Markov Chains and Student Academic Progress

Jessica Porath ❖ Department of Mathematics ❖ University of Wisconsin-Eau Claire
Faculty Mentor: Dr. Don Reynolds

Abstract

With data obtained from enrollment records, absorbing Markov chains are used to model the academic progress of students attending the University of Wisconsin-Eau Claire over a specific period of time. Useful statistics, such as the amount of time a student enrolling in UWEC as a freshman can expect to spend before graduating, are derived from the Markov model. With this information, the University may more accurately measure the academic progress of its students, and thus better reflect on its own institutional effectiveness.

Background

An **absorbing Markov chain** consists of:

- A list of a finite number, r, of **states** of existence from which there are periodic transitions
  - At least one of these states is an **absorbing state** where, once reached, it is impossible to leave.
  - Transition from each non-absorbing state to some absorbing state is possible and the probability that the process will eventually reach an absorbing state is 1.
- A **transition matrix** $P = [p_{ij}] \in M(r,r)$
  - The entry p_{ij} is the probability of moving from the i^{th} state to the j^{th} state.
  - If the states are numbered so that the absorbing states are listed first then the transition matrix will be of the following form:

$$P = \left[\begin{array}{c|c} I & R \\ \hline \mathbf{0} & Q \end{array}\right]$$

  - Then the matrix $P^n = \left[\begin{array}{c|c} I & R + RQ + \cdots + RQ^{n-1} \\ \hline \mathbf{0} & Q^n \end{array}\right]$ gives the probabilities of moving between states after n steps.
  - Thus, the Q^n submatrix is the portion of the transition matrix corresponding to the nonabsorbing states such that each entry gives the frequency of a visit to that position after n steps.
  - In order to calculate the total number of visits to a specific state until absorption, we need to sum the same entry for every possible power of Q. Thus, if the system begins in the j^{th} nonabsorbing state, the expected number of visits to the i^{th} nonabsorbing state is the *ij*-entry of $I + Q + Q^2 + Q^3 + \ldots$
  - This series of matrices will converge in the sense that the entries of the partial sums each converge.
  - In strict analogy with the convergence of the geometric series of real numbers, this geometric series of matrices converges to $(I-Q)^{-1}$ (called the **fundamental matrix**) provided that each entry in Q is less than 1.

Developing the Transition Matrix

- Four **non-absorbing states**:
  - Freshman Class (fewer than 30 credits)
  - Sophomore Class (30 to 59 credits)
  - Junior Class (60 to 89 credits)
  - Senior Class (90 or more credits)
- One **absorbing state** (way a student may leave the system):
  - Graduation
- Data files were obtained from the UWEC Registrar's Office that identified the entrance year and the class rank for every student enrolled from 1997 through 2005. The entries in P were determined by analyzing these data.
- The following assumptions were then made:
  - Only full-time students were considered. A student in the same class (except for the Senior class) for three years was assumed not to be full-time.
  - Students who transferred or dropped out permanently were not considered.
  - Senior class students who failed to return were assumed to have graduated.
- The entries in the **transition matrix** are the weighted averages of the probabilities of moving from one state to another from 1999 to 2005.

P =

		s_5 Gr.	s_1 Fr.	s_2 So.	s_3 Jr.	s_4 Sr.
s_5	Gr.	1	0	0	0	0.6535
s_1	Fr.	0	0.1930	0	0	0
s_2	So.	0	0.7478	0.1391	0	0
s_3	Jr.	0	0.0591	0.7804	0.1049	0
s_4	Sr.	0	0	0.0801	0.8919	0.3465

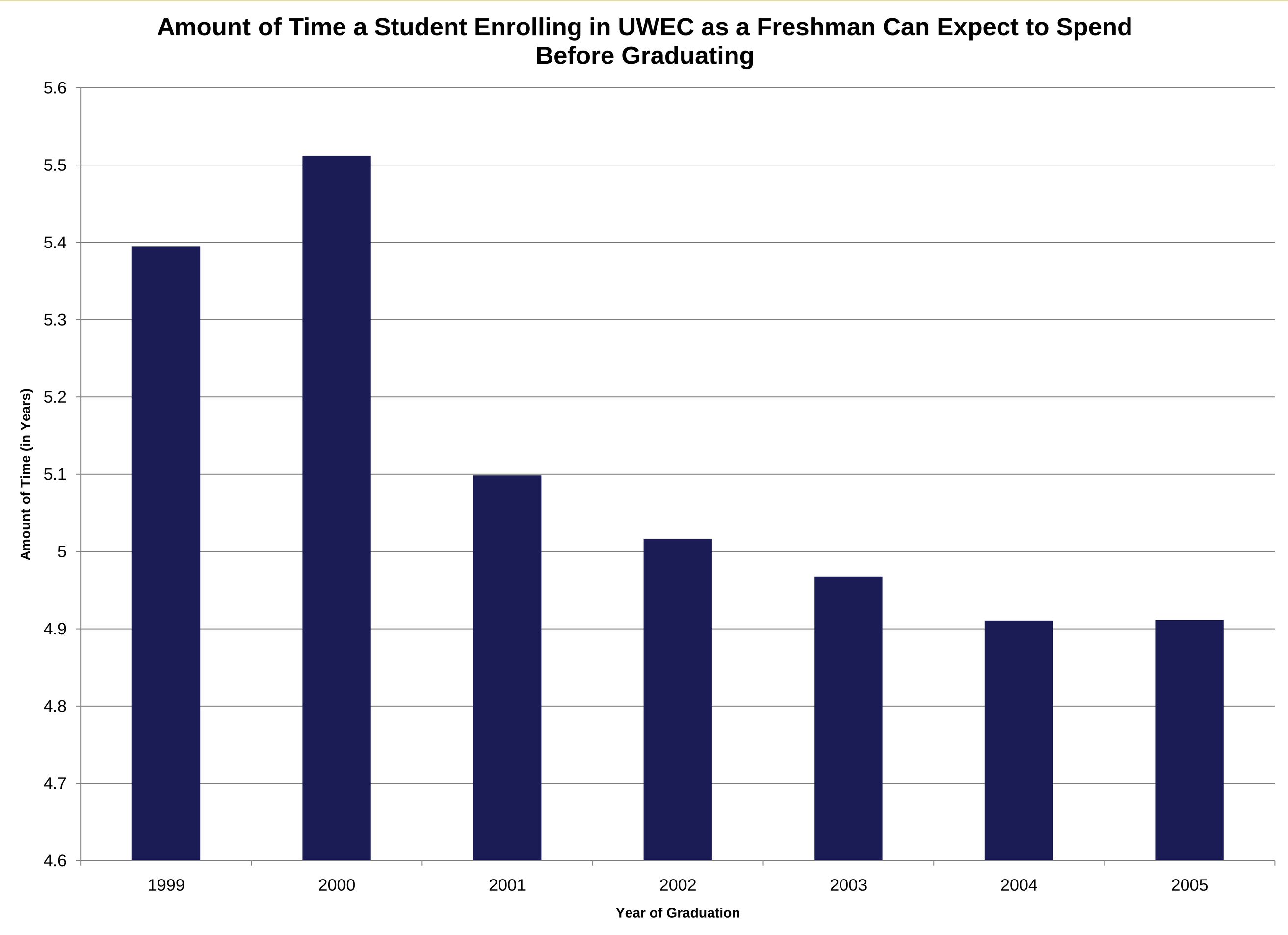
- Then the **fundamental matrix** is:

$$(I-Q)^{-1} = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.1930 & 0 & 0 & 0 \\ 0.7478 & 0.1391 & 0 & 0 \\ 0.0591 & 0.7804 & 0.1049 & 0 \\ 0 & 0.0801 & 0.8919 & 0.3465 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} 1.2392 & 0 & 0 & 0 \\ 1.0764 & 1.1616 & 0 & 0 \\ 1.0203 & 1.0127 & 1.1172 & 0 \\ 1.5244 & 1.5246 & 1.5248 & 1.5302 \end{bmatrix}$$

- The sum of the entries in the diagonal of the fundamental matrix will give us the average amount of time a student enrolling in UWEC as a freshman can expect to spend before graduating.
- 1.2392 + 1.1616 + 1.1172 + 1.5302 = 5.0482 years

Results and Conclusions



- For a student who graduated from 1999 though 2005, the average expected amount of time spent at UWEC was generally decreasing.
- Based on the mean of the data for graduates between 1999 and 2005, a student enrolling in UWEC as a freshman can expect to spend an average of 5.0482 years before graduating.
- The fundamental matrix also gives information about the expected time until graduation for students entering UWEC as a sophomore, junior, or senior.

Future Research Directions

Further studies are being done to investigate how demographics affect a student's academic progress. We are using the same Markov model, but instead of looking at the entire group of full-time students, we will examine specific groups based on gender and ethnicity. Lastly, we will compare the statistics derived from the models of these groups with each other and with the general results we previously calculated.

Sources

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- Messer, Robert. Linear Algebra: The Gateway to Mathematics. New York: Harper Collins College, 1993.
- Williams, Gareth. Linear Algebra with Applications. Massachusetts: Jones and Bartlett Publishers, 2005.

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