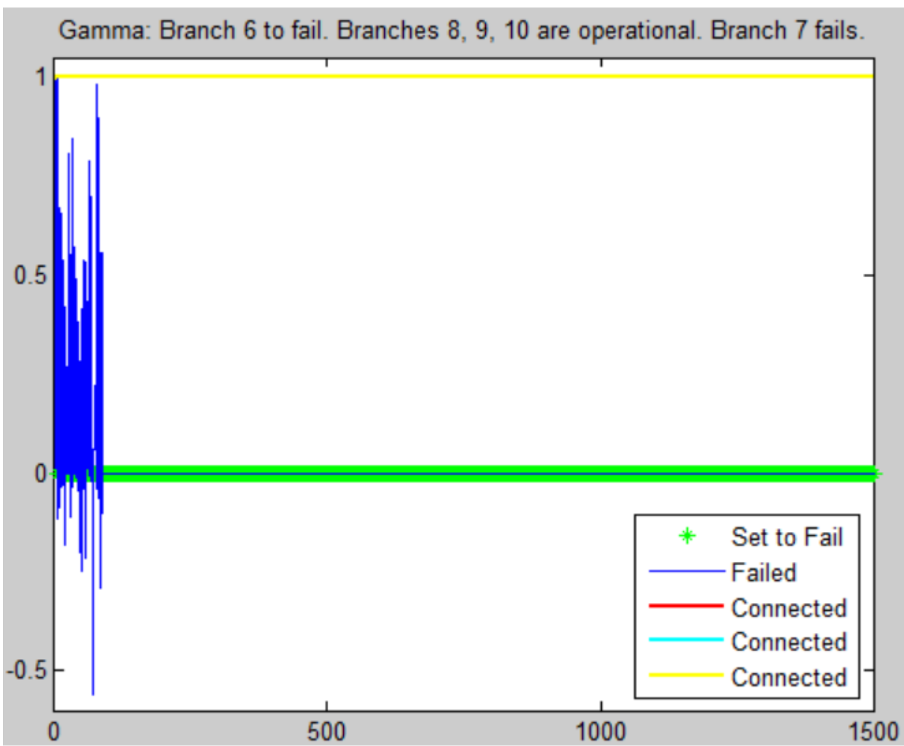

Figure 4.3.5: Line Failure Indicator γ vs. time (secs)

In the previous two figures, setting one transmission line to fail results in another transmission line failure. However, the other state variables still converge to an equilibrium. For instance:

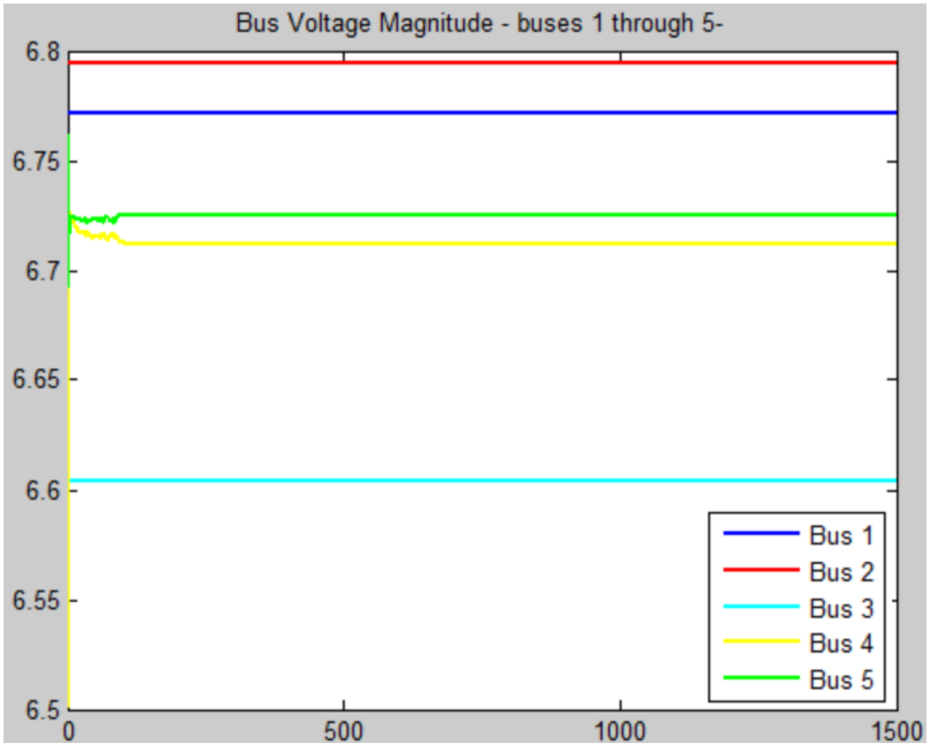

Figure 4.3.6: Bus Voltage Magnitude $|v|$ vs. time (secs)

Case 3: The k^{th} branch successfully disconnect, but leads to the failure of other transmission lines, causing the overall system to be unstable.

Example: Setting branch 16 to fail leads to an unstable system. The simulation terminates at t=400 secs.

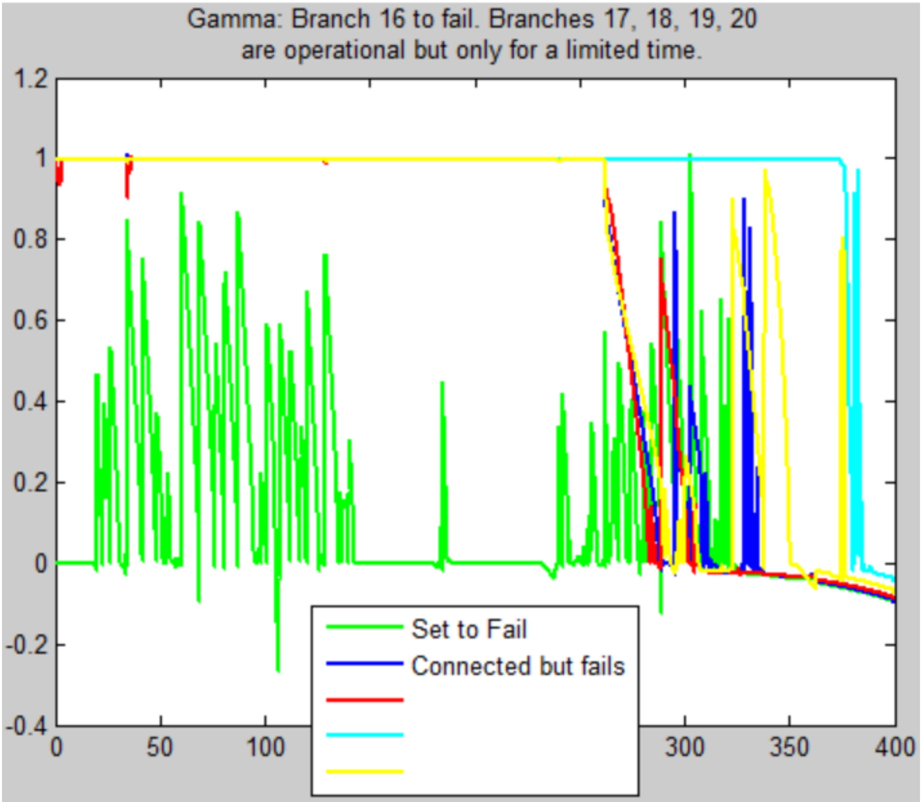

Figure 4.3.7: Line Failure Indicator γ vs. time (secs)

The remaining state variables $(\omega, \alpha, |v|)$ do not converge, but run off to infinity. For instance:

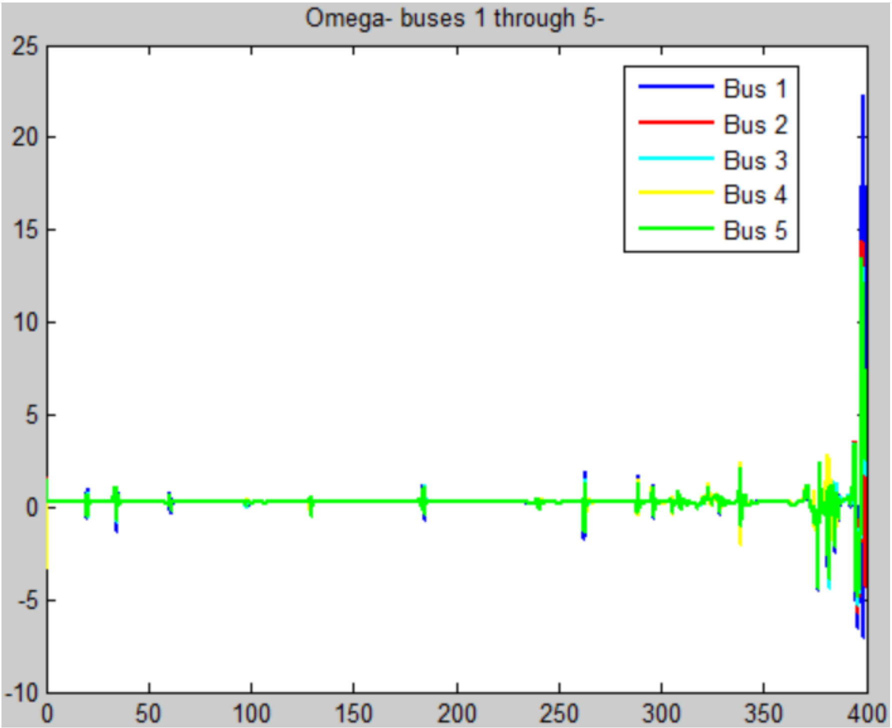

Figure 4.3.8: Bus Frequency ω vs. time (secs)

B. Disconnecting Two Branches

We run the ode solver with $\gamma_{k1} = \gamma_{k2} = 0$, i.e., we select the initial conditions which disconnect branches $k1$, and $k2$. Given the role different transmission lines play in the topology of the network, results vary considerably depending on which combination of branches we choose to disconnect.

Case 1: Successfully disconnect both transmission lines. Other transmission lines remain connected.

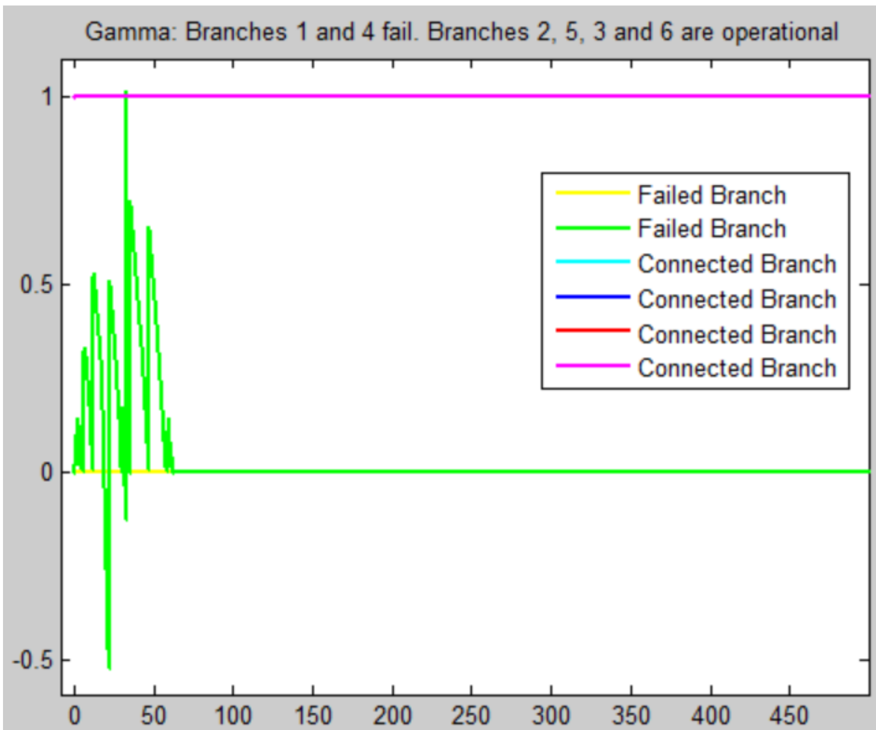

Figure 4.3.9: Line Failure Indicator γ vs. time (secs)

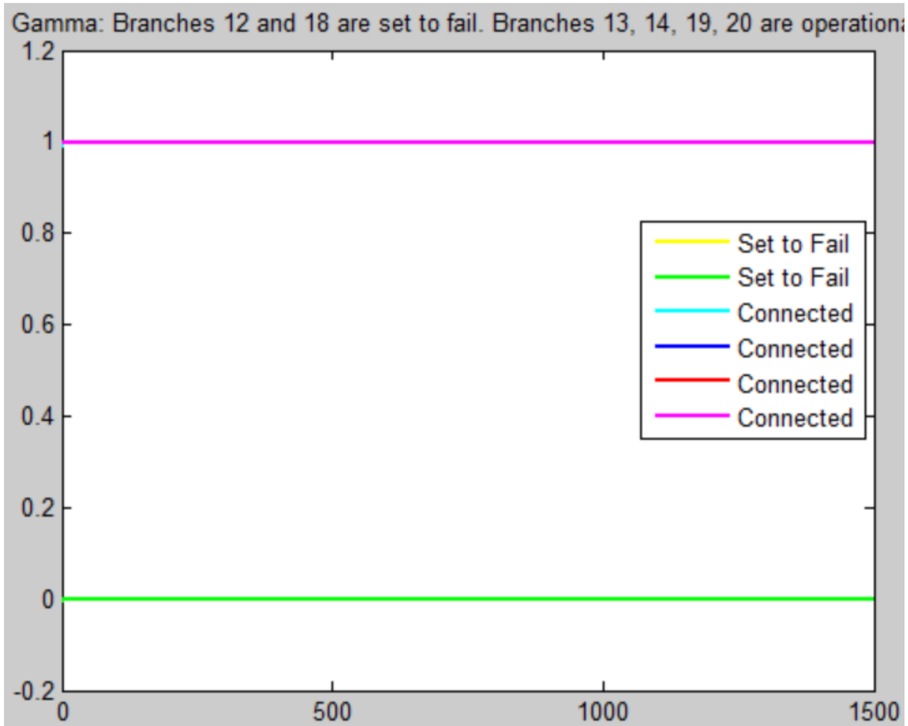

Figure 4.3.10: Line Failure Indicator γ vs. time (secs)

Case 2: Successfully disconnect both transmission lines. One or more transmission lines fail as a result.

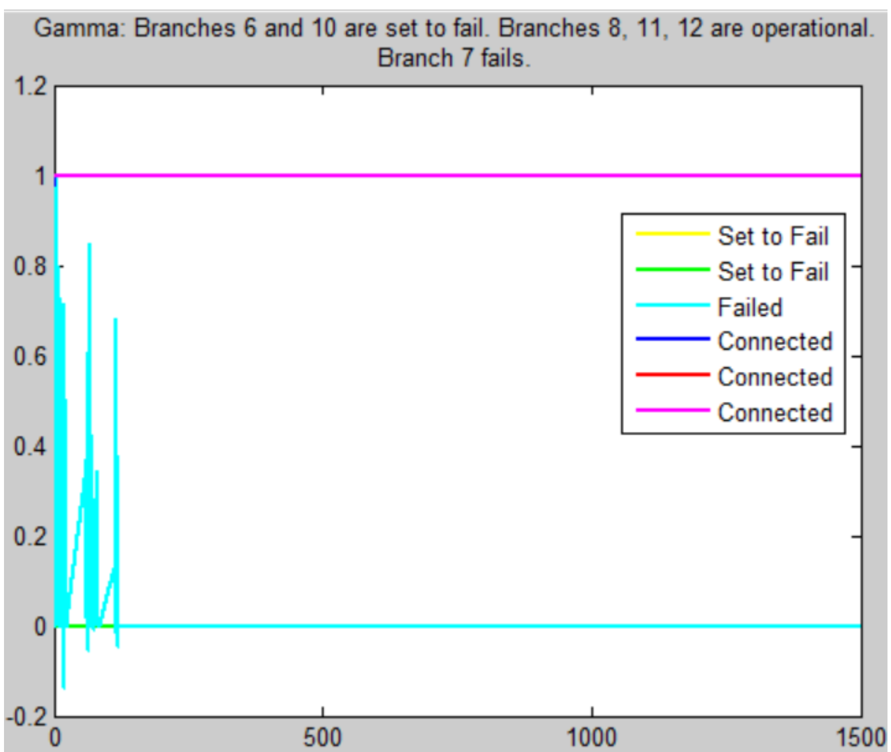

Figure 4.3.11: Line Failure Indicator γ vs. time (secs)

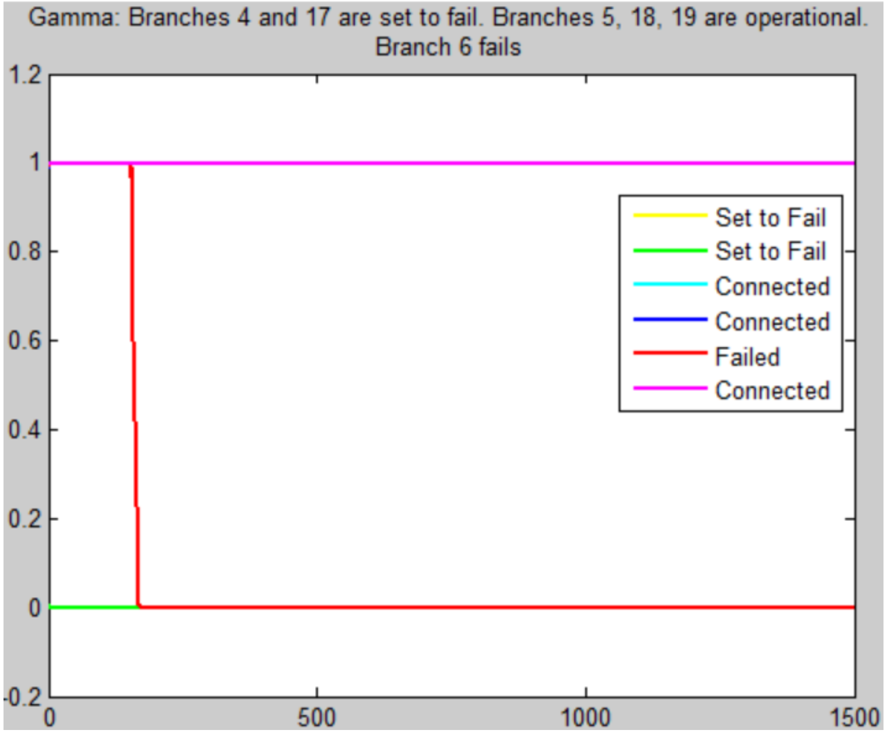

Figure 4.3.12: Line Failure Indicator γ vs. time (secs)

When we set branches 4 and 17 to fail; branch 6 fails as well. This doesn't impact the other state variables. For example:

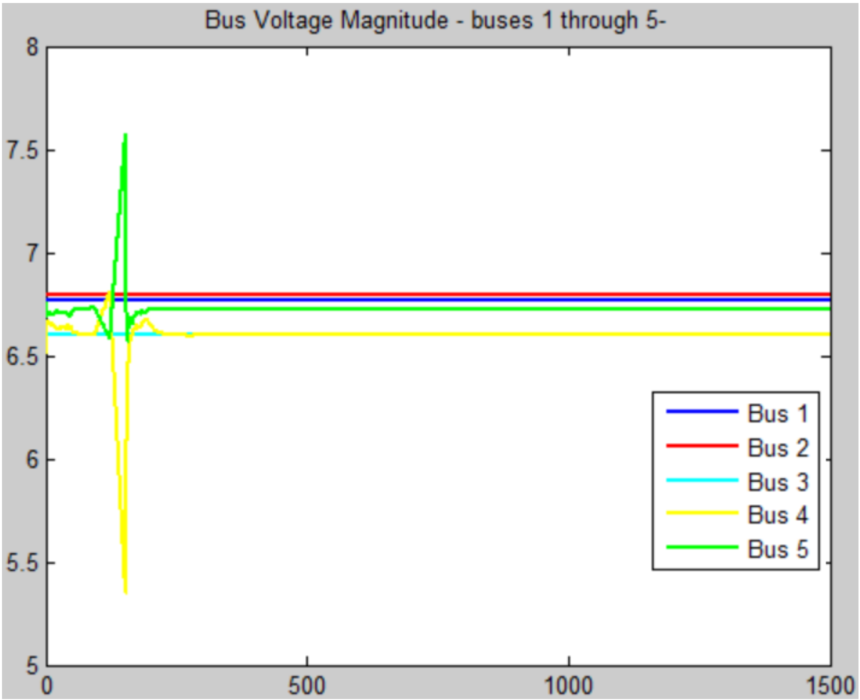

Figure 4.3.13: Bus Voltage Magnitude $|v|$ vs. time (secs)

Case 3: Two transmission lines are set to disconnect but only 1 remains disconnected; γ for one of the branches reverts to a value of ~ 1.

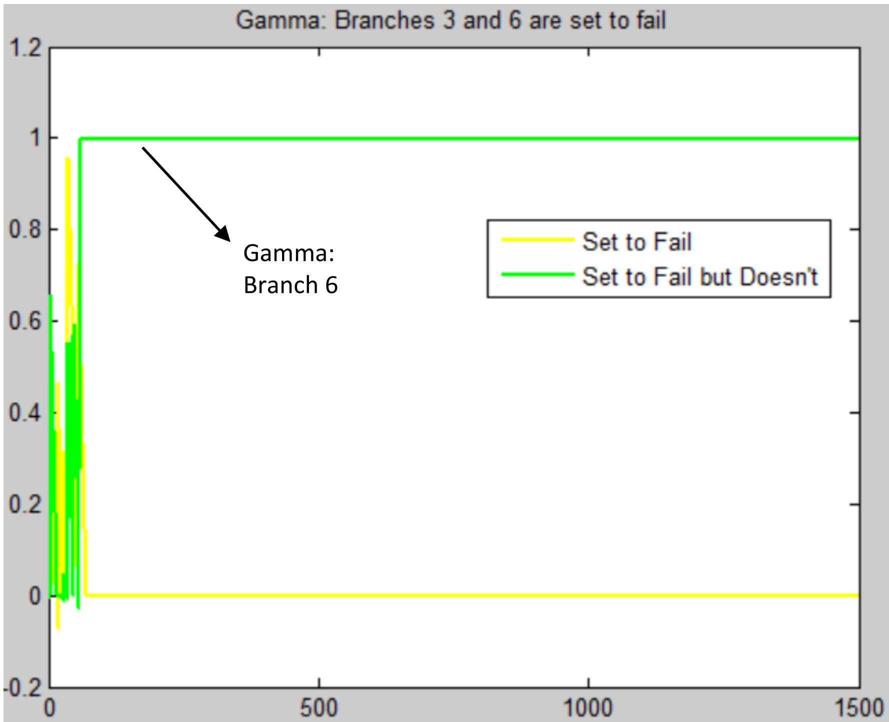

Figure 4.3.14: Line Failure Indicator γ vs. time (secs)

Examining the connectivity of the transmission lines neighboring branches 3 and 6, we note that branch 7 disconnects temporarily:

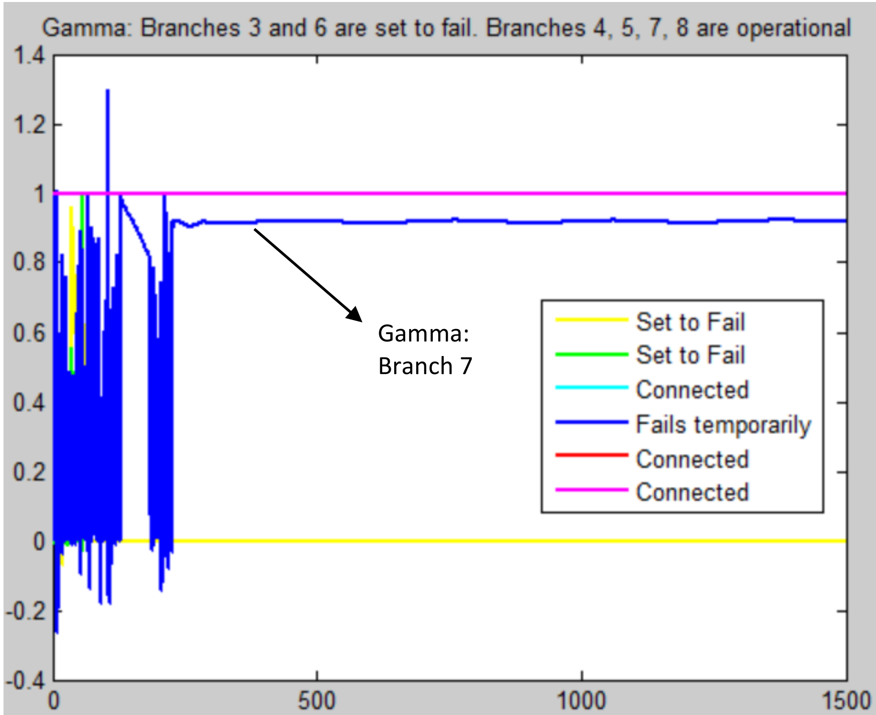

Figure 4.3.15: Line Failure Indicator γ vs. time (secs)

In this case, all the other state variables still reach an equilibrium, for instance:

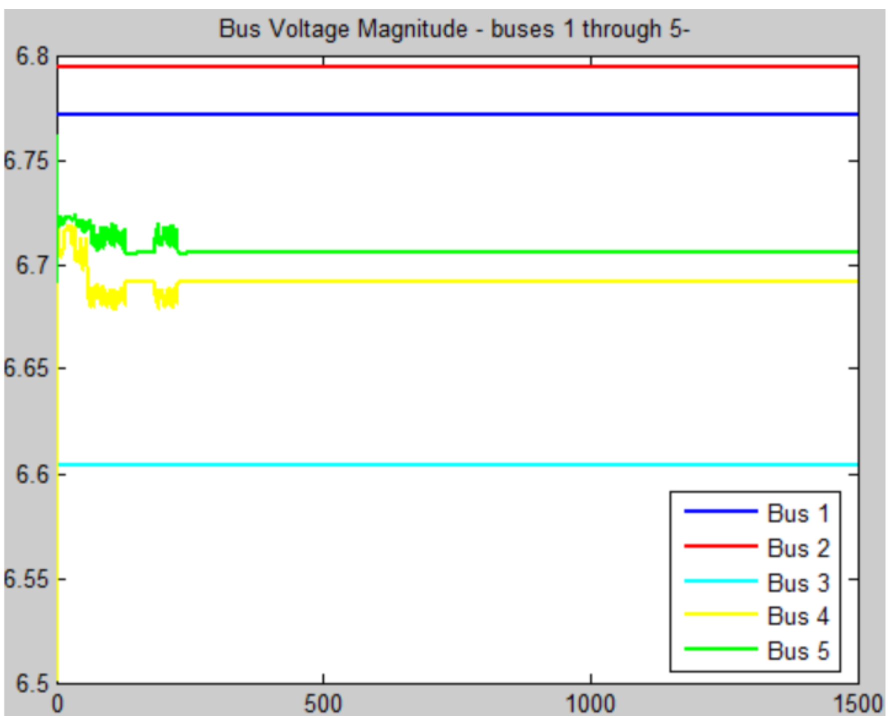

Figure 4.3.16: Bus Voltage Magnitude $|v|$ vs. time (secs)

Case 4: Two transmission lines are set to disconnect; several branches fail as a result making the system unstable.

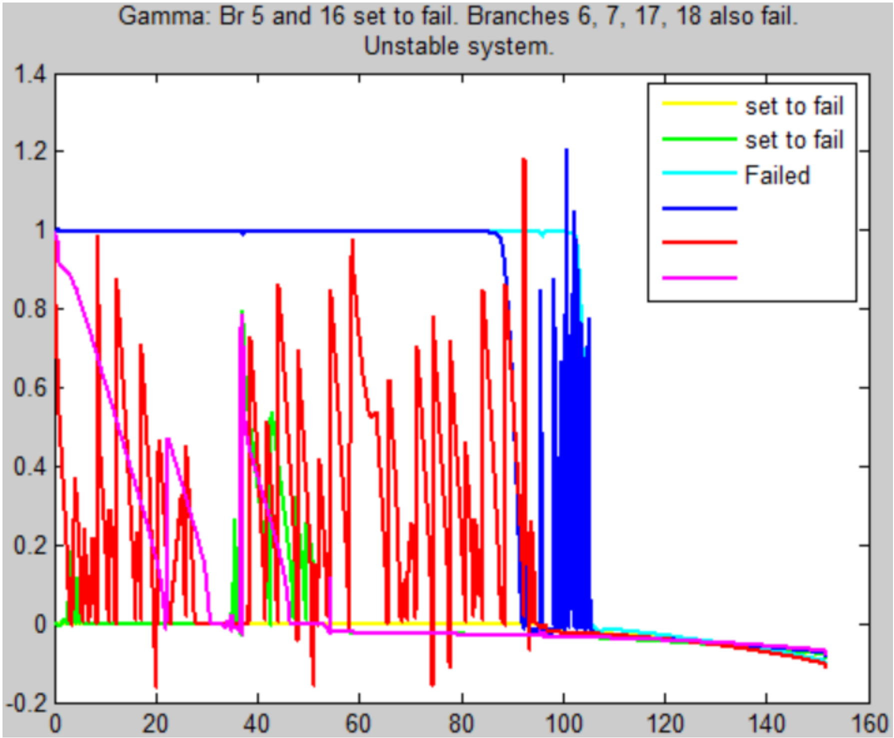

Figure 4.3.17: Line Failure Indicator γ vs. time (secs)

However, for the same two branches set to disconnect, if the equilibrium vector is used as initial condition, we obtain a stable system where all of the other branches are operational:

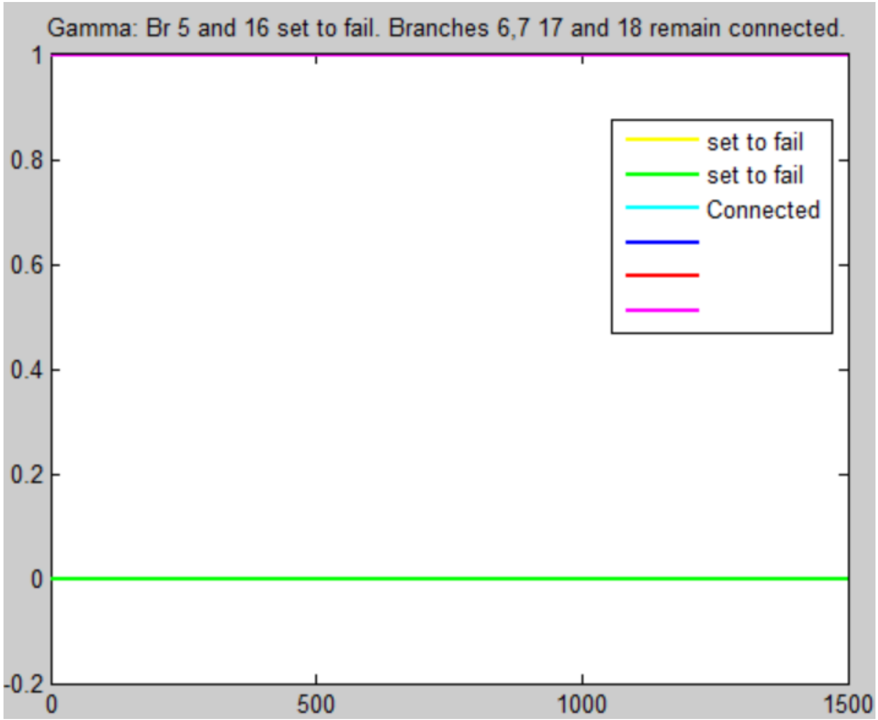

Figure 4.3.18: Line Failure Indicator γ vs. time (secs)

4.4 Right Hand Side of $\dot{\gamma}$

The differential equation for the state variable γ is:

$$\dot{\gamma} = -\nabla\theta(\gamma, thresh) + \frac{1}{2}\left(AA'\vec{v}_{bus}\right)^{*} AA'\vec{v}_{bus}\, bb \tag{4.3}$$

We can distinguish between two parts in the right hand side of the above equation. The first is the gradient of $\theta(\gamma)$ and the other is related to Φ or the power flow equations.

Let $f_1 = \nabla\theta(\gamma, thresh)$

And $f_2 = \frac{1}{2}\,(AA'\vec{v}_{bus})^{*}\, AA'\vec{v}_{bus}\, bb$ (4.4)

We examine f_1 and f_2 as a function of time with two initial conditions: equilibrium vector, and vector of values perturbed away from equilibrium. We note whether γ reverting back to 1 is reflected by the RHS of its differential equation.

Case 1: γ_k is set to 0 and doesn't revert back to 1.

<u>Example</u>: Branches 3 and 10 are set to disconnect

With the initial conditions being the equilibrium vector, the values for f_1 and f_2 are close to zero, as expected (Fig. 4.4.1). However when we use a column from the vecIC matrix (values perturbed from equilibrium), the term f_2 is significantly larger (Fig. 4.4.2).

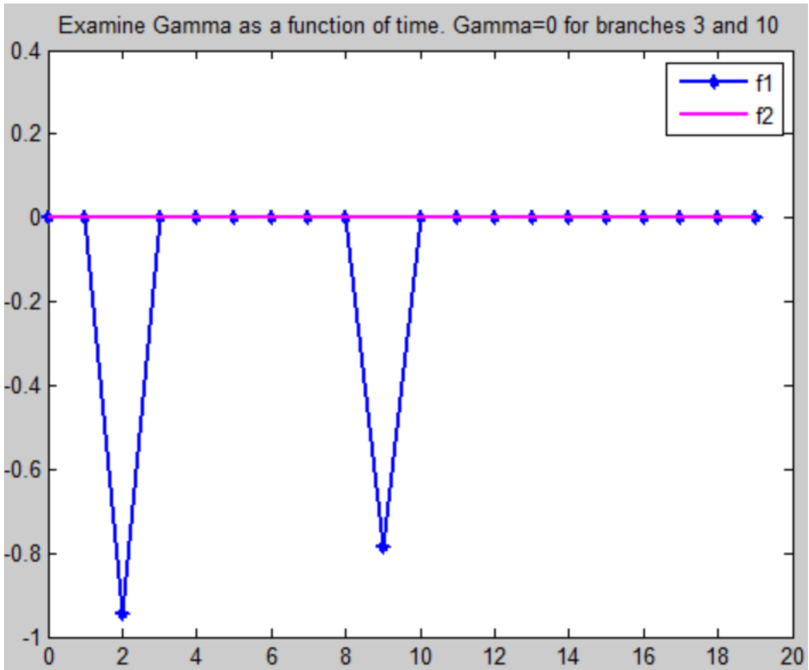

Figure 4.4.1: RHS of $\dot{\gamma}$ vs. time (secs)

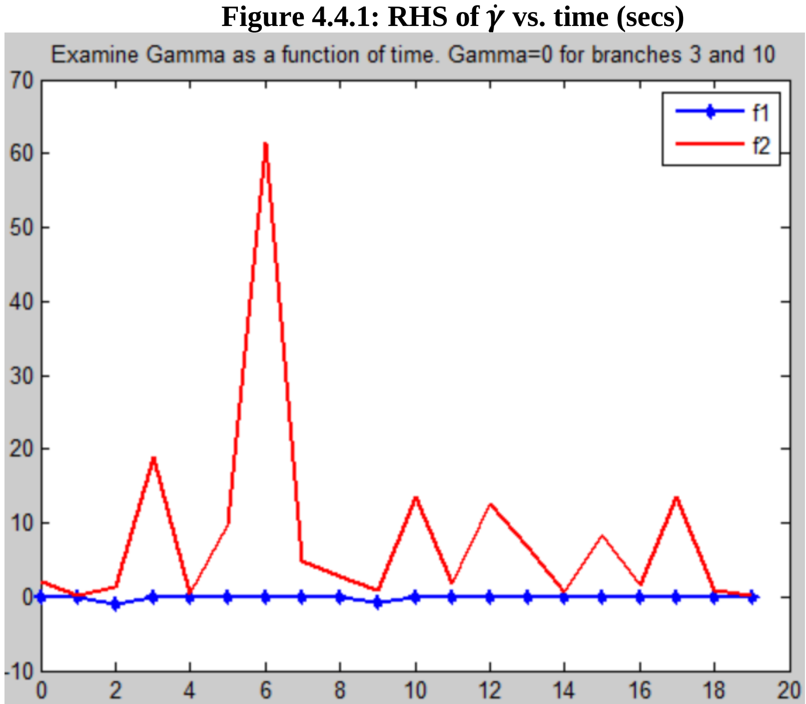

Figure 4.4.2: RHS of $\dot{\gamma}$ vs. time (secs)

Case 2: γ_k is set to zero but reverts to a value ~ 1 during the simulation.

Example: Branches 3 and 6 are set to disconnect. We show f_1 and f_2 as functions of time with initial conditions set as **vecIC(: , 19)**, i.e. vector of values perturbed from equilibrium.

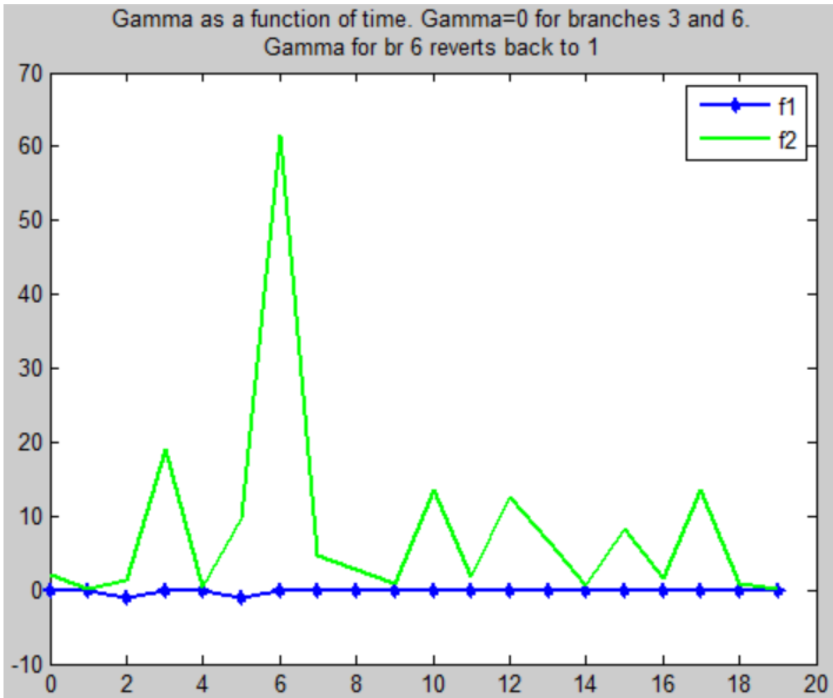

Figure 4.4.3: RHS of $\dot{\gamma}$ vs. time (secs)

5 CONCLUSION

A power system consists of many components that can experience disturbances, which then push the system away from its initial equilibrium, and raise the question of whether the system state will converge back to an acceptable operating condition. This makes the stability analysis and control of the power system challenging. We simulated the occurrence of a fault within a small power network, consisting of 14 buses and 20 transmission lines. When a branch is disconnected, our simulations revealed that several outcomes are possible. Depending on which transmission line(s) are set to disconnect, we can drive the system away from an initial stable operating trajectory. In many cases, disconnecting a branch has led to a cascade of transmission line failures. In order to provide further insight into system's ability to reach a stable operating point following a fault, we will examine a larger bus system next. The model that we developed using function Φ to examine the 14-bus system can be applied to a larger and more realistic power network. The analysis of a larger system (e.g. 145-bus system) will likely shed light on other issues regarding stability and cascading failures.

6 APPENDIX

A. Hamiltonian Formulation

This formulation relates to the Lagrangian via the following Legendre transformation:

$$\mathcal{H}(q_i, p_i, t) = \sum \dot{q}_i p_i - L(q_i, \dot{q}_i, t) \tag{A.1}$$

In the notation corresponding to our n-bus system:

$$\mathcal{H}\left(\tilde{\delta}_i, \widetilde{\omega}_i\right) = \sum \widetilde{\omega}_i M_i \widetilde{\omega}_i - L(\tilde{\delta}_i, \widetilde{\omega}_i) \tag{A.2}$$

B. A_{weight} Matrix Configurations

First, diagonal A_{weight} with all negative entries; this simplifies the state space significantly insuring a solution that converges.

		nbus	nbus	nbus	nline	
	nbus	$-\varepsilon_0 * I$	O	O	O	
A_{weight} =	nbus	O	$-\varepsilon_1 * I$	O	O	(B.1)
	nbus	O	O	$-\varepsilon_2 * I$	O	
	nline	O	O	O	$-\varepsilon_3 * I$	

Where: $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.05$

Secondly, A_{weight} is non diagonal, with values of epsilon $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.9$. The larger values of epsilon would lead to a more stable system; this is expected as they represent the damping of the system.

		nbus	nbus	nbus	nline	
	nbus	$-\varepsilon_0 * I$	$-MM^{-1}$	O	O	
A_{weight} =	nbus	MM^{-1}	$-\varepsilon_1 * I$	O	O	(B.2)
	nbus	O	O	$-\varepsilon_2 * I$	O	
	nline	O	O	O	$-\varepsilon_3 * I$	

7 REFERENCES

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