IMPACT LOCATION IN AN ISOTROPIC PLATE WITHOUT TRAINING

by

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ABSTRACT

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Unexpected impacts are major concerns in the aerospace industry that can cause difficulty to detect damage. Techniques have been developed to determine the impact location using piezoelectric sensors. Most existing systems require training data to develop a database of known structural responses and properties that can be referenced for location of impacts. This data collection is time consuming and if an impact and corresponding sensor data is outside the range of training data, the system may not be able to analyze it correctly. Some methods use specific sensor positions to reduce this phenomenon. Current systems typically utilize data from 3 or 4 sensors and are dependent on the knowledge of the speed of wave propagation in the material or reference data.

This thesis develops a method of impact detection and location based on hyperbolic positioning suitable for isotropic homogenous plates that does not require training or knowledge of wave speed in the material. This derivation is not dependent on specific sensor layouts though sensor locations must be known and potential certain degenerate cases should be avoided. Equations were developed based on the time difference of arrival of strain waves at sensors with known location for impact location. This technique utilizes data from additional sensors to eliminate the need for training data or known propagation velocity. This technique translates Matlab code that was written based on these equations to automate the calculation and experimental validation that was performed using data from real specimens. Impact position error comparable to prior existing systems was verified.

To

my parents,

my sister,

and my friends

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CHAPTER I: INTRODUCTION

Motivation for Impact Detection and Location:

Impact location and detection has been an important topic for aerospace industry. Impacts can cause local changes in materials properties and hard to detect damage. If not detected and left uninspected, these can lead to catastrophic failure. Therefore, systems to detect and locate impacts are of great interest, because they can instigate and guide inspection to detect damage. The area of research is multi-disciplinary involving theory from Mechanical, Materials, and Electrical Engineering fields.

The aerospace industry is particularly interested in this work as impacts from foreign objects kicked up from a runway or incurred in flight can lead to significant damage. For example, during the launch of the space shuttle Columbia, STS-107 a piece of foam insulation broke off from the Space Shuttle external tank and struck the left wing of the orbiter. The resulting damage resulted in the failure of the shuttles thermal protection panels and the orbiter broke apart on re-entry. A few previous shuttle launches had seen minor damage from foam shedding [1]. An embedded impact detection system could have confirmed the impact and provided the location guiding inspection and preventing the loss.

Impact detection and location systems also have the potential to reduce the weight of aerospace structures, increasing fuel efficiency and capabilities, by reducing the design margins necessary to tolerate damage from undetected impacts.

Background:

Because of the high interest and consequence of this field, a lot of research has already been done on impact detection and location. The most common techniques used are Time difference of Arrival (TDOA) and Hyperbolic Positioning. Time difference of arrival methods are based on measuring the difference in time of the arrival of a strain wave, generated by the impact, at multiple sensors. This difference is the result of differing propagation paths and distances. Based on this difference, information from 2 sensors can map out a hyperbola of possible locations on a plane. Adding a 3rd sensor generates another hyperbola of possible positions that intersects the first hyperbola, potentially at multiple points. Adding sensors adds hyperbolas reducing possible locations. Prior work has depended on knowledge of the strain wave propagation velocity in the structure to generate these hyperbolas to form a unique solution.

The velocity of the signal is typically found beforehand based on an experimental impact.

The technique outlined by Melkonyan, Arsen [2] in his thesis Impact Detection for Structural Health Monitoring, used two sensors to determine the wave velocity in an isotropic plate; an impact was made on one of the sensor, the time of arrival in that sensor was instantaneous and there was a delay for the second sensor. The distance between the sensors was known. Based on these, speed was calculated. A typical impact error of 0.024 m was achieved through this experiment. Data from 50 trials was collected and the Root mean square was used to calculate the location of the impact.

Another experiment was performed in anisotropic in-homogeneous plate using an alternate algorithm. Kundu et al. [3] has proposed a different approach method based on optimizing an objective function. The objective function was defined using the location of the sensors, impact location, times of arrival of the signals and the wave propagation of the signals [3]. The times of arrival were the relative time the strain wave reached each sensor. All of the variables were known except the impact location. A gird of potential impact locations was created and used as a range in the objective function to minimize the error. A metallic ball was used to impact the structure. Altogether 17 different sensors were used, placed in a semi-circle, but only using data from four sensors at a time to calculate the impact location. Speed was calibrated beforehand similarly to what had been done in the thesis before. An impact was made on the center of the plate, time of arrival is instantaneous for that impact, but delayed for the other sensors. Based on the 17 sensors, speed was calculated for each, in each direction, using the time of arrival and the distance between the impact and the sensors. This algorithm was verified experimentally which produced an error of 0.05m. This algorithm was also introduced for anisotropic material because the velocity of the signal is dependent of the direction of strain wave propagation in the composite material.

Another article written by Kundu et al. [4], for an isotropic plate, using three sensors, the source of an acoustic wave can be determined. The three sensors were placed in an L-shaped pattern. With distance between the sensors on the legs of the L being the same. One of the requirements was that the distance between the sensors should be much smaller than the distance between the acoustic sources [4]. The inclination angle between the source and the sensors was calculated and then the wave velocity calculated, and finally the location identified. This technique functionally created a directional sensor system to perform triangulation and ranging simultaneously. The sensor layout and relatively large distance from the impact/emitter are critical to the function of this system. This technique works best on isotropic plates application to anisotropic plates is still under development.

Challenges:

All of the current, existing TDOA systems reviewed require training data or specific sensor layouts. As stated in the background, time difference of arrival and hyperbolic positioning has been the main technique in most of the works to locate impact. But all of them used either three or four sensors. Some used different techniques like triangulation technique or optimizing an objective function to minimize the error in impact location. Training based systems are susceptible to changes in the properties of the material, like temperature [5], that would change wave propagation properties and require new training data. Based on existing systems hyperbolic positioning systems, which utilize either three or four sensors, impact location can be determined. But, wave propagation speed has to be known and is often calculated through calibration and collection of training data.

Collecting accurate training data is an issue because the impacts of interest are potentially damaging to the structure, therefore impacting the structure with similar force and energy would have the potential to damage the structure in training. Differing impacts have the potential to induce waves that propagate at differing speeds. In addition, environmental variations like changes in temperature can cause variations in wave propagation velocity in the same structure. Further complicating the matter, for some modes of wave propagation the propagation velocity is frequency dependent so different impacts have a potential to generate waves propagating at different speeds. Eliminating the need for training data would save significant time and effort and has the potential to overcome the need for compensation for environmental effects like temperature changes.

CHAPTER II: PROBLEM STATEMENT

The goal of this work is to detect and locate impacts on an isotropic plate based on strain wave propagation without foreknowledge of the plate properties or reference data.

- The location of the sensors is known but not specific and velocity of strain wave propagation is assumed to be constant

CHAPTER III: APPROACH

Algorithm Development: An algorithm was developed based on TDOA equations to solve for both position and wave velocity eliminating the need for knowledge of the wave propagation velocity in the plate. This required adding an equation to solve for the additional unknown which was achieved by integrating an additional sensor into the system. The key to this approach is that the fundamental theory is the same as prior work, TDOA methods, but wave propagation velocity is calculated such that the need for calibration or test data is eliminated. The primary assumption was that strain wave travel at the same speed in all direction because of the isotropic nature of the plate.

Experimental Validation: Matlab version 2014b [6] was used to write code based on the algorithm and it was tested on two plates with different thickness and different sensor locations.

CHAPTER IV: MAJOR TASKS

Major tasks for the project falls under two main categories: Algorithm Development and Experimental Validation.

Algorithm Development:

Time Difference of Arrival (TDOA):

A signal and multiple receivers will be applied in this technique. The difference in arrival times of the signals at each receiver are estimated. This is called TDOA. Equations based on the time difference of arrival at two receivers creates a hyperbola of possible locations in 2D. With one additional receiver, another equation is generated and thus another hyperbola. The intersection of the two hyperbolas will result in finite location solutions. Adding sensors add hyperbolas of possible locations and their intersections provide for solution to the system. This method is commonly called hyperbolic positioning. Fig 1 shows an example of a hyperbolic equation solution.

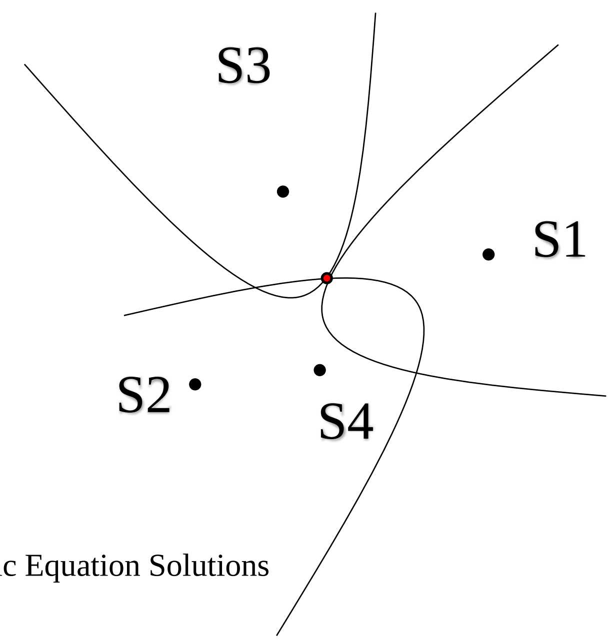

Fig 1: Hyperbolic Equation Solutions

TDOA Position Location Method or Hyperbolic Positioning

Hyperbolic Positioning is done in two phases [7]. During the first phase, the TDOAs of the signals are estimated by the time delay technique between the receivers. During the second phase, TDOAs compared to other receivers and hyperbolic equations are generated. The solutions of the equations generate a calculated position of the impact. The TDOA of a signal can be estimated by subtracting time of arrival measurements from two base stations to produce a relative TDOA [2]. Once the equations and TDOAs have been obtained, algorithms were generated to solve these equations. The initial and final equations are presented here and the full manipulation to produce the final equations is presented in Appendix A.

The derivation began with the relation between time distance and velocity in the form of equation 4.1

$$t_i = t_0 + \frac{D_i}{V} \qquad \text{…………………………………………………………} 4.1$$

Where,

t_i = time of arrival at receiver i

t0 = time at which emitter emits a signal

Di = Distance between emitter and receiver

V = Velocity of the signal

This is taken for two sensors, 1 and 2,

$$t_1 - t_0 = \frac{D_1}{V} \qquad \text{…………………………………………………………}4.2$$

$$t_2 - t_0 = \frac{D_2}{V} \qquad \text{…………………………………………………………}4.3$$

Combining these two and eliminating, t_0 results in 4.4,

$$t_2 - t_1 = \frac{D_2 - D_1}{V} \quad \text{.. 4.4}$$

This is the common form of Time Difference of Arrival equation.

If we use the co-ordinate system now, the equations will be long, so for simplicity, we will use vector form and get two equations in the end which can be solved by the matrix equation solver. Here, we have V, X_0, Y_0, t_0 as unknown.

$$D_i = \left|\vec{P_i} - \vec{P_0}\right| \quad \text{.. 4.5}$$

D_i is the Norm of difference between Position vector of receiver i and emitter 0.

$$\vec{P_i} = (X_i, Y_i) \qquad \vec{P_0} = (X_0, Y_0)$$

Vector P_i and vector P_0 are the co-ordinates of the receiver i and impact source 0 respectively. Now using these equations 4.2 and 4.3 we first square these equations, take the difference and use norm of D, each expanded term here was shown and replaced in equation 4.6, after algebraic manipulation, we get equation 4.7,

$$t_1 - t_0 = \frac{D_1}{V} \qquad t_2 - t_0 = \frac{D_2}{V}$$

$$(t_2 - t_0)^2 - (t_1 - t_0)^2 = \frac{\left|\vec{P_2} - \vec{P_0}\right|^2 - \left|\vec{P_1} - \vec{P_0}\right|^2}{V^2} \quad \text{.................. 4.6}$$

$$(t_i - t_0)^2 = {t_i}^2 - 2t_i t_0 + {t_0}^2$$

$$\left|\vec{P_i} - \vec{P_0}\right|^2 = (X_i - X_0)^2 + (Y_i - Y_0)^2$$

$$\left|\vec{P_i} - \vec{P_0}\right|^2 = ({X_i}^2 + {X_0}^2) + ({Y_i}^2 + {Y_0}^2) - 2(X_i X_0 + Y_i Y_0)$$

$$\left|\vec{P_i} - \vec{P_0}\right|^2 = \left|\vec{P_i}\right|^2 + \left|\vec{P_0}\right|^2 - 2\vec{P_i}^{T} P_0$$

$$\frac{\left|\vec{P_2}\right|^2 - \left|\vec{P_1}\right|^2}{(t_2 - t_1)} - \frac{2\vec{P}_0\left(\vec{P}_2^{\,T} - \vec{P}_1^{\,T}\right)}{(t_2 - t_1)} = V^2(t_2 + t_1) - 2t_0 V^2 \quad \text{...............4.7}$$

This is the final Time Difference of Arrival equation with receivers 1 and 2. With 5 receivers, there are three more equations for a total of 4 equations. The unknowns t_0 and V can be eliminated by combining equations and finally, solve for the vector P_0 using the Matrix Method.

The full derivation of the equations are in Appendix A.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Here,

$$a_{11} = \frac{2(X_2-X_1)}{(t_2-t_1)(t_3-t_2)} - \frac{2(X_3-X_1)}{(t_3-t_1)(t_3-t_2)} - \frac{2(X_2-X_1)}{(t_2-t_1)(t_4-t_2)} + \frac{2(X_4-X_1)}{(t_4-t_1)(t_4-t_2)}$$

$$a_{12} = \frac{2(Y_2-Y_1)}{(t_2-t_1)(t_3-t_2)} - \frac{2(Y_3-Y_1)}{(t_3-t_1)(t_3-t_2)} - \frac{2(Y_2-Y_1)}{(t_2-t_1)(t_4-t_2)} + \frac{2(Y_4-Y_1)}{(t_4-t_1)(t_4-t_2)}$$

$$a_{21} = \frac{2(X_2-X_1)}{(t_2-t_1)(t_3-t_2)} - \frac{2(X_3-X_1)}{(t_3-t_1)(t_3-t_2)} - \frac{2(X_2-X_1)}{(t_2-t_1)(t_5-t_2)} + \frac{2(X_5-X_1)}{(t_5-t_1)(t_5-t_2)}$$

$$a_{22} = \frac{2(Y_2-Y_1)}{(t_2-t_1)(t_3-t_2)} - \frac{2(Y_3-Y_1)}{(t_3-t_1)(t_3-t_2)} - \frac{2(Y_2-Y_1)}{(t_2-t_1)(t_5-t_2)} + \frac{2(Y_5-Y_1)}{(t_5-t_1)(t_5-t_2)}$$

$$b_1 = \frac{(X_2{}^2+Y_2{}^2)-(X_1{}^2+Y_1{}^2)}{(t_2-t_1)(t_3-t_2)} - \frac{(X_2{}^2+Y_2{}^2)-(X_1{}^2+Y_1{}^2)}{(t_2-t_1)(t_4-t_2)} - \frac{(X_3{}^2+Y_3{}^2)-(X_1{}^2+Y_1{}^2)}{(t_3-t_1)(t_3-t_2)} + \frac{(X_4{}^2+Y_4{}^2)-(X_1{}^2+Y_1{}^2)}{(t_4-t_1)(t_4-t_2)}$$

$$b_2 = \frac{(X_2{}^2+Y_2{}^2)-(X_1{}^2+Y_1{}^2)}{(t_2-t_1)(t_3-t_2)} - \frac{(X_2{}^2+Y_2{}^2)-(X_1{}^2+Y_1{}^2)}{(t_2-t_1)(t_5-t_2)} - \frac{(X_3{}^2+Y_3{}^2)-(X_1{}^2+Y_1{}^2)}{(t_3-t_1)(t_3-t_2)} + \frac{(X_5{}^2+Y_5{}^2)-(X_1{}^2+Y_1{}^2)}{(t_5-t_1)(t_5-t_2)}$$

Here, X_0 and Y_0 represents the position of the Impact. Now since the impact location has been calculated, we can find the speed of the signal using,

$$V = \sqrt{\frac{1}{(t_3-t_2)}\left[\frac{2\{(X_2X_0+Y_2Y_0)-(X_1X_0+Y_1Y_0)\}}{(t_2-t_1)} - \frac{2\{(X_3X_0+Y_3Y_0)-(X_1X_0+Y_1Y_0)\}}{(t_3-t_1)} - \frac{(X_2{}^2+Y_2{}^2)-(X_1{}^2+Y_1{}^2)}{(t_2-t_1)} + \frac{(X_3{}^2+Y_3{}^2)-(X_1{}^2+Y_1{}^2)}{(t_3-t_1)}\right]}$$

Based on these equations, a Matlab code was written to verify the computational system.

Experimental Validation:

Two metal plates with the same lateral dimension and different thickness were tested. APC 850 PZT sensors measuring 6 mm in diameter and 0.5 mm thick were place at various random locations but it was ensured no 3 sensors were in a single line to promote unique location solutions [8]. Piezoelectric materials have the property that, when mechanically strained, they produce an electrical charge displacement, known as the piezoelectric effect, and when subjected to an electrical field they mechanically strain, known as the reverse piezoelectric effect [9]. Therefore these piezoelectric transducers can be used both to actuate and sense strain waves. The sensors were epoxied to the plate with a conductive epoxy. Wires were also epoxied to the sensors and wired in through a Tektronix MDO3014 oscilloscope [10]. Figure 2 shows an example of the plate and the sensors and Figure 3 shows an example of an impact on the real plate. The plate is made up of 6061-T651 aluminium and is 0.304 m by 0.304 m in size.

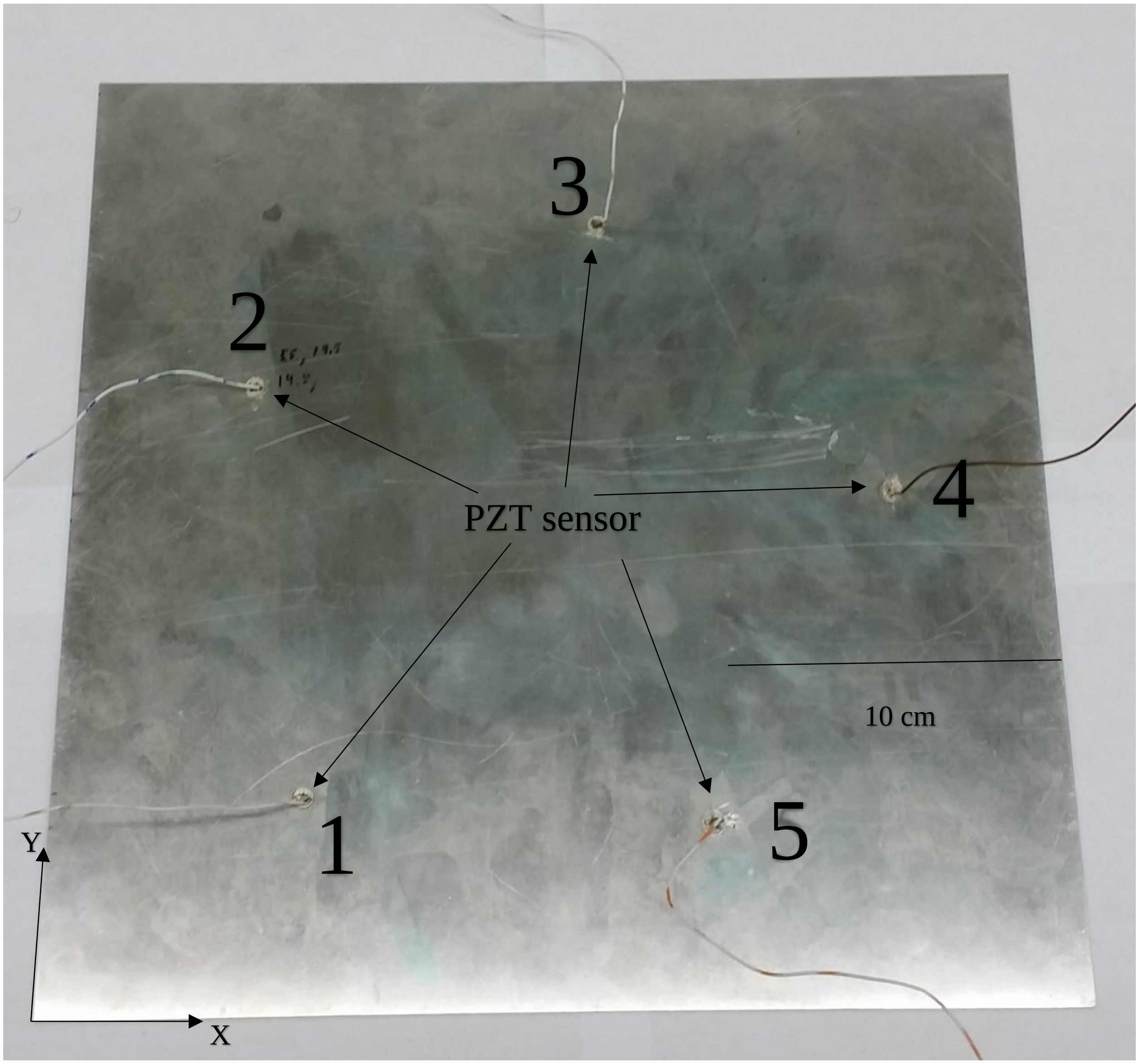

Fig 2: Aluminium Plate with five pizeoelectric sensors.

PZT Sensor Location:

1) 0.035 m by 0.098 m

2) 0.051 m by 0.254 m

3) 0.220 m by 0.240 m

4) 0.248 m by 0.077 m

5) 0.100 m by 0.024 m

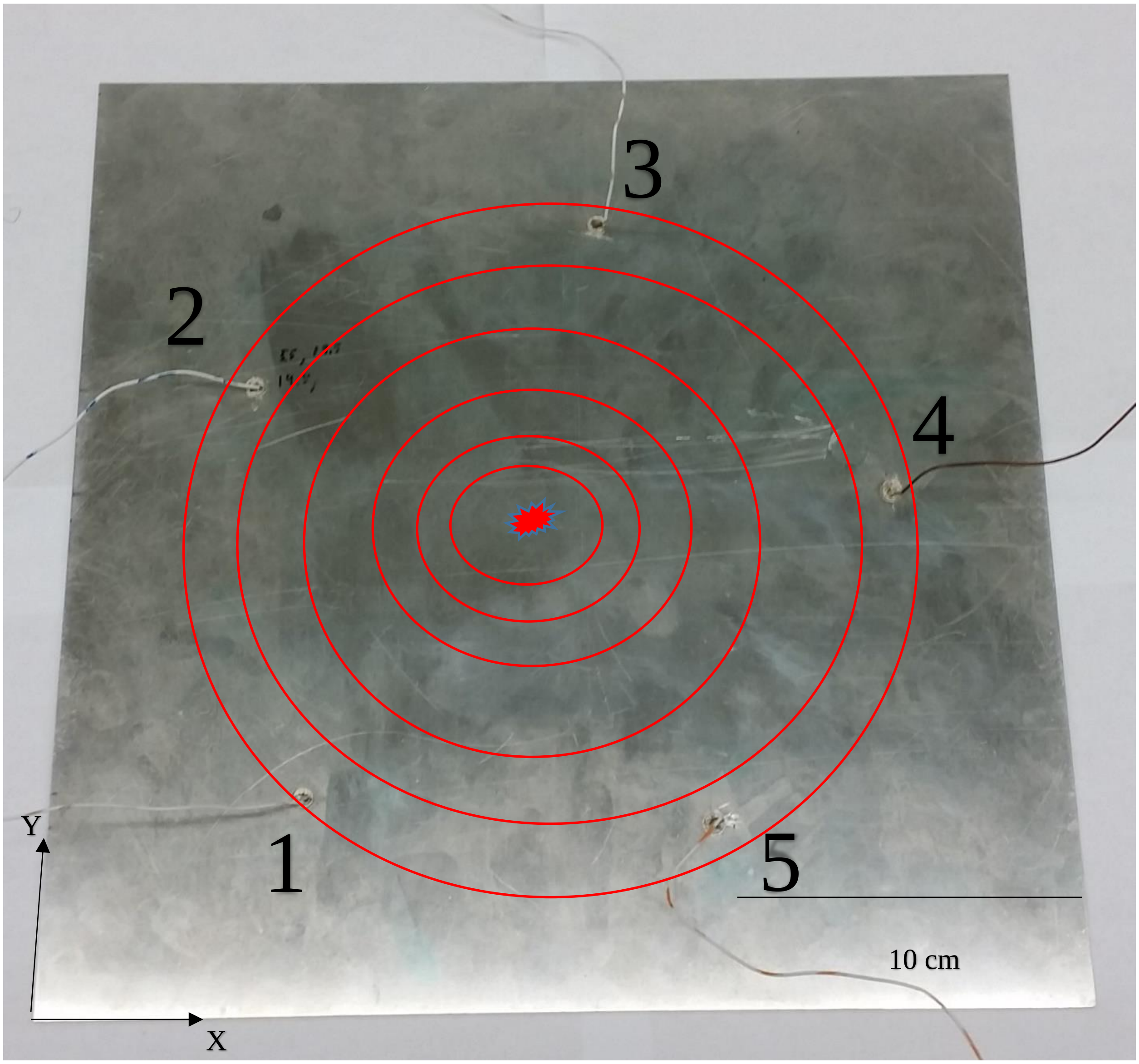

Fig 3: When an impact occurs, strain waves travel with same speed in all direction.

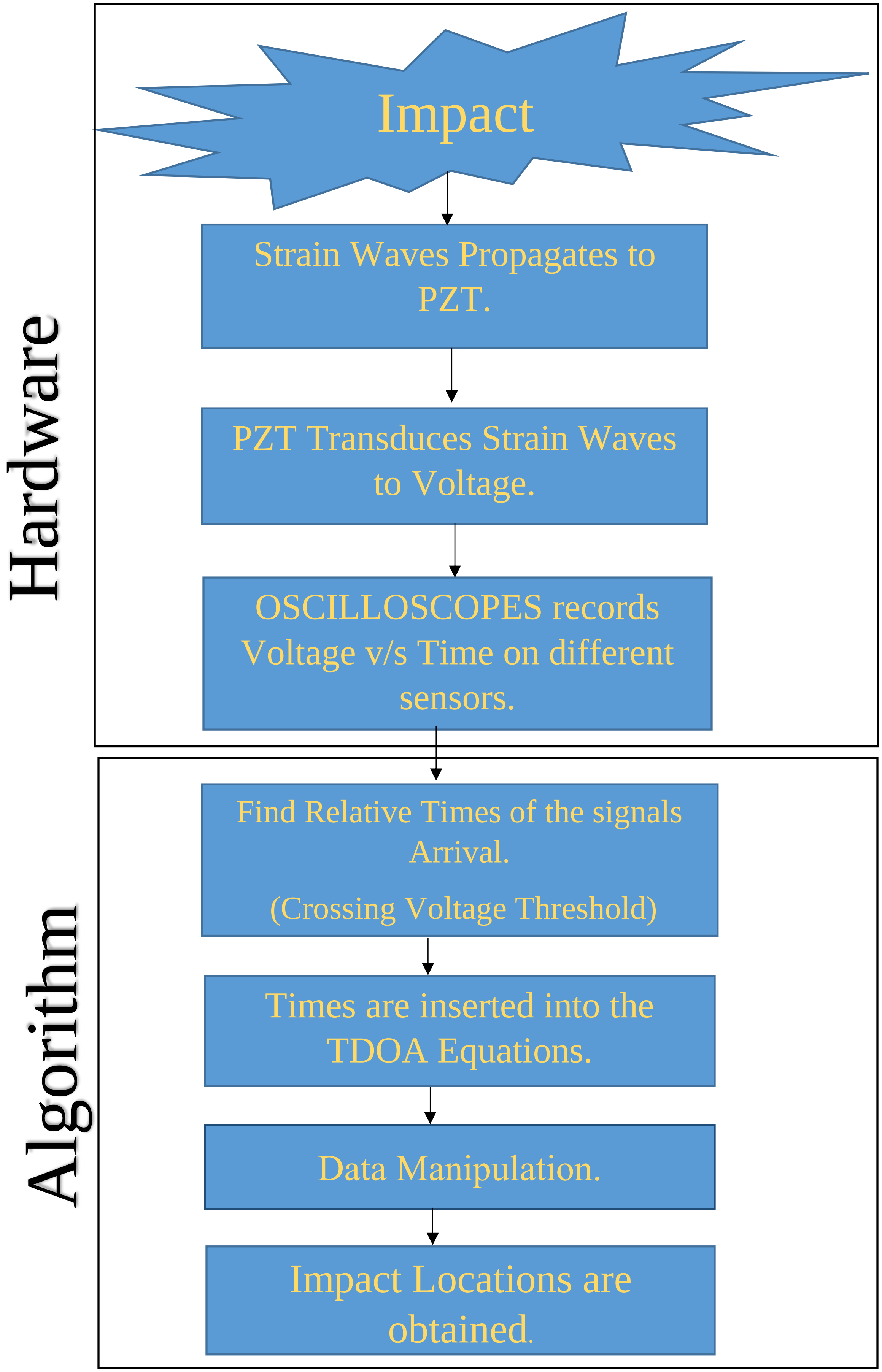

Fig 4: Flowchart of how the system works and what's happening.

Fig 4 shows a flowchart of how the system works. After setup, the plates were impacted at various locations. Stronger impacts were found to produce better results. As shown in figure 3, Strain waves were assumed to propagate with the same speed in all direction and reaches the PZT where the strain waves were transduced to voltages by the PZTs. Signals were recorded by the oscilloscope. Time of arrivals were extracted from the voltage signals and entered into the TDOA algorithm to produce a result but to do this some signal processing and data manipulation was necessary.

Because, only four channels were available on an oscilloscope, and 5 sensor signals were necessary two oscilloscopes were used to collect the data. To align the data in time channel 1 was shared between the two oscilloscopes. The data from oscilloscope 2 was time shifted such that the channel 1 data on both oscilloscopes matched, and all of the other data from oscilloscope 2 was shifted the same amount. This aligned all of the data to a single timeline. Once the channels were shifted, relative times of the signals were located by finding the time at which the signal crossed threshold voltages. Data was acquired at a sampling rate of 10 MHz and voltage precision of 100 mV was taken. Whenever the signal crossed positive or negative the threshold value, the time was found and fed into the algorithm in the Matlab code. Different thresholds resulted in different impact locations, so to minimize the error and find a single location, signal processing and data manipulation had to be done and how it's done is shown in figure 5. An example of the signal data is shown in figure 6.

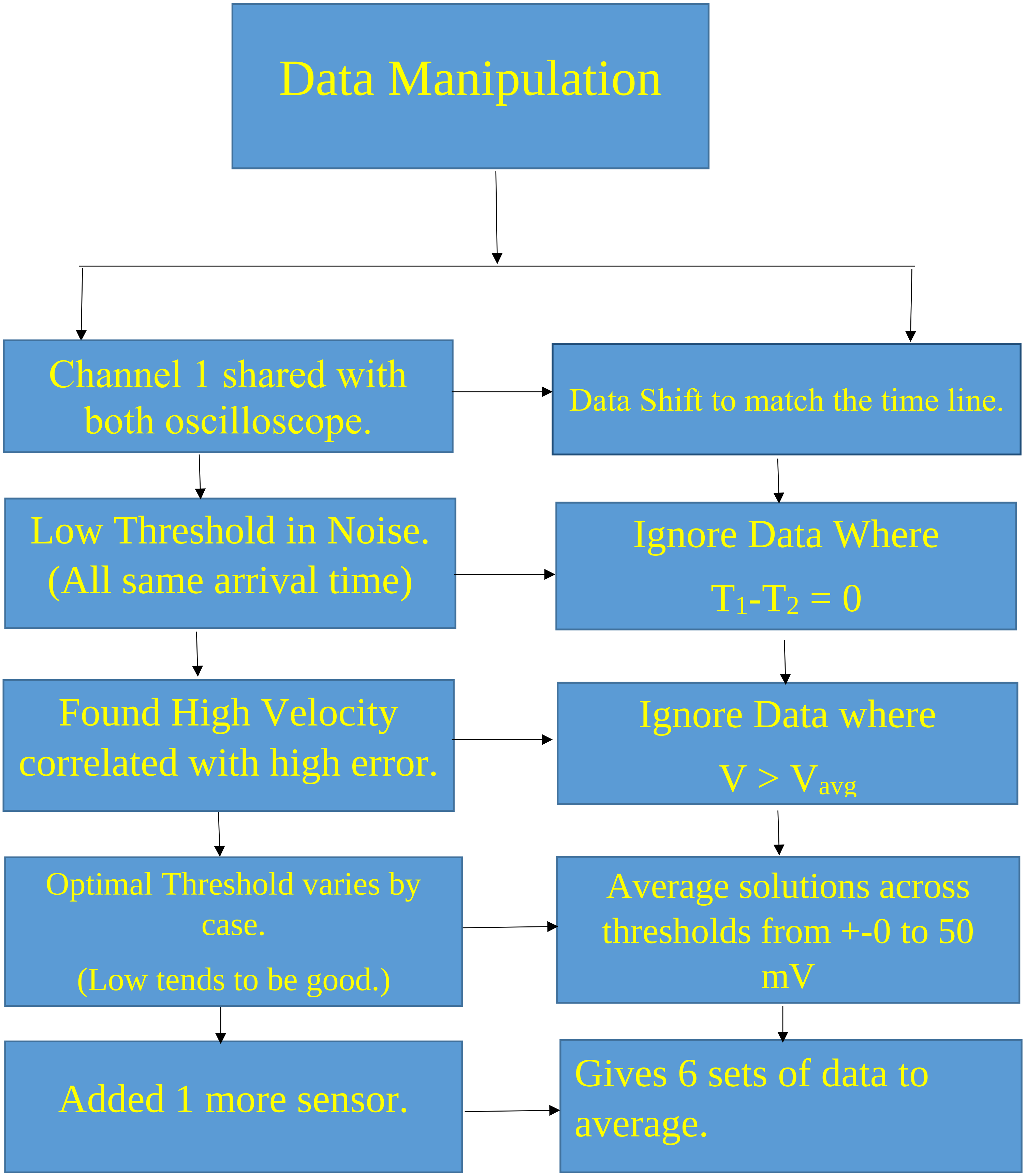

Fig 5: Flowchart of how Data Manipulation works.

The above flow chart discusses how the data manipulation works. Low thresholds were found to correlate with low error therefore low thresholds were sought, though no single threshold was found to be optimal across multiple tests. Therefore impact locations and wave velocities were calculated using thresholds from 0 to 50 mV in 1 mV increments. Thresholds below the noise floor could be identified in that they resulted in the same time of arrival at all sensors, when this was found to be the case the data set was ignored. After many experiments, high calculated velocity of the signal was found to correlate with high error too. To address this, from, the fifty velocities (minus thresholds in the noise), the mean was taken and all the results which had velocity higher than the average were eliminated. The mean of the remaining data was taken which produced decent results and worked for every impact location tested on either plate. To reduce the error more and make the system more efficient, one more sensor was added. Taking the combination of any five out of six sensors, six permutations were obtained. Six times the data sets were produced to take the mean. Doing this, significantly decreased the error of the system as a whole. Figure 6 shows the plate with six different sensors.

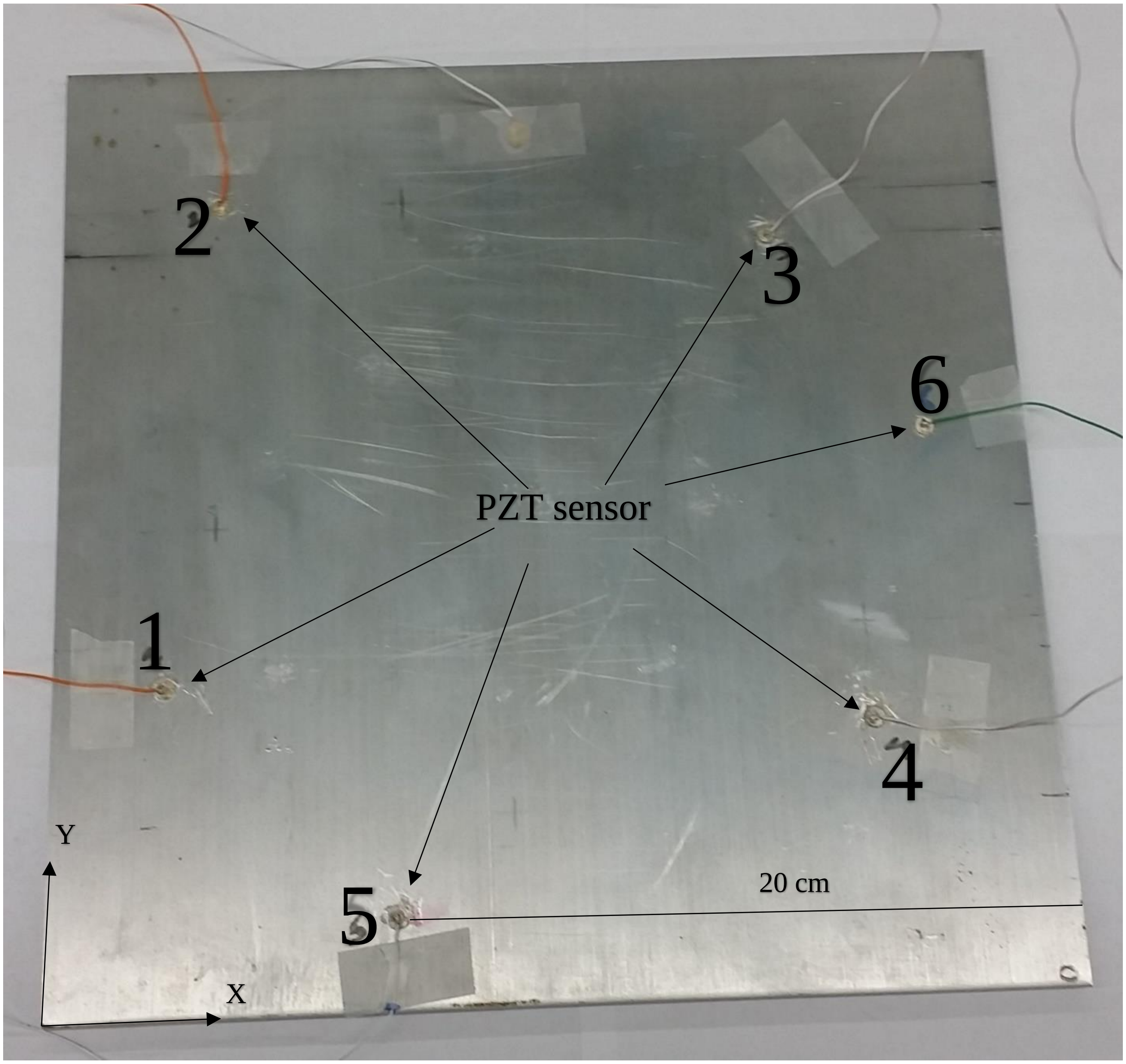

Fig 6: Aluminum Plate with six piezoelectric sensors

Sensor Location:

1) 0.035 m by 0.098 m

2) 0.051 m by 0.254 m

3) 0.220 m by 0.240 m

4) 0.248 m by 0.077 m

5) 0.100 m by 0.024 m

6) 0.275 m by 0.175 m

CHAPTER V: RESULTS

The experiment was performed on two different plates with different thickness. One was 0.01 m thick and another was 0.05 m thick. Figure 7 shows an example of a signal after an impact was made on the thin plate at 0.100 m by 0.150 m, multiple locations were impacted to demonstrate general functionality. After an impact occurred, signals were generated. The times of arrival were extracted using range of voltage thresholds. Figure 8 and 9 shows the range of raw impact data and mean impact location data respectively for an impact occurring at 0.100 m by 0.150 m. All of the fifty location data aren't shown. An example of an impact, Real impact location: 0.100 m by 0.150 m, Calculated Average Location: 0.112 m by 0.147 m which has an error of 0.012 m. The average velocity is 2.1370 X 10^3 m/s. The results for many other impact locations are in Appendix B. For the second plate of thickness 0.05 m, a sixth sensor was added. So, taking five at a time, six combinations were possible. Those six combinations gave six data sets at each thresholds. When averaged across, error was reduced significantly. Table 1 shows the error between calculated impact location and measured impact location with different combination. It also compares the error of the new method with the previous ones. The overall average of the new system was 0.029 m, which when compared to the old systems of 0.024 m and 0.05 m. However the new algorithm has the advantage of not requiring training data at the cost of an added sensor.

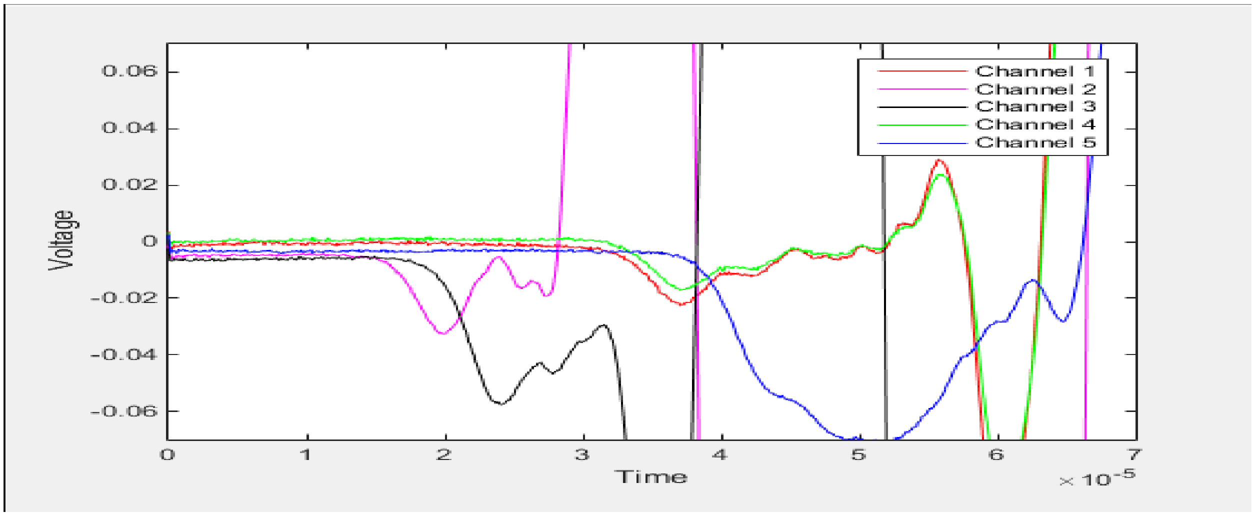

Fig 7: Signal from 5 different channels for impact location 0.100 m by 0.150 m.

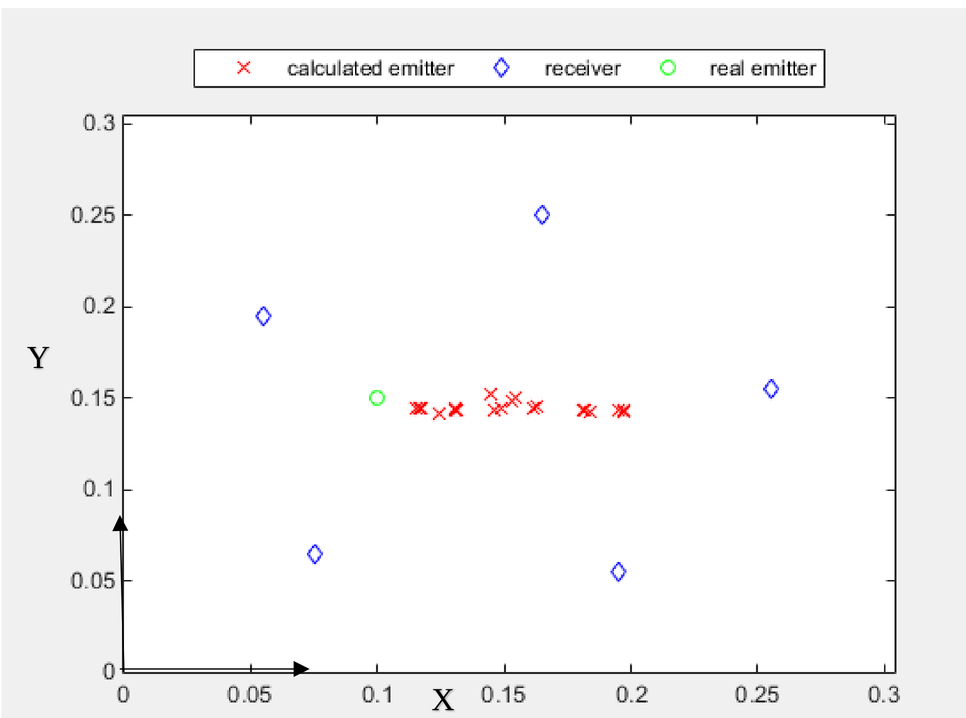

Fig 8: Impact Locations with range of threshold

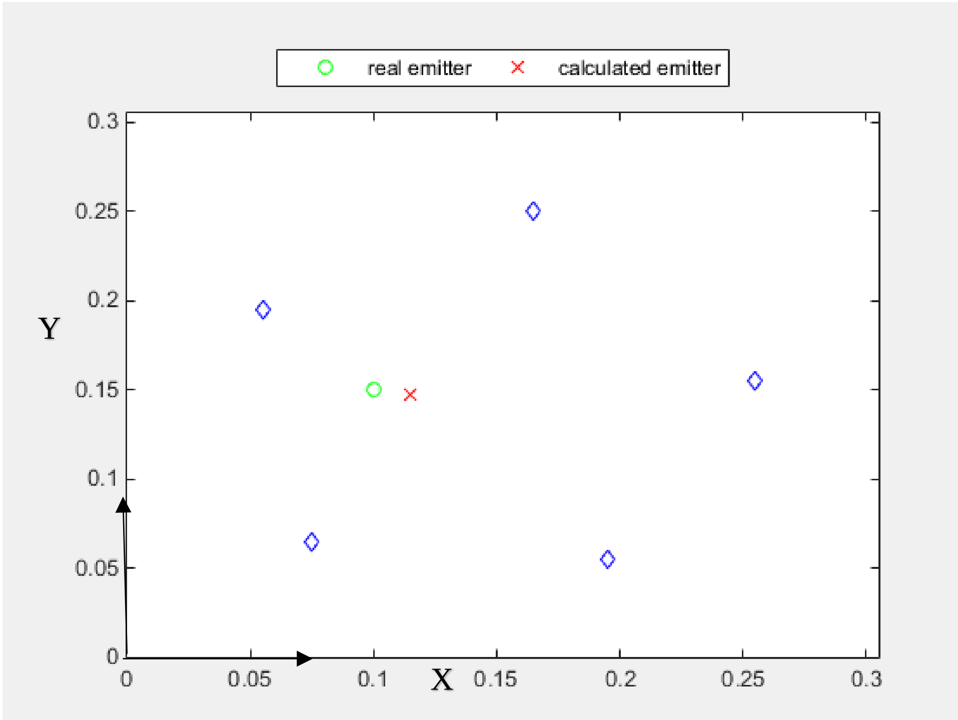

Fig 9: Impact Location with Averaged data.

		Combination (Error in meters)							
New Method	Impact Location	1	2	3	4	5	6	Average across threshold	Averaged sensor combination
	0.200 m by 0.150 m	0.024	0.027	0.022	0.002	0.011	0.026	0.019	0.029
	0.063 m by 0.170 m	0.01	0.04	0.023	0.059	0.06	0.042	0.023	
	0.100 m by 0.200 m	0.04	0.034	0.043	0.043	0.041	0.033	0.039	
	0.135 m by 0.240 m	0.047	0.037	0.044	0.026	0.061	0.044	0.043	
	0.182 m by 0.100 m	0.009	0.0085	0.023	0.0089	0.05	0.04	0.022	
Prior methods	Isotropic 4 sensor								0.024
	Anisotropic 4 sensor								0.05

Table 1: Error between calculated impact location and measured impact location

CHAPTER VI: DISCUSSION AND CONCLUSION

The newly developed system demonstrated reasonable accuracy in testing on multiple plates, with multiple sensor locations, and of differing thicknesses, without training data. The system required data conditioning to address common issues in impact location. The system was demonstrated with multiple impact locations, multiple sensor locations, and multiple plates of different geometries and produced an overall average error of 0.029 m. This error is comparable to the error published in prior work summarized in Table 2.

System	Average Error (m)
4 sensor isotropic plate with training [2]	0.024
4 sensor anisotropic plate with training [3]	0.05
New five sensor training free algorithm	0.029

Table 2: Comparison of Average Error with prior work and current work

The new system has the advantages of working without training data or defined sensor geometries. Wave propagation velocity is calculated simultaneously with impact location.

Future Works:

Future work building on this thesis has the potential to improve the results or broaden applications. Some of these are already under way. Potential future works includes:

Sensor Location Optimization: Instead of placing the sensors at random locations, simulation based analysis of error sensitivity due to geometry can be used for optimization of the sensor location.

Validate Temperature Independence: Since, this system doesn't rely on the speed of the signal, temperature variation shouldn't make a difference, but this should to be verified experimentally.

Validate Material Independence: Because this system didn't use material properties, beyond being isotropic, this system should work in any isotropic material. But this should be verified experimentally.

Expand capabilities to orthotropic materials: This system could potentially be expanded to be used on materials where the wave speed is not same in all direction using similar addition of data/equations to solve for unknowns. This could pose a major challenge but broadly increase applicability.

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Appendix A:

Starting off with the basic TDOA equation:

$$t_i = t_0 + \frac{D_i}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 4.1$$

Where,

t_i = time of arrival at receiver i

t0 = time at which emitter emits a signal

Di = Distance between emitter and receiver

V = Velocity of the signal

This is the general time difference of arrival equation. When we start off with two sensors, 1 and 2,

$$t_1 - t_0 = \frac{D_1}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 4.2$$

$$t_2 - t_0 = \frac{D_2}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 4.3$$

Combining these two, taking t_0 on one side and equating them,

$$t_2 - t_1 = \frac{D_2 - D_1}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 4.4$$

This is a form of Time Difference of Arrival equation.

If we use the co-ordinate system now, the equations will be long, so for simplicity, we will use vector form and get two equations in the end which can be solved by the matrix equation solver. Here, we have V, X0, Y0, t0 as unknown.

$$D_i = \left|\overrightarrow{P_i} - \overrightarrow{P_0}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 4.5$$

D_i is the Norm of difference between Position vector of receiver i and emitter 0.

$$\overrightarrow{P_i} = (X_i, Y_i) \qquad \overrightarrow{P_0} = (X_0, Y_0)$$

Vector P_i and vector P_0 are the co-ordinates of the receiver i and impact source 0 respectively. Now using these equations 4.2 and 4.3 we first square these equations, take the difference and use norm of D, the expanded term of equation 4.6 is shown below.

$$t_1 - t_0 = \frac{D_1}{V} \qquad t_2 - t_0 = \frac{D_2}{V}$$

$$(t_2 - t_0)^2 - (t_1 - t_0)^2 = \frac{|\vec{P_2} - \vec{P_0}|^2 - |\vec{P_1} - \vec{P_0}|^2}{V^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots 4.6$$

$$(t_i - t_0)^2 = t_i^2 - 2t_i t_0 + t_0^2$$

$$|\vec{P_i} - \vec{P_0}|^2 = (X_i - X_0)^2 + (Y_i - Y_0)^2$$

$$|\vec{P_i} - \vec{P_0}|^2 = (X_i^2 + X_0^2) + (Y_i^2 + Y_0^2) - 2(X_i X_0 + Y_i Y_0)$$

$$|\vec{P_i} - \vec{P_0}|^2 = |\vec{P_i}|^2 + |\vec{P_0}|^2 - 2\vec{P_i}^T P_0$$

Replacing all the term is in equation 4.6 we get above equation and after algebraic term manipulation, we get equations 4.7

$$(t_2^2 - 2t_2 t_0 + t_0^2) - (t_1^2 - 2t_1 t_0 + t_0^2) = \frac{(|\vec{P_2}|^2 + |\vec{P_0}|^2 - 2\vec{P_2}^T P_0) - (|\vec{P_1}|^2 + |\vec{P_0}|^2 - 2\vec{P_1}^T P_0)}{V^2}$$

$$V^2(t_2^2 - t_1^2) - 2t_0(t_2 - t_1)V^2 = |\vec{P_2}|^2 - |\vec{P_1}|^2 - 2\vec{P_2}^T P_0 + 2\vec{P_1}^T P_0$$

$$\frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_2 - t_1)} - \frac{2\vec{P_0}\left(\vec{P_2}^T - \vec{P_1}^T\right)}{(t_2 - t_1)} = V^2(t_2 + t_1) - 2t_0 V^2 \quad \ldots\ldots 4.7$$

This is the final Time Difference of Arrival equation with receivers 1 and 2.

Here we have 5 receivers, so, we will get three more equations with 3-1, 4-1, 5-1 receiver combination. Total of 4 equations are:

$$\frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_2 - t_1)} - \frac{2\vec{P_0}(\vec{P_2}^T - \vec{P_1}^T)}{(t_2 - t_1)} = V^2(t_2 + t_1) - 2t_0V^2 \quad \text{.........4.8}$$

$$\frac{|\vec{P_3}|^2 - |\vec{P_1}|^2}{(t_3 - t_1)} - \frac{2\vec{P_0}(\vec{P_3}^T - \vec{P_1}^T)}{(t_3 - t_1)} = V^2(t_3 + t_1) - 2t_0V^2 \quad \text{.........4.9}$$

$$\frac{|\vec{P_4}|^2 - |\vec{P_1}|^2}{(t_4 - t_1)} - \frac{2\vec{P_0}(\vec{P_4}^T - \vec{P_1}^T)}{(t_4 - t_1)} = V^2(t_4 + t_1) - 2t_0V^2 \quad \text{.........4.10}$$

$$\frac{|\vec{P_5}|^2 - |\vec{P_1}|^2}{(t_5 - t_1)} - \frac{2\vec{P_0}(\vec{P_5}^T - \vec{P_1}^T)}{(t_5 - t_1)} = V^2(t_5 + t_1) - 2t_0V^2 \quad \text{.........4.11}$$

Now, we have unknown t0, so combine the equations 4.8 and 4.9, 4.8 and 4.10 and 4.8 and 4.11 to eliminate t0 and get three equations,

$$\frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_2 - t_1)} - \frac{|\vec{P_3}|^2 - |\vec{P_1}|^2}{(t_3 - t_1)} + V^2(t_3 - t_2) = \vec{P_0}[\frac{2(\vec{P_2}^T - \vec{P_1}^T)}{(t_2 - t_1)} - \frac{2(\vec{P_3}^T - \vec{P_1}^T)}{(t_3 - t_1)}] \quad \text{...4.12}$$

$$\frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_2 - t_1)} - \frac{|\vec{P_4}|^2 - |\vec{P_1}|^2}{(t_4 - t_1)} + V^2(t_4 - t_2) = \vec{P_0}[\frac{2(\vec{P_2}^T - \vec{P_1}^T)}{(t_2 - t_1)} - \frac{2(\vec{P_4}^T - \vec{P_1}^T)}{(t_4 - t_1)}] \quad \text{...4.13}$$

$$\frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_2 - t_1)} - \frac{|\vec{P_5}|^2 - |\vec{P_1}|^2}{(t_5 - t_1)} + V^2(t_5 - t_2) = \vec{P_0}[\frac{2(\vec{P_2}^T - \vec{P_1}^T)}{(t_2 - t_1)} - \frac{2(\vec{P_5}^T - \vec{P_1}^T)}{(t_5 - t_1)}] \quad \text{...4.14}$$

Since, the velocity of the signal is also unknown, we will again eliminate V by combining the equations 4.12 and 4.13 and 4.12 and 4.14. After manipulation, put all the equation with vector P_0 on one side and rest on the other. So, then we get two equations and two unknowns, which we will solve by Matrix Method.

$$\frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_3 - t_2)(t_2 - t_1)} - \frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_4 - t_2)(t_2 - t_1)} - \frac{|\vec{P_3}|^2 - |\vec{P_1}|^2}{(t_3 - t_1)(t_3 - t_2)} + \frac{|\vec{P_4}|^2 - |\vec{P_1}|^2}{(t_4 - t_2)(t_4 - t_1)} =$$
$$\vec{P_0}[\frac{2(\vec{P_2}^T - \vec{P_1}^T)}{(t_2 - t_1)(t_3 - t_2)} - \frac{2(\vec{P_3}^T - \vec{P_1}^T)}{(t_3 - t_1)(t_3 - t_2)} - \frac{2(\vec{P_2}^T - \vec{P_1}^T)}{(t_2 - t_1)(t_4 - t_2)} + \frac{2(\vec{P_4}^T - \vec{P_1}^T)}{(t_4 - t_1)(t_4 - t_2)}] \quad \text{..4.15}$$

$$\frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_3 - t_2)(t_2 - t_1)} - \frac{|\vec{P_2}|^2 - |\vec{P_1}|^2}{(t_5 - t_2)(t_2 - t_1)} - \frac{|\vec{P_3}|^2 - |\vec{P_1}|^2}{(t_3 - t_1)(t_3 - t_2)} + \frac{|\vec{P_5}|^2 - |\vec{P_1}|^2}{(t_5 - t_2)(t_5 - t_1)} = \vec{P_0}\left[\frac{2(\vec{P_2}^T - \vec{P_1}^T)}{(t_2 - t_1)(t_3 - t_2)} - \frac{2(\vec{P_3}^T - \vec{P_1}^T)}{(t_3 - t_1)(t_3 - t_2)} - \frac{2(\vec{P_2}^T - \vec{P_1}^T)}{(t_2 - t_1)(t_5 - t_2)} + \frac{2(\vec{P_5}^T - \vec{P_1}^T)}{(t_5 - t_1)(t_5 - t_2)}\right] \quad ..4.16$$

Now, solving them by the Matrix Equation Solver technique and get the location of the emitter. We make these two equations in Matrix form and solve using X= a\b,

$$a_{11}X_0 + a_{12}Y_0 = b_1$$

$$a_{21}X_0 + a_{22}Y_0 = b_2$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Here,

$$a_{11} = \frac{2(X_2 - X_1)}{(t_2 - t_1)(t_3 - t_2)} - \frac{2(X_3 - X_1)}{(t_3 - t_1)(t_3 - t_2)} - \frac{2(X_2 - X_1)}{(t_2 - t_1)(t_4 - t_2)} + \frac{2(X_4 - X_1)}{(t_4 - t_1)(t_4 - t_2)}$$

$$a_{12} = \frac{2(Y_2 - Y_1)}{(t_2 - t_1)(t_3 - t_2)} - \frac{2(Y_3 - Y_1)}{(t_3 - t_1)(t_3 - t_2)} - \frac{2(Y_2 - Y_1)}{(t_2 - t_1)(t_4 - t_2)} + \frac{2(Y_4 - Y_1)}{(t_4 - t_1)(t_4 - t_2)}$$

$$a_{21} = \frac{2(X_2 - X_1)}{(t_2 - t_1)(t_3 - t_2)} - \frac{2(X_3 - X_1)}{(t_3 - t_1)(t_3 - t_2)} - \frac{2(X_2 - X_1)}{(t_2 - t_1)(t_5 - t_2)} + \frac{2(X_5 - X_1)}{(t_5 - t_1)(t_5 - t_2)}$$

$$a_{22} = \frac{2(Y_2 - Y_1)}{(t_2 - t_1)(t_3 - t_2)} - \frac{2(Y_3 - Y_1)}{(t_3 - t_1)(t_3 - t_2)} - \frac{2(Y_2 - Y_1)}{(t_2 - t_1)(t_5 - t_2)} + \frac{2(Y_5 - Y_1)}{(t_5 - t_1)(t_5 - t_2)}$$

$$b_1 = \frac{(X_2^2+Y_2^2)-(X_1^2+Y_1^2)}{(t_2-t_1)(t_3-t_2)} - \frac{(X_2^2+Y_2^2)-(X_1^2+Y_1^2)}{(t_2-t_1)(t_4-t_2)} - \frac{(X_3^2+Y_3^2)-(X_1^2+Y_1^2)}{(t_3-t_1)(t_3-t_2)} + \frac{(X_4^2+Y_4^2)-(X_1^2+Y_1^2)}{(t_4-t_1)(t_4-t_2)}$$

$$b_2 = \frac{(X_2^2+Y_2^2)-(X_1^2+Y_1^2)}{(t_2-t_1)(t_3-t_2)} - \frac{(X_2^2+Y_2^2)-(X_1^2+Y_1^2)}{(t_2-t_1)(t_5-t_2)} - \frac{(X_3^2+Y_3^2)-(X_1^2+Y_1^2)}{(t_3-t_1)(t_3-t_2)} + \frac{(X_5^2+Y_5^2)-(X_1^2+Y_1^2)}{(t_5-t_1)(t_5-t_2)}$$

Here, X_0 and Y_0 represents the position of the Impact. Now since the impact location has been calculated, we can find the speed of the signal using, vector form,

$$V = \sqrt{\frac{1}{(t_3-t_2)}(\vec{P}_0[\frac{2(\vec{P}_2^{\,T} - \vec{P}_1^{\,T})}{(t_2-t_1)} - \frac{2(\vec{P}_3^{\,T} - \vec{P}_1^{\,T})}{(t_3-t_1)}] - \frac{|\vec{P}_2|^2 - |\vec{P}_1|^2}{(t_2-t_1)} + \frac{|\vec{P}_3|^2 - |\vec{P}_1|^2}{(t_3-t_1)})}$$

Cartesian co-ordinate form,

$$V = \sqrt{\frac{1}{(t_3-t_2)}\left[\frac{2\{(X_2X_0 + Y_2Y_0) - (X_1X_0 + Y_1Y_0)\}}{(t_2-t_1)} - \frac{2\{(X_3X_0 + Y_3Y_0) - (X_1X_0 + Y_1Y_0)\}}{(t_3-t_1)} - \frac{(X_2^2 + Y_2^2) - (X_1^2 + Y_1^2)}{(t_2-t_1)} + \frac{(X_3^2 + Y_3^2) - (X_1^2 + Y_1^2)}{(t_3-t_1)}\right]}$$

Appendix B:

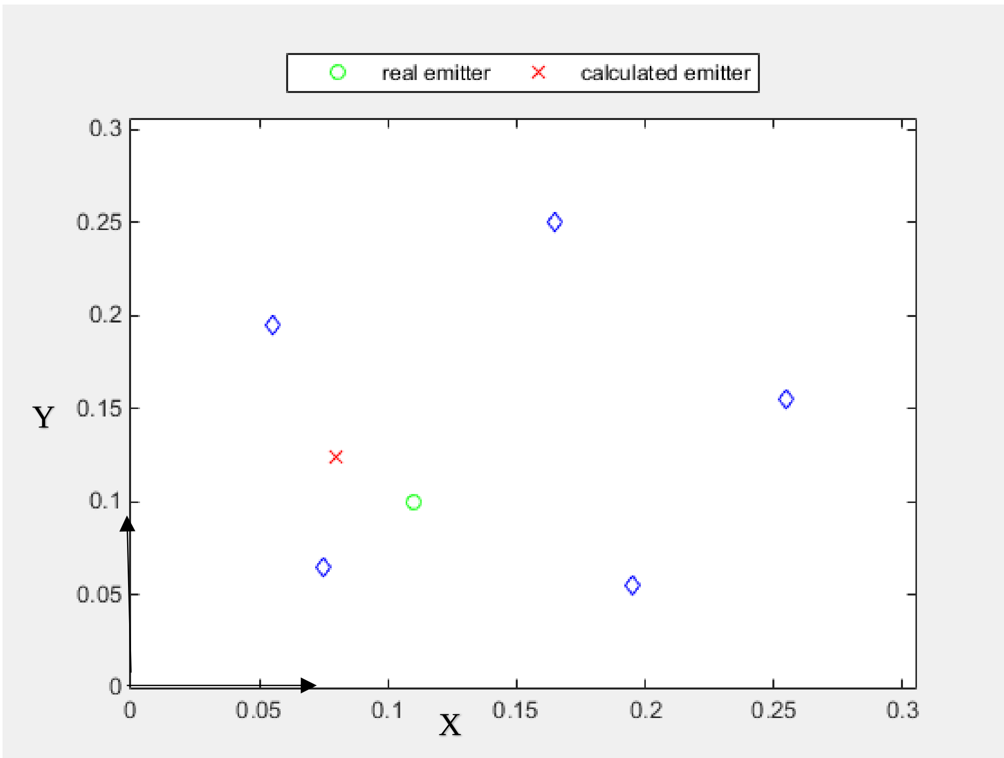

Fig 10: Average Impact Location for 0.110 m by 0.100 m

Impact Location: 0.110 m, 0.100 m

Calculated Location: 0.080 m, 0.124 m

Error: 0.038 m

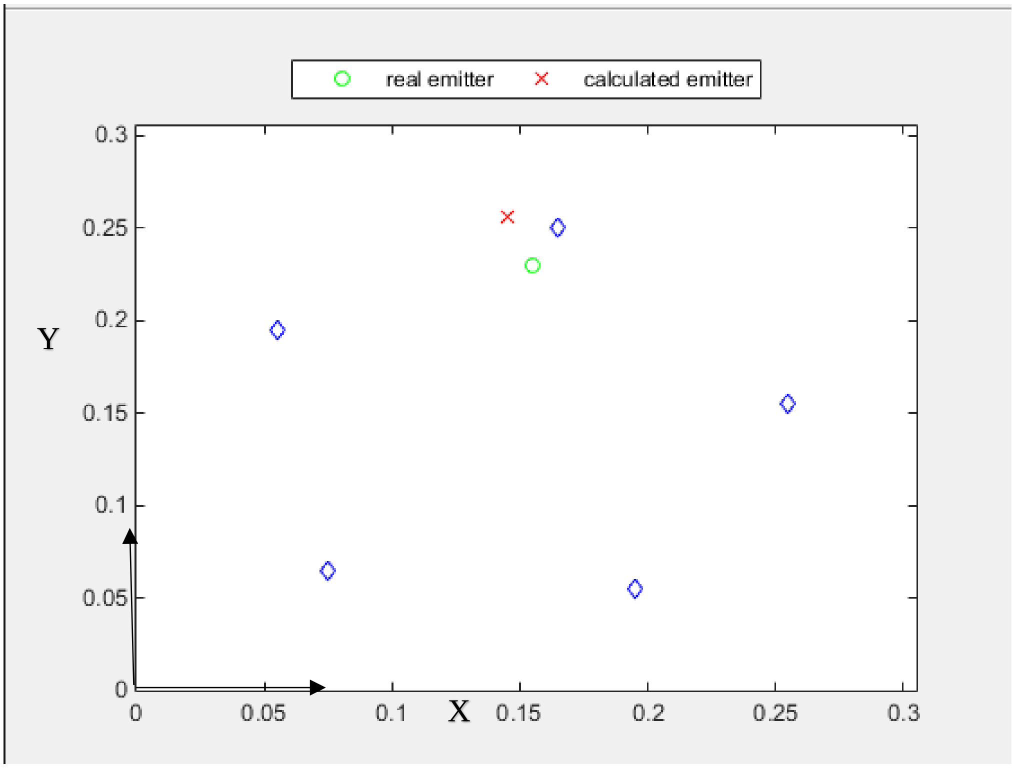

Fig 11: Average Impact Location for 0.155 m by 0.230 m

Impact Location: 0.155 m, 0.230 m

Calculated Location: 0.145 m, 0.256 m

Error: 0.028 m

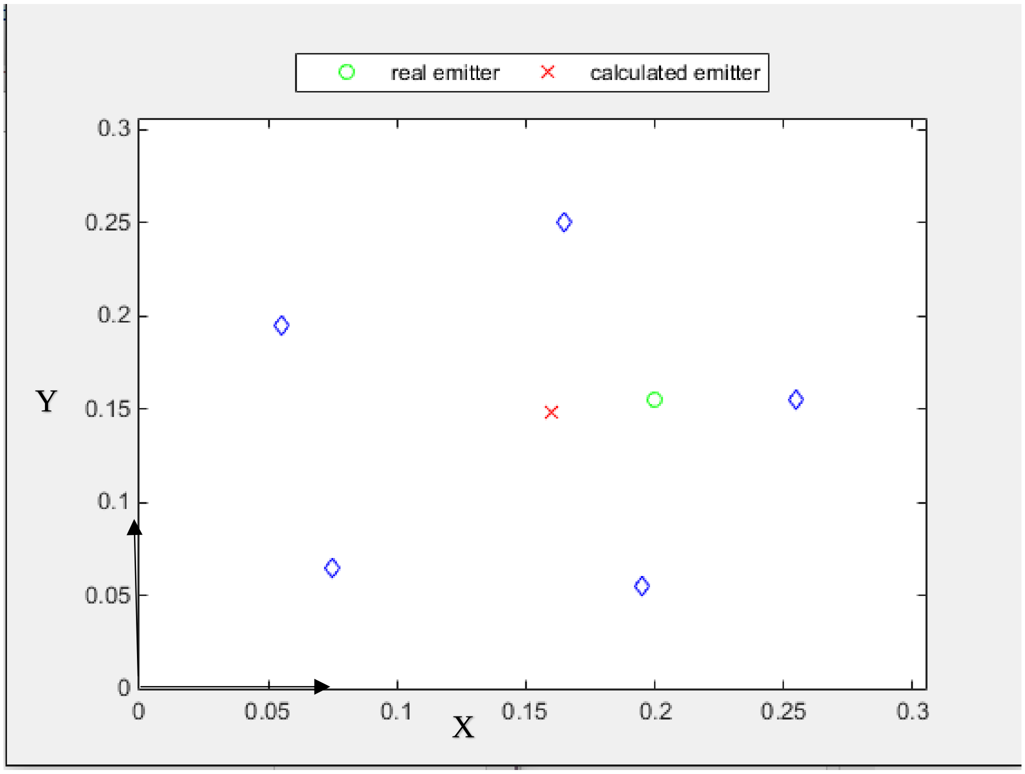

Fig 12: Average Impact Location for 0.200 m by 0.155 m

Impact Location: 0.200 m, 0.155 m

Calculated Location: 0.160 m, 0.148 m

Error: 0.040 m

Thick Plate:

Thickness of 0.05 m

The first five sensors were the initially to begin with. Taking combination of any five, we get better results.

Impact Location: 0.200 m by 0.150 m

For the First combination, Sensor 1,2,3,4,5,

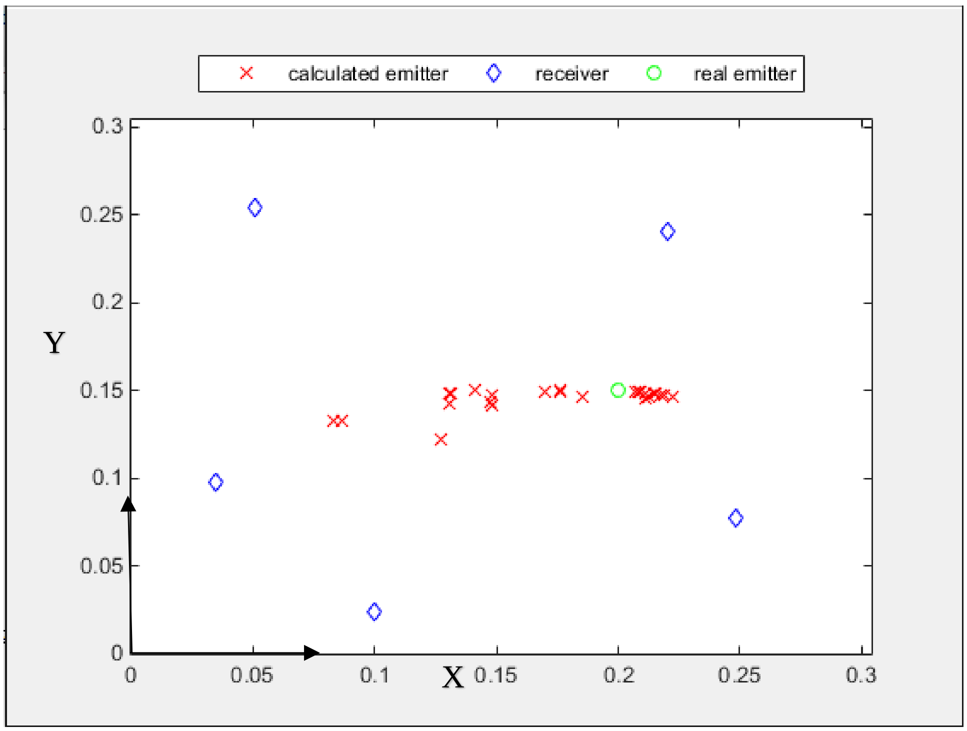

Fig13: Impact Location with range of threshold

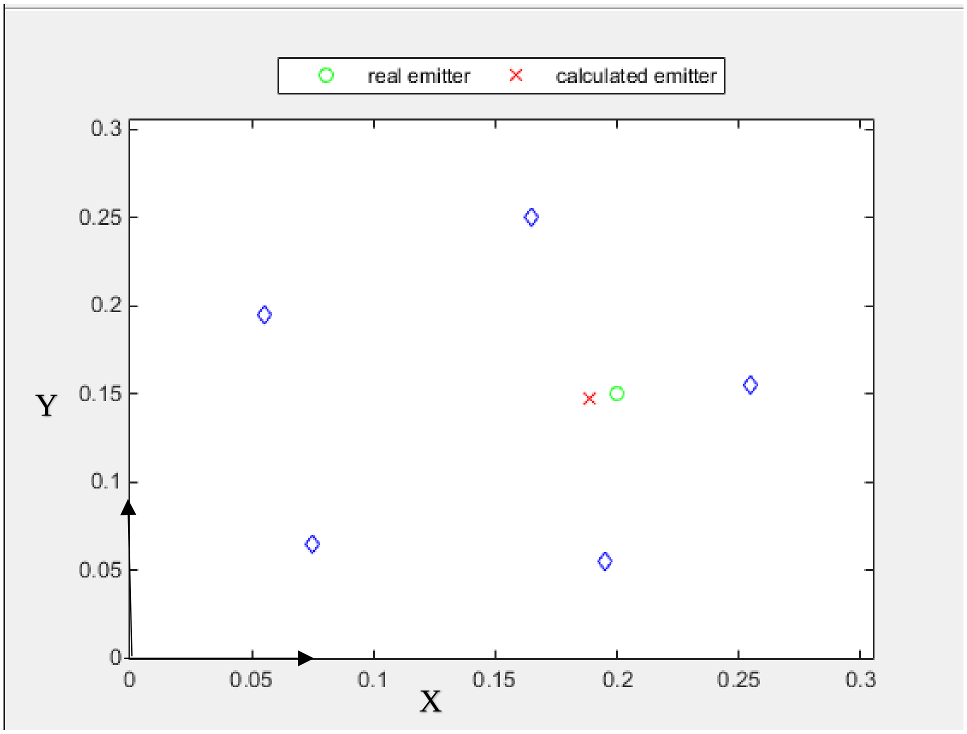

Fig 14: Average location of the Impact for combination 1

Calculated Average Location: 0.189 m by 0.147 m

Error: 0.011 m

Velocity: 2.3484 X 10^4 m/s

For the Second combination, Sensor 1,2,3,4,6

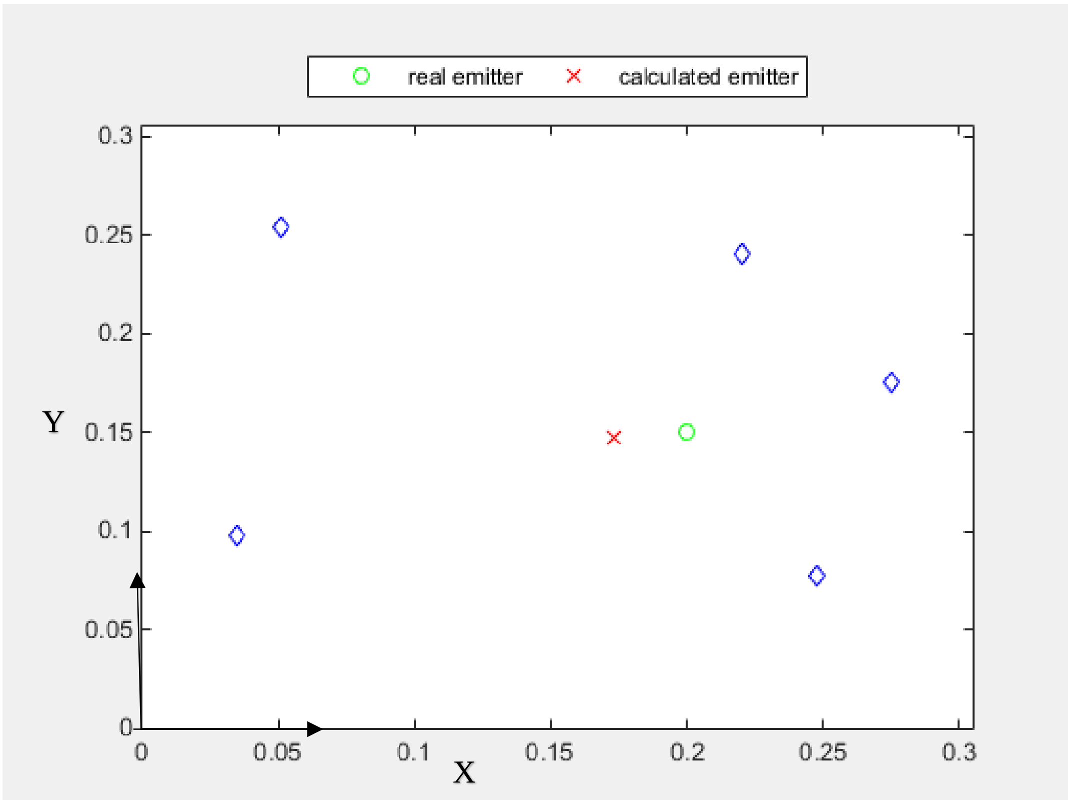

Fig 15: Average location of the Impact for combination 2

Calculated Average Location: 0.173 m by 0.147 m

Error: 0.027 m

Velocity: 2.2417 X 10^4 m/s

For the Third combination, Sensor 1,2,3,5,6

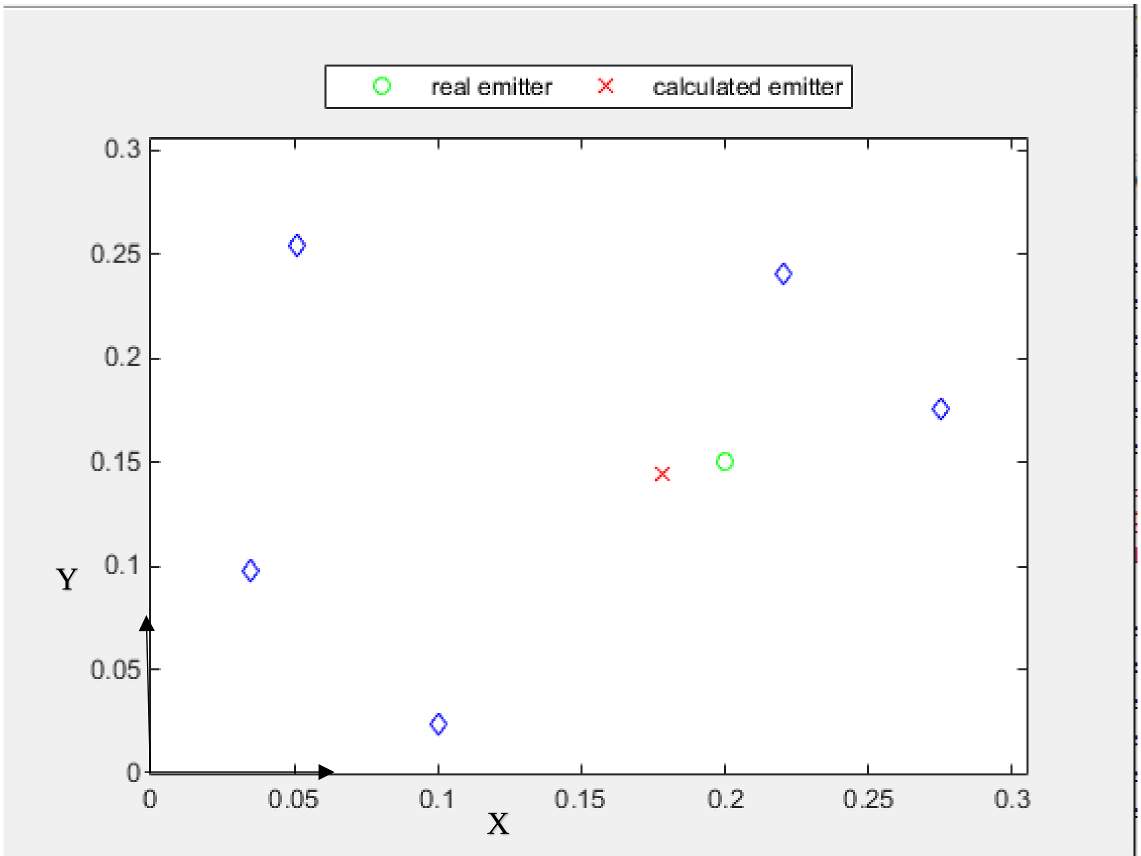

Fig 16: Average location of the Impact for combination 3

Calculated Average Location: 0.178 m by 0.144 m

Error: 0.022 m

Velocity: 2.3484 X 10^4 m/s

For the Fourth combination, Sensor 1,2,4,5,6

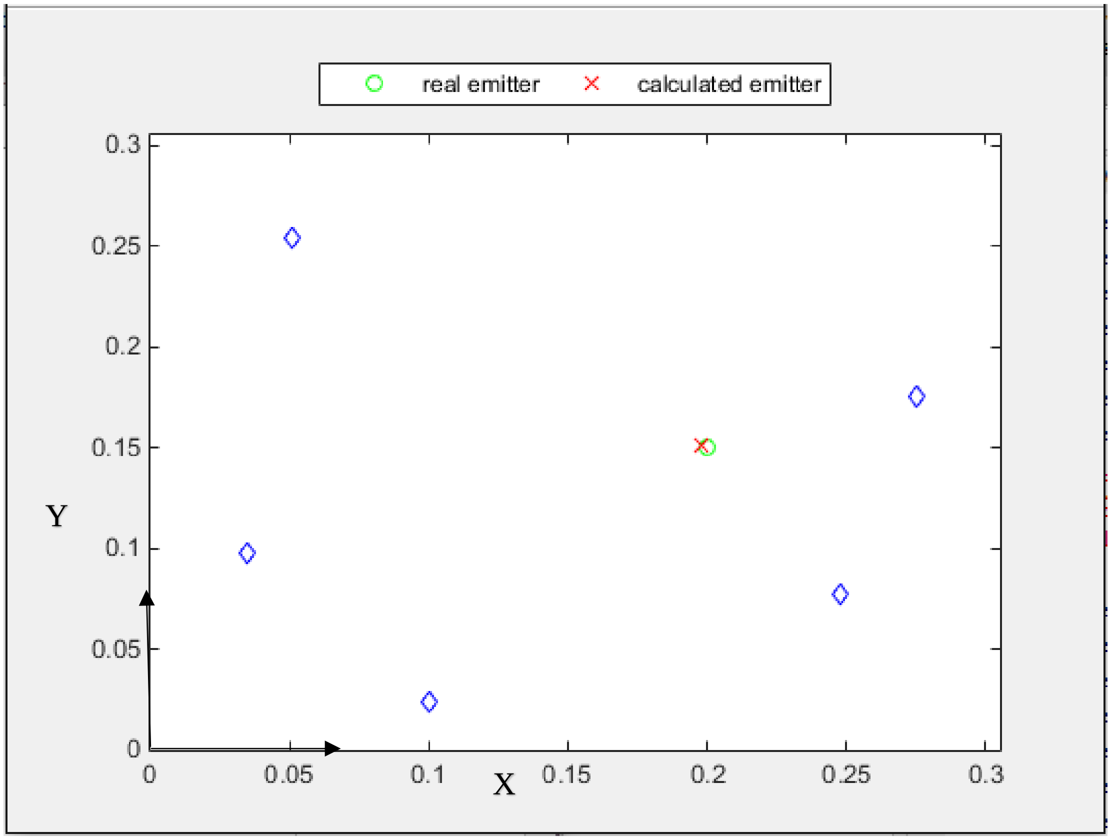

Fig 17: Average location of the Impact for combination 4

Calculated Average Location: 0.198 m by 0.151 m

Error: 0.002 m

Velocity: 2.3484 X 10^4 m/s

For the Fifth combination, Sensor 1,3,4,5,6

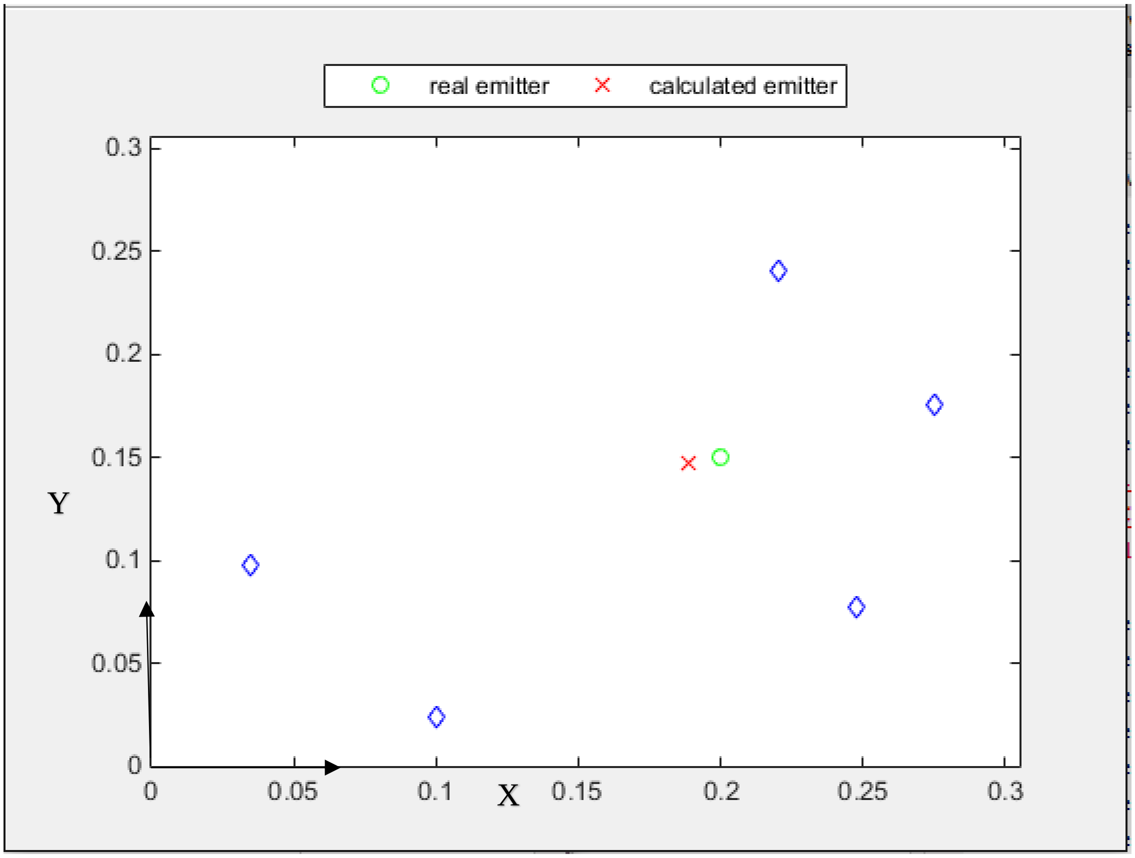

Fig 18: Average location of the Impact for combination 5

Calculated Average Location: 0.189 m by 0.147 m

Error: 0.011 m

Velocity: 2.1919 X 10^4 m/s

For the Sixth combination, Sensor 2,3,4,5,6

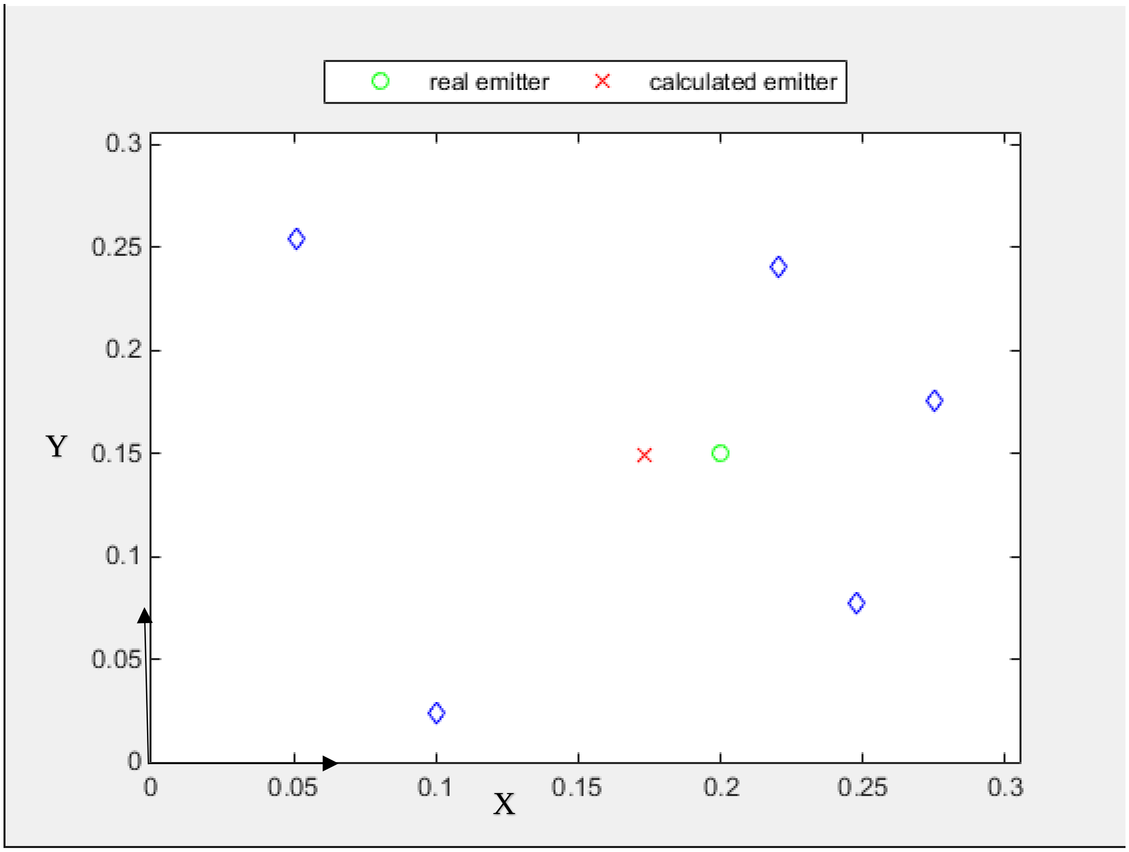

Fig 19: Average location of the Impact for combination 6

Calculated Average Location: 0.173 m by 0.149 m

Error: 0.026 m

Velocity: 2.3484 X 10^4 m/s

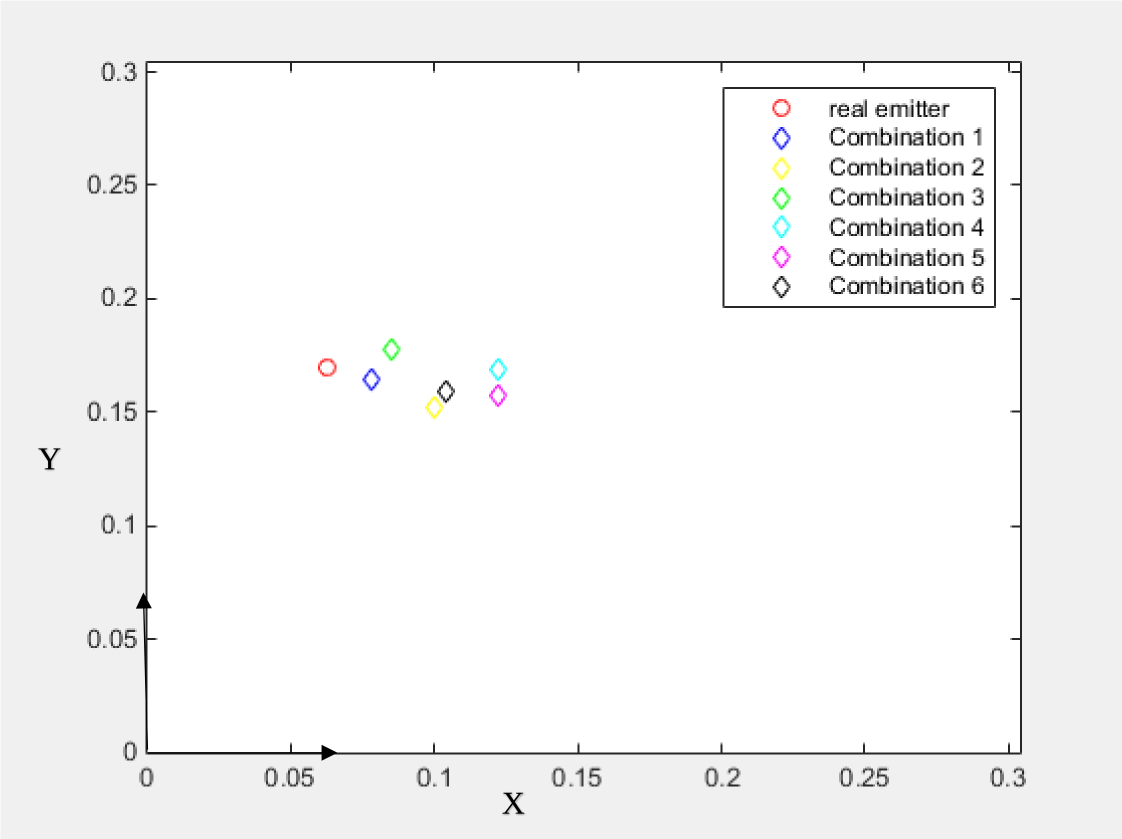

Fig 20: Average Impact location for all combination

Impact Location: 0.063 m, 0.170 m

Combination 1: 0.078 m by 0.164 m
Error:0.01 m

Combination 2: 0.100 m by 0.152 m
Error:0.04 m

Combination 3: 0.085 m by 0.178 m
Error:0.023 m

Combination 4: 0.122 m by 0.169 m
Error:0.059 m

Combination 5: 0.122 m by 0.157 m
Error:0.06 m

Combination 6: 0.104 m by 0.159 m
Error:0.042 m

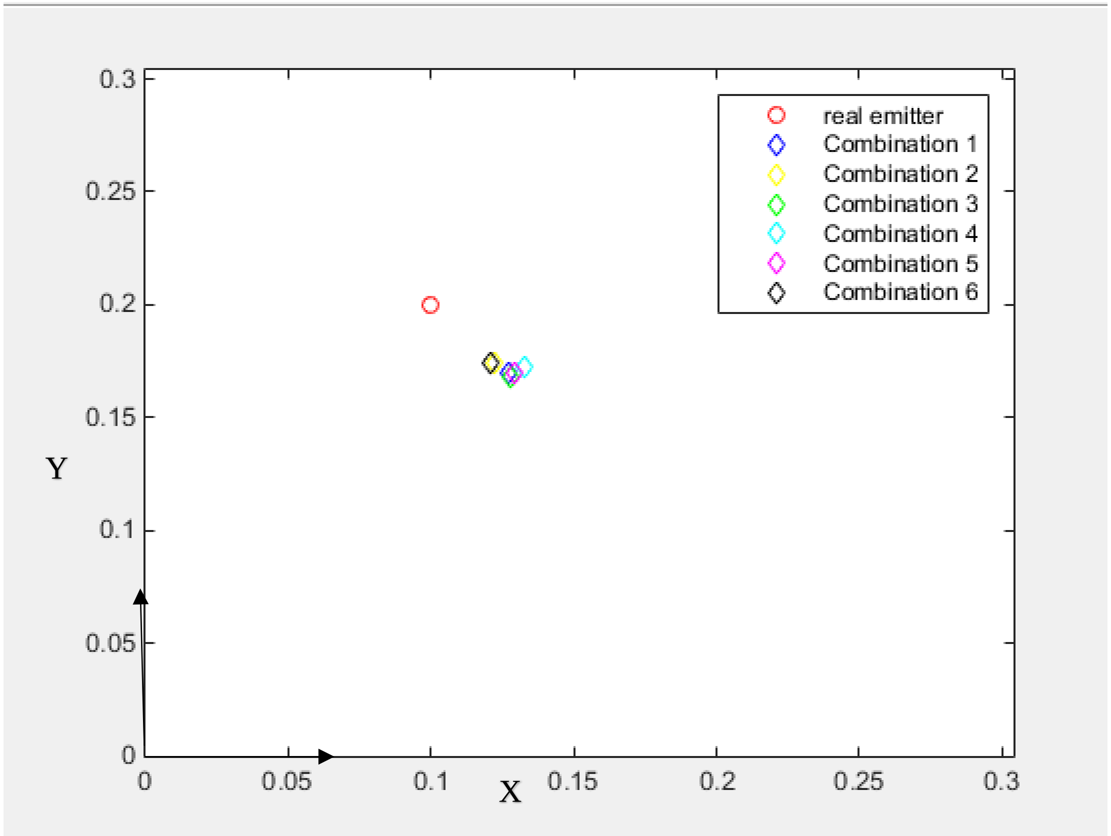

Fig 21: Average Impact location for all combination

Impact Location: 0.100 m, 0.200 m

Combination 1: 0.127 m by 0.170 m
Error:0.04 m

Combination 2: 0.122 m by 0.174 m
Error:0.034 m

Combination 3: 0.128 m by 0.168 m
Error:0.043 m

Combination 4: 0.133 m by 0.172 m
Error:0.043 m

Combination 5: 0.129 m by 0.170 m
Error:0.041 m

Combination 6: 0.121 m by 0.174 m
Error:0.033 m

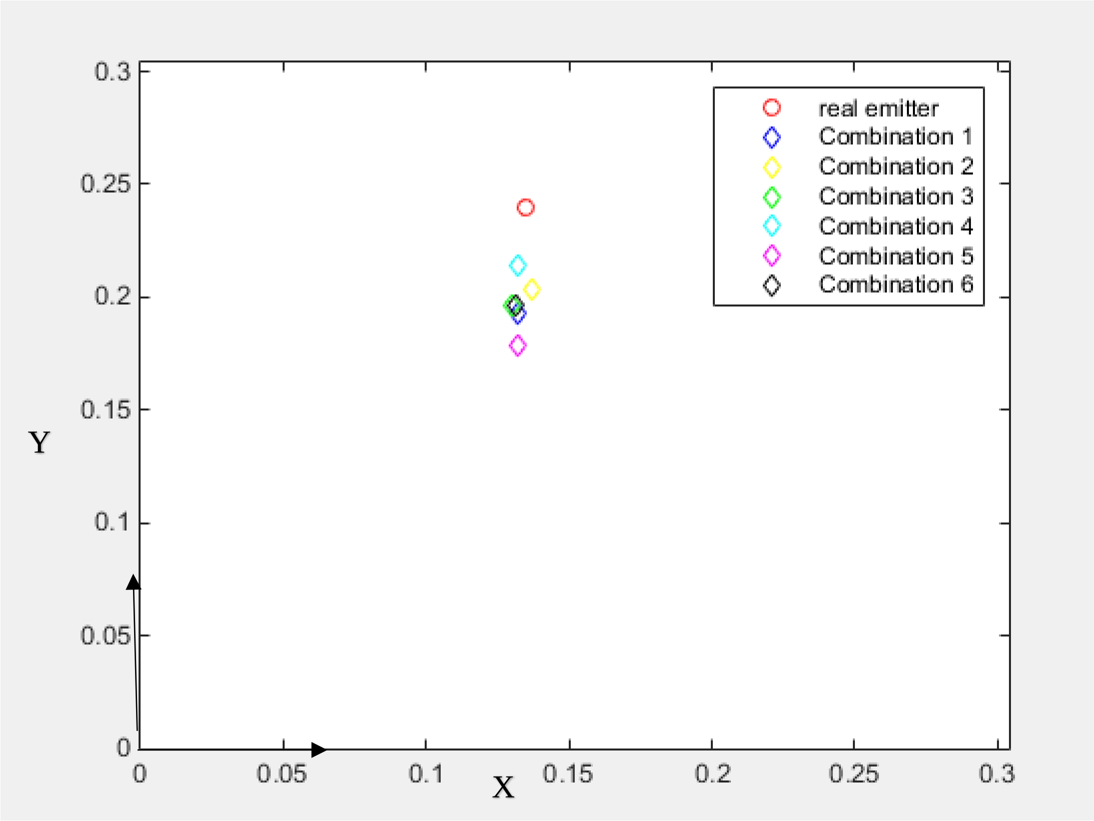

Fig 22: Average Impact location for all combination

Impact Location: 0.135 m, 0.240 m

Combination 1: 0.132 m by 0.193 m
Error:0.047 m

Combination 2: 0.137 m by 0.203 m
Error:0.037 m

Combination 3: 0.130 m by 0.196 m
Error:0.044 m

Combination 4: 0.132 m by 0.214 m
Error:0.026 m

Combination 5: 0.132 m by 0.179 m
Error:0.061 m

Combination 6: 0.131 m by 0.196 m
Error:0.044 m

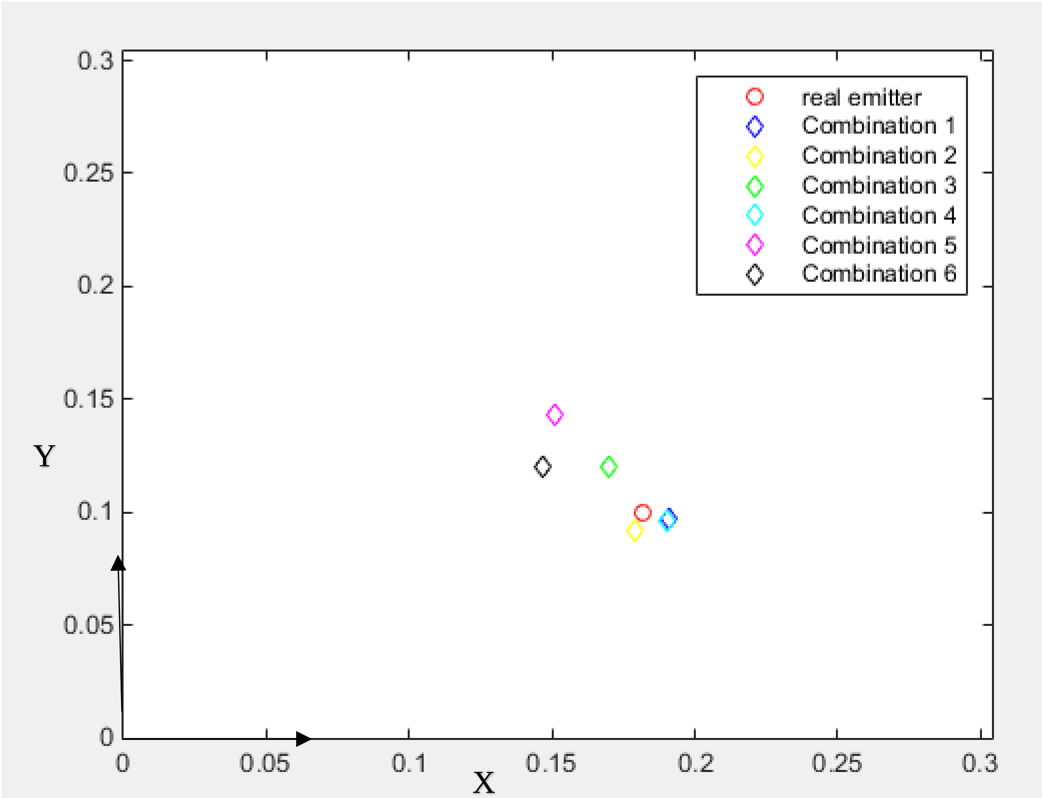

Fig 23: Average Impact location for all combination

Impact Location: 0.182 m, 0.100 m

Combination 1: 0.191 m by 0.097 m
Error:0.009 m

Combination 2: 0.179 m by 0.092 m
Error:0.0085 m

Combination 3: 0.170 m by 0.120 m
Error:0.023 m

Combination 4: 0.190 m by 0.096 m
Error:0.0089 m

Combination 5: 0.151 m by 0.143 m
Error:0.05 m

Combination 6: 0.147 m by 0.120 m
Error:0.04 m