Non-Zero Time-Averaged Thermoacoustic Effects, Linear or Nonlinear?

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ABSTRACT

N.Rott, et al.[1,2] has developed a so-called linear thermoacoustic theory (LTAT) to explain non-zero, time-averaged thermoacoustic effects under acoustical approximation assumption. However, two important facts, which are given in the main text of this paper, have been a shortcoming of the LTAT for a long time. Here, the author proposes a distinguished viewpoint: that is, the nonzero, time-averaged thermoacoustic effects are basically nonlinear. With this new recognition, the author solves the two shortcomings encountered by the LTAT. Then, a framework of weakly nonlinear thermoacoustic theory is developed. For strongly nonlinear thermoacoustic systems, the full Computational Fluid Dynamics (CFD) method is a more powerful and feasible theoretical tool.

INTRODUCTION

Classical regenerative machines or recently developed thermoacoustic machines operate on complicated interactions between oscillating flow and its contacted solids, the so-called thermoacoustic effects. Rott et al. have developed linear thermoacoustic theory (LTAT) to explain the effects and the working mechanism of related machines.[1,2,3,4] One of the main viewpoints of the linear thermoacoustic theory is that the nonzero time-averaged thermoacoustic effects such as time-averaged enthalpy flux, heat flux and acoustical power flux are linear thermoacoustic effects under acoustical approximation assumption. However, there are two important examples which have seriously doubted the main standpoints of the LTAT.

Now let us look at the first example. Figure 1 is the schematic of an orifice pulse tube refrigerator (OPTR). It is straightforward to have the following energy balance relationship between the time-averaged cooling power and the time-averaged enthalpy fluxes in the regenerator and pulse tube:

$$Q_c = <\dot{E}_p> - <\dot{E}_R> \qquad (1)$$

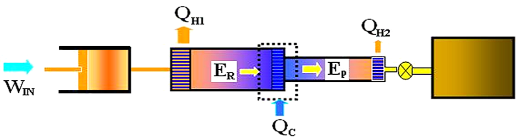

Figure 1. Schematic of an orifice pulse tube refrigerator.

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For simplicity, we make the following assumptions: i) the regenerator is ideal so that the time-averaged enthalpy flux in the regenerator is null, ii) the flow in the pulse tube is adiabatic and inviscous, and iii) the working gas is an ideal gas. From the first assumption, the cooling power is simply equal to the time-averaged enthalpy flux of the pulse tube. By means of the relationship of thermodynamic variables of ideal gas, the following expression can be obtained:

$$\begin{aligned} Q_c &=< \bar{E}_p >= \frac{1}{\tau}\oint(\int_A \hat{m} i dA)dt = \frac{1}{\tau}\oint(\int_A \rho u C_p T dA)dt = \frac{C_p A}{\tau R}\oint p u dt = \frac{C_p A}{\tau R}\oint(p_0 + \hat{p}_1 + ...)(\hat{u}_1 + ...)dt \\ &= \frac{C_p A}{R}\frac{1}{2}\mathrm{Re}[\tilde{p}_1 \tilde{u}_1^*] = \frac{C_p}{R}\frac{1}{2}\mathrm{Re}[\tilde{p}_1 \tilde{U}_1^*] \end{aligned} \tag{2}$$

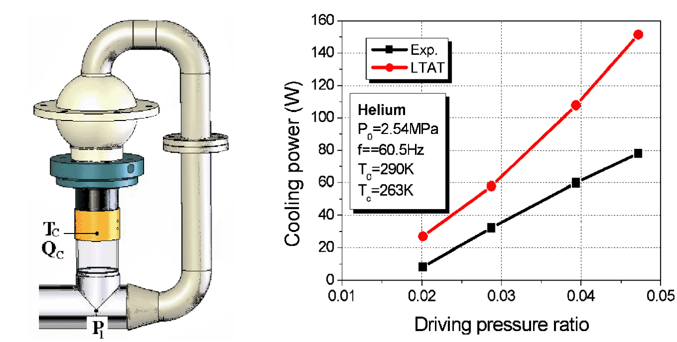
With the expression, Storch et al.[5] developed a phase shifting theory to explain the fundamental principle of the OPTR. Indeed, this expression correctly reflects some important physical aspects of the OPTR operation. The expression can also be taken as a result of the LTAT by adopting one of several deductive processes. If we use another deduction process, we will obtain a completely different result as follows:

$$\begin{aligned} Q_c &=< \bar{E}_p >=< \bar{E}_p >= \frac{1}{\tau}\oint(\int_A \hat{m} i dA)dt = \frac{1}{\tau}\oint[\int_A (\rho_0 + \hat{\rho}_1 + ...)(\hat{u}_1 + ...)C_p(T_0 + \hat{T}_1 + ...)dA]dt = \frac{1}{2}\rho_0 C_{p0}\mathrm{Re}[\tilde{T}_1 \tilde{U}_1^*] \\ &= \frac{1}{2}\rho_0 C_{p0}\mathrm{Re}[(\frac{1}{\rho_0 C_{p0}}\tilde{p} - \frac{\tilde{u}_1}{j\omega}\frac{dT_0}{dx})\tilde{U}_1^*] = \frac{1}{2}\mathrm{Re}[\tilde{p}_1 \tilde{U}_1^*] \end{aligned} \tag{3}$$

In the second deduction process, we have used the oscillating pressure and velocity to substitute for the oscillating temperature based on the second and third assumption. Now we have two completely different answers for the same question. Both deduction processes are strictly correct within acoustical approximation assumption, but why are the final results different? This is the first shortcoming of the linear thermoacoustic theory.

Now let us look at the second example. Figure 2a is the schematic diagram of a thermoacoustic-Stirling refrigerator that has been tested in our laboratory. Its structure parameters can be found elsewhere.[6] The LTAT has declared that it can reasonably describe the time-averaged thermoacoustic effects well within the acoustical approximation assumption. However, our experiments have shown that the LTAT can not reasonably predict the performance of the looped system, when even the acoustical variables are very small. Figure 2b gives the comparison of the cooling powers between the LTAT and the experiment. As can be seen from this figure, there is a substantial difference between the LTAT and the experiment no matter how small the driving pressure ratio is. The author thinks that this is the second shortcoming of the LTAT.

These are two important examples. One is a non-loop system which should have no cross-sectional DC mass flow, while the other is a looped system that usually has a cross-sectional DC mass flow. Moreover, no matter how small the oscillating pressure and velocity in both systems are, the LTAT fails to provide a robust prediction. In the past years, the author has carefully examined these questions and thinks that the time-averaged thermoacoustic effects, which have been stated in



a) Schematic diagram b) Cooling powers vs. driving pressure ratio.

Figure 2. Schematic diagram of the thermoacoustic-Stirling refrigerator and its cooling powers.

the LTAT, should be nonlinear in physical essence. Probably, the LTAT fails to provide a robust prediction of the nonzero, time-averaged thermoacoustic effects. If the LTAT is correct, it should be applicable when the acoustical variables are adequately small. However, it has failed to explain the phenomenon.

NONLINEAR THERMOACOUSTIC VIEWPOINT

The LTAT can not be corrected by simple modification. In fact, the energy effects in the regenerative or thermoacoustic systems can be the zeroth order, the first-order, the second-order, and higher orders. For widely-developed regenerative or thermoacoustic systems, the zeroth order energy fluxes do not exist because the particle velocity does not achieve sound velocity. The first-order energy fluxes do exist and are huge, but their time-averaged values are equal to zero; that is, it can not produce nonzero, time-averaged thermoacoustic effects. From a mathematical viewpoint, the first-order energy flows are really linear. For the second-order energy fluxes, they are the products of two acoustical or thermodynamic variables and should be considered as nonlinear. Usually, the second-order energy fluxes are composed of two parts: a nonzero time-averaged part and a time-dependent part. It is the second-order nonzero time-averaged item that the regenerative or thermoacoustic machines must operate on. Now that the nonzero, time-averaged energy effects are second-order and nonlinear, we must incorporate those nonzero time-averaged acoustical and thermodynamic variables with second-order accuracy, because they are actually parasitic in oscillating systems and can also produce nonzero time-averaged energy effects with second-order magnitude. For a small oscillating amplitude, the controlling equations of mass, momentum and energy conservation may be expanded to the second order, as do the nonzero time-averaged thermoacoustic effects. For a large oscillating amplitude, both of them may be expanded to higher orders such as a fourth order or even higher; however, its mathematical treatment would be too difficult to be realized.

Let us look at the first example again with the nonlinear viewpoint mentioned above. In the following mathematical treatments, all acoustical, thermodynamic and energy effect are kept with second-order accuracy.

$$
\begin{aligned}
p &= p_0 + \hat{p}_1 + p_{20} + \hat{p}_2 + \ldots \\
u &= \hat{u}_1 + u_{20} + \hat{u}_2 + \ldots \\
\rho &= \rho_0 + \hat{\rho}_1 + \rho_{20} + \hat{\rho}_2 + \ldots \\
T &= T_0 + \hat{T}_1 + T_{20} + \hat{T}_2 + \ldots
\end{aligned}
\tag{4}
$$

$$
<\vec{\dot{E}}> = <\vec{\dot{E}}_1> + <\vec{\dot{E}}_2> + \ldots = <\vec{\dot{m}}i> = \dot{m}_{20} C_p T_0 + C_p \hat{\dot{m}}_1 \hat{T}_1 \tag{5}
$$

Then, we perform the first and second deduction processes again. Firstly, we conduct the first deduction process.

$$
\begin{aligned}
Q_c &= <\overline{E}_p> = \frac{1}{\tau}\oint\left(\int_A \hat{\dot{m}} i dA\right)dt = \frac{1}{\tau}\oint\left(\int_A \rho u C_p T dA\right)dt = \frac{C_p A}{\tau R}\oint p u dt = \frac{C_p A}{\tau R}\oint (p_0 + \hat{p}_1 + \ldots)(\hat{u}_1 + u_{20} + \hat{u}_2 + \ldots)dt \\
&= \frac{C_p A}{R}\frac{1}{2}\mathrm{Re}[\tilde{p}_1 \tilde{u}_1^* + p_0 u_{20}]
\end{aligned}
\tag{6}
$$

Because the DC mass flow rate in the non-loop system is equal to zero, there is the following expression for u_{20} :

$$
u_{20} = -\frac{1}{2\rho_0}\mathrm{Re}(\tilde{\rho}_1 \tilde{u}_1^*) \tag{7}
$$

Substituting Equation (7) and making use of the thermodynamic relationship of an ideal gas, it is straightforward to obtain the following expression:

$$
\begin{aligned}
Q_c &= \frac{C_p A}{R}\frac{1}{2}\mathrm{Re}[\tilde{p}_1 \tilde{u}_1^* + p_0 u_{20}] = \frac{C_p A}{R}\frac{1}{2}\mathrm{Re}[(R T_0 \tilde{\rho}_1 + R \rho_0 \tilde{T}_1)\tilde{u}_1^* - \frac{p_0}{\rho_0}\tilde{\rho}_1 \tilde{u}_1^*] \\
&= \frac{1}{2}\rho_0 C_p \,\mathrm{Re}(\tilde{T}_1 \tilde{u}_1^*) = \frac{1}{2}\mathrm{Re}(\tilde{p}_1 \tilde{U}_1^*)
\end{aligned}
\tag{8}
$$

Now let us perform the second deduction process.

$$
\begin{aligned}
Q_c &= <\bar{E}_p> = <\bar{E}_p> = \frac{1}{\tau}\oint\left(\int_A \hat{m}idA\right)dt \\
&= \frac{1}{\tau}\oint\left[\int_A(\rho_0+\hat{\rho}_1+\rho_{20}+\hat{\rho}_2+\ldots)(\hat{u}_1+u_{20}+\hat{u}_2+\ldots)C_p(T_0+\hat{T}_1+T_{20}+\hat{T}_2+\ldots)dA\right]dt \\
&= \frac{1}{2}\rho_0 C_{p0}\,\mathrm{Re}[\tilde{T}_1\tilde{U}_1^*] + <\rho_0 u_{20}+\hat{\rho}_1\hat{u}_1> C_p T_0 = \frac{1}{2}\rho_0 C_{p0}\,\mathrm{Re}[\tilde{T}_1\tilde{U}_1^*] + \dot{m}_{20} C_p T_0 \\
&= \frac{1}{2}\rho_0 C_{p0}\,\mathrm{Re}[\tilde{T}_1\tilde{U}_1^*] = \frac{1}{2}\mathrm{Re}[\tilde{p}_1\tilde{U}_1^*]
\end{aligned}
\tag{9}
$$

In making a comparison between Equations (8) and (9), they are completely identical. This reasonable result gives the first support for our nonlinear viewpoint.

WEAKLY NONLINEAR THERMOACOUSTIC THEORY

Nonlinear Controlling Equations for Simple Channels (Parallel Plate as Example)

The second example is not as simple as the first one, and it needs a more complicated deduction process. For simplicity, we assume that the working gas is ideal and the heat capacity of the solid contacted by the working gas is infinitely large. Then, we can write the nonlinear controlling equations of mass, momentum and energy conservation of oscillating gas. With the relationships of thermodynamic properties of ideal gas, the following equations for pressure, velocity and temperature can be given as follows:

Pressure equation:

$$\frac{\partial p}{\partial t} + \vec{V}\cdot\nabla p + \gamma p\nabla\cdot\vec{V} = (\gamma-1)[\nabla\cdot(k\nabla T)+\phi] \tag{10a}$$

Velocity equation:

$$\rho\frac{\partial \vec{V}}{\partial t} + \rho\vec{V}\cdot\nabla\vec{V} + \nabla[p-(\mu_v+\frac{4}{3}\mu)\nabla\cdot\vec{V}] + \nabla\times(\mu\nabla\times\vec{V}) = 0 \tag{10b}$$

Temperature equation:

$$\rho C_p\frac{\partial T}{\partial t} + \rho C_p\vec{V}\cdot\nabla T - (\frac{\partial p}{\partial t}+\vec{V}\cdot\nabla p) = [\nabla.(k\nabla T)+\phi] \tag{10c}$$

State equation of ideal gas:

$$p = R\rho T \tag{10d}$$

Using nonlinear perturbation method, we can obtain the following zeroth-, first- and second-order equations.[7,8]

Zeroth-, First- and Second-Order Equations in Time Domain. We first give the following zeroth-, first- and second-order perturbation equations in time domain.

Zeroth-order equations:

Only state equation has zeroth-order equation as follows:

$$p_0 = R\rho_0 T_0 \tag{11}$$

First-order equations:

$$\frac{\partial p_1}{\partial t} + \gamma p_0\nabla\bullet\vec{V}_1 = (\gamma-1)k_0\frac{\partial^2 T_1}{\partial y^2} \tag{12a}$$

$$\rho_0\frac{\partial u_1}{\partial t} + \frac{dp_1}{dx} - \mu_0\frac{\partial^2 u_1}{\partial y^2} = 0 \tag{12b}$$

$$\rho_0 C_p\frac{\partial T_1}{\partial t} + \rho_0 C_p u_1\frac{dT_0}{dx} - \frac{\partial p_1}{\partial t} = k_0\frac{\partial^2 T_1}{\partial y^2} \tag{12c}$$

$$p_1 = R(\rho_0 T_1 + \rho_1 T_0) \tag{12d}$$

Boundary conditions:

$$y=0,\frac{\partial u_1}{\partial y}=0;\frac{\partial T_1}{\partial y}=0 \text{ and } y=b,\vec{V}_1=0;T_1=0 \tag{12e}$$

Second-order equations:

$$\frac{\partial p_2}{\partial t}+u_1\frac{dp_1}{dx}+\gamma(p_0\nabla\bullet\vec{V}_2+p_1\nabla\bullet\vec{V}_1)=(\gamma-1)[k_0\frac{\partial^2 T_2}{\partial y^2}+\frac{\partial}{\partial}(k_1\frac{\partial T_1}{\partial y})] \tag{13a}$$

$$\rho_0\frac{\partial u_2}{\partial t}+\rho_1\frac{\partial u_1}{\partial t}+\rho_0\vec{V}_1\bullet\nabla u_1+\frac{dp_2}{dx}+\frac{\partial}{\partial y}(\mu_0\frac{\partial u_2}{\partial y})+\frac{\partial}{\partial y}(\mu_1\frac{\partial u_1}{\partial y})=0 \tag{13b}$$

$$(\rho_0 C_p\frac{\partial T_2}{\partial t}+\rho_1 C_p\frac{\partial T_1}{\partial t})+\rho_0 C_p u_2\frac{dT_0}{dx}+\rho_0 C_p\vec{V}_1\bullet\nabla T_1-(\frac{\partial p_2}{\partial t}+u_1\frac{dp_1}{dx})$$

$$=[k_0\frac{\partial^2 T_2}{\partial y^2}+\frac{\partial}{\partial y}(k_1\frac{\partial T_1}{\partial y})] \tag{13c}$$

$$p_2=R(\rho_0 T_2+\rho_2 T_0+\rho_1 T_1) \tag{13d}$$

Boundary conditions:

$$y=0,\frac{\partial u_2}{\partial y}=0;\frac{\partial T_2}{\partial y}=0 \text{ and } y=b,\vec{V}_2=0;T_2=0 \tag{13e}$$

Zeroth-, First- And Second-Order Equations in Frequency Domain. Using the following complex notation for acoustical and thermoacoustic variables can lead to the controlling equations in frequency domain. Because the oscillating part of the second-order acoustical variables have no contribution to nonzero, time-averaged thermoacoustic effects, we won't write out their frequency-domain equations.

$$\begin{aligned}p&=p_0+\mathrm{Re}(\tilde{p}_1 e^{j\omega t})+p_{20}+\mathrm{Re}(\tilde{p}_2 e^{j2\omega t})+\ldots\\ u&=\mathrm{Re}(\tilde{u}_1 e^{j\omega t})+u_{20}+\mathrm{Re}(\tilde{u}_2 e^{j2\omega t})+\ldots\\ \rho&=\rho_0+\mathrm{Re}(\tilde{\rho}_1 e^{j\omega t})+\rho_{20}+\mathrm{Re}(\tilde{\rho}_2 e^{j2\omega t})+\ldots\\ T&=T_0+\mathrm{Re}(\tilde{T}_1 e^{j\omega t})+T_{20}+\mathrm{Re}(\tilde{T}_2 e^{j2\omega t})+\ldots\end{aligned} \tag{14}$$

First-order controlling equations in frequency domain:

$$j\omega\tilde{p}_1+\gamma p_0\nabla\bullet\tilde{\vec{V}}_1=(\gamma-1)k_0\frac{\partial^2\tilde{T}_1}{\partial y^2} \tag{15a}$$

$$j\rho_0\omega\tilde{u}_1+\frac{d\tilde{p}_1}{dx}-\mu_0\frac{\partial^2\tilde{u}_1}{\partial y^2}=0 \tag{15b}$$

$$j\omega\rho_0 C_p\tilde{T}_1+\rho_0 C_p\tilde{u}_1\frac{dT_0}{dx}-j\omega\tilde{p}_1=k_0\frac{\partial^2\tilde{T}_1}{\partial y^2} \tag{15c}$$

$$\tilde{p}_1=R(\rho_0\tilde{T}_1+\tilde{\rho}_1 T_0) \tag{15d}$$

Boundary conditions:

$$y=0,\frac{\partial\tilde{u}_1}{\partial y}=0;\frac{\partial\tilde{T}_1}{\partial y}=0 \text{ and } y=b,\tilde{\vec{V}}_1=0;\tilde{T}_1=0 \tag{15e}$$

Second-order controlling equations:

$$\frac{1}{2}\mathrm{Re}(\tilde{u}_1^*\frac{d\tilde{p}_1}{dx})+\gamma[p_0\nabla\bullet\vec{V}_{20}+\frac{1}{2}\mathrm{Re}(\tilde{p}_1^*\nabla\bullet\tilde{\vec{V}}_1)=(\gamma-1)[\frac{\partial^2 T_{20}}{\partial y^2}+\frac{1}{2}\mathrm{Re}[\frac{\partial}{\partial}(\tilde{k}_1^*\frac{\partial\tilde{T}_1}{\partial y})]] \tag{16a}$$

$$\frac{1}{2}\mathrm{Re}(\tilde{\rho}_1^* j\omega\tilde{u}_1)+\frac{1}{2}\mathrm{Re}[\rho_0\tilde{\vec{V}}_1^*\bullet\nabla\tilde{u}_1]+\frac{dp_{20}}{dx}+\frac{\partial}{\partial y}(\mu_0\frac{\partial u_{20}}{\partial y})+\frac{1}{2}\mathrm{Re}[\frac{\partial}{\partial y}(\mu_1^*\frac{\partial\tilde{u}_1}{\partial y})=0 \tag{16b}$$

$$\frac{1}{2}\mathrm{Re}(\tilde{\rho}_1^* C_p j\omega\tilde{T}_1)+\rho_0 C_p u_{20}\frac{dT_0}{dx}+\frac{1}{2}\rho_0 C_p\mathrm{Re}(\tilde{\vec{V}}_1^*\ \nabla\tilde{T}_1)-\frac{1}{2}\mathrm{Re}(\tilde{u}_1^*\frac{d\tilde{p}_1}{dx})$$

$$=[k_0\frac{\partial^2 T_{20}}{\partial y^2}+\frac{1}{2}\mathrm{Re}[\frac{\partial}{\partial y}(\tilde{k}_1^*\frac{\partial \tilde{T}_1}{\partial y})]] \tag{16c}$$

$$p_{20}=R[\rho_0 T_{20}+\rho_{20}T_0+\frac{1}{2}\mathrm{Re}(\tilde{\rho}_1\tilde{T}_1^*)] \tag{16d}$$

Boundary conditions:

$$y=0,\frac{\partial u_{20}}{\partial y}=0;\frac{\partial T_{20}}{\partial y}=0 \text{ and } y=b,\vec{V}_{20}=0;T_{20}=0 \tag{16e}$$

Obviously, solving the first-order equations is the key to the problem. Fortunately, the first-order equations are still the same as the LTAT, and their solution were already obtained by Rott[1], and were nicely summarized by Swift[2], Xiao[3] and Tominaga[4]. Substituting the first-order solutions into the second-order Equations (16a) to (16e) and integrating the second-order time-averaged variables can give the final equations that is independent of y coordinate. The final nonlinear controlling equations are as follows:

$$\frac{d\tilde{p}_1}{dx}=-\frac{j\omega\rho_0}{A(1-f_\mu)}\tilde{U}_1 \tag{17a}$$

$$\frac{d\tilde{U}_1}{dx}=-\frac{j\omega A}{\gamma p_0}[1+(\gamma-1)f_\mu]\tilde{p}_1+\frac{f_k-f_\mu}{(1-\mathrm{Pr})(1-f_\mu)}\frac{1}{T_0}\frac{dT_0}{dx}\hat{U}_1 \tag{17b}$$

$$\frac{dp_{20}}{dx}=-\frac{1}{A^2}\iint\{\mu_0\frac{\partial^2 u_{20}}{\partial y^2}+\frac{1}{2}\mathrm{Re}[\tilde{\rho}_1^* j\omega\tilde{u}_1+\rho_0\tilde{\vec{V}}_1^*\bullet\nabla\tilde{u}_1+\frac{\partial}{\partial y}(\mu_1^*\frac{\partial\tilde{u}_1}{\partial y})]\}dA^2 \tag{17c}$$

$$\frac{du_{20}}{dx}=\frac{1}{A}\int\{-\frac{1}{2}\mathrm{Re}(\tilde{u}_1^*\frac{d\tilde{p}_1}{dx}+\tilde{p}_1^*\nabla\bullet\tilde{\vec{V}}_1)+\frac{(\gamma-1)}{\gamma p_0}[\frac{1}{2}\mathrm{Re}(\tilde{\rho}_1^* C_p j\omega\tilde{T}_1+\rho_0 C_p\vec{V}_1^*\bullet\nabla\tilde{T}_1-\tilde{u}_1^*\frac{d\tilde{p}_1}{dx})+\rho_0 C_p u_{20}\frac{dT_0}{dx}]\}dA \tag{17d}$$

$$\frac{dT_0}{dx}=-\frac{{}^1\!/_2\mathrm{Re}[\tilde{U}_1\tilde{p}_1^*(1-f_{qx})-(<\dot{E}_x>-C_p\int T_0\dot{m}_{20}dA)}{k_{eff}A_f} \tag{17e}$$

Nonlinear Controlling Equations for Porous Regenerator

We use one-dimensional models for oscillating regenerator as follows:

Pressure equation:

$$\frac{\partial p}{\partial t}+\vec{V}\bullet\nabla p+\gamma p\nabla\bullet\vec{V}=(\gamma-1)[\nabla\bullet(k\nabla T)+\phi+\phi_m+\frac{ha_H}{\delta}(T_s-T)] \tag{18a}$$

Velocity equation:

$$\rho\frac{\partial\vec{V}}{\partial t}+\rho\vec{V}\bullet\nabla\vec{V}+\nabla p=-\alpha\mu\vec{V}-\rho\beta_f|\vec{V}|\vec{V}-\beta_t\frac{\partial\vec{V}}{\partial t} \tag{18b}$$

Temperature equation of gas:

$$\rho C_p\frac{\partial T}{\partial t}+\rho C_p\vec{V}\bullet\nabla T-(\frac{\partial p}{\partial t}+\vec{V}\bullet\nabla p)=[\nabla.(k\nabla T)+\phi+\phi_m+\frac{ha_H}{1-\delta}(T_s-T)] \tag{18c}$$

Temperature equation for solid matrix:

$$\rho_s C_s\frac{\partial T_s}{\partial t}=[\nabla.(k_s\nabla T)+\frac{ha_H}{1-\delta}(T-T_s)] \tag{18d}$$

State Equation of ideal gas:

$$p=R\rho T \tag{18e}$$

Zeroth-, First- And Second-Order Equations in Frequency Domain. With a similar deduction process, we can obtain the final controlling equations of a regenerator in frequency domain as follows:

$$\frac{d\tilde{p}_1}{dx}=[-\frac{j\omega\rho_0}{A_f}(1+\beta_t)+\frac{\alpha\mu_0}{A_f}]\tilde{U}_1 \tag{19a}$$

$$\frac{d\tilde{U}_1}{dx}=-\frac{j\omega A_f}{\gamma p_0}[1+(\gamma-1)f_h]\tilde{p}_1+f_{WT}\frac{1}{T_0}\frac{dT_0}{dx}\hat{U}_1$$

$$\frac{dp_{20}}{dx} = -\alpha\mu_0\tilde{u}_1 - \frac{1}{2}\mathrm{Re}(\tilde{\rho}_1^* j\omega\tilde{u}_1 + \rho_0\tilde{u}_1^*\frac{\partial\tilde{u}_1}{\partial x}) \tag{19b}$$

(19c)

$$\frac{du_{20}}{dx} = \frac{1}{\gamma p_0}\{-\frac{1}{2}\mathrm{Re}(\tilde{u}_1^*\frac{d\tilde{p}_1}{dx}) + (\gamma-1)[\frac{1}{2}\mathrm{Re}(C_p\tilde{\rho}_1^* j\omega\tilde{T}_1 + C_p\rho_0\tilde{u}_1^*\frac{d\tilde{T}_1}{dx} - \tilde{u}_1^*\frac{d\tilde{p}_1}{dx}) + \rho_0 C_p u_{20}\frac{dT_0}{dx}]\}$$

(19d)

$$\frac{dT_0}{dx} = -\frac{\frac{1}{2}\mathrm{Re}[\tilde{U}_1\tilde{p}_1^*(1-f_{qx}) - (<\dot{E}_x> - C_p\int T_0\dot{m}_{20}dA)}{k_{eff}A_f}$$

(19e)

Nonlinear Controlling Equations for Jet Pump

A jet pump is the representative element where pressure drop and energy losses produce an abrupt change. The following expressions provide the estimates for the drop and loss.[9]

$$\Delta p_{20} = -\frac{1}{8}\rho_0 |\tilde{u}_1|^2 (\kappa_{out} - \kappa_{in}) \tag{20a}$$

$$\Delta <\dot{W}_x> = -\frac{8}{3\pi}\Delta p_{20} |A\tilde{u}_1| \frac{\kappa_{out} + \kappa_{in}}{\kappa_{out} - \kappa_{in}} \tag{20b}$$

Combining the above nonlinear equations for the thermoacoustic-Stirling refrigerator shown in Figure 2, we recalculated the cooling powers under different driving pressure ratio. In the meantime, we conducted an additional experiment by using an elastic membrane to suppress the DC mass flow rate in the system, which was pointed out by Gedeon[10], and also finished an additional calculation with the new nonlinear theory. All these results are given in Figure 3. A good agreement between the nonlinear theory and the experiment is achieved. Thus, we think that this is the second strong support for our nonlinear viewpoint.

However, for more strong nonlinear oscillation, the weakly nonlinear thermoacoustic theory developed here may not be applicable. Although we can expand the acoustical variable and energy fluxes up to higher orders with the above nonlinear perturbation method, it will be extremely complicated in the mathematical treatment. For this case, we recommend using a complete CFD method, which is more realistic and feasible. In fact, we have already successfully simulated two- and three-dimensional problems of various thermoacoustic systems by using a CFD method which has shown a good prospect in simulating oscillating thermoacoustic systems.[11]

SUMMARY

Two important examples of the shortcomings of LTAT are given. Then the author declared that nonzero, time-averaged thermoacoustic effects, which occurs in oscillating thermodynamic systems, should be better considered as nonlinear effects in physical essence. In fact, there is streaming in the oscillating flow systems; including mass, momentum and energy streaming phenomenon. The three acoustic streamings occur simultaneously in coupled ways. Introducing the new standpoint, it is reasonable to explain the two difficult questions encountered by the LTAT and reach a

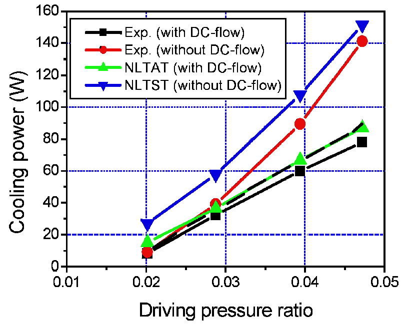


Figure 3. Comparison between the nonlinear thermoacoustic model and the experiment

self-contained conclusion. For small oscillating flow system, a weakly nonlinear thermoacoustic theory can be founded by using the nonlinear perturbation method. And, a second-order models for the simple flow channel, regenerator and jet pump are developed or summarized, which may provide a deep insight and powerful tool to understand and design thermoacoustic and regenerative systems. For strong oscillating flow systems, a fully CFD method could be a realistic and powerful theoretical tool.

NOMENCLATURE

A Cross-sectional area
a_H Specific heat exchange area of regenerator
C_p Isobaric specific capacity of gas
E Total energy or enthalpy flux
f Complex thermoacoustic factor
h Convective heat exchange coefficient
I Enthalpy per unit kilogram
J Imaginary unit
k Thermal conductivity
Pr Prandtl number
p Pressure
Q Cooling power
R Gas constant
Re[] Real part of
T Temperature
t Time
U Volume velocity
u Velocity of x component
V Velocity vector
W Acoustical power or consumption power
x Longitudinal direction
y Transversal direction
<> Time-averaged

Greek letters
α Flowing resistance factor of regenerator
β_f Nonlinear resistance factor
β_t Additional mass factor
γ Adiabatic index
δ Porosity
ϕ Dissipation function of flowing
μ Dynamic viscosity
ρ Density of gas
ω Angular frequency
κ Local pressure loss factor
τ Cycle period

Subscripts and superscripts
0 Zeroth order
1 First order (acoustical approximation)
2 Second order
f Fluid
s Solid
κ Thermal
μ Viscous
^ Fluctuation quantity
* Complex conjugation of
~ Amplitude in frequency domain

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REFERENCES

1. Rott, N., "Thermoacoustics," *Adv.Appl.Mech* , Vol. 20 (1980), pp.135-175.
2. Swift, G.W., "Thermoacoustic Engines," *J.Acoust.Soc.Am.*, Vol.84 (1988), pp.1145-1180.
3. Xiao, J.H., "Thermoacoustic Heat Transportation and Energy Transformation," *Cryogenics,* Vol. 35, Issue: 1 (January 1995), pp.15-29.
4. Tominaga, A., "Thermodynamic Aspects of Thermoacoustic Theory," *Cryogenics*, Vol. 35, Issue: 7 (July 1995), pp. 427-440.
5. Storch, P., and Radebaugh, R., "Development and Experimental Test of an Analysis Model of the Orifice Pulse Tube Refrigerator," *Adv. in Cryogenic Engineering*, Vol. 33 (1987), Plenum Press, New York, pp. 851-859.
6. Luo, E.C., Dai, W., Zhang, Y., et al., "Thermoacoustically Driven Thermoacoustic Refrigerator with Double Thermoacoustic-Stirling Cycles," *Applied Physic Letters*, vol. 88 (2006), pp.74-102.
7. Olson, J.R., and Swift, G.W., "Acoustic Streaming in Pulse Tube Refrigerators: Tapered Pulse Tubes," *Cryogenics*, Vol. 37, Issue: 12 (December 1997), pp.769-776.

8. Gusev, V. et al., “Acoustic Streaming in Annular Thermoacoustic Prime-Movers,” *J.Acoust.Soc.Am.*, Vol. 108, No. 3 (2000), pp. 934-945.

9. Petculescu, A., Wilen, L.A., “Oscillatory Flow in Jet Pumps: Nonlinear Effects and Minor Losses,” *J.Acoust.Soc.Am.*, Vol. 113, No. 32 (2003), pp.1282-1290.

10. Gedeon, D., “DC Gas Flows in Stirling and Pulse-Tube Cryocoolers,” *Cryocooler*s 9, Plenum Press, New York (1997), pp.85-392. 9.

11. Yu, G.Y., et al., "Study on Several Important Nonlinear Processes of a Large Experimental Thermoacoustic-Stirling Heat Engine using Computational Fluid Dynamics," *Cryogenics* (2006, in review).