RELIABILITY OF BRIDGE SUPERSTRUCTURES IN WISCONSIN

by

Azam Nabizadehdarabi

A Thesis Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Master of Science

in Engineering

at

The University of Wisconsin-Milwaukee

August 2015

ABSTRACT

RELIABILITY OF BRIDGE SUPERSTRUCTURES IN WISCONSIN

by

Azam Nabizadehdarabi

The University of Wisconsin-Milwaukee, 2015

Under the Supervision of Professor Habib Tabatabai

Bridge reliability analyses form the basis of modern bridge design provisions. Such analyses are also used to evaluate bridge condition and assess future maintenance needs. The majority of these reliability assessments are based on evaluating risks associated with the various load effects exceeding their corresponding resistance. However, bridge components such as decks and superstructures typically do not reach the end of their service lives by exceeding the strength limit states (structural failure). Survival analyses, which are commonly used in biomedical research, consider different factors affecting survival time (or service life) by analyzing large-scale data on survival time and corresponding factors. The survival (reliability) and hazard (failure rate) functions of the bridge (elements) are then determined by fitting the data to an appropriate statistical distribution. Only a few bridge reliability evaluations have been based on survival analyses.

In this research, reliability of bridge superstructures in Wisconsin was investigated through the hypertabastic accelerated failure time survival model. The 2012 National Bridge Inventory (NBI) data were used in the survival analysis. The parameters of the model were determined using the maximum likelihood method.

The type of bridge superstructure, bridge age, maximum span length (MSL) and average daily traffic (ADT) were taken into account to evaluate superstructure survival time. A recorded NBI superstructure condition rating of 5 was considered to be the end of service life for the superstructure.

The results show that the type of superstructures, (ADT), and (MSL) are important factors in survival time of bridge superstructures. The mean age of steel superstructures at the end of service life was larger than that for concrete superstructures. At a given age, as the maximum span length increases, the reliability of the superstructure decreases and the failure rate increases. Similarly, increasing ADT has a significant effect on reducing the superstructure reliability and increasing the failure rate. Although the code-specified bridge design life is 75 years, results showed a very small level of superstructure reliability at that age.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1. Introduction

1.1 Background

Based on the National Bridge Inventory database, there are approximately 600,000 public road bridges in the United States transportation network (Kirt, 2007), of which more than 9700 are located in the State of Wisconsin. Most of these bridges were built between 1930s and 1960s. According to the American Association of State Highway and Transportation Officials (AASHTO) Standard Specification for Highway Bridges, the design life for bridges is 75 years. This indicates that many U.S bridges are at or near the end of their design service life.

In addition to serving as economic lifelines, bridges also provide crucial evacuation linkages at the time of natural disasters and extreme conditions. Bridge decks and superstructures deteriorate due to long-term effects of environmental exposure, traffic load (live load), and deficient maintenance. Engineers have an interest in seeking better maintenance and repair methods to achieve a desirable bridge conditions. Each bridges must be evaluated by inspectors at least once every two years. Inspectors give numerical ratings to different components of each bridge through visual inspection and sometimes NDT[1] methods.

[1] Nondestructive Testing consists of techniques used to evaluate material properties and structure condition without causing damage or altering the structure (Yehia et al, 2007).

Occasional catastrophic bridge collapses refocus bridge engineers' keen attention on the topic of bridge reliability and safety assessment. Numerous research effects have focused in recent years on reliability and probability of failure of bridge structures.

Reliability of bridge structure have been frequently studied using the strength limit states as the governing criterion. Serviceability limit states have also been used as the basis for reliability analysis of bridges and bridge elements, but to a much smaller extent than the strength limit states.

Reliability analyses can be performed based on time-dependent or time-independent approaches. A general reliability-based model uses different random variables to assess various strength limit states. Reliability analysis using strength limit states can be used to calculate a reliability index β, which is a relative indication of the probability of load exceeding resistance (failure).

In recent years, the concept of survival analysis, which is used mostly in bio-medical research, has been introduced into bridge reliability engineering. The term "survival" commonly used in biomedical research is synonymous with "reliability" used in engineering. Similarly, the term "hazard" has the same meaning as "failure rate". Some research has been done to develop survival models for bridges decks and superstructures through survival analysis (Tabatabai et al, 2011; Beng et al, 2012; and Yang et al, 2013). Survival analysis takes into account the time that takes it to reach an event such as corrosion initiation, cracking, or the end of service life. Time is a characteristic variable in this approach. In general, "survival time" is a non-negative random variable indicating the elapsed time from a reference time to the occurrence of a given event (Lee and Go, 1997). Examples of survival time are: time to recurrence of cancer after administration of a drug, time until an electronic device breaks down, time until a bridge deck is no longer serviceable, etc. In general, survival analyses are based on an assumption of the type and shape of the hazard function.

Survival analyses typically require availability of large datasets. The survival time as well as other variables of interest (covariates) are included in the analyses. Results of survival analyses can be used to determine the probability of survival (reliability) and instantaneous failure rates (hazard).

1.2 Objective

This research is aimed at developing survival models for bridge superstructures in Wisconsin using data obtained from the NBI database. In this regard, survival (reliability) and hazard (failure rate) equations were developed for bridge superstructures as a function of maximum span length (MSL) and average daily traffic (ADT).

1.3 Outline of Thesis

This thesis is organized into five chapters.

Chapter 1 provides a brief background about reliability and survival analyses, objective of the research, and an outline of the thesis.

Chapter 2 reviews reliability analyses and their application to bridge engineering. Different reliability methods are presented through literature review of research in the field of reliability of bridges.

Chapter 3 discusses conventional reliability modeling, reliability index β, time dependent and independent reliability, and survival models. The basic attributes of each method are presented.

Chapter 4 includes results of survival analyses for Wisconsin's bridge superstructures using the hypertabastic model and discusses different factors affecting the survival time.

Finally, Chapter 5 summarizes the research, presents conclusions, and outlines ideas for future studies.

Chapter 2. Literature Review

2.1 Introduction

Although extensive studies have been performed on bridge reliability engineering, the great majority of such works are reliability assessments that are based on strength limit states (load exceeding resistance). However, many bridge components (such as decks and girders) typically do not reach the end of their service lives by exceeding a strength limit state.

Survival analysis, which is a type of reliability analysis, has its root in biomedical applications and has not been widely utilized in bridge engineering. In this chapter, a review of general reliability engineering is presented followed by discussion of strength-based, time-dependent, and serviceability-based approaches including survival analysis.

In section 2.2 of this chapter, the general concept of reliability in engineering is introduced. In section 2.3, a very brief introduction to reliability of bridges is presented based on the safety index β. Section 2.4 discusses studies on reliability methods versus load rating. Other studies involving application of strength-based or serviceability-based reliability are summarized in section 2.5. Finally, in section 2.5, the survival model used in this study is discussed and distinguished from other approaches.

2.2 General Concept of reliability

As a general term, reliability is a desirable characteristic of any system, which is associated with trustworthiness and dependability. However, in engineering applications, reliability is a parameter that can be evaluated, and predicted for various engineering elements or systems. Although a system may be designed for a specific level of performance, it may not reach or sustain those performance targets under some circumstances. Engineering reliability analysis is an important tool that is commonly used to estimate the likelihood of a system or component ceasing to perform its intended function (i.e. failure) under different circumstances. Reliability engineering helps product developers, engineers, and maintenance personal understand and quantify the potential for failure. This can help improve safety and effectiveness of products and systems in various fields including aerospace, transportation, etc (Bazovsky, 2005; and Elsayed, 2012).

Any system may fail to operate properly under one of three different circumstances. First, early failures can occur due to quality issues in manufacturing or construction. Second, longer-term failures can occur as a consequence of component aging and wear, or due to inadequate maintenance. And finally, there are also “chance” or random failures (Bazovsky, 2005).

Reliability can be defined as probability of a product or system continuing to operate without failure for a given time period (Bazovsky, 2005).

Considering a time interval (t-Δt, t), reliability R(t) at any time t can be formulated as (Elsayed, 2012):

$$R(t) = \frac{n_s(t)}{n_s(t)+n_f(t)} = \frac{n_s(t)}{n_o} \qquad (2.1)$$

Where $n_s(t)$ is quantity of surviving or operating components at the time t, $n_f(t)$ is the number of failed components during the time interval (t- Δt, t), and n_o is the total number of identical elements in the time interval (t- Δt, t).

2.3 Bridge reliability

Modern safety and reliability models has been extensively applied to the highway bridge engineering systems in past decades. The reliability models have been the foundation for development of the Load and Resistance Factor Design (LRFD) (Frangopol, 1999). However, reliability aspects are not explicitly treated in the LRFD bridge specifications (Frangopol, 1999). Thus, bridge engineers may not be aware of the underlying reliability concepts for bridge design provisions (Frangopol, 1999).

Due to uncertainties in design, loading, construction procedures, material properties and strength parameters, there is always a slight risk of failure in structure. Although absolute safety is not realistic, an acceptable risk level consistent with safety and economic considerations is inherent in the design provisions for bridges (AASHTO, 2012), buildings (AISC, 14th edition), and offshore platforms (API, 7th edition) (Frangopol, 1999).

Developments in probability theory and risk analysis along with available statistical data on load and resistance has changed the traditional approach for structural design. In the traditional design method, a single safety factor is used to determine allowable stresses. The traditional allowable stress design, however, generally resulted in a widely non-uniform level of reliability across all elements of a structure. The newer reliability-based approaches aim for a more uniform level of reliability cross all elements and components of the bridge (M. Frangopol, 1999).

2.3.1 Reliability index, β

The basic random variables, for a strength-based reliability model, are resistance (R) and load or load effect (L). Each of these two parameters may be dependent on other random variables. For example, resistance is dependent on a structural member's dimensions, material properties, reinforcement detail, and construction procedures. Variable L consists of several load types including dead load, live load, wind load, etc. Live load has uncertainties related to magnitude of truck loads and positions of those loads on a bridge. A function representing each random variable can be expressed based on available statistical information, (Frangopol, 1999).

In general, a failure function g is defined as follows (Frangopol, 1999):

$$g = R - L \tag{2.2}$$

If $g > 0$, resistance of the element under consideration exceeds the corresponding load effect, and thus failure would not occur. When $g < 0$, the applied load exceeds the resistance of the element under consideration and the element would fail.

The failure probability may be written as (P_f):

$$P_f = P[g < 0] \tag{2.3}$$

Strength-based reliability depends on evaluating the risk associated with load exceeding resistance considering the variability of both parameters. The probability of failure can be controlled through the choice of load and resistance factors in the specifications. Risks are measured based on a comparison of demand and capacity and the uncertainties related to these parameters (Frangopol, 1999). This approach is not intended to completely eliminate the risk of failure, but to realize an "acceptable" level of risk.

Strength-based reliability in structures including bridges is usually calculated through an assumption of normal (or log-normal) distributions for random variables. The reliability index, β, can be determined using the following equation (Nowak, 2000):

$$\beta = {}^{\overline{g}}/_{\sigma_g} \qquad (2.4)$$

Where, $\overline{g}$ is the mean of the failure function g and σ_g is standard deviation of g. The reliability index β indicates the number of standard deviations that the mean of the failure function is distanced from g=0 (failure). A larger β value is representative of higher reliability.

2.4 Load rating versus reliability analysis

Wardhana and Hadipriono (2008) investigated factors contributing to failure in more than 500 bridges between 1989 and 2000. They determined that the most frequent causes of bridge failure were floods and collisions, overloading of trucks on bridges and impact forces from trucks and barges. Bridge owners perform periodic load ratings of bridges to assess strength and permissible truck weights (Estes and Frangopol, 2005; Hwang, 2008). Akgul and Frangopol (2003) explored load ratings to determine the bridge reliability index based on AASHTO LRFD bridge design specifications and AASHTO Guide for Condition Evaluation of Bridges. Two load rating criteria – inventory and operating – are typically considered. The inventory rating is associated with the maximum load that is expected to be applied on a routine basis. The operating rating indicates the maximum load that can be applied during the lifetime of a bridge.

Akgul and Frangopol (2004) showed interdependence between load-rating and reliability indices through analysis of a network of bridges for different limit states. They reported that correlation existed between load rating and reliability indices for some of limit states in steel or concrete superstructures.

Akgul and Frangopol (2003) utilized load ratings and illustrated reliability assessment procedures at different stages from design through construction to maintenance. By comparing bridge ratings at discrete time intervals with probability-based prediction models, they related rating and reliability results (Hwang, 2008). Akgul and Frangopol (2004) also proposed a basis for allocating federal highway funds to bridge maintenance and rehabilitation based on NBI bridge ratings.

2.5 Practical application of conventional bridge reliability analysis

Most recent reliability-based optimization of bridge management tasks such as inspection, maintenance, and rehabilitation strategies for deteriorated bridges are based on serviceability and strength limit states (e.g., Mori and Ellingwood 1994a,b; Thoft-Christensen 1995; Estes and Frangopol 1999; Faber and Sorensen 1999) (Stewart. et al, 2004). Although most bridge reliability research is based on time-independent analyses, some have focused on time-dependent reliability analysis. Furthermore, some researchers took into account both strength and serviceability limit states when considering time-dependent reliability (e.g., Stewart and Rosowsky 1998; Vu and Stewart 2000). Others worked on the serviceability limit alone in reliability assessments (e.g., Troive and Sundquist 1998; Holicky and Mihashi 1999; Estes and Frangopol 2000).

According to Lu et al. (2012), there are four types of time-dependent reliability assessment: time-integrated method; time discretized method; time discretized-integrated method; and first passage probability method.

In the time-integrated approach, resistance, R(t), and load changes, S(t), are evaluated within the service time period of 0 to T years (entire standard service period). In this method, the resistance parameter is considered to be the minimum resistance in the full service period, while the load or load effect is the maximum applied in the same service period.

In the time integrated method, the system failure probability can be written as:

$$p_f(T) = \mathrm{P}[R_{min} < S_{max}] \qquad (2.7)$$

Whereas, $S_{max} = \max_{0 \le t \le T} S(t)$ indicates the maximum load effect in the entire service life and $R_{min} = \min_{0 \le t \le T} R(t)$ indicates the minimum resistance in the service period.

In the time-discretized approach, the entire service time is split into discrete periods, and the resistance within each period is estimated and compared with corresponding maximum load effects within the same period. Thus, the time-dependent evaluation is converted into a series of time-independent problems (Lu et al., 2012).

Gong and Zhao (1998) proposed a combination of time-integrated and time-discretized methods described as time integrated-discretized approach (Lu et al., 2012).

In the time integrated-discretized approach, the failure probability would be (Lu et al., 2012):

$$Z(t) = R(t) - S_G - S_Q\,(t) \qquad (2.13)$$

Where, R (t) is time-dependent resistance, S_G is dead load effect, and $S_Q\,(t)$ is the live load effect. Therefore, failure probability in any desired period during structure lifetime:

$$p_f(T) = \mathrm{P}\left[\min_{0\le t\le T}\left[R(t) - S_G - S_Q\,(t)\right] < 0\right] \qquad (2.14)$$

The design period is then divided into m equal periods and the structural failure probability is calculated.

Kong et al. (2003) used the reliability index β (t) to optimize life-cycle cost and evaluate the reliability of deteriorating structures. Instead of considering a fixed value for the reliability index at a certain point in time, the reliability index profile is defined as a probability distribution. The quality and quantity of the information used as input data is very important in this approach (Kong et al., 2003).

Lee (2011), used FRP composites to rehabilitate bridges. In this work, a time-dependent reliability method was used to evaluate safety and approximate remaining service life of the strengthened deck. The β index for a reinforced concrete bridge deck strengthened with FRP was determined.

Lu et al. (2011) conducted time-dependent reliability assessments of deteriorated reinforced concrete bridges under service load. To calculate the reliability index, the first-passage probability method was used for the concrete crack and deflection limit states. They compared the calculated reliability index to the target reliability index. The remaining bridge service life was predicted and compared based on both concrete crack and component deflection criteria.

In 2004, Stewart et al performed research on the effect of strength and serviceability limit states on life-cycle costs and bridge replacement strategies related to corrosion (Akgul and Frangopol, 2004). They concluded that considering strength limit state without serviceability analysis would result in unrealistic repair cost estimates. They concluded that multiple limit states should be considered under aggressive environments when analyzing bridge deck life-cycle costs (Akgul and Frangopol, 2004).

In 2001, Sun and Hong performed time-dependent reliability research on the effect of corrosion on reliability of bridge girders. They included the effects of parameters uncertainties on bridge girder's reliability. Reinforcement corrosion's influence on the bridge lifetime was established through a corrosion model in two phases: 1) time to corrosion initiation, and 2) corrosion development or propagation. They used strength limit states in their research, but took into account time-variant parameters in failure probability functions. The effects of corrosion initiation, corrosion growth, and modelling error were incorporated in the flexural and shear capacity (resistance) of the members.

They concluded that ignoring the uncertainties in surface chloride concentration and diffusion coefficient do not significantly vary the reliability estimation. The most important factor dominating the outcomes related to the nonlinear behavior of the corrosion growth model.

Giorgio and Frangopol (2013) investigated probabilistic approaches based on the annual reliability index, annual risk, and two lifetime distributions (reliability and hazard function) in order to determine the optimized maintenance time for aging structures. They investigated correlations between failure modes and concluded that for correlated failure modes, both annual reliability index and annual risk approaches result in approximately the same time for maintenance of deteriorating components.

Saydam and Frangopol (2011) studied time-dependent performance indicators on deteriorated bridge superstructures. They investigated vulnerability, redundancy and robustness of bridge superstructures. The developed framework was then applied to a bridge located in Wisconsin. Results showed that the main factors affecting reliability are due to corrosion and live load effects. At early years of service life, live load effects have a greater effect on reliability while at late stages, corrosion would become a more significant factor.

In 1998, Stewart and Rosowsky conducted a study on time-dependent reliability of deteriorating reinforced concrete bridge decks. They used a reliability model to assess the probability of flexural failure of a concrete slab bridge under corrosion. The Monte Carlo simulation was used to simulate 75 years of service life. The authors showed that reinforcement corrosion could cause a significant drop in resistance, and long-term reduction in safety.

2.6. Survival analysis

In spite of substantial research performed on the topic of reliability of bridges, limited studies exist regarding time-dependent survival analysis on bridge superstructures.

The reliability index approach (resistance versus load) is commonly utilized for future (remaining) service life prediction. However, the vast majority of bridge deterioration / end of service life cases do not relate to exceeding the strength limit state. Bridge structures commonly reach the end of their service life without any structural failure. The extent of deterioration renders the bridge not serviceable. Therefore, survival analysis may be more appropriate as the contributing factors are studied without a direct causal relationship between one factor (load) and failure.

Survival analyses could be used through non-parametric, semi-parametric, and parametric approaches. For simple data and failure patterns, non-parametric approach would work efficiently. However, as variables in the failure process increase, the need for parametric approach becomes more evident.

In survival analyses, the time to any event is considered a random variable T, (survival time) with probability density function $f(t)$, where t is time. Survival function, $S(t)$, and hazard function, $h(t)$ are defined below (Elsayed, 2012):

$$S(t) = P(T \geq t) = 1 - F(t) \qquad (2.17)$$

Where $F(t)$ is defined as:

$$F(t) = P(T < t) = \int_0^t f(t)dt \qquad (2.18)$$

$$h(t) = \lim_{\delta t \to 0} \left\{ \frac{P(t \leq T \leq t + \delta t | T \geq t)}{\delta t} \right\} \qquad (2.19)$$

Hazard function, survival function, and probability function are related to each other as below:

$$h(t) = \frac{f(t)}{S(t)} \qquad (2.20)$$

Hazard is referred to as probability of failure per unit time at any given time assuming survival up to that time. Commonly used survival models include Weibull, log-normal, and log-logistic distributions. Each model follows a specific probability distribution as discussed in the chapter 3.

Yang et al. (2013) and Beng et al. (2012) used survival analysis to evaluate bridge infrastructure performance. They used Weibull probability distribution and Kaplan-Meier approaches for censored survival data.

Sobanjo et al. (2010) conducted research on reliability-based modelling of bridge deterioration in Florida. They used Florida's bridge NBI data to predict the "natural deterioration" for bridges without rehabilitation during their lifetime, with uncensored and right censored data. Unlike reliability studies discussed earlier, this study followed the survival analysis approach. The authors conducted reliability assessments of bridge deck and superstructure based on the type of roadway (interstate roadway or non-interstate roadway) and materials (steel or concrete). They determined that the Weibull distribution was the best fit for the data. All Weibull reliability parameters were established through the maximum likelihood estimation method. The NBI rating range used was 7 to 9 (utilizing the 1992 to 2005 NBI data), and the effective mean service life estimated based on the NBI condition rating of 7. Based on their results, most bridges remain a minimum time of 1 or 2 years in their excellent condition (rating 9), 5 to 10 years in the rating 8 (very good condition), and usually below 6 years in the rating 7 (good condition). It was also concluded that interstate highway bridges deteriorate more rapidly than non-interstate roadway bridges.

Tabatabai et al. (2011) used the hypertabastic survival model, which has been used in medical research, for survival analysis of bridge decks in Wisconsin. This model is a two-parameter continuous probability distribution, first proposed by Tabatabai et al. (2007). A special feature of the hypertabastic distribution model is its varying hazard shapes that can be compatible with different failure patterns. These hazard shapes include monotonically increasing, monotonically decreasing, and single humped shapes.

In this research, the performance of superstructures of Wisconsin bridges is evaluated through the hypertabastic survival model. The NBI records for Wisconsin is used as input data as described in chapter 3 of this thesis.

Chapter 3. Reliability and Survival Methods

3-1 Strength based reliability methods:

Strength-based reliability methods depend on assessing risks associated with loads exceeding resistance capacity of a member in light of the variability of both parameters. Typically, the risk of failure is controlled in the design specifications using prescribed load and resistance factors. Risks are measured based on a comparison of demand (applied load or load effect) with capacity (resistance) and considering uncertainties related to these parameters (Frangopol, 1998). The general approach is to identify an "acceptable" level of risk. This acceptable risk is managed by using appropriate safety and resistance factors, which are based on a knowledge of load and resistance variability.

Strength-based reliability in structures including bridges is usually calculated through an assumption of normal (or log-normal) distributions for random variables, and using the mean and variance of the failure function g described previously. The probability of failure can be determined from (Frangopol, 1998),

$$P_f = \Phi \left[\overline{g} / \sigma_g \right] \qquad (2.4)$$

Where, Φ is a cumulative standard normal distribution function, $\overline{g}$ is the mean of failure function, and σ_g is standard deviation of g. The risk of failure is measured through a safety index, β, where:

$$\beta = \overline{g} / \sigma_g \qquad (2.5)$$

The reliability index β is an indication of the number of standard deviation that the mean of the failure function is distanced from the failure condition (g=0). A larger β value is representative of higher reliability.

3-2 Survival Analysis

Survival analysis is a technique primarily developed and utilized for bio-medical research. However, this method has also been used in other fields including social studies, economics, engineering, etc. The term "Survival" which is mostly used in health-related fields, is equivalent to the term "reliability" used in engineering. The key variable is the "survival time", which does not necessarily mean time to death. In general, "survival time" is a non-negative random variable indicating the elapsed time from a reference time to the occurrence of a given event (Lee and Go, 1997). Examples of survival time are: time to recurrence of cancer after administration of a drug, lifetime of an electronic component, etc.

In survival analysis, the study time may not cover the entire survival time. For instance, a patient may leave the clinical investigation early and the researchers are unable to follow up and determine the actual survival time. In other cases, reasons unrelated to the study may lead to the end of survival. These kinds of observations are called "censored" observation. Censoring corresponds to missing data within the observation time. When survival extends beyond the observation period, this is referred to as right censored data. When a component fails before the observation interval begins, the associated data is called "left censored". The right censored data are more common (Sobanjo et al. 2010).

A normal distribution is not appropriate for survival analysis because, survival data are usually censored and incomplete, and the shape of the survival time distribution is skewed. Therefore, distributions such as exponential, Weibull, lognormal, gamma, Gompertz, and log logistic are typically considered for survival analyses (Lee and Go, 1997).

Three distinct functions are commonly used in survival analysis. These functions are defined below (Lee and Go, 1997):

1) The survival function is represented by:

$$S(t) = P(T > t) = 1 - F(t) \tag{3.1}$$

Where T indicates the survival time as a random variable, t is the time, and $F(t)$ denotes the cumulative probability of failure at various times.

$S(t) = 1\ at\ t = 0\ and\ S(t) = 0\ at\ t = \infty.$

2) The probability density function represents the unconditional failure rate at any given time, and is defined as:

$$f(t) = \lim_{\Delta t \to 0} P(t < T < t + \Delta t)/\Delta t \tag{3.2}$$

3) The hazard function is a conditional failure rate (failure rate at any given time assuming survival up to that time), and is represented as:

$$h(t) = \lim_{\Delta t \to 0} p(t < T < t + \Delta t | T > t)/\Delta t \tag{3.3}$$

Parametric, nonparametric, and semi-parametric approaches can be utilized in survival analyses. A brief description of each approach is given below (Lee and Go, 1997).

3.2.1 Nonparametric Model - The Kaplan-Meier or Product Limit Method

The Kaplan-Meier method is one of the most common methods used to estimate the empirical distribution of survival time. This non-parametric method is mostly used in medical research. This is called non-parametric approach because potential parameters contributing to outcomes are not considered. In this method, the observation time is divided into a series of time intervals such that only one failure occurs at the beginning of each time interval (survival times are first sorted, and ranked from lowest to highest).

The probability of survival at time t, can be estimated using the Kaplan-Meier method as follows (Lee and Go, 1997):

$$\hat{S}(t) = \prod_{t_i<t}\left(\frac{n-r_i}{n-r_i+1}\right)^{\delta_i}, t \leq t_{(n)} \quad (3.4)$$

Where t_i displays the *i*th survival time (can be censored or uncensored), δ_i is a parameter taken as 0 for censored data and 1 for uncensored data, r_i is the rank of t_i, n is the total number of observation intervals, and $t_{(n)}$ indicates the longest survival time (Lee and Go, 1997).

3.2.2 Semi-Parametric Model - Cox Regression Model

It is important to consider risk factors that may affect a certain survival time in both medical research and engineering failure analyses. For example, in the health sciences, it is usually of interest to understand how covariates such as smoking, cholesterol amount, and genetic characteristics (referred to as risk factors) may be related to heart disease. In bridge engineering, we may be interested in in the influence of factors such as, average daily traffic, span length, span width, type of the superstructure, etc. on service life of bridge components. In addition to having a complete set of risk factors, it is important to determine the most influential risk factors. The Cox Regression Model is an effective method to determine how a specific risk factor is related to probability of survival at various times. In the Cox Regression Model, there is no limitation regarding the presence of censored data. For a total of p risk factors (covariates) , $\boldsymbol{X} = (X_1, X_2, \ldots, X_p)$, the hazard function would be given as shown below (Lee and Go, 1997):

$$h(t, \boldsymbol{X}) = h_0(t) \exp(\boldsymbol{b}^T \boldsymbol{X}) \quad (3.5)$$

Where, $h_0(t)$ is the baseline hazard function, and $\boldsymbol{b}$ is a vector of multipliers for various covariates**.** The Cox model is defined as a semi-parametric model because it does not have a specified survival function.

The hazard function above can be shown in logarithmic form (Lee and Go, 1997):

$$\ln h(t, \boldsymbol{X}) = \ln h_0(t) + \boldsymbol{b}^T \boldsymbol{X} \quad (3.6)$$

In this equation, the ratio of two hazard functions with risk factor $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ is calculated as (Lee and Go, 1997):

$$h_1(t, X_1)/h_2(t, X_2) = \exp[\boldsymbol{b}^T (X_1 - X_2)] \quad (3.7)$$

From the above, it could be concluded that the ratio of the two hazard functions are independent of the baseline hazard function, and the two hazard functions would not intersect each other. This condition is referred to as proportional hazard. However, the Cox Models are not restricted to proportional hazard models and non-proportional models can be developed through time-dependent covariates. For more details refer to Lee and Go, (1997).

3.2.3. Parametric method:

The parametric method is the most elaborate form of survival analysis. This was developed as a consequence of computer technology advancements and promotion of statistical data analyses. A comparison of parametric and non-parametric methods are summarized below:

Biomedical researchers have been using non-parametric survival analyses because there are fewer assumptions made in non-parametric approaches compared to parametric approaches. In non-parametric methods, there are no assumed baseline hazard functions (hazard shapes).

The parametric method can be a good substitute for the Cox model when the proportional hazard assumptions is not valid, and a distribution function is available for the survival time and the baseline hazard. In such cases, parametric calculations are more descriptive and concise in comparison to non-parametric and semi-parametric methods. In addition, the maximum likelihood[2] approach could be employed for calculating survival function parameters (Lee and Go, 1997).

The six commonly used distribution functions used in survival analyses are shown in Table 3.1.

---

[2] Maximum Likelihood Estimation (MLE) is an approach to estimate parameters for a statistical model. Basically maximum likelihood method estimates a set of model parameters to maximize the likelihood of selected model to fit a dataset (Myung, 2002).

Table 3.1 Common distribution functions in survival analysis (Lee and Go, 1997)

Distribution	Parameter	Probability density function	Hazard function
Exponential	$\lambda > 0$	$f(t) = \lambda \exp(-\lambda t)$ $S(t) = \exp(-\lambda t)$	$h(t) = \lambda$
Weibull	$\lambda, \gamma > 0$	f(t) = λγ(λt)^(γ−)1 exp(−λt)^γ S(t) = exp(−λt) γ	h(t) = λγ(λt)^(γ−1)
Gamma	$\lambda, \gamma > 0$	f(t) = [λ/Γ(γ)](λγ)^(γ−1) exp(−λt) $S(t) = \int_t^{\infty} f(x)dx$	h(t) = f(t)/S(t)
Gompertz	$\lambda, \gamma > 0$	f(t) = exp[(λ + γt) − 1/γ(e^λ+γt – e^λ)] S(t) = exp[−e^λ /γ(e^γt − 1)]	h(t) = exp(λ + γt)
Lognormal	$\mu, \sigma > 0$ a = exp(−μ)	$G(y) = \frac{1}{\sqrt{2\pi}} \int_0^y exp(-u\ 2\ /2)du$ f(t) = 1/tσ √ 2π exp[−1/2σ^2 (ln at)^2] S(t) = 1 − G(ln at/σ)	h(t) = {1/tσ √ 2π × exp[−1/2σ 2 (ln at)^2]} × {1 − G(ln (at/σ))} −1
Log-logistic	$\lambda, \gamma > 0$	f(t) = λγ(λt)^(γ−1) [1 + (λt)^γ]^−2 S(t) = [1 + (λt)^γ]^−1	h(t) = λγ(λt)^γ−1 [1 + (λt)^γ] −1

The exponential distribution model is a special case of more complex distribution functions such as Weibull and Gamma distributions.

For the exponential distribution, the natural logarithm of the survival function is $\ln S(t) = -\lambda t$, which is a linear function of time. Therefore, the exponential distribution could be determined by plotting ln S (t) against t. If the plot has a constant slope, the slope represents the hazard rate λ. This kind of distribution has had vast applications in medical research such as cancer studies where effects of different variables (risk factors) are evaluated.

The Weibull distribution has two characteristic parameters, γ and λ, which are the shape and scale parameters, respectively. The hazard rate is constant if $\gamma=1$ (the same as exponential function). The hazard rate would increase with time when $\gamma>1.0$, and decrease with time when $\gamma<1.0$. The Weibull model could be effective in modelling data sets with constant, increasing, or decreasing hazards. It is thus broader than the Exponential distribution. The Weibull distribution has been used expansively in medical research and other applications. Examples include (Go et al, 1997) carcinogenesis tests by Williams (1978), radiation response characterization by Scott et al (1980), time to return to prison by released convicts by Schmidt and Witte (1988), and human death modelling by Juckett et al (1993).

The Gamma distribution is another parametric model that has both shape (γ) and scale (λ) parameters. The Gamma function shows a decreasing hazard rate when $0<\gamma<1$, and increasing hazard rate when $\gamma>1$. It also shows a constant hazard rate (the exponential distribution) when $\gamma=1$. A large numbers of studies have used this distribution ((Lee and Go (1997), Galli et al (1983), Niederjohn et al (1986), Bolin et al (1986), Meyer et al (1991), Hendriks (1993)).

The Gompertz distribution also uses the λ and γ parameters as in the two distributions discussed earlier. The special characteristics of this function are: constant hazard rate of exp (λ) when $\gamma=0$, increasing hazard rate when $\gamma>0$ and decreasing when $\gamma<0$. The Gompetz distribution has been used to investigate mortality due to lung cancer, prostate cancer, stroke and so on, (Lee and Go, 1997).

When the logarithm of a variable approximates a normal distribution, the distribution is known as a lognormal distribution. The lognormal hazard function first follows an increasing pattern up to a maximum, followed by a decreasing hazard. Researchers have widely used lognormal distribution in reliability engineering and biomedical studies. Ahmed et al (1993) used the lognormal distribution to illustrate the effect of consumption of chemically contaminated sea-foods on human health.

The Log-logistic distribution is used when the logarithm of T follows a logistic distribution. Similar to other survival model distributions, the log-logistic model has two parameters as λ and γ. The hazard function would be decreasing from infinity to zero when $\gamma<1$, and decreasing from λ to 0 when $\gamma=1$. When $\gamma>1$, the hazard function first increases and then decreases.

The log-logistic distribution can be a substitute for other distribution functions such as Weibull, lognormal and Gamma distribution. The log-logistic distribution has been used in studies on recidivism of released prisoners (Lee and Go (1997), Schmidt et al (1988)) and time to resumption of smoking after quitting (Lee and Go (1997), Elketroussi et al (1991).

3.2.3.1 Goodness of fit:

An important step in survival analyses is to determine how well a dataset follows a particular distribution. Goodness-of-fit is a test that uses the Anderson Darling (AD) statistic to identify how well a data meets assumption of a probability distribution. The AD statistic evaluates difference between hypothesized (null hypothesis) and empirical cumulative distribution. A smaller AD statistic represents better fit to the data.

A null hypothesis (H_0) can also be evaluated through P-value, which has a value between 0 and 1. P-value shows the strength of evidence provided by the sample against a null hypothesis. The smaller the P-value, the stronger the evidence to reject a null hypothesis. In other words, the smaller the P-value, the better the data fits the assumed distribution.

The maximum likelihood method can be used to estimate parameters of the selected distribution function. The maximum likelihood method is a widely used approach that determine parameters such that the likelihood of the selected model fitting the data is maximized.

The likelihood function is shown in equation (3-8) (Lee and Go, 1997).

$$L = \prod_{i=1}^{r} f(t) \ \prod_{i=1}^{n-r} S(t) \qquad (3.8)$$

Where:

$f(t)$: Probability density function

$S(t)$: Survival function

n: Total number of observations

r: Number of uncensored observations

3.2.4 Hypertabastic distribution:

The hypertabastic distribution is a relatively new type of distribution, introduced by Tabatabai et al. (2007). It has been used in several applications including studying the effect of covariates on the survival time of cancer patients (Tabatabai et al, 2007). The random variable T is following hypertabastic distribution if its cumulative distribution function could be formulated as shown below (Tabatabai, 2011):

$$\begin{cases} 1 - Sech\left[\alpha(1 - t^{\beta} Coth(t^{\beta})) / \beta\right] & \text{for } t > 0 \\ \end{cases}$$

$F(t) =$ (3.9)

$0 \qquad \text{for } t \leq 0$

The parameters α and β are both positive and $Sech[\bullet]$ and $Coth[\bullet]$ are hyperbolic secant and hyperbolic cotangent functions, respectively. The probability density function of hypertabastic distribution is given as below (Tabatabai, 2011):

$$f(t) = \begin{cases} Sech[W(t)]\left(\alpha t^{2\beta-1} Csch^2\left(t^{\beta}\right) - \alpha t^{\beta-1} Coth\left(t^{\beta}\right)\right) Tanh[W(t)] & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases} \quad (3.10)$$

Where, $Csch[\bullet]$ is hyperbolic cosecant and $W(t) = \alpha\left(1 - t^{\beta} Coth\left(t^{\beta}\right)\right)/\beta$

Considering continuous random variable t representative of time to event (waiting time for the occurrence of the event), the Hypertabastic survival function is defined as (Tabatabai, 2011):

$$S(t) = Sech\left[\alpha(1 - t^{\beta} Coth\left(t^{\beta}\right)) / \beta\right] \quad (3.11)$$

Where, *S(t)* (survival function) is the probability that waiting time exceeds *t*.

The hypertabastic hazard function $h(t)$, which represents the instantaneous failure rate at time *t*, given survival up to time *t*, is defined as (Tabatabai, 2011):

$$h(t) = \alpha\left(t^{2\beta-1} Csch^2\left(t^{\beta}\right) - t^{\beta-1} Coth\left(t^{\beta}\right)\right) Tanh[W(t)] \quad (3.12)$$

And the cumulative hazard function H(t) is defined as :

$$H(t) = -\ln(Sech[W(t)]) \quad (3.13)$$

As discussed earlier, the commonly used survival models are Weibull, lognormal, log logistic, etc. An important consideration affecting the choice of a distribution type is related to the shape of the baseline hazard function. The Weibull hazard function has monotonically increasing, monotonically decreasing or constant shapes as shown in Figure 3.1, depending on the choice of parameters used.

The lognormal hazard function increases with time until it reaches a maximum point and then decreases. The log logistic hazard function either monotonically decreases or has single-mode shapes. Unlike the distributions described above, a significant feature of the hypertabastic hazard function is its capability to model a wide variety of hazard shapes. These diverse hazard shapes offer versatility to apply the model to different biological and engineering problems when conventional hazard models do not properly represent the real hazard patterns. A list of possible shapes for the hypertabastic hazard function using equation (3.12) is described below (Tabatabai at al, 2011):

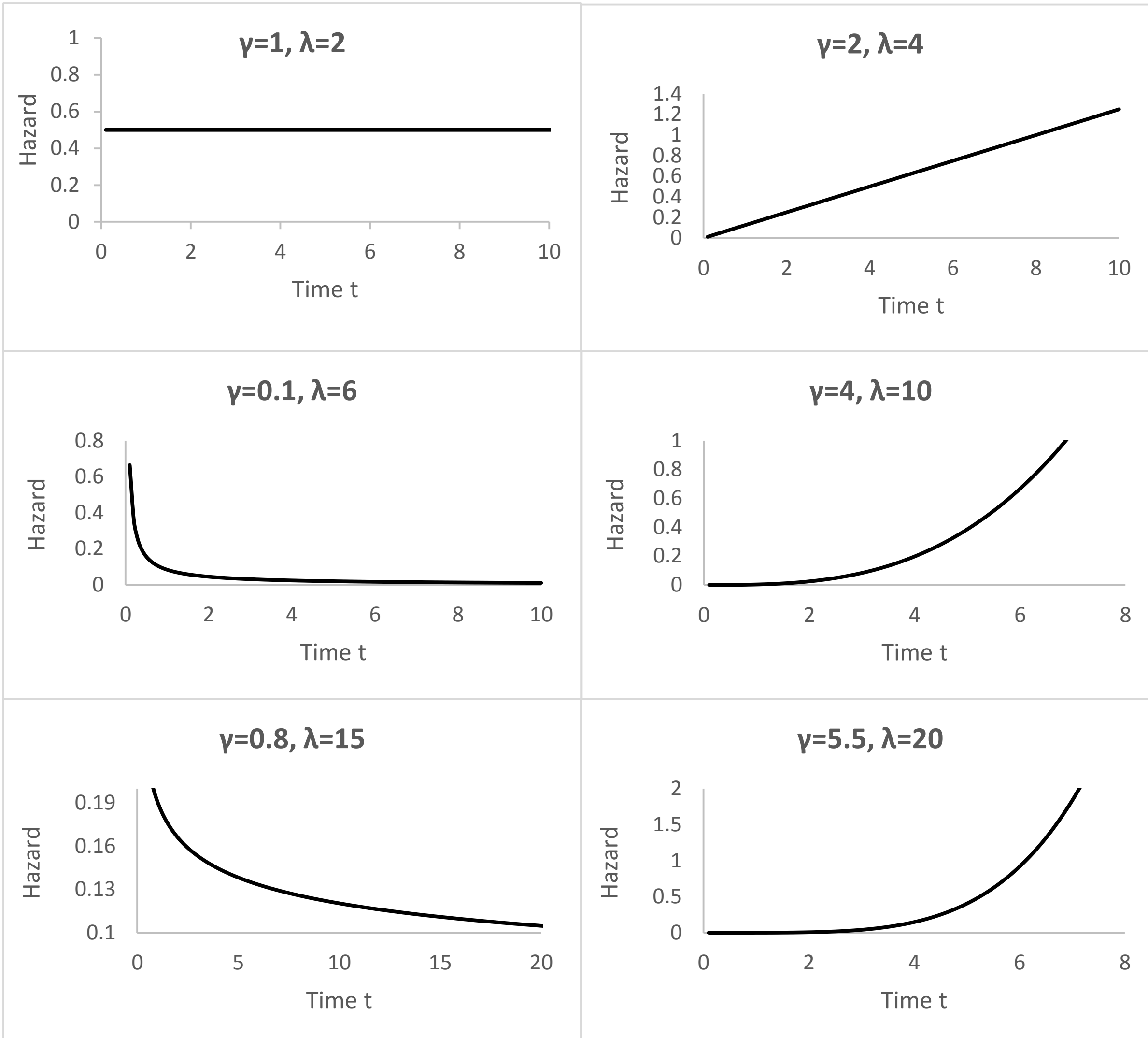

Figure 3.1 Weibull distribution with different values of γ and λ

1. When $0 < \beta \leq 0.25$, the hypertabastic baseline failure rate function follows monotonically decreasing pattern (Fig. 3.2a).
2. When $0.25 < \beta < 1$, the hypertabastic baseline hazard function first follows increasing pattern with time up to a maximum point and then decreases (uni-modal) (Fig. 3.2b).
3. When $\beta = 1$, the hypertabastic baseline hazard function increases with time at the start and then approaches a horizontal asymptote α (Fig. 3.2c).
4. When $1 < \beta < 2$, the hypertabastic baseline hazard function increases with upward concavity until it reaches an inflection point at which point it increases continuously with downward concavity thereafter (Fig. 3.2d).
5. When $\beta = 2$, the hypertabastic baseline hazard function increases with upward concavity a followed by a linear increase with slope α. (Fig. 3.2e).
6. When $\beta > 2$, the hypertabastic baseline hazard function increases monotonically with upward concavity. (Fig. 3.2f).

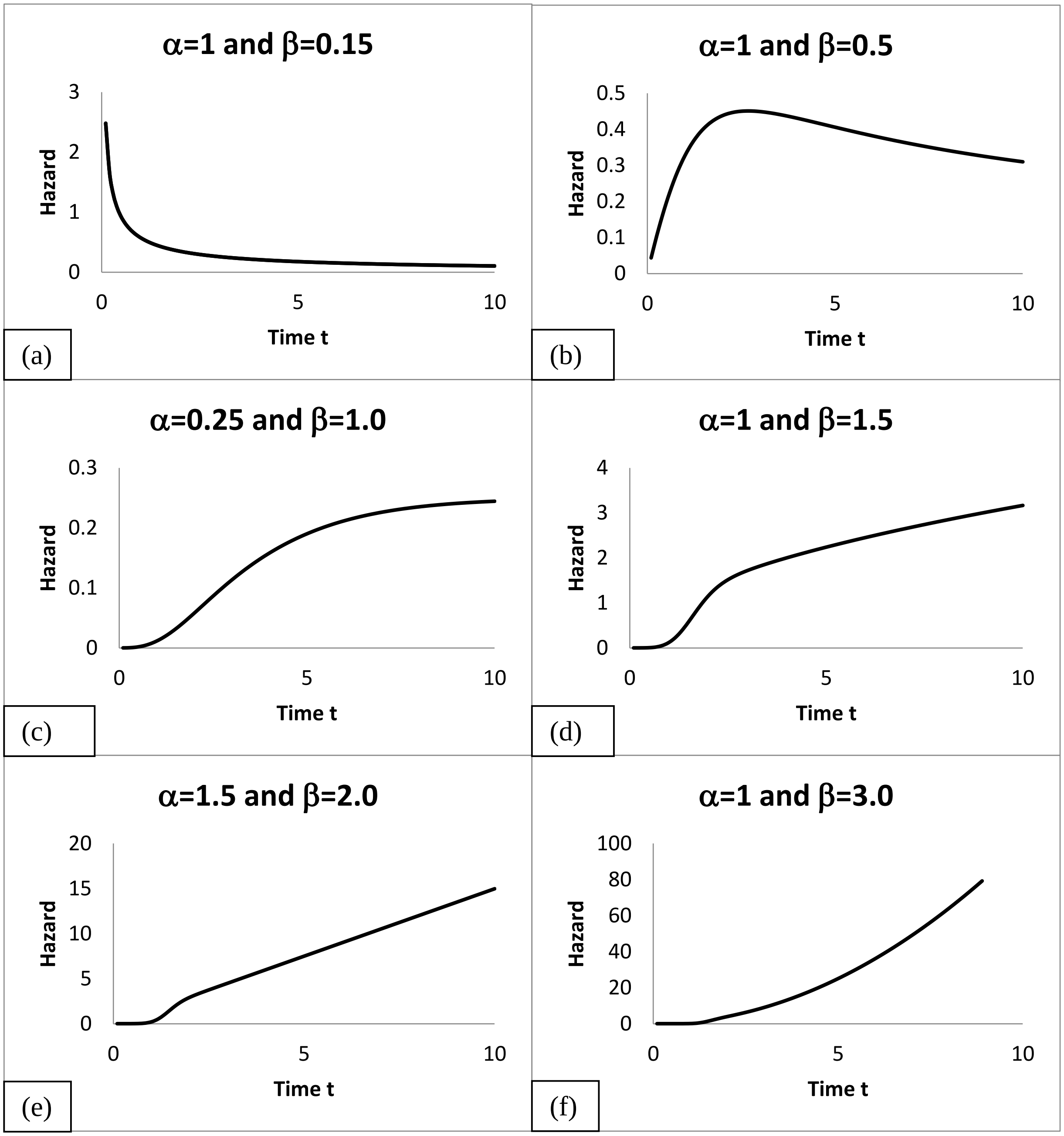

Figure 3.2 Hypertabastic hazard function with different values of parameters α and β (Tabatabai et al, 2011)

As described earlier, in proportional hazard models, it is assumed that the hazard functions for two groups of risk factors are proportional within the observation time, which means that hazard function curves are not intersecting over time. Accelerated hazard functions, as an alternative for proportional hazard models, are introduced where the effect of covariates on the failure time is multiplicative with time.

3.2.5 Hypertabastic Proportional Hazard Models:

The hypertabastic proportional hazard function is defined as (Tabatabai et al, 2011):

$$h(t|x,\theta) = h_0(t)g(x|\theta) \qquad (3.14)$$

Whereas θ is a vector of unknown parameters, vector **x** is a vector containing p covariates, and $h_0(t)$ is the baseline hazard function, equation (3.12).

It is assumed that $g(x|\theta)$ is a non-negative function of x defined as (Tabatabai et al, 2011):

$$g(x|\theta) = e^{\sum_{k=1}^{p}\theta_k x_k} \qquad (3.15)$$

The hypertabastic survival function $S(t|x,\theta)$ and the probability density function $f(t|x,\theta)$ for the proportional hazards function could be defined respectively in equations (3-16) and (3-17) (Tabatabai et al, 2011):

$$S(t|x,\theta) = [S_0(t)]^{g(x|\theta)} \qquad (3.16)$$

$$f(t|x,\theta) = f_0(t)[S_0(t)]^{g(x|\theta)-1} g(x|\theta) \qquad (3.17)$$

Where $f_0(t)$ and $S_0(t)$ were introduced earlier in equations (3-10) and (3-11), as the baseline probability density function, and the baseline survival function, respectively. To estimate the model parameters, the method of maximum likelihood and a log-likelihood function are used based on the type of censoring.

The log-likelihood function for right censored data is shown below (Tabatabai et al, 2011):

$$LL(\theta, \alpha, \beta : x) = \sum_{i=1}^{n} (\ln[\mathrm{Sech}(\frac{\alpha(1 - t_i^{\beta} Coth\left(t_i^{\beta}\right))}{\beta})] g(x_i|\theta)$$

$$+ \delta_i \ln[t_i((\alpha t_i^{-1+2\beta} Csch\left(t_i^{\beta}\right)^2 - \alpha t_i^{-1+\beta} Coth\left(t_i^{\beta}\right))$$

$$* \tanh(\frac{\alpha\left[1 - t_i^{\beta} Coth\left(t_i^{\beta}\right)\right]}{\beta})) g(x_i|\theta)] \qquad (3.18)$$

Where, $\delta_i = \begin{cases} 0 \; if \; t_i \; is \; a \; right \; censored \; observation \\ 1 \qquad otherwise \end{cases}$

3.2.6 Hypertabastic Accelerated Failure Time Model:

The hypertabastic accelerated failure hazard function is defined as (Tabatabai et al, 2011):

$$h(t|x,\theta) = h_0(tg(x|\theta))g(x|\theta) \qquad (3.19)$$

The hypertabastic survival function $S(t|x,\theta)$ for the accelerated failure hazards function is defined as (Tabatabai et al, 2011):

$$S(t|x,\theta) = S_0(tg(x|\theta)) \qquad (3.20)$$

The hypertabastic probability density function for the accelerated failure time model is (Tabatabai et al, 2011):

$$f(t|x,\theta) = f_0(tg(x|\theta))g(x|\theta) \qquad (3.21)$$

When right censored data is used, the log-likelihood function for the hypertabastic accelerated failure time model is defined as (Tabatabai et al, 2011):

$$LL(\theta,\alpha,\beta:x)$$
$$= \sum_{i=1}^{n}(\ln[\mathrm{Sech}(\frac{\alpha(1-[Z(t_i)]^{\beta}Coth([Z(t_i)]^{\beta}))}{\beta})]$$
$$+ \delta_i \ln[t_i((\alpha[Z(t_i)]^{-1+2\beta}Csch([Z(t_i)]^{\beta})^2 - \alpha[Z(t_i)]^{-1+\beta}Coth([Z(t_i)]^{\beta}))$$
$$* \tanh(\frac{\alpha[1-[Z(t_i)]^{\beta}Coth([Z(t_i)]^{\beta})]}{\beta}))g(x_i|\theta)] \qquad (3.22)$$

Where $Z(t_i) = t_i g(X_i|\theta)$

Tabatabai et al (2011) report the log likelihood functions for data with other types of censoring.

Using the proposed hypertabastic accelerated failure model, the calculations of survival time and failure rates are performed using Eq. (3-23) and Eq. (3-24). Since the first term inside the parenthesis in Eq. (3-12) is very close to zero (for bridge deck data), it is neglected for parameter calculations in this dataset. Therefore, the equations used for this dataset are as follows:

$$S(t_g) = Sech[\alpha(1-t_g^{\beta}Coth(t_g^{\beta}))/\beta] \qquad (3.23)$$

$$h(t_g) = \alpha(-t_g^{\beta-1}Coth(t_g^{\beta}))Tanh[W(t_g)]e^{[c(SPAN)+d(ADT)+h(TYPE)]} \qquad (3\text{-}24)$$

$$t_g = (AGE)e^{[c(MSL)+d(ADT)+h(TYPE)]} \tag{3-25}$$

AGE: age of bridge superstructure,

MSL: maximum span length

ADT: average daily traffic

TYPE: type of bridge superstructure, equal to 1 for steel, 0 for concrete (when steel and concrete superstructures are distinguished), and 2 for both types of superstructures combined (when superstructure types not distinguished)

c: numerical coefficient for variable MSL

d: numerical coefficient for variable ADT

h: numerical coefficient for variable TYPE

The parameters α, β, c, d, and h are determined using the maximum likelihood. These parameters and the results of analyses are presented in Chapter 4.

Chapter 4. Results and Discussion

In this chapter, the effect of various factors (covariates) on bridge superstructure reliability and failure rates are studied. These covariates included maximum span length, average daily traffic (ADT), and the type of superstructure (steel or concrete). The hypertabastic survival model (Tabatabai et al., 2011) was used to develop the governing equations for reliability (survival) and failure rate. The 2012 NBI bridge data for Wisconsin were used in the analysis. A description of the data selection and processing is given in section 4.1.

4.1 National Bridge Inventory (NBI) data

In this research, the basic data are derived from the National Bridge inventory database (NBI) for Wisconsin bridges. NBI was established in 1967 after the catastrophic failure of the Silver River Bridge (Shenton and Seymour, 2013). NBI database includes numerical ratings for the condition of major bridge components. The numerical ratings range from 0 to 9, and are given to individual components by bridge inspectors based on the Federal Highway Administration (FHWA) guidelines (Tabatabai et al, 2011). The 0 rating for a bridge component indicates a failed condition, while other ratings are described as follows: imminent failure condition (1), critical condition (2), serious condition (3), poor condition (4), fair condition (5), satisfactory condition (6), good condition (7), very good condition (8), and excellent condition (9).

The inspections are usually conducted once every 2 years, and the results of these inspections form the basis for some of the 117 data entries in the NBI records. These include descriptive bridge information such as bridge location and geometry as well as recorded ratings for various bridge components.

The 2012 NBI data record was used in this study to assess the reliability (survival) of bridge superstructures.

The following procedures were used to filter the 2012 NBI data for Wisconsin for use in this study:

1. Bridge records without a superstructure rating and/or construction date were eliminated. Item 27 of NBI database provides the year when construction was completed. Item 59 provides the superstructure rating.
2. All reconstructed or rehabilitated superstructures were removed. Item 106 provides the reconstruction data, when applicable.
3. Uncommon bridge types and superstructures were excluded. Steel superstructures (NBI item 43A, code 1, 2, 5, or 6), concrete and pre-stressed concrete superstructures (NBI item 43A, code 3 or 4) were retained for analysis. All other structural materials and superstructures were excluded from the data.
4. Bridges with more common structural systems such as slab, multi-beam, girder, tee beam, floor beam, and box beam (NBI item 43B, code 1 to 6) were retained for the analysis. Data from bridges with less common structural systems such as trusses, arches, and cable-stayed were removed.
5. The target superstructure rating representing the end of service life was chosen as rating 5. Bridges with other superstructure ratings were thus excluded from the data set. The deck or superstructure ratings of 4 or 5 are generally considered to be the practical end of service life for that element (Tabatabai et al, 2011).
6. The retained bridge records were then classified based on their superstructure material type (NBI, 59) (steel or concrete).

There are several parameters that could potentially affect the bridge superstructure performance. These include: deck area, which is the product of structure length (NBI item 49) times bridge roadway width curb to curb (NBI item 51); ADT (NBI item 29); and MSL (NBI item 48).

Tabatabai et al. (2011) showed that deck area was an important factor in reliability of bridge decks. This parameters was initially considered for inclusion in this study because a large deck area indicates a large superstructure member length. With a large length, the probability of appearance of defects would increase. The ADT was also included because of its direct influence on superstructure load. The maximum span length could also be a relevant parameters as it has direct influence on bending moment. The type of superstructure (steel or concrete) can also be a parameter with possible impact on superstructure reliability. As discussed earlier, the deck area was removed from consideration as a covariates because of its moderate correlation with MSL.

4.2. Analysis of bridge superstructure ratings

The data extracted from the NBI database were used as the required statistical data for the survival analysis. Table 4.1 shows basic statistical information on the NBI data for the steel and concrete superstructure as well as combination of all superstructures types.

Table 4.1 Statistical information on the Wisconsin NBI data used in the analyses

	Wisconsin bridge with superstructure rating of 5								
	Steel superstructure			Concrete superstructure			Both superstructure types		
	ADT	Max Span (m)	Age (year)	ADT	Max span(m)	Age (year)	ADT	Max Span (m)	Age (year)
Mean	1911	14.02	61.8	6014	14.54	50.9	3879	14.2	56.6
Median	210	9.8	60	1200	12.8	47	450	11.9	52
Mode	40	9.4	60	80	12.2	47	80	9.4	47
Standard deviation	4971	13.0	17.0	11035	8.43	20.6	8682	11.06	19.6
Kurtosis	33.6	102.3	-0.3	16.0	49.7	-0.5	26.0	109.64	-0.6
Skewness	4.9	8.2	0.21	3.6	4.8	0.5	4.4	7.96	0.2
No. of bridges	472	472	472	435	435	435	907	907	907

Tabatabai et al. (2011) compared a number of survival models such as Weibull, hypertabastic, lognormal, etc. with respect to their closeness of fit to the NBI bridge deck data. The Akaike Information Criterion (AIC) and the maximum log-likelihood method were used. This best fit model was determined to be the hypertabastic accelerated failure model (Tabatabai et al, 2011). The same model is used here for the superstructure reliability analysis in this thesis.

The Kaplan-Meier nonparametric method was first applied to the NBI dataset. Figures 4.1 and 4.2 show the non-parametric reliability and failure rate for Wisconsin bridge superstructures using the Kaplan-Meier method. The failure rate function for steel, concrete and both superstructure type intersect each other in several points, which is an indication that the accelerated failure time model should be used in lieu of the proportional hazard model.

Figure 4.1 Kaplan-Meier cumulative reliability for Wisconsin bridge superstructures

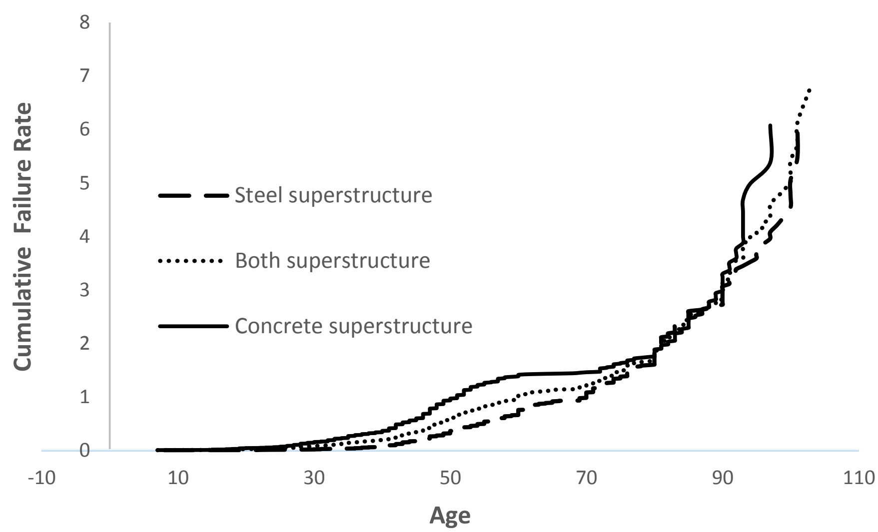

Figure 4.2 Kaplan-Meier cumulative failure rate for Wisconsin bridge superstructures

For the bridge superstructure parameters, a discrete covariate is assigned with a value of 0 for the case of concrete superstructures and 1 for the steel superstructures. When both superstructure types are combined, the value of this parameter is 2. The other covariates considered were ADT, and MSL (m). Initially, the deck area was also included as a parameter. However, a correlation analysis was performed among all covariates to determine if they are correlated, as shown in table 4.2.

Table 4.2 Correlation evaluation between covariates

	ADT	*Maximum span length*	*Age*	*Deck area*
ADT	1			
Maximum span length	0.240	1		
Age	-0.229	-0.207	1	
Deck area	0.385	0.665	-0.217	1

The correlation analysis between maximum span length and deck area showed that there is moderate correlation between deck area and maximum span length (Pearson correlation coefficient of 0.665), and therefore, the two parameters are not independent of each other. This indicates that both parameters cannot be used simultaneously and one has to be selected. In the preceding study on reliability of bridge decks (Tabatabai et al. 2011), the deck area was used and maximum span length was not used. The magnitude of maximum span length can significantly affect the overall structural response (including dynamic response) of the bridge. Therefore, in this study, the maximum span length is included as a parameter (in lieu of deck area) for the survival analysis of bridge superstructures.

As discussed in chapter 3, the reliability and failure rate for the hypertabastic accelerated failure time model can be calculated through equations (3.23) through (3.25).

$$S(t_g) = Sech\left[\alpha\left(1 - t_g^{\beta} Coth\left(t_g^{\beta}\right)\right) / \beta\right] \quad (3.23)$$

$$h(t_g) = \alpha\left(-t_g^{\beta-1} Coth\left(t_g^{\beta}\right)\right) Tanh\left[W\left(t_g\right)\right] e^{\left[c(SPAN)+d(ADT)+h(TYPE)\right]} \quad (3.24)$$

$$t_g = (AGE)e^{[c(SPAN)+d(ADT)+h(TYPE)]} \tag{3.25}$$

In the above equations parameters α, β, c, d, and h are determined through the maximum likelihood method. This was determined using the Mathematica software program. The code used is shown in the Appendix B. The units for the maximum span length and age are meter (m) and year, respectively. All calculated parameters for the three superstructure types (steel, concrete and both superstructure types) are shown in the Table 4.3.

Table 4.3 Reliability and Failure rate parameters

Parameters used in survival analysis equation			
Type (superstructure)	**Steel**	**Concrete**	**Both**
Parameters	1	0	2
α	0.000583	0.000583	0.000536
β	2.11745	2.11745	2.09096
c	0.00374	0.00374	0.00337
d	6.2848e-6	6.2848e-6	8.308e-6
h	-0.14984	-0.14984	0.00

It is important to emphasize that all the parameters shown in Table 4.3 are calculated for Wisconsin bridges only. In table 4.3, the parameters α, β, c, d, and h are identical for steel and concrete superstructures. The value of type parameter (0 or 1) describes the different between the two.

The calculated P-values are shown in Table 4.4. Smaller P-values corresponding to a variable indicates that the effect of that parameters is statistically significant. The P- values in Table 4.3 indicate that covariates type, maximum span length (MSL) and ADT are all statistically significant parameters.

4-2. Results

The developed hypertabastic survival model (equations 3.23, 3.24, 3.25) can estimate the reliability and failure rates at any age as a function of superstructure type, ADT and MSL. The estimated age (corresponding to a reliability of 0.5) at the end of service life for steel and concrete superstructures was 59 and 51 years, respectively, when means of covariates were used. Table 4.5 shows bridge corresponding to various reliability levels when either mean and median values of covariates (ADT and MSL) are used for concrete and steel bridges.

Table 4.4 Parameter and Standard Error Estimate for hypertabastic Accelerated Failure Time model

Parameter	**Estimate**	**Standard error**	**Wald**	***p*-value**
α	0.000583336	0.000117414	24.683	6.75767×10^{-7}
β	2.11745	0.0535047	1566.19	1.624625188330933×10-342
Type	-0.14984	0.0225231	44.259	2.87676×10-11
Max span length	0.00374751	0.000661487	32.0953	1.4679×10-8
ADT	6.28482×10-6	1.1636×10-6	29.1729	6.61971×10-8

Table 4.5 Deciles for Estimated Age corresponding to different reliability levels at the End of Service Life Using hypertabastic Accelerated Failure Time Model

Reliability Percentiles	**Mean (ADT and MSL)**		**Median (ADT and MSL)**	
	Concrete(years)	**Steel(years)**	**Concrete(years)**	**Steel(years)**
0.1	83	86	77	89
0.2	66	76	68	79
0.3	60	69	62	72
0.4	55	64	57	66
0.5	51	59	52	61
0.6	47	54	48	56
0.7	42	49	44	51
0.8	38	44	39	45
0.9	31	36	32	37

The reliability curves associated with steel, concrete, and all superstructures are shown as a function of age in Figure 4.3 and 4.4. In Figure 4.3, the ADT and maximum span length are assumed to be at their mean values (ADT=3878 and maximum span length=14.27 m). In Figure 4.4, these covariates are assumed to be at their median values (ADT=450 and maximum span length=11.9 m).

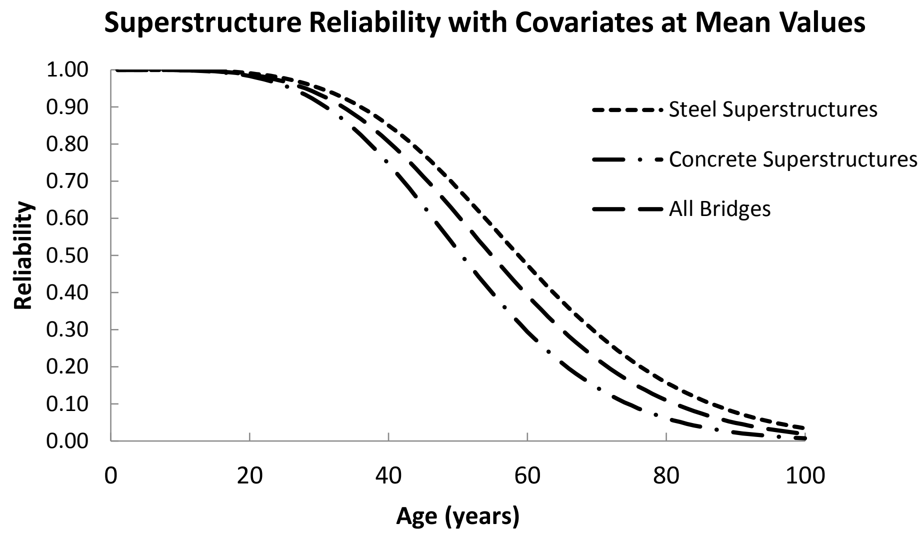

Figure 4.3 Hypertabastic reliability curves for bridge superstructures (ADT=3878 and Maximum Span length= 14.2 m)

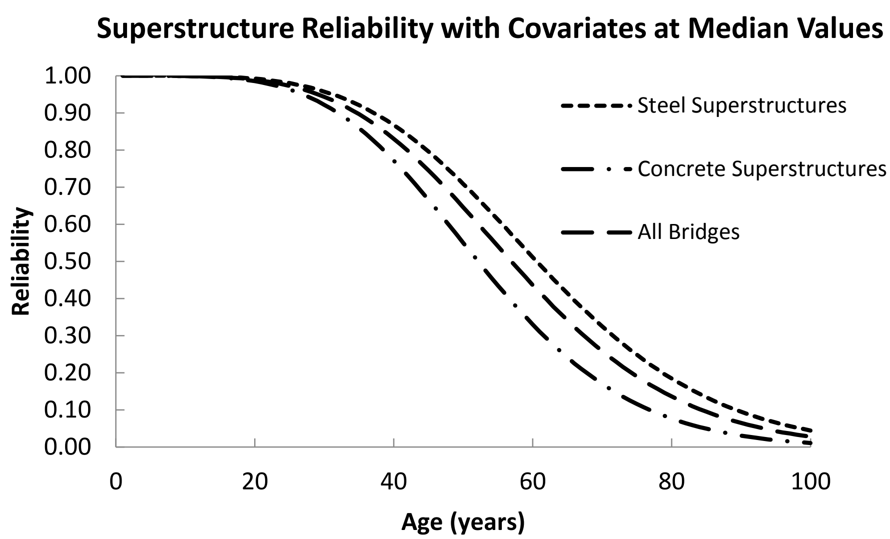

Figure 4.4 Hypertabastic reliability curves for bridge superstructures (ADT=450 and Maximum Span length= 11.9 m)

Based on the results shown in the Figure 4.3 and 4.4, the reliability (survival) of steel superstructure are slightly higher than concrete superstructures in Wisconsin. Reliability starts from a value of 1 or 100% at the beginning of the service life and reduces as the age increases. At 75 years of age, the reliability of steel, concrete, and all bridges (with mean of covariates) are 0.22, 0.10, and 0.16, respectively. The reliability results at the age of 75 years (with covariates at their median values) are 0.25, 0.12, and 0.19 for steel, concrete, and both superstructures combined, respectively.

Figure 4.5 and 4.6 show failure rate curves for three superstructures type with covariates at their mean and median values, respectively.

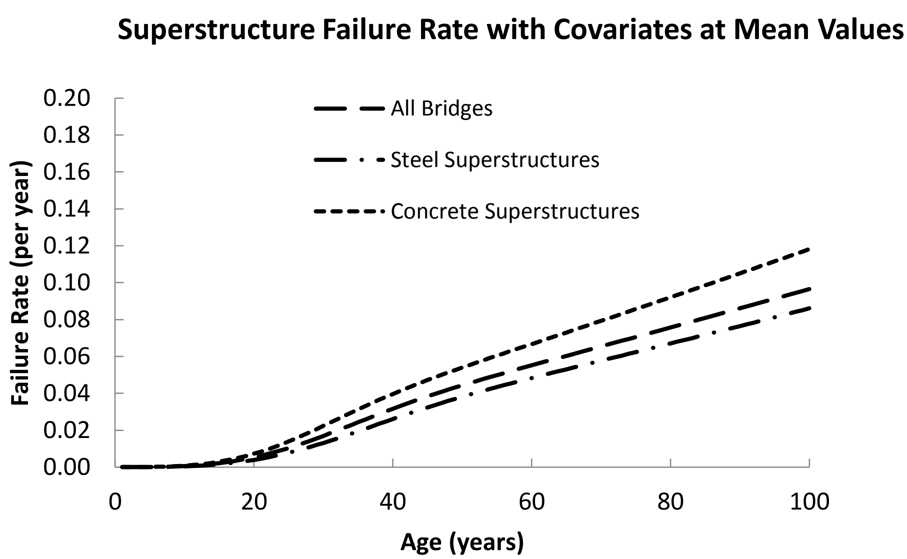

Figure 4.5 Hypertabastic failure curves for steel and concrete bridges (ADT=3878 and Maximum Span length= 14.2)

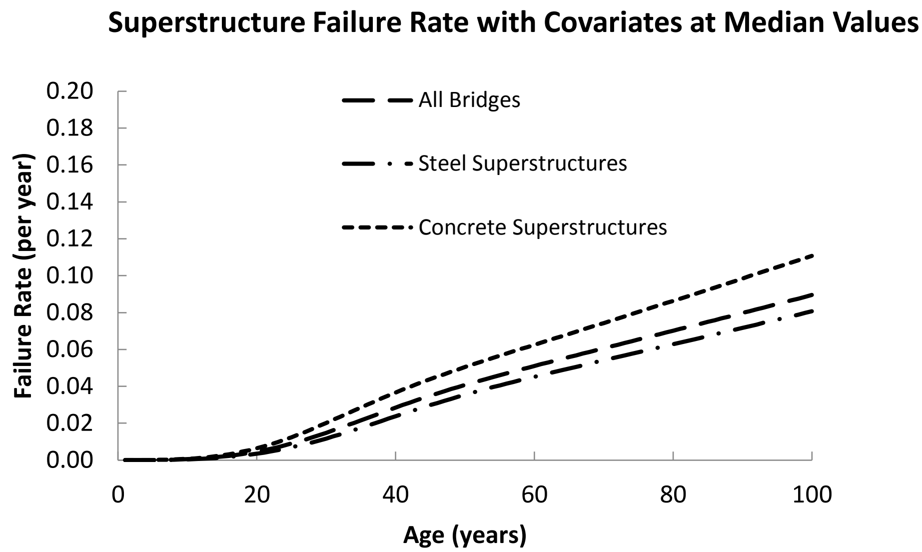

Figure 4.6 Hypertabastic failure curves for steel and concrete bridges (ADT=450and Maximum Span length= 11.9)

The failure rates are generally slightly larger for the concrete superstructures for the covariate values considered. Failure rate is relatively small prior to 20 years of age, after which the rate rises more rapidly. During the age of 20 to 40 the failure rate increases more rapidly. At more advanced ages, the superstructure failure rate increases are nearly linear with respect to time.

Figure 4.7 and 4.8 illustrate the probability density functions (pdf) for the steel, concrete, and both superstructure types as a function of age.

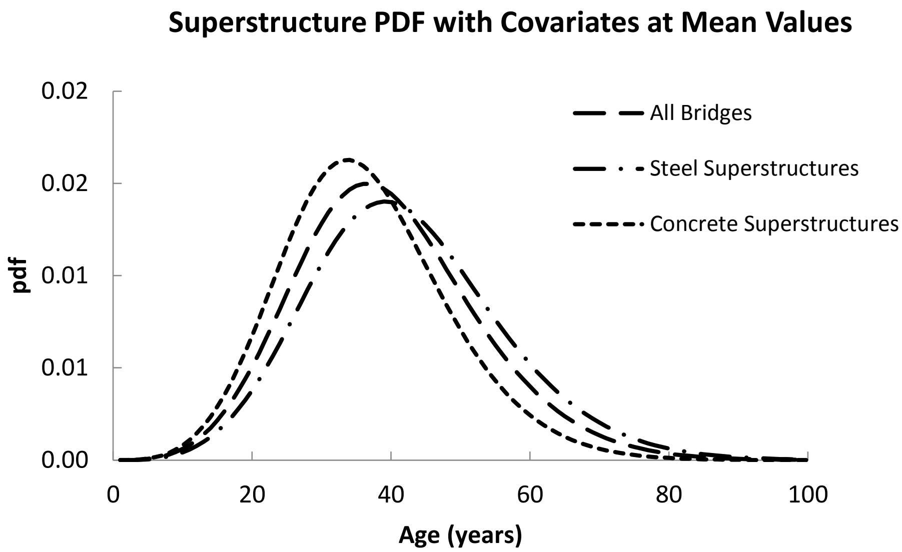

Figure 4.7 Hypertabastic PDF for steel and concrete bridges

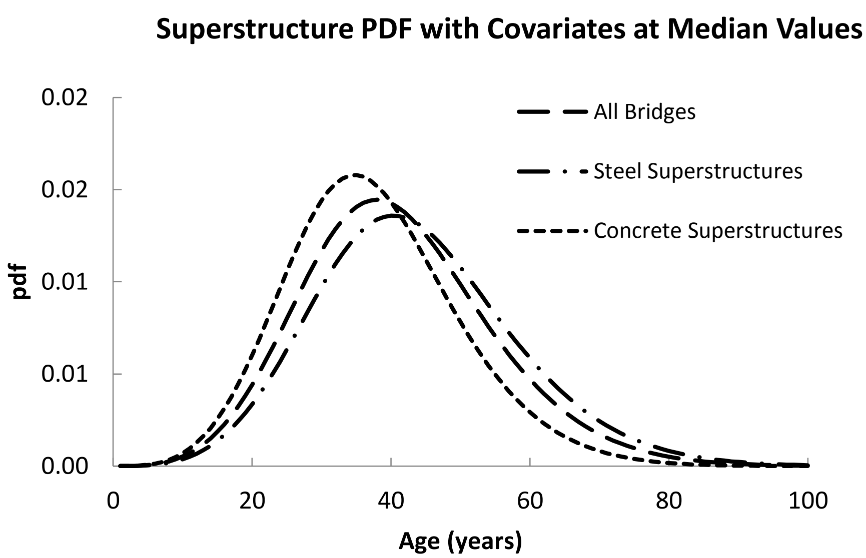

Figure 4.8 Hypertabastic PDF for steel and concrete bridges

4-3. Discussion

To investigate the effect of MSL on the reliability and failure rate of superstructures, the maximum span lengths of 10m to 50m, were analyzed for a constant ADT of 5000. The results are shown in the figures 4.9 through 4.12. As expected, the reliability values decrease and failure rates increase at a given age as the MSL is increased.

The importance of the maximum span length is clearly indicated in these figures. As the maximum span length increases, the reliability of the superstructures decreases and the failure rate increases.

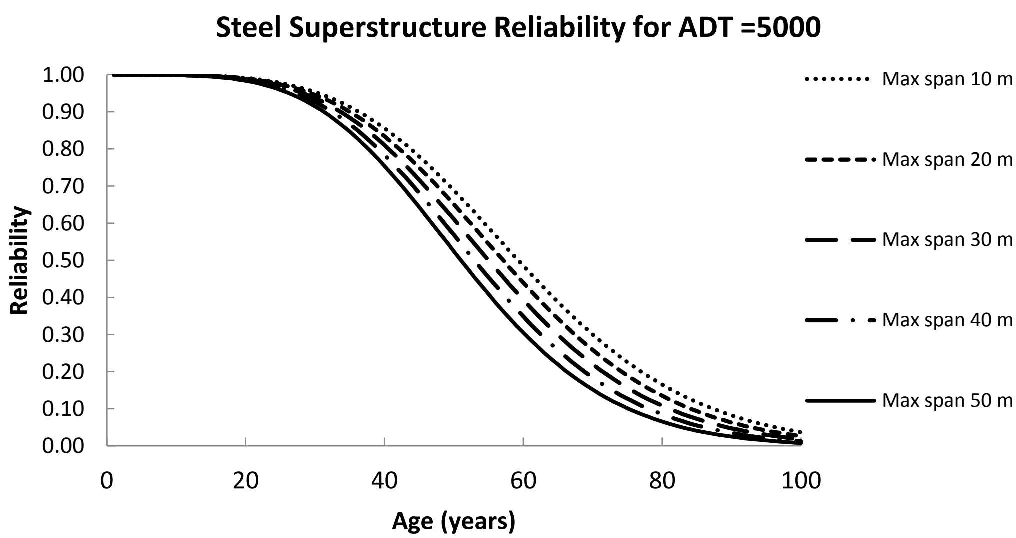

Figure 4.9 Steel superstructure reliability versus age at different maximum span lengths with ADT=5000

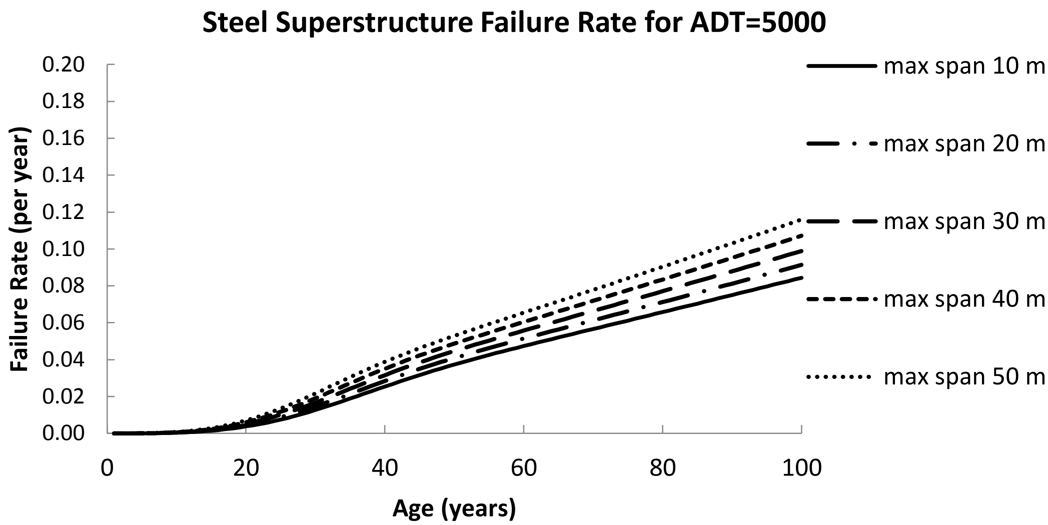

Figure 4.10 Steel superstructure failure rate versus age at different maximum span length with ADT=5000

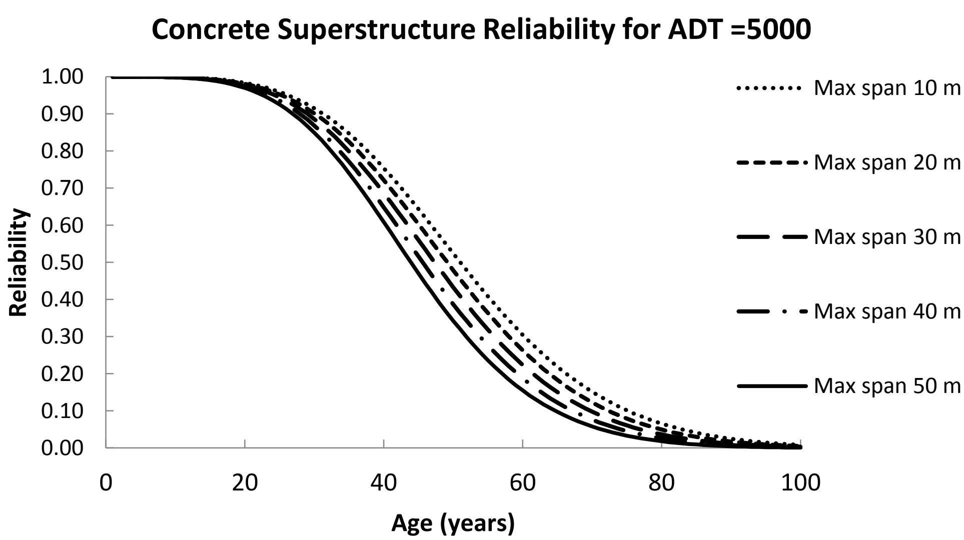

Figure 4.11 Concrete superstructure reliability versus age at different maximum span length with ADT=5000

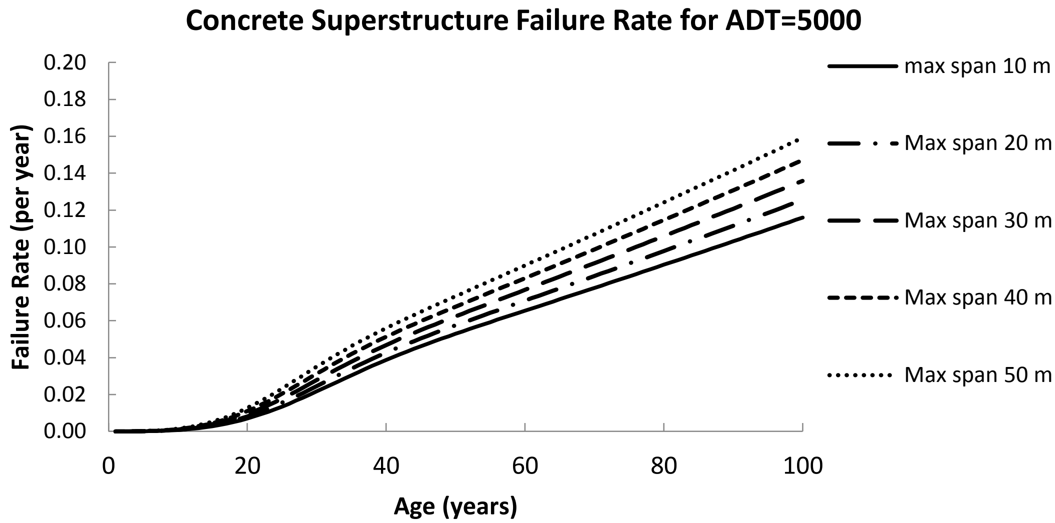

Figure 4.12 Concrete superstructure failure rate versus age at different maximum span length with ADT=5000

Figure 4.13 through 4.16 show the reliability and failure rate of steel and concrete superstructures as a function of ADT at the bridge age of 50 years. The analysis was run for different MSL values.

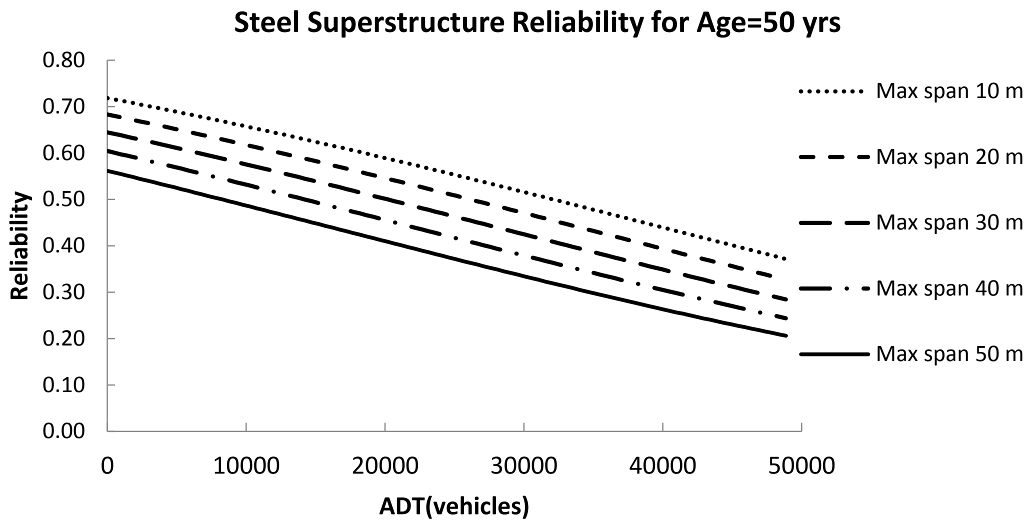

Figure 4.13 Steel superstructure reliability versus ADT at different maximum span length at the age of 50 years

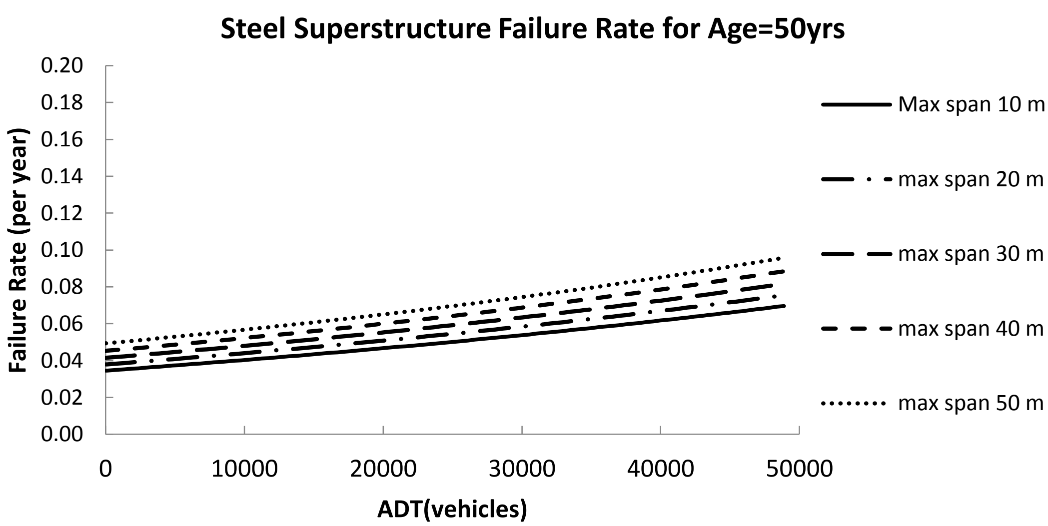

Figure 4.14 Steel superstructure failure rate versus ADT at different maximum span length at the age of 50 years

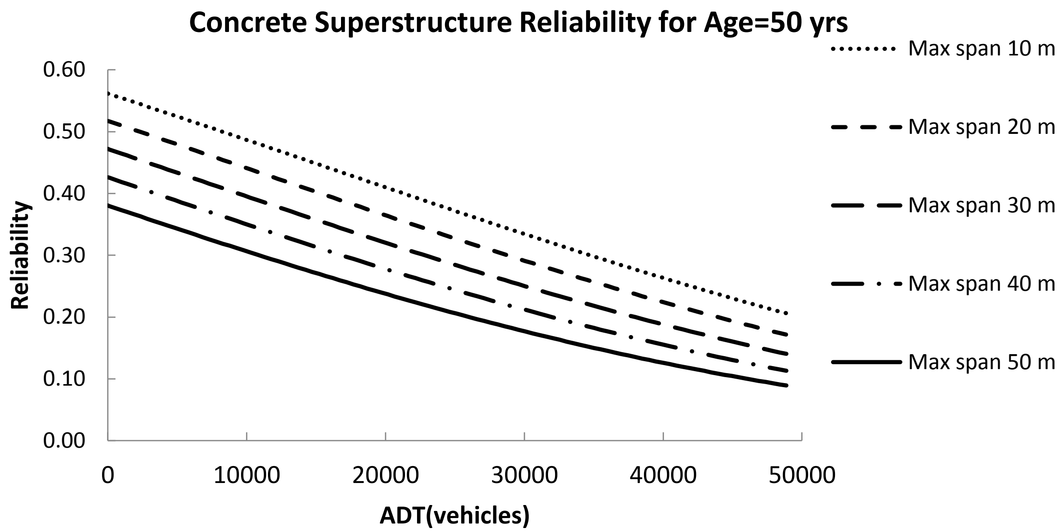

Figure 4.15 Concrete superstructure reliability versus ADT at different maximum span length at the age of 50 years

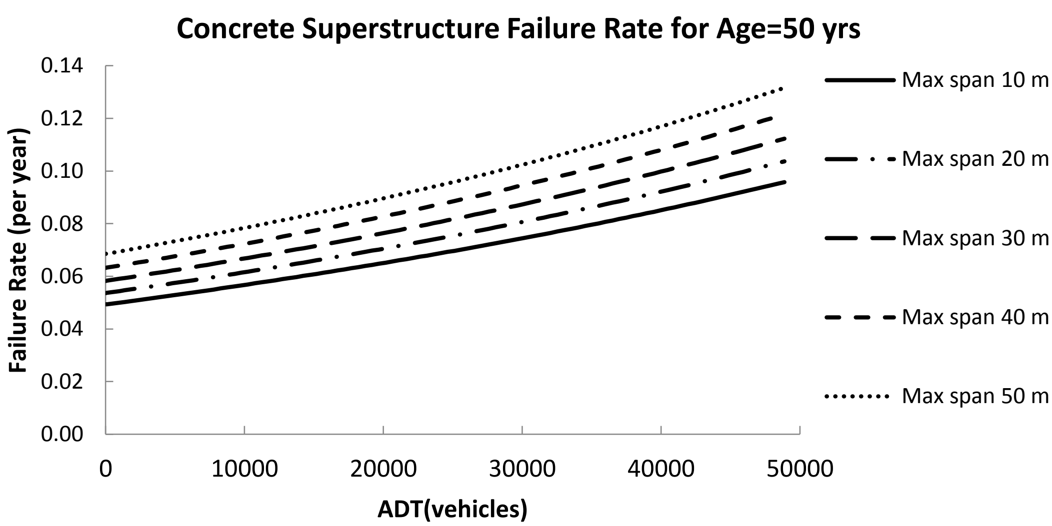

Figure 4.16 Concrete superstructure failure versus ADT at different maximum span length at the age of 50 years

As expected, at the age of 50 years, the reliability decreases rapidly as ADT increases. Similarly, increasing ADT increases the failure rate, as well. Increasing ADT means increasing traffic load and deicing salt usage during the winter (chloride exposure). The reinforcing steel bars embedded in concrete are at increased risk of corrosion due to exposure to chlorides. In figures 4.13 and 4.15, reliability values range from 0.72 to 0.21, and from 0.56 to 0.09 for steel and concrete superstructures, respectively depending on the ADT and MSL.

Figures 4.13 through 4.16 show a nearly linear relationship for reliability and failure rate versus ADT (with different MSL) at the age of 50 years. The slope of the reliability decrement for steel superstructure is approximately 0.8% per 1000 vehicles (Figure 4.13). The slope of the failure rate for the same superstructures is estimated as 0.08% per 1000 vehicles (Figure 4.14).

Figure 4.17 through 4.20 show the reliability and failure rate of steel and concrete superstructures as a function of Maximum span length at the bridge age of 50 years. The analysis was run for different ADT ranges.

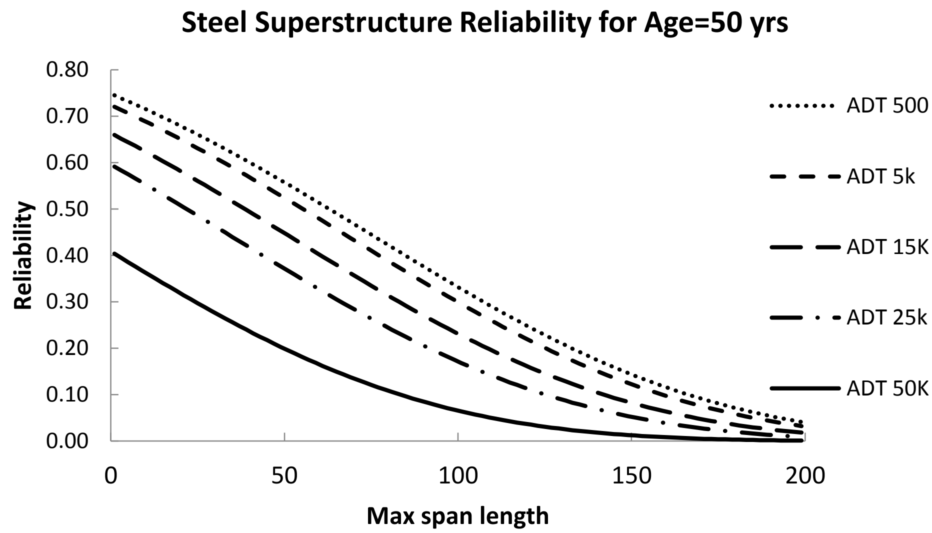

Figure 4.17 Steel superstructure reliability versus maximum span length with different ADT at the age of 50 years

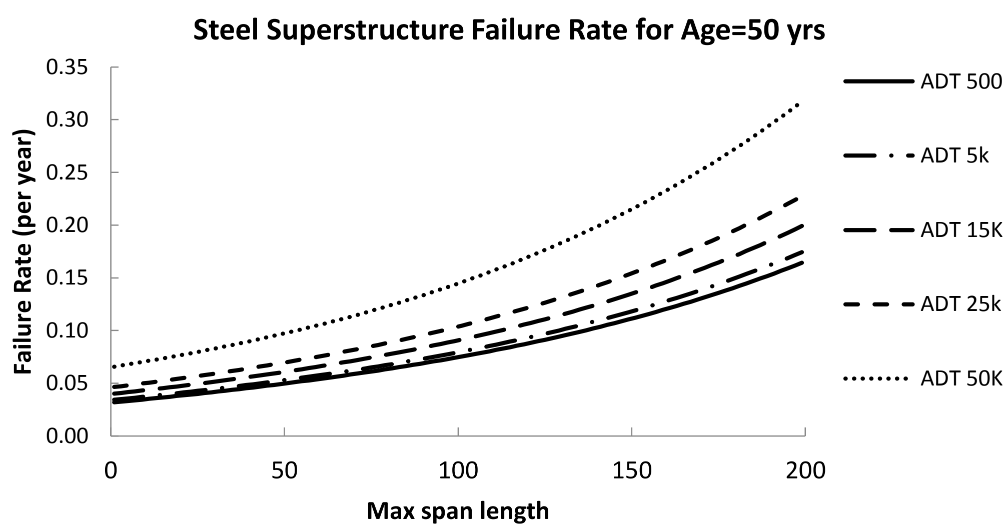

Figure 4.18 Steel superstructure failure versus maximum span length with different ADT at the age of 50 years

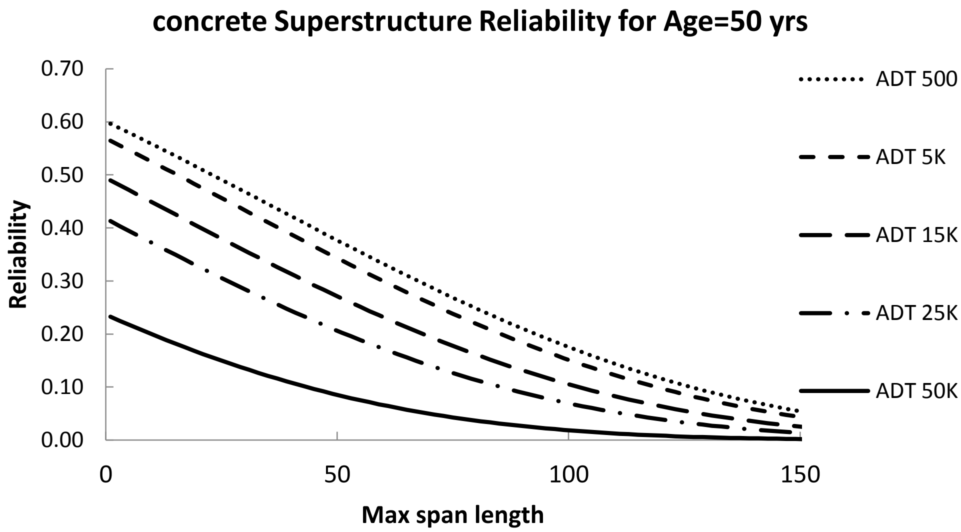

Figure 4.19 Concrete superstructure reliability versus maximum span length with different ADT at the age of 50 years

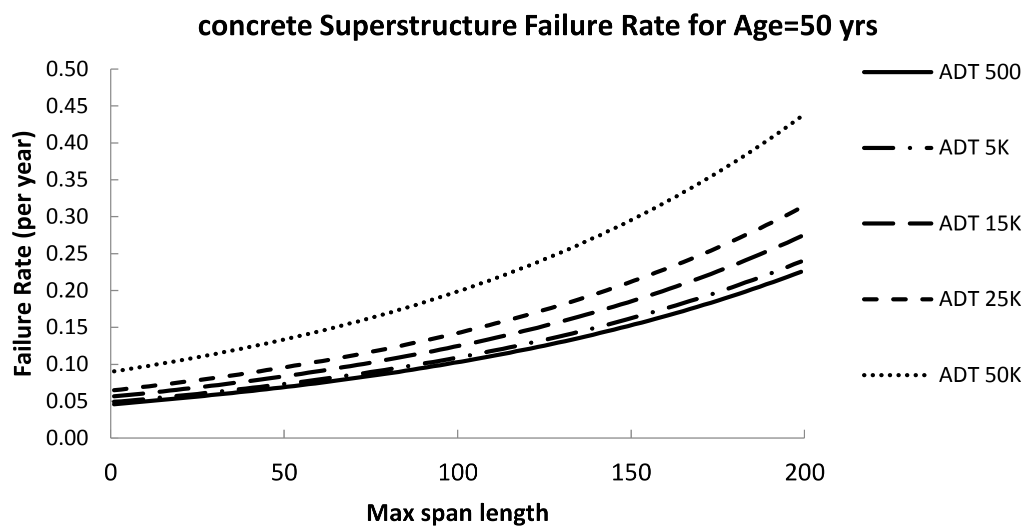

Figure 4.20 Concrete superstructure failure versus maximum span length with different ADT at the age of 50 years

The reliability and failure rate of bridge superstructures are also examined at the age of 75 (years). The analyses were done for different maximum span length and different ADTs. The results for steel and concrete superstructures are shown in the figures 4.21 through 4.24.

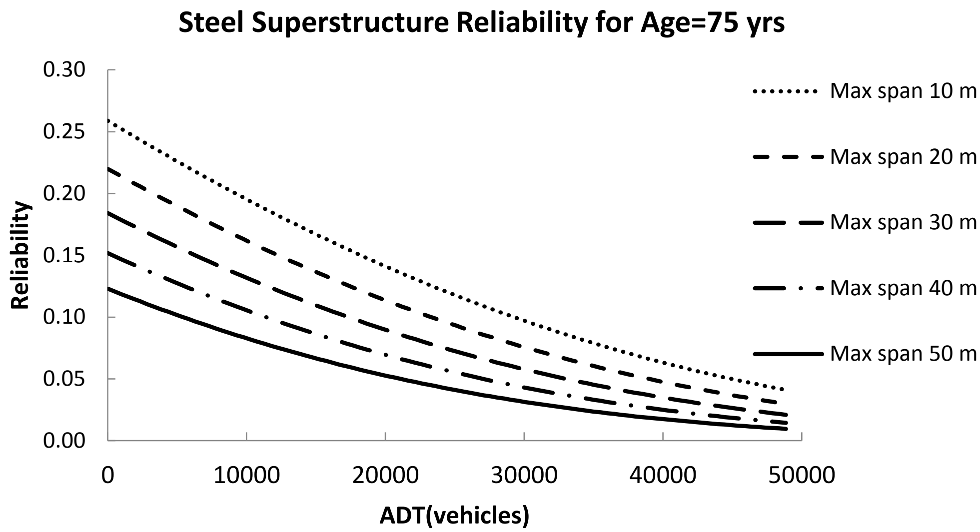

Figure 4.21 Steel superstructure reliability versus ADT at different maximum span length at the age of 75 years

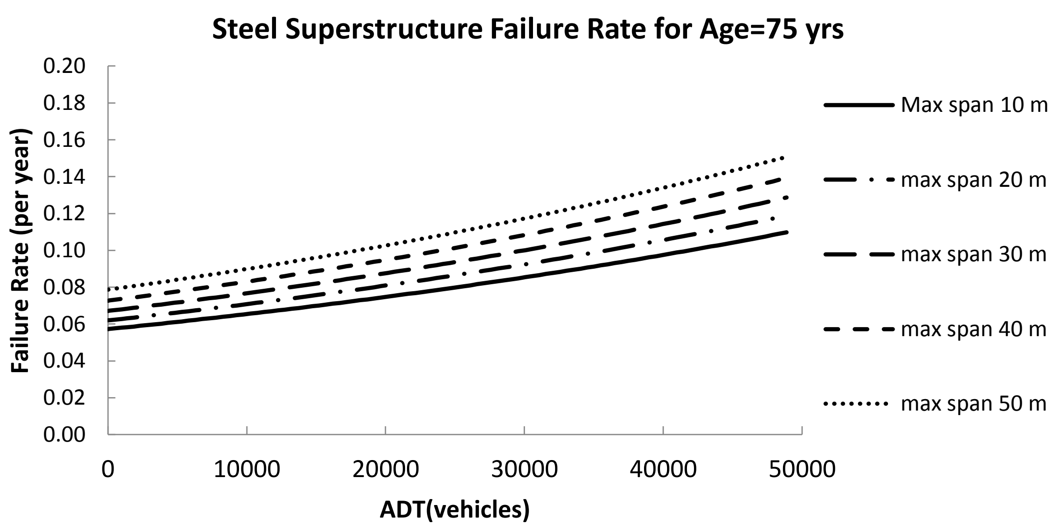

Figure 4.22 Steel superstructure failure rate versus ADT at different maximum span length at the age of 75 years

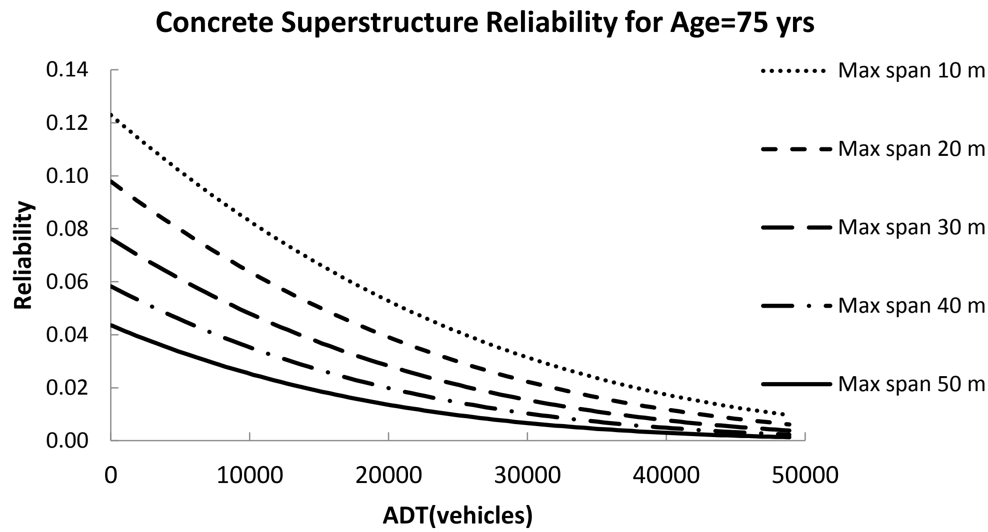

Figure 4.23 Concrete superstructure reliability versus ADT at different maximum span length at the age of 75 years

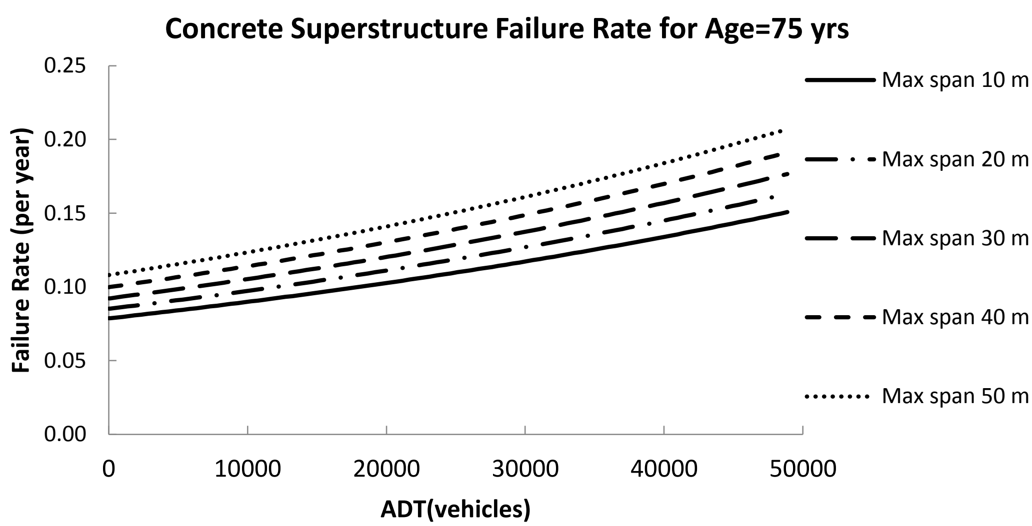

Figure 4.24 Concrete superstructure failure versus ADT at different maximum span length at the age of 75 years

Figure 4.25 through 4.28 show the reliability and failure rate of steel and concrete superstructures as a function of maximum span length at the bridge age of 75 years. The analysis was run for different ADT ranges.

The reliability and failure rate change with respect to the maximum span length. As expected, reliability decreases rapidly with increases in the maximum span length, and failure rate increases as maximum span length increases. Increasing ADT has a significant negative effect on reliability, as shown in the figures 4.25 and 4.27. At the age of 75 years, concrete superstructure reliability can range between 0.15 to less than 0.01 for ADT of 500 and 50k, respectively. Similarly, for steel superstructures, reliability ranges from 0.29 to less than 0.05 as ADT increases from 500 to 50k.

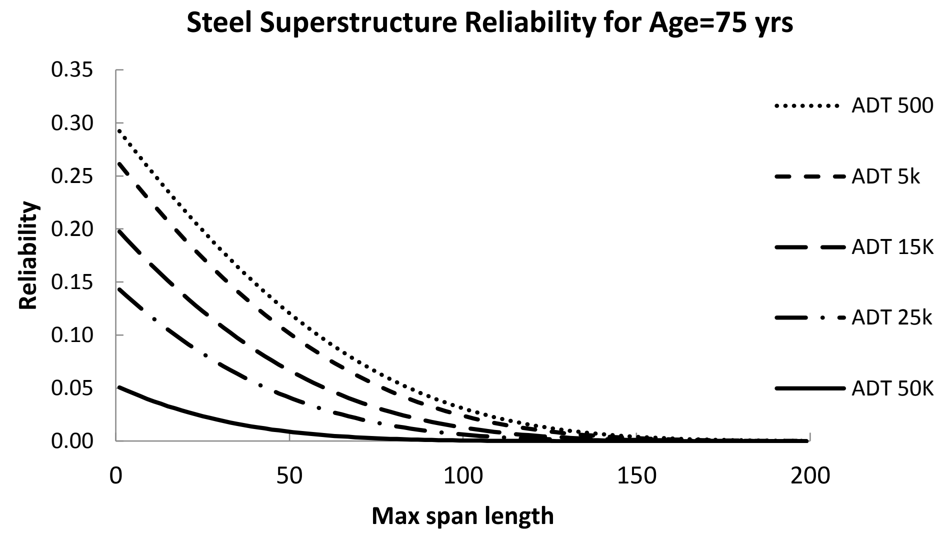

Figure 4.25 Steel superstructure reliability against maximum span length with different ADT at the age of 75 years

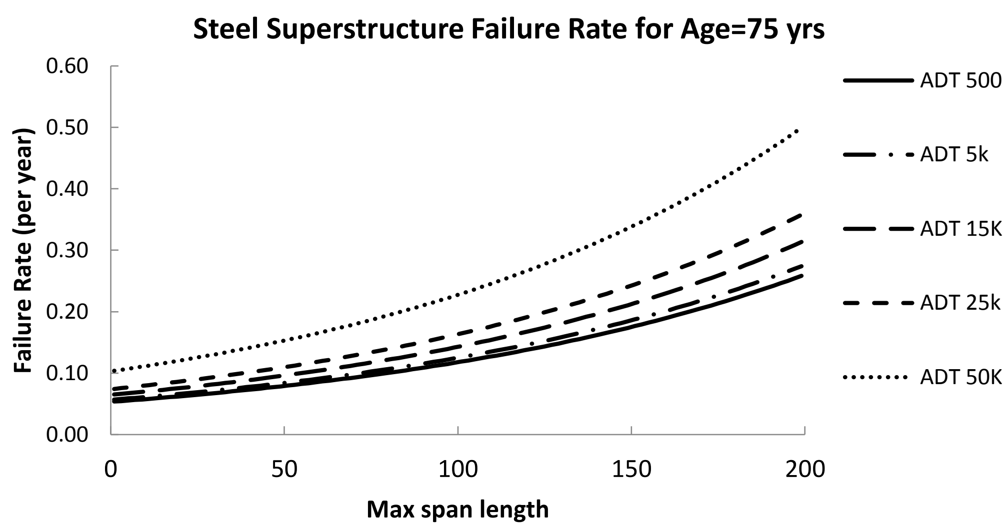

Figure 4.26 Steel superstructure failure rate against maximum span length with different ADT at the age of 75 years

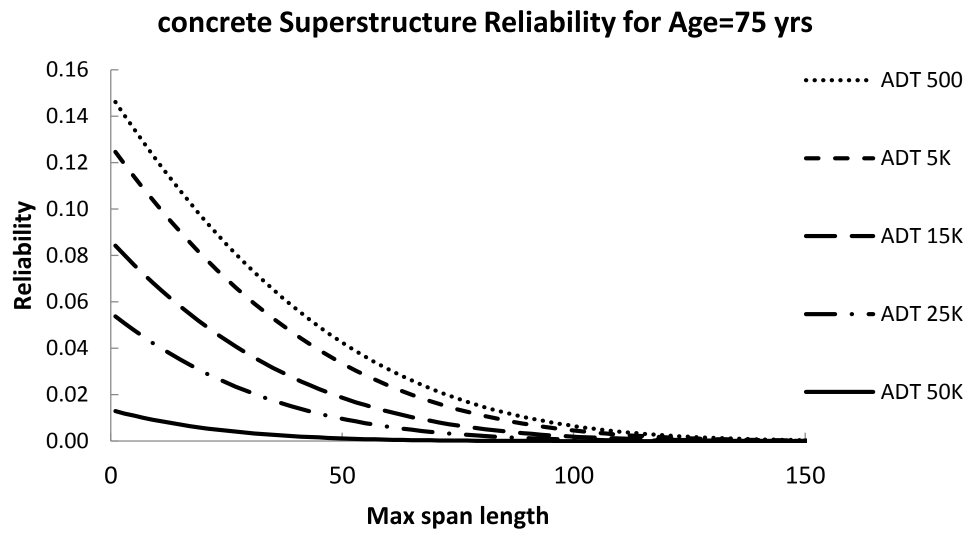

Figure 4.27 Concrete superstructure reliability against maximum span length with different ADT at the age of 75 years

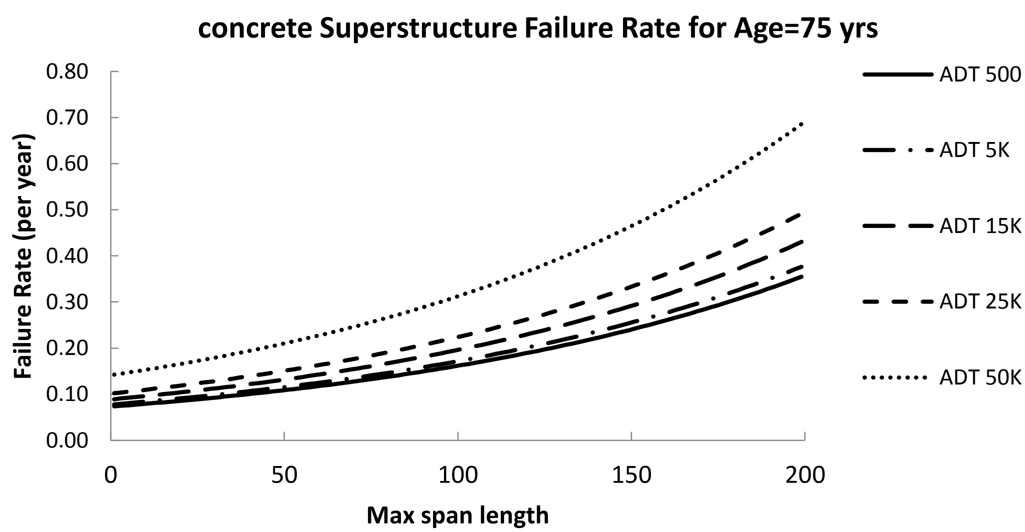

Figure 4.28 Concrete superstructure failure against maximum span length with different ADT at the age of 75 years

As expected, comparison of 50 and 75 years results show a higher range of reliability for both concrete and steel superstructure at the age of 50 years. For example, maximum reliability values at the age of 75 years with respect to ADT and maximum span length of 20 m are 22% and 10% for steel and concrete superstructures, respectively. Whereas, the corresponding reliability values increase to 70% for steel superstructure and 52% for concrete superstructure at the age of 50 years.

Chapter 5. Summary and conclusion

5.1 Summary

Reasonably accurate estimates of service lives of bridge components are crucial for effective long-term bridge management and timely repair and rehabilitation. Utilization of available information on bridges allows the development of survival analysis tools that can provide statistical information on reliability (survival) and failure rates (hazard) at various ages of bridge. Survival analyses have long been used in medical research, but have recently found their ways into bridge engineering.

In this research, the hypertabastic survival model was used to conduct analyses of bridge superstructure reliability and failure rates in Wisconsin. The 2012 NBI data was used for statistical analysis and parameter estimations for the survival model. The included variables were: average daily traffic (ADT), type of superstructure (steel or concrete), age of superstructures (last inspection year minus the year built), and maximum span length. The NBI superstructure rating of 5 was considered as the end of service life. According to Tabatabai et al (2011), a recorded NBI bridge rating of 5 is an indicator of end of service life representing the time bridge rehabilitation or repair would be required. Data were filtered for reconstructed bridges and less common bridge types.

In a study designed to develop survival models for bridge decks in Wisconsin. Tabatabai et al. (2011) determined that the hypertabastic accelerated failure time model was the best fit model compared to other models based on the AIC criterion. The most important feature of the hypertabastic survival model is its capability to represent a variety of hazard shapes.

Survival analysis were run through the Mathematica software. The model parameters were determined using the method of maximum likelihood.

5.2 Conclusions

Survival time of steel and concrete superstructure were evaluated through the hypertabastic accelerated failure time model with covariates (ADT and maximum span length) set at their mean and median values for Wisconsin. Steel superstructures (distinguished by code 1) showed relatively higher levels of reliability at ages of 50 and 75 years, while concrete superstructure (distinguished by code 0) presented higher level of failure rate at those same ages.

Age is an important factor in bridge superstructures reliability. As bridges age, the reliability decreases and failure rate increases. Failure rate is relatively small prior to 20 years of age, after which the rate rises more rapidly. During the age of 20 to 40 the failure rate increases more rapidly. At more advanced ages, the superstructure failure rate increases are nearly linear with respect to time.

Results indicates that both ADT and maximum span length influence the bridge survival time and failure rate significantly. Effect of increasing maximum span length were studied for a fixed ADT of 5000. Results showed that with increase in maximum span length, the superstructures reliability at ages of 50 and 75 years decreases, and the failure rate increases. Similarly, effect of varying ADT was studied for a fixed MSL. Results indicated that superstructure reliability decreased while failure rate increased at ages of 50 and 75 years.

Results for superstructures show a nearly linear relationship between reliability and ADT (with different MSL) at the age of 50 years. The reliability for distinguished steel superstructure decreases by the rate of approximately 0.7% per 1000 vehicles. Similarly, the slope of the failure rate for the same superstructures is estimated as 0.08% per 1000 vehicles.

References:

- Ahmed FE, Hattis D, Wolke RE, Steinman D. (1993), *“Human health risks due to consumption of chemically contaminated fishery products”*, Environ. Health Perspect. Suppl. 101:297–302.
- Akgul, F., Frangopol D. M. (2003), *“Rating and reliability of existing bridges in a network.”* Journal of Bridge Engineering, 8(6), 383–393.
- Akgul F., Frangopol D. M. (2004), *“Bridge Rating and Reliability Correlation: Comprehensive Study for Different Bridge Types*”, Journal of Structural Engineering. 2004.130:1063-1074.
- Bazovsky I. (2005), *“Reliability Theory and Practice”*, Dover Publication, Inc., N. Y.
- Barone G., Frangopol D. M. (2014), *“Reliability, risk and lifetime distributions as performance indicators for life-cycle maintenance of deteriorating structures”*, Reliability Engineering and System Safety, 123 (2014) 21–37.
- Beng S. S., Matsumoto T. (2012), *“Survival Analysis on Bridges for Modeling Bridge Replacement and Evaluating Bridge Performance”,* Journal of Structure and Infrastructure Engineering, Vol. 8, No 3, March 2012, 251-268.
- Bolukbasi M., Mohammadi J., Arditi D. (2004), *“Estimating the Future Condition of Highway Bridge Components Using National Bridge Inventory Data”*, Practice Priodical on Structural Design and Construction, 2004.9:16-25.
- Elsayed A. E. (2012), “Reliability Engineering”, 2nd Edition, A JOHN WILEY &INC., Publication.

- Estes A. C., Frangopol D. M. (2005), *“Load Rating versus Reliability Analysis”,* Technical Notes, Journal of Structural Engineering. 2005.131:843-847.
- Frangopol M., *“Bridge Safety and Relaibility”*, ASCE, SEI, 1999.
- Hesse A. (2013), *“U sing Expert Opinion to Quantify Accuracy of Nondestructive Evaluation on Bridges”* (Master Thesis), Colorado State University.
- Hwang S. N. “*Risk Based Relaibility Assessment Incorporating Factor of Safety, Rating Factors and Systematic Approaches”*, 2008.
- Kong J. S., Frangopol D. M. (2003), *“Life-Cycle Reliability-Based Maintenance Cost Optimization of Deteriorating Structures with Emphasis on Bridges”*, Journal of Structural Engineering, Vol. 129(6), pp. 818–828.
- Kottegoda N. T., Rosso R. (1997), *“Statistics, Probability, and Reliability for Civil and Environmental Engineers”,* McGraw-Hill Companies, Inc. ISBN 0-07-035965-2.
- Lee E. T., Go O. T. (1997), *“Survival Analysis in Public Health Research”,* Annu. Rev. Public Health. 1997. 18:105–34.
- LEE L. S. (2011), “Rehabilitation and service life estimation of bridge *superstructures”*, 4, E-Book, Woodhead Publishing Limited.
- Lu D. G., Jiang W., Fan X. P., and Yang Y.-X. (2011), *“Reliability assessment and life prediction for serviceability of concrete bridge”*, Applications of Statistics and Probability in Civil Engineering, Taylor & Francis Group, London, ISBN 978-0-415-66986-3.
- Lu D.-G., Jiang W., Fan X. P.(2012), *“Comparisons of four time-dependent reliability approaches for safety assessment of deteriorated concrete bridges”,*

Bridge Maintenance, Safety, Management, Resilience and Sustainability – Biondini & Frangopol (Eds), Taylor & Francis Group, London, ISBN 978-0-415-62124-3.

- Juckett DA, Rosenberg B. (1993), *"Comparison of the Gompertz andWeibull functions as descriptors for human mortality distributions and their intersections"*, Mech. Ageing Dev. 69:1–31.
- Myung I. J. (2003), *"Tutorial on maximum likelihood estimation",* Journal of Mathematical Psychology 47 (2003) 90–100.
- Nowak A. S., Collins K. R. (2000), *"Reliability of Structures",* McGraw-Hill Companies, Inc. ISBN 0-07-048163-6.
- Saydam D., Frangopol D. M. (2011), *"Time-dependent performance indicators of damaged bridge superstructures"*, Engineering Structures, 33 (2011) 2458–2471.
- Schmidt P, Witte DA. (1988), *"Predicting Recidivism Using Survival Models",* NY: Springer-Verlag.
- Scott BR, Hahn FF. (1980), *"A model that leads to theWeibull distribution function to characterize early radiation response probabilities"*, Health Phys 39:521–30
- Sobanjo J, Mtenga P., Rambo-Roddenberry M. (2010), *"Reliability-Based Modeling of Bridge Deterioration Hazards",* Journal of Bridge Engineering. 2010.15:671-683.
- Sun J. Hong H. P. (2001), *"Effect of Reinforcement Corrosion on Reliability of Bridge Girders",* Civil Engineering and Environmental Systems, Vol. 19(1), pp. 67-85.

- Steward M. G., Estes A. C., Frangopol D. M. (2004), “Bridge Deck Replacement for Minimum Expected Cost under Multiple Reliability Constraints”, Journal of Structural Engineering, Vol. 130(9), pp 1414-1419.
- Steward M. G, Rosowzky D. V. (1998), *“Time-Dependent Reliability of Deteriorating Reinforcement Concrete Bridge Decks”,* Structural Safety, 20 (1998) 91-109.
- Robert S. Kirt, William J. Mallett, CRS report for congress (2007). “*Highway Bridges: Conditions and the Federal/State Role*”, 2007, August 10.
- Tabatabai M. A., Bursac Z., Williams D. K., Singh K. P. (2007), *“Hypertabastic Servival Model”,* Theoretical Biology and Modelling, doi: 10.1186/1742-4682-4-40.
- Tabatabai H., Tabatabai M., Lee C. W. (2011), “*Reliability of Bridge Decks in Wisconsin”*, Journal of Bridge Engineering. 2011.16:53-62.
- Wang N. (2010), *“Reliability-Based Condition Assessment of Existing Highway Bridge”* (Doctoral dissertation), Georgia Institute of Technology.
- Williams JS. (1978), *“Efficient analysis of Weibull survival data from experiments on heterogeneous patient populations”*, Biometrics 34:209–22.
- Wu J. (2010), *“Reliability-Based Lifetime Performance Analysis of Long-Span Bridges”* (Doctoral dissertation), Colorado State University.
- Wis Dot Brief, *“Better Ways to Preserve Wisconsin’s Concrete Bridges”*, Wisconsin of Transportation, No. 0092-00-17.

- Yang Y. N., Kumaraswamy M. M., Pam H. J., Xie H. M. (2013), *"Integrating Semiparametric and Parametric Models in Survival Analysis of Bridge Element Deterioration",* Journal of Infrastructure System. 2013.19:176-185.
- Yehia S., Abudayyeh O., Nabulsi S., Abdelqader I. (2007), *"Detection of Common Defects in Concrete Bridge Decks Using Nondestructive Evaluation Techniques"*, Journal of Bridge Engineering. 2007.12:215-225.

Appendix A

Notations

The following symbols are used in this thesis:

ADT = Average Daily Traffic (vehicles per day)

MSL = Maximum Span Length (m)

c = Numerical coefficient for variable MSL

d = Numerical coefficient for variable ADT

h = Numerical coefficient for variable TYPE

F(t) = hypertabastic distribution function

f(t) = hypertabastic probability density function

$f_0(t)$ = Baseline hypertabastic probability density function

$g(x|\theta)$ = A non-negative function of x and θ

h(t) = hypertabastic hazard (failure rate) function

$h_0(t)$ = Baseline hypertabastic proportional hazard function

$LL(\theta,\alpha,\beta|x)$ = The log-likelihood function for the hypertabastic proportional or accelerated failure models

n = Number of observations

p = Number of covariates

$S(t)$ = hypertabastic survival (reliability) function

$S_0(t)$ = Baseline hypertabastic survival function

x = A p-dimensional vector of covariates

θ = A p-dimensional vector of constant multipliers for the covariates

t = Age (years)

t_i = Age for the i^{th} observation (years)

$t_g = Z(t)$, a function of θ, x, and t

TYPE = Type of bridge superstructure [steel (1) or concrete (0)]

$W(t)$ = A function of α, β, and t

$Z(t)$ = A function of θ, x, and t

α = A positive constant

β = A positive constant

δ_i = constant (0 or 1); depends on whether t_i is a right censored observation

Appendix B

The log-likelihood equation coded in Mathematica software is shown in this appendix. The code was used to find the parameters for hypertabastic survival model using NBI 2012 dataset.

NBI Data Analysis for Superstructures with Covariate Type Value of 1 for Steel and 0 for Concrete (When distinguished by type)

This is an example of Accelerated failure Model Using the Hypertabastic Model with Log time .Bridge Superstructure Rating of 5 Wisconsin Data where "w" is the bridge type variable, "t" is the age of the bridge , "x" is the maximum span length and "z" is traffic intensity . x, z and w are included in the model. No observation is censored, thus status is set to be 1 for all observations .

Accelerated an Data example failure Hypertabastic is
Log Model[2] of Rate5 the This Using where Wisconsin with time.Bridge

```
n = 907;
Status = ConstantArray[1, n];
ID = {4, 201, 337, 346, 412, 417, 429, 437, 439, 469, 471, 474, 475, 680, 952, 965, 966, 967,
   1015, 1049, 1050, 1057, 1063, 1077, 1085, 1091, 1092, 1106, 1107, 1117, 1118, 1123, 1133,
   1152, 1310, 1312, 1313, 1322, 1326, 1334, 1340, 1495, 1507, 1516, 1523, 1533, 1582, 1640,
   1647, 1669, 1686, 1753, 1811, 1830, 1831, 1835, 1840, 1841, 1849, 1854, 1855, 1860, 1863,
   1866, 1869, 1872, 1899, 1900, 1912, 1913, 1915, 1918, 1927, 1929, 1932, 1936, 1945,
   1947, 1950, 1952, 1955, 1965, 1966, 1984, 1987, 1993, 1995, 1997, 2013, 2017, 2018,
   2032, 2045, 2074, 2191, 2266, 2268, 2289, 2295, 2303, 2328, 2330, 2331, 2333, 2334,
   2442, 2448, 2543, 2599, 2775, 2785, 2786, 2792, 2796, 2806, 2807, 2808, 2818, 2860,
   2864, 2922, 2923, 2964, 2973, 2975, 2992, 2993, 2998, 3010, 3040, 3046, 3184, 3200,
   3219, 3450, 3454, 3465, 3478, 3602, 3641, 3650, 3651, 3775, 3779, 3784, 3795, 3827,
   3845, 3846, 3853, 3854, 3864, 3867, 3873, 3889, 3912, 3966, 3983, 3994, 4005, 4007,
   4010, 4127, 4167, 4270, 4346, 4349, 4350, 4351, 4375, 4427, 4430, 4439, 4443, 4465,
   4468, 4469, 4559, 4600, 4716, 4753, 4759, 4778, 4804, 4834, 4926, 4931, 5114, 5117,
   5118, 5120, 5121, 5123, 5124, 5127, 5131, 5134, 5144, 5151, 5152, 5153, 5157, 5164,
   5171, 5225, 5247, 5369, 5476, 5483, 5486, 5493, 5582, 5586, 5591, 5661, 5676, 5686,
   5687, 5697, 5699, 5710, 5717, 5722, 5730, 5733, 5740, 5741, 5758, 5765, 5766, 5778,
   5786, 5796, 5797, 5798, 5799, 5804, 5814, 5846, 5847, 5857, 5858, 5860, 5873, 5882,
   5899, 5900, 5901, 5902, 5905, 5918, 5927, 5931, 5938, 5939, 5940, 5941, 5999, 6002,
   6003, 6004, 6005, 6006, 6007, 6013, 6020, 6033, 6034, 6069, 6085, 6108, 6112, 6127,
   6132, 6136, 6192, 6193, 6349, 6416, 6443, 6444, 6447, 6448, 6452, 6457, 6458, 6462,
   6625, 6643, 6735, 6736, 6741, 6743, 6762, 6791, 6793, 6810, 6819, 6963, 6997, 7011,
   7013, 7046, 7051, 7052, 7128, 7138, 7149, 7163, 7165, 7178, 7293, 7326, 7352, 7353,
   7358, 7362, 7366, 7374, 7376, 7381, 7393, 7403, 7423, 7500, 7518, 7547, 7566, 7568,
   7575, 7577, 7578, 7617, 7623, 7680, 7688, 7698, 7706, 7707, 7945, 7982, 8047, 8049,
   8163, 8167, 8179, 8266, 8267, 8276, 8292, 8311, 8320, 8330, 8481, 8501, 8503, 8507,
   8637, 8642, 8643, 8644, 8645, 8654, 8681, 8690, 8715, 8721, 8725, 8729, 8732, 8820,
   8824, 8825, 8828, 8835, 8838, 8856, 8907, 8912, 8932, 8938, 8953, 8954, 8957, 8996,
   9005, 9006, 9007, 9008, 9013, 9055, 9084, 9116, 9131, 9164, 9166, 9172, 9292, 9304,
```

9306, 9321, 9516, 9518, 9526, 9551, 9577, 9602, 9615, 9683, 9705, 9722, 9736, 9755,
9757, 9758, 9815, 9874, 9875, 9876, 9890, 9903, 9920, 9921, 9940, 9941, 9942, 9943,
9947, 9948, 9949, 9952, 9953, 9979, 10 040, 10 048, 10 051, 10 162, 10 173, 10 276, 10 322,
10 324, 10 337, 10 338, 10 346, 10 347, 10 354, 10 388, 10 586, 10 592, 10 769, 10 773, 10 778,
10 803, 10 822, 10 827, 10 838, 10 867, 10 898, 10 903, 10 923, 10 937, 10 938, 10 940, 10 941,
10 942, 10 943, 10 951, 10 958, 10 961, 10 966, 10 969, 10 976, 10 982, 10 988, 10 991, 10 996,
10 999, 11 014, 11 015, 11 018, 11 023, 11 025, 11 026, 11 028, 11 029, 11 031, 11 033, 11 034,
11 035, 11 039, 11 042, 11 044, 11 046, 11 047, 11 049, 11 053, 11 056, 11 057, 11 058, 11 061,
11 068, 11 069, 11 086, 11 087, 11 111, 11 125, 11 141, 11 142, 11 143, 11 145, 11 155, 11 176,
11 177, 11 185, 11 187, 11 190, 11 191, 11 201, 11 212, 11 213, 11 214, 11 216, 11 218, 11 227,
11 236, 11 237, 11 240, 11 255, 11 263, 11 286, 11 290, 11 308, 11 357, 11 360, 11 361, 11 362,
11 363, 11 364, 11 365, 11 370, 11 371, 11 379, 11 381, 11 382, 11 384, 11 386, 11 387, 11 388,
11 389, 11 391, 11 393, 11 401, 11 404, 11 407, 11 412, 11 425, 11 445, 11 449, 11 481, 11 515,
11 517, 11 518, 11 521, 11 523, 11 525, 11 527, 11 528, 11 530, 11 532, 11 533, 11 534, 11 540,
11 546, 11 548, 11 550, 11 553, 11 555, 11 563, 11 564, 11 566, 11 567, 11 570, 11 574, 11 610,
11 625, 11 630, 11 633, 11 648, 11 658, 11 662, 11 665, 11 669, 11 671, 11 673, 11 675, 11 676,
11 692, 11 717, 11 725, 11 734, 11 736, 11 751, 11 765, 11 788, 11 790, 11 802, 11 810, 11 811,
11 813, 11 825, 11 828, 11 830, 11 841, 11 846, 11 858, 11 860, 11 869, 11 872, 11 881, 11 882,
11 890, 11 899, 11 903, 11 907, 11 919, 11 923, 11 925, 11 933, 11 934, 11 945, 11 948, 11 965,
11 975, 11 986, 11 991, 12 007, 12 011, 12 013, 12 020, 12 021, 12 037, 12 038, 12 045, 12 046,
12 052, 12 077, 12 112, 12 130, 12 131, 12 133, 12 138, 12 156, 12 157, 12 194, 12 205, 12 234,
12 242, 12 253, 12 267, 12 282, 12 295, 12 297, 12 303, 12 310, 12 318, 12 327, 12 341, 12 348,
12 381, 12 390, 12 413, 12 414, 12 415, 12 428, 12 432, 12 452, 12 458, 12 461, 12 468, 12 471,
12 473, 12 479, 12 481, 12 488, 12 495, 12 504, 12 513, 12 514, 12 519, 12 520, 12 598, 12 602,
12 614, 12 647, 12 653, 12 654, 12 656, 12 662, 12 665, 12 675, 12 677, 12 703, 12 711, 12 713,
12 718, 12 723, 12 740, 12 748, 12 760, 12 764, 12 766, 12 767, 12 768, 12 772, 12 773, 12 786,
12 787, 12 793, 12 794, 12 797, 12 798, 12 801, 12 802, 12 808, 12 814, 12 818, 12 822, 12 830,
12 831, 12 832, 12 833, 12 836, 12 851, 12 870, 12 878, 12 907, 12 944, 12 979, 12 992, 13 005,
13 008, 13 013, 13 016, 13 024, 13 027, 13 034, 13 042, 13 046, 13 059, 13 077, 13 079, 13 099,
13 110, 13 112, 13 125, 13 140, 13 142, 13 144, 13 146, 13 151, 13 152, 13 158, 13 165, 13 172,
13 175, 13 198, 13 199, 13 209, 13 224, 13 225, 13 226, 13 235, 13 243, 13 244, 13 247, 13 258,
13 259, 13 260, 13 263, 13 264, 13 272, 13 276, 13 297, 13 302, 13 344, 13 392, 13 396, 13 406,
13 417, 13 422, 13 429, 13 433, 13 434, 13 441, 13 445, 13 458, 13 461, 13 471, 13 494, 13 495,
13 499, 13 503, 13 521, 13 525, 13 526, 13 528, 13 529, 13 533, 13 537, 13 538, 13 542, 13 556,
13 559, 13 560, 13 562, 13 564, 13 568, 13 575, 13 577, 13 578, 13 582, 13 583, 13 600, 13 602,
13 605, 13 609, 13 622, 13 636, 13 637, 13 656, 13 665, 13 669, 13 677, 13 678, 13 680, 13 681,
13 682, 13 688, 13 693, 13 694, 13 695, 13 696, 13 697, 13 701, 13 706, 13 708, 13 709, 13 710,
13 713, 13 714, 13 716, 13 723, 13 727, 13 729, 13 733, 13 736, 13 737, 13 738, 13 741, 13 750,
13 752, 13 756, 13 757, 13 758, 13 768, 13 777, 13 779, 13 782, 13 789, 13 791, 13 793, 13 804,
13 805, 13 807, 13 808, 13 809, 13 810, 13 813, 13 817, 13 821, 13 855, 13 873, 13 876, 13 880,
13 882, 13 895, 13 899, 13 913, 13 914, 13 934, 14 008, 14 014, 14 033, 14 037, 14 040, 14 041,
14 043, 14 048, 14 060, 14 061, 14 070, 14 072, 14 073, 14 081, 14 084, 14 087, 14 089);

```
x = {300, 4700, 7000, 180, 3300, 1400, 19 700, 220, 6400, 20 890, 71 900, 71 900, 57 200, 3060, 2150,
  280, 370, 230, 6132, 1270, 6900, 650, 400, 310, 1600, 8140, 7160, 8140, 3720, 160, 2000,
  2600, 160, 2910, 8100, 300, 490, 900, 500, 1190, 290, 50, 340, 430, 660, 390, 500, 460,
  2500, 190, 460, 390, 1700, 1480, 1740, 4230, 1460, 1460, 4285, 655, 280, 1370, 830,
  4930, 400, 2400, 24 900, 24 900, 20 200, 15 000, 592, 16 400, 23 900, 21 120, 450, 430, 470,
  675, 570, 21 600, 770, 800, 375, 10 900, 10 900, 350, 170, 300, 12 500, 715, 9926, 6600,
  23 300, 1350, 1310, 395, 685, 3780, 870, 510, 8050, 9600, 19 900, 10 300, 11 000, 7300,
  12 500, 140, 70, 200, 610, 200, 400, 3400, 11 300, 11 900, 11 300, 8080, 400, 2910, 800,
  800, 400, 600, 7400, 9930, 15, 2600, 460, 1400, 1800, 250, 100, 600, 5900, 560, 290,
  5400, 1200, 190, 4600, 4400, 1400, 550, 770, 1100, 1200, 10 900, 10 300, 11 935, 11 935,
  120, 3320, 1360, 190, 720, 1340, 1800, 1150, 16 500, 16 500, 15 800, 4600, 3400, 725,
  1500, 5700, 3330, 440, 805, 830, 4200, 4400, 2500, 26 000, 11 550, 11 550, 880, 17 250,
  430, 360, 190, 930, 250, 5000, 10 600, 4458, 490, 3790, 3790, 560, 710, 1010, 1670, 470,
  930, 1980, 4900, 1080, 300, 1080, 270, 830, 3500, 880, 800, 90, 2200, 4600, 7200, 7600,
  1700, 5000, 1100, 780, 69 750, 69 750, 74 900, 9000, 15 800, 6600, 4700, 11 080, 9600,
  5800, 52 900, 49 700, 11 100, 9900, 9900, 34 600, 15 000, 8700, 4100, 1100, 7400, 7300,
  20 400, 17 960, 9740, 20 300, 10 300, 21 400, 1400, 4500, 27 100, 12 500, 10 100, 10 500,
  11 600, 5885, 37 700, 15 100, 6270, 6110, 14 500, 1870, 8300, 4000, 9500, 9500, 9500,
  9500, 9500, 39 700, 6800, 11 900, 11 900, 12 800, 50 600, 12 800, 7670, 10 200, 4300, 1200,
  9000, 9000, 18 700, 4875, 10 600, 10 600, 10 600, 10 600, 10 050, 7900, 7900, 80, 400, 950,
  460, 280, 175, 3400, 330, 340, 3300, 10 200, 310, 275, 10 900, 500, 15 200, 8800, 220,
  100, 1990, 390, 430, 260, 17 430, 430, 29, 320, 620, 2130, 5100, 15 300, 12 100, 21 300,
  23 100, 23 100, 520, 1100, 160, 5600, 1600, 2500, 322, 730, 850, 30 800, 15 300, 8900,
  10 600, 1400, 780, 3800, 2300, 3900, 8800, 22 550, 21 600, 9900, 290, 510, 820, 22 900,
  22 900, 20 730, 2900, 56 800, 5800, 6280, 2400, 160, 140, 1731, 3700, 680, 710, 680, 450,
  1000, 700, 11 600, 310, 1600, 6800, 2820, 390, 400, 490, 530, 680, 110, 925, 100, 8513,
  5120, 55, 2260, 2000, 2000, 2000, 1080, 290, 540, 440, 350, 540, 65, 1100, 1300, 500,
  100, 80, 200, 730, 100, 240, 1900, 80, 180, 210, 1100, 4600, 3550, 275, 1700, 1500,
  50, 210, 200, 13 850, 12 800, 7800, 1700, 1900, 6300, 17 200, 9000, 10 300, 5300, 22 200,
  25 250, 22 200, 25 250, 11 100, 17 300, 16 600, 12 200, 12 700, 14 850, 530, 12 700, 4242,
  320, 8220, 3600, 1050, 10 600, 33 300, 33 300, 33 300, 33 300, 16 000, 13 800, 3000, 2100,
  20, 20, 1200, 80, 40, 75, 1100, 2180, 80, 680, 40, 25, 25, 45, 50, 40, 40, 390, 425, 250,
  2000, 80, 80, 70, 4000, 3200, 40, 80, 280, 280, 500, 40, 40, 40, 80, 50, 50, 10, 60, 175,
  20, 150, 20, 20, 10, 20, 20, 20, 175, 120, 990, 10, 20, 40, 40, 1000, 640, 850, 210, 220,
  650, 180, 2160, 3000, 160, 150, 660, 170, 80, 670, 50, 50, 50, 50, 50, 100, 100, 50, 160,
  50, 80, 180, 50, 180, 250, 40, 200, 80, 175, 80, 300, 300, 160, 45, 20, 40, 350, 10, 260,
  10, 20, 2430, 40, 4475, 75, 27 950, 2100, 400, 4100, 20, 280, 270, 140, 100, 50, 80, 70,
  80, 75, 100, 100, 100, 750, 1100, 80, 100, 25, 50, 200, 171, 280, 120, 50, 25, 50, 1400,
  2090, 300, 21, 50, 50, 50, 3600, 6500, 350, 5300, 500, 40, 50, 100, 100, 50, 50, 100,
  50, 50, 200, 800, 700, 35, 75, 50, 50, 50, 760, 75, 530, 410, 100, 100, 75, 50, 150, 100,
  530, 100, 50, 250, 580, 1500, 330, 2600, 100, 75, 140, 75, 27, 225, 35, 130, 20, 40, 80,
  20, 230, 80, 6700, 20, 320, 40, 45, 250, 1800, 400, 400, 45, 80, 75, 5, 710, 500, 1500,
  1000, 34, 14, 390, 270, 50, 1200, 750, 2500, 3600, 3800, 3800, 490, 40, 80, 80, 90, 40,
```

```
80, 80, 50, 610, 40, 640, 20, 175, 80, 40, 40, 10, 40, 25, 10 200, 1460, 1480, 1000, 40,
1500, 50, 1600, 5700, 9200, 3500, 6000, 6500, 8200, 6200, 170, 80, 230, 230, 230, 50,
210, 40, 50, 80, 40, 20, 350, 80, 10, 500, 2020, 230, 175, 25, 40, 420, 1100, 500, 370,
20, 60, 63, 800, 1900, 175, 75, 210, 30, 80, 75, 50, 50, 320, 50, 300, 740, 20, 125, 80,
1250, 252, 375, 1500, 350, 80, 500, 16 000, 2200, 10, 20, 150, 160, 80, 80, 70, 1, 450,
7800, 90, 20, 150, 490, 150, 610, 720, 130, 500, 75, 40, 190, 80, 450, 650, 50, 15, 175,
75, 215, 190, 50, 25, 280, 670, 20, 100, 40, 100, 100, 20, 1050, 230, 175, 175, 20, 40,
40, 80, 100, 50, 40, 360, 175, 300, 80, 500, 400, 20, 175, 50, 80, 80, 50, 25, 75, 75,
20, 80, 100, 400, 75, 20, 80, 20, 80, 30, 75, 175, 50, 40, 130, 20, 20, 100, 25, 20, 40,
40, 230, 200, 200, 310, 600, 200, 40, 190, 80, 40, 40, 30, 20, 40, 80, 80, 1070, 80, 20,
80, 480, 20, 260, 210, 200, 270, 2100, 220, 40, 40, 30, 40, 40, 50, 3400, 300, 80, 80,
13 000, 200, 1800, 590, 660, 20, 415, 800, 40, 40, 450, 660, 80, 40, 20, 40, 40, 450};

w = {0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0,
   0, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 1,
   0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1,
   1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0,
   0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1,
   1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0,
   0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1,
   1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
   0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1,
   1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0,
   0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0,
   0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1, 0,
   0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0,
   0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0,
   0, 0, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1,
   0, 0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0,
   1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0,
   1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
   1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0,
   0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1,
   1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1,
   1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1,
   1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1,
   1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 1,
   1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
   1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1};
```

```
t = {34, 39, 57, 39, 62, 57, 55, 51, 48, 46, 44, 45, 45, 27, 72, 82, 83, 33, 42, 59, 55, 49, 48, 42,
    40, 40, 40, 40, 40, 39, 39, 76, 29, 24, 73, 71, 51, 83, 51, 47, 35, 73, 62, 52, 40, 50, 25,
    57, 80, 31, 25, 14, 72, 59, 60, 58, 57, 57, 55, 53, 54, 53, 53, 53, 52, 51, 50, 51, 48,
    48, 48, 47, 50, 50, 49, 49, 49, 49, 49, 50, 47, 46, 47, 41, 42, 44, 44, 43, 40, 42, 41,
    40, 23, 28, 71, 71, 71, 69, 83, 60, 47, 47, 48, 48, 47, 62, 33, 17, 59, 73, 76, 76, 57,
    54, 46, 46, 46, 46, 35, 32, 74, 74, 71, 62, 37, 37, 74, 56, 48, 33, 30, 22, 62, 54, 61,
    34, 47, 32, 51, 47, 32, 33, 74, 71, 62, 29, 46, 45, 45, 43, 43, 31, 46, 41, 23, 19, 69,
    57, 47, 46, 46, 45, 72, 47, 74, 55, 81, 61, 58, 26, 64, 55, 57, 53, 43, 44, 44, 22, 15,
    74, 88, 47, 20, 57, 28, 54, 52, 57, 56, 56, 56, 55, 54, 54, 54, 53, 52, 47, 51, 51, 50,
    49, 47, 38, 32, 25, 73, 55, 40, 38, 34, 79, 44, 35, 74, 52, 51, 52, 51, 50, 49, 51, 49,
    47, 48, 47, 47, 49, 49, 49, 46, 47, 43, 43, 43, 43, 43, 44, 46, 46, 46, 46, 46, 46, 44,
    42, 42, 42, 42, 42, 7, 45, 42, 43, 43, 43, 43, 35, 35, 35, 36, 35, 35, 35, 36, 35, 35, 35,
    39, 37, 34, 34, 23, 28, 27, 17, 17, 76, 47, 42, 42, 42, 42, 42, 42, 42, 81, 81, 37, 72,
    73, 82, 51, 27, 63, 63, 49, 47, 76, 45, 39, 40, 29, 24, 24, 57, 52, 41, 31, 28, 26, 8, 25,
    56, 54, 48, 41, 41, 42, 41, 41, 30, 39, 15, 62, 36, 74, 73, 72, 53, 52, 41, 42, 22, 56,
    55, 49, 44, 46, 56, 51, 28, 27, 60, 51, 33, 52, 52, 52, 52, 38, 33, 28, 50, 32, 29, 27,
    74, 71, 70, 69, 69, 38, 19, 15, 63, 61, 58, 52, 43, 71, 76, 87, 61, 54, 76, 46, 21, 32,
    26, 26, 19, 19, 19, 80, 51, 49, 48, 46, 41, 20, 7, 62, 47, 28, 28, 26, 91, 55, 53, 41, 69,
    69, 80, 62, 44, 43, 18, 27, 72, 32, 18, 57, 58, 58, 29, 58, 58, 82, 55, 48, 51, 51, 47,
    46, 47, 46, 45, 47, 47, 46, 46, 41, 27, 28, 26, 61, 45, 73, 62, 59, 53, 53, 43, 43, 38,
    33, 42, 75, 95, 85, 70, 76, 45, 76, 60, 80, 50, 44, 65, 88, 88, 86, 60, 60, 65, 56, 60,
    50, 55, 85, 85, 93, 42, 46, 50, 55, 41, 93, 93, 65, 37, 95, 31, 83, 88, 84, 85, 80, 80,
    34, 60, 42, 60, 60, 42, 39, 39, 36, 50, 30, 28, 90, 81, 80, 82, 75, 70, 70, 41, 76, 94,
    76, 60, 63, 71, 90, 80, 56, 49, 60, 50, 36, 36, 96, 83, 81, 44, 82, 70, 86, 80, 74, 76,
    84, 49, 81, 80, 80, 47, 81, 92, 40, 60, 80, 44, 49, 51, 44, 83, 96, 37, 60, 87, 85, 90,
    80, 46, 29, 71, 76, 37, 94, 85, 85, 93, 42, 93, 47, 71, 43, 41, 44, 97, 50, 76, 66, 70,
    76, 66, 42, 53, 81, 43, 39, 77, 80, 80, 83, 45, 35, 55, 42, 43, 45, 83, 91, 72, 50, 61,
    92, 101, 81, 76, 61, 87, 49, 49, 61, 76, 61, 91, 40, 77, 75, 41, 49, 74, 51, 71, 80, 73,
    80, 54, 69, 57, 49, 57, 53, 85, 81, 58, 57, 81, 80, 53, 83, 97, 47, 101, 50, 50, 70, 55,
    80, 72, 75, 62, 48, 82, 84, 61, 55, 80, 93, 81, 80, 80, 41, 87, 75, 72, 81, 88, 59, 47,
    38, 48, 38, 70, 52, 52, 52, 52, 81, 64, 74, 83, 81, 70, 69, 83, 46, 79, 81, 90, 73, 81,
    80, 64, 90, 49, 90, 26, 60, 60, 42, 75, 77, 54, 58, 53, 76, 77, 54, 52, 85, 55, 47, 77,
    70, 70, 77, 32, 65, 78, 82, 60, 51, 85, 47, 78, 90, 90, 80, 70, 70, 90, 70, 47, 55, 51,
    84, 97, 33, 93, 92, 48, 45, 87, 41, 90, 92, 30, 99, 45, 32, 51, 69, 92, 58, 80, 72, 103,
    90, 45, 43, 45, 37, 90, 74, 45, 84, 81, 60, 32, 31, 48, 92, 100, 123, 97, 73, 81, 33, 48,
    53, 54, 52, 46, 54, 51, 69, 41, 30, 48, 59, 80, 70, 51, 81, 78, 55, 72, 81, 30, 33, 74,
    88, 100, 90, 85, 83, 92, 81, 72, 51, 90, 90, 74, 72, 89, 91, 43, 64, 53, 68, 37, 83, 58,
    85, 80, 84, 65, 36, 51, 91, 80, 80, 70, 42, 98, 51, 85, 49, 46, 44, 89, 84, 42, 38, 82,
    80, 56, 42, 70, 60, 50, 49, 47, 48, 60, 36, 85, 50, 50, 56, 44, 47, 75, 41, 85, 77, 60,
    60, 35, 83, 77, 60, 30, 70, 77, 60, 47, 47, 47, 50, 37, 39, 50, 90, 60, 80, 83, 84, 36,
    48, 46, 81, 48, 89, 89, 58, 54, 90, 50, 43, 70, 31, 30, 37, 65, 60, 100, 41, 60, 46};

x = {7.6, 26.8, 8.2, 21.3, 15.2, 21.6, 13.7, 21.9, 27.7, 28, 21, 25.6, 24.4, 6.7, 7.5, 9.6, 9.6,
```

18.2, 21.5, 6.7, 24.8, 23.6, 11, 32.9, 35.1, 16, 16, 16, 47.5, 45.6, 36.4, 15.2, 18.3,
22.9, 12.6, 42.7, 25.9, 7.8, 12.8, 12.2, 10.1, 18.3, 23.2, 20.7, 14.5, 22.4, 7.6, 10.9,
199, 12.2, 11.9, 13.1, 7.3, 12.6, 20.7, 22.9, 8.5, 5.9, 13, 18.3, 18.3, 15.7, 15.8, 9.9,
23.7, 22.9, 15.2, 15.2, 12.8, 14.5, 21, 14.9, 13.1, 19.4, 20.7, 12.2, 6.4, 6.9, 18.3,
8.2, 10.7, 8.4, 9.4, 17.4, 16.2, 9.8, 9.8, 10.7, 24.4, 18.1, 11, 16.8, 29.7, 8.5, 8.2,
6.9, 7.9, 9.4, 12.2, 8.8, 18.3, 14.6, 15.2, 11.9, 11.9, 13.9, 36.4, 12.8, 21.3, 7.6, 7.5,
7.4, 12.2, 10.7, 15.8, 18.6, 18.6, 20.7, 12.5, 13.7, 7.5, 11, 7.6, 20.4, 27.5, 22.2,
12.6, 10.1, 7.5, 13.1, 16.5, 13.4, 18.3, 24.4, 7.6, 8.4, 17.4, 17.7, 13.7, 18, 18.1,
18.1, 12.3, 13.1, 18.7, 13, 14.3, 18.7, 17.2, 21.6, 21.6, 14.6, 15.8, 16.6, 12.2, 13.4,
9.6, 9.6, 20.7, 13.9, 16.2, 9.9, 14.2, 21, 18.7, 13.7, 8.1, 32, 12.8, 20.4, 12.6, 21.3,
34.1, 12.2, 137.2, 16.8, 16.8, 11.9, 20.1, 8.6, 8.1, 11, 14.9, 19.4, 16.3, 29.5, 12.8,
8.4, 9.4, 9.4, 15.5, 19.4, 13.4, 13.4, 6.4, 28, 9.4, 25.5, 13.7, 8.2, 9.5, 11, 14.3, 40.5,
11.7, 15.2, 24.4, 8.8, 21, 40.2, 34.2, 9.6, 16.5, 16.9, 10.7, 14, 21.3, 17.2, 20.1, 22,
22.1, 11, 24.8, 20.4, 19.4, 21.5, 15.8, 19.7, 20.7, 20.7, 13.4, 9.7, 19.5, 21.6, 23.7,
29, 20.4, 13, 20.7, 20.7, 23.5, 25, 28.7, 11.9, 20, 33.8, 36.6, 35.5, 34, 35.4, 9.2,
17.5, 13.7, 15.1, 15.1, 26.5, 13.4, 36.1, 19.4, 35.7, 35.4, 35.4, 35.7, 32.6, 35.4,
24.4, 35.5, 31.9, 15.9, 13.4, 16.3, 18.3, 25.1, 116.9, 16.2, 9.4, 9.4, 20.7, 25.1, 23.2,
23.2, 11.9, 11.9, 12.2, 15.8, 15.8, 8.4, 9.4, 39.6, 14.2, 14.2, 14.3, 15.8, 32, 17.6,
7.9, 20.7, 12.8, 16, 29.1, 39.6, 25.3, 9.4, 12.8, 12.2, 11.9, 16.8, 12.2, 13.3, 39.6,
13.7, 3.5, 12.8, 21.8, 14.9, 22.4, 25.5, 9.6, 23.9, 18.4, 18.4, 9.1, 16.9, 15.2, 24.4,
23.1, 14.2, 17.9, 7.9, 9.5, 17.6, 23.2, 6.7, 33.6, 22.1, 18.3, 12.2, 17.1, 16.2, 7.6,
25.8, 8.1, 15.2, 18.7, 12.2, 18.9, 12.2, 12.2, 12.2, 18.7, 21.3, 18.3, 14.6, 21.2, 13.7,
12.2, 10.7, 6.2, 14.3, 14.6, 6.2, 7.3, 24.4, 9.8, 17.1, 9.3, 7.6, 8.2, 8.1, 11.4, 21.6,
20.5, 7.3, 12.6, 14.2, 14.6, 10.7, 38.6, 16.2, 17.6, 16.5, 9.8, 8.5, 12.2, 6.9, 13.7,
20.7, 9.4, 13.9, 11.3, 10.4, 14.3, 12.8, 15.8, 14, 11, 15.2, 11.3, 30.5, 12.8, 13.7, 8.4,
7.8, 11, 18.9, 32.3, 29.4, 13.4, 12.8, 18.4, 9, 16.8, 6.4, 9.6, 7.7, 12.2, 20.1, 20.1,
12.8, 11.6, 21.3, 21, 20.4, 14.5, 14.5, 12.6, 12.6, 29.7, 10.4, 10.4, 25.9, 26.5, 17.5,
17.1, 20.7, 7.6, 21.3, 13.9, 11.8, 7.7, 32.8, 14, 14, 29.6, 25.3, 33.5, 11, 21.5, 13.7,
8.1, 12.2, 9.4, 10.4, 5.8, 10.5, 6.3, 7.6, 7.2, 10.8, 7, 9.8, 9.4, 9.4, 7.2, 8.8, 8.8,
6.5, 14.3, 12.5, 7.3, 7.8, 7.8, 8.4, 6.6, 13.6, 7.4, 6.8, 7.3, 10.9, 9.4, 9.1, 7.1, 8.1,
7.2, 11.3, 7, 8.5, 7.3, 9.4, 9.4, 7.1, 9.8, 10.1, 3.2, 6.5, 4.7, 9.4, 6.9, 7.1, 7, 6.7,
6.4, 6.7, 11.9, 4.2, 9.1, 11.9, 14.6, 5.8, 8.9, 5.8, 28.4, 39.7, 9, 6.9, 8.7, 8.8, 6.6,
17.1, 14.9, 18, 18, 17.4, 11.3, 8.7, 21.3, 15, 9, 7.3, 41.5, 6.5, 6.9, 6.6, 6.6, 8.2, 6.8,
9, 8.2, 7.3, 23.9, 8.8, 5.8, 6.5, 7.3, 12, 9.1, 7.2, 9.8, 8.5, 6.9, 8.5, 9.5, 4.5, 6.6,
20.9, 11.9, 7.3, 10.8, 15.2, 9.7, 11.4, 22.3, 9.4, 5.2, 10.1, 13.5, 8.4, 14.2, 17.8, 9.9,
10.4, 6.6, 15.2, 9.4, 9.4, 7.3, 14.8, 4.5, 10.5, 8.8, 15.2, 5.9, 6.6, 15.5, 21.6, 9.4,
6.7, 18.3, 9.5, 7.8, 9, 12.6, 24.3, 18.8, 24.8, 18.9, 7.9, 9.5, 18.6, 11.7, 15.2, 7.2,
6.3, 30.5, 8.6, 8.7, 8.5, 6.3, 8.9, 7.6, 8, 6.5, 11, 15.5, 10.4, 9.8, 9.3, 11.9, 9.5,
12.6, 6.7, 12.1, 7, 6.9, 6, 6.9, 7.7, 7.7, 8.8, 6.9, 7.6, 18.2, 4.6, 15.7, 15.6, 8.5, 10,
7, 9.4, 11, 10.9, 7.3, 19.2, 9.4, 6.6, 15.2, 3.7, 6.2, 11.8, 9.6, 12.7, 12.8, 9.4, 9.4,
9.6, 6.9, 3.6, 8.1, 8.7, 8.2, 12, 11.7, 18, 9.1, 7.6, 9.4, 9.1, 11.3, 26.5, 8.5, 11.2,
9.6, 11.2, 7.6, 6.9, 11.1, 18.3, 8.9, 9.7, 11, 14.1, 7.1, 6.7, 9.5, 9.6, 9.4, 12.5, 7.9,
18.3, 12.4, 6.8, 9.4, 6.9, 39.3, 3.7, 3.9, 9.1, 12.6, 9.1, 8.4, 21.2, 14.6, 7.9, 11.9,

```
27.4, 27.7, 29.3, 15.7, 8.5, 9.3, 9, 11.9, 16, 15.7, 7.6, 7.7, 8.2, 8.9, 8.8, 10.9, 7.6,
12.6, 11.4, 7.4, 10.8, 11.7, 7.2, 9.4, 6.9, 8, 12.6, 8.7, 6.6, 8.8, 25.6, 9.7, 9.6, 6.4,
6.4, 9.4, 21.7, 9.6, 9.4, 9.4, 4.3, 8.1, 13.1, 7, 7.6, 10.7, 7.6, 9.4, 10.3, 7.5, 14.4,
13.8, 21.2, 21.2, 11.7, 9.8, 25, 6.9, 9.8, 8.2, 10.7, 6.9, 7, 9, 7.2, 36.6, 43.1, 17.4,
7.1, 7.1, 9.5, 15.5, 15.5, 11.9, 15.7, 10.7, 18.7, 6.6, 6.9, 7.7, 6.2, 7.5, 9, 11.9, 7.2,
9.1, 3.2, 6.8, 9.6, 10.4, 9.1, 9.1, 14.9, 17.9, 18, 10.7, 14.3, 12.6, 6.6, 8.4, 14.3,
14.2, 18.3, 8, 11.3, 7.8, 14.3, 14.3, 9.7, 9.6, 8.3, 12.8, 14.3, 6.7, 6.9, 6.6, 6.9, 6.9,
8.9, 7.8, 17.7, 14.9, 6.7, 6.7, 6.7, 7, 24.1, 8.5, 8.7, 8.1, 7.1, 7.1, 6.4, 7.4, 8.6,
8.9, 15.5, 8.2, 6.9, 7.6, 7, 8.8, 7.3, 7.6, 8.8, 8.5, 5.8, 6.7, 7.4, 7.1, 15, 13, 12.5,
20.1, 8.1, 4.4, 11.9, 7.2, 24.4, 8.9, 15.3, 19.7, 10.5, 16.5, 8.9, 7.7, 7, 4.4, 10.4,
7.1, 7.1, 7, 10.6, 16.3, 8.2, 6.8, 8.6, 3.4, 8.8, 8.4, 9.3, 17.7, 15.2, 9, 7.7, 7.9, 7.7,
6.9, 12.6, 8.5, 7, 13.4, 8.6, 6.7, 21.3, 7.3, 17.9, 17.7, 7.6, 8.8, 8.5, 9.3, 6.5};
```

```
f = Sum[(Log[Sech[a * (1 - (t[[i]] * Exp[c * x[[i]] + d * z[[i]] + h * w[[i]]])^b *
        Coth[(t[[i]] * Exp[c * x[[i]] + d * z[[i]] + h * w[[i]]])^b]) / b]] +
    Status[[i]] * Log[t[[i]] * (((a (- (t[[i]] * Exp[c * x[[i]] + d * z[[i]] + h * w[[i]]]))^(-1+b)
          Coth[(t[[i]] * Exp[c * x[[i]] + d * z[[i]] + h * w[[i]]])^b] +
         (t[[i]] * Exp[c * x[[i]] + d * z[[i]] + h * w[[i]]])^(-1+2b)
          Csch[(t[[i]] * Exp[c * x[[i]] + d * z[[i]] + h * w[[i]]])^b]^2)
        Tanh[1/b (a (1 - (t[[i]] * Exp[c * x[[i]] + d * z[[i]] + h * w[[i]]])^b
            Coth[(t[[i]] * Exp[c * x[[i]] + d * z[[i]] +
                  h * w[[i]]])^b]))])) * Exp[c * x[[i]] + d * z[[i]] + h * w[[i]]]]), {i, 1, n}];
k = D[f, {{a, b, c, d, h}, 2}];
FindMaximum[{f, a > 0, b > 0}, {a, .0074913915321},
  {b, 2.98612131}, {c, 0}, {d, 0}, {h, 0}, MaxIterations → 250]
```

{-318.748, {a → 0.000583336, b → 2.11745, c → 0.00374751, d → 6.28482 × 10^-8, h → -0.14984}}

```
a = 0.0005833358146616144` ;
b = 2.117453901656705` ;
c = 0.003747505602254099 4` ;
d = 6.284823595800878`*^-6 ;
h = -0.1498402105824965` ;
Coef = {a, b, c, d, h};
VarianceCovariance = Inverse[- k]
SD = √Diagonal[VarianceCovariance]
(Coef / SD) ^2
```

{{1.3786 × 10^-8, -6.15423 × 10^-6, -2.20673 × 10^-11, -1.6012 × 10^-11, -5.55058 × 10^-7},
{-6.15423 × 10^-6, 0.00286275, -2.92535 × 10^-6, 3.3547 × 10^-9, 0.000101055},
{-2.20673 × 10^-11, -2.92535 × 10^-6, 4.37565 × 10^-7, -1.29898 × 10^-10, -1.46317 × 10^-6},
{-1.6012 × 10^-11, 3.3547 × 10^-9, -1.29898 × 10^-10, 1.35396 × 10^-12, 6.87393 × 10^-9},
{-5.55058 × 10^-7, 0.000101055, -1.46317 × 10^-6, 6.87393 × 10^-9, 0.000507288}}

{0.000117414, 0.0535047, 0.000661487, 1.1636 × 10^-6, 0.0225231}

{24.683, 1566.19, 32.0953, 29.1729, 44.259}

```
Needs["HypothesisTesting`"]
ChiSquarePValue[24.68302703519825`, 1]
ChiSquarePValue[1566.1883915069363`, 1]
ChiSquarePValue[32.09532622015964`, 1]
ChiSquarePValue[29.17294706090572`, 1]
ChiSquarePValue[44.25901765975128`, 1]
```

OneSidedPValue → 6.75767 × 10^-7

OneSidedPValue → 1.6246251883309 33 × 10^-342

OneSidedPValue → 1.4679 × 10^-8

OneSidedPValue → 6.61971 × 10^-8

OneSidedPValue → 2.87676 × 10^-11

NBI Data Analysis for Combined Steel and Concrete Superstructures With Value of 2 for The Variable Type.

This is an example of Accelerated failure Model Using the Hypertabastic Model with Log time .Bridge Superstructure Rating of 5 Wisconsin Data where "w" is the bridge type variable, "t" is the age of the bridge , "x" is the maximum span length and "z" is traffic intensity . x, z and w are included in the model "The type value of 2 has no effect in this set of data analysis since there is no effective parameter h." No observation is censored, thus status is set to be 1 for all observations .

Accelerated an Data example failure Hypertabastic is
Log Model[2] of Rate5 the This Using where Wisconsin with time.Bridge

```
n = 907;
Status = ConstantArray[1, n];
ID = {4, 201, 337, 346, 412, 417, 429, 437, 439, 469, 471, 474, 475, 680, 952, 965, 966, 967,
   1015, 1049, 1050, 1057, 1063, 1077, 1085, 1091, 1092, 1106, 1107, 1117, 1118, 1123, 1133,
   1152, 1310, 1312, 1313, 1322, 1326, 1334, 1340, 1495, 1507, 1516, 1523, 1533, 1582, 1640,
   1647, 1669, 1686, 1753, 1811, 1830, 1831, 1835, 1840, 1841, 1849, 1854, 1855, 1860, 1863,
   1866, 1869, 1872, 1899, 1900, 1912, 1913, 1915, 1918, 1927, 1929, 1932, 1936, 1945,
   1947, 1950, 1952, 1955, 1965, 1966, 1984, 1987, 1993, 1995, 1997, 2013, 2017, 2018,
   2032, 2045, 2074, 2191, 2266, 2268, 2289, 2295, 2303, 2328, 2330, 2331, 2333, 2334,
   2442, 2448, 2543, 2599, 2775, 2785, 2786, 2792, 2796, 2806, 2807, 2808, 2818, 2860,
   2864, 2922, 2923, 2964, 2973, 2975, 2992, 2993, 2998, 3010, 3040, 3046, 3184, 3200,
   3219, 3450, 3454, 3465, 3478, 3602, 3641, 3650, 3651, 3775, 3779, 3784, 3795, 3827,
   3845, 3846, 3853, 3854, 3864, 3867, 3873, 3889, 3912, 3966, 3983, 3994, 4005, 4007,
   4010, 4127, 4167, 4270, 4346, 4349, 4350, 4351, 4375, 4427, 4430, 4439, 4443, 4465,
   4468, 4469, 4559, 4600, 4716, 4753, 4759, 4778, 4804, 4834, 4926, 4931, 5114, 5117,
   5118, 5120, 5121, 5123, 5124, 5127, 5131, 5134, 5144, 5151, 5152, 5153, 5157, 5164,
   5171, 5225, 5247, 5369, 5476, 5483, 5486, 5493, 5582, 5586, 5591, 5661, 5676, 5686,
   5687, 5697, 5699, 5710, 5717, 5722, 5730, 5733, 5740, 5741, 5758, 5765, 5766, 5778,
   5786, 5796, 5797, 5798, 5799, 5804, 5814, 5846, 5847, 5857, 5858, 5860, 5873, 5882,
   5899, 5900, 5901, 5902, 5905, 5918, 5927, 5931, 5938, 5939, 5940, 5941, 5999, 6002,
   6003, 6004, 6005, 6006, 6007, 6013, 6020, 6033, 6034, 6069, 6085, 6108, 6112, 6127,
   6132, 6136, 6192, 6193, 6349, 6416, 6443, 6444, 6447, 6448, 6452, 6457, 6458, 6462,
   6625, 6643, 6735, 6736, 6741, 6743, 6762, 6791, 6793, 6810, 6819, 6963, 6997, 7011,
   7013, 7046, 7051, 7052, 7128, 7138, 7149, 7163, 7165, 7178, 7293, 7326, 7352, 7353,
   7358, 7362, 7366, 7374, 7376, 7381, 7393, 7403, 7423, 7500, 7518, 7547, 7566, 7568,
```

7575, 7577, 7578, 7617, 7623, 7680, 7688, 7698, 7706, 7707, 7945, 7982, 8047, 8049,
8163, 8167, 8179, 8266, 8267, 8276, 8292, 8311, 8320, 8330, 8481, 8501, 8503, 8507,
8637, 8642, 8643, 8644, 8645, 8654, 8681, 8690, 8715, 8721, 8725, 8729, 8732, 8820,
8824, 8825, 8828, 8835, 8838, 8856, 8907, 8912, 8932, 8938, 8953, 8954, 8957, 8996,
9005, 9006, 9007, 9008, 9013, 9055, 9084, 9116, 9131, 9164, 9166, 9172, 9292, 9304,
9306, 9321, 9516, 9518, 9526, 9551, 9577, 9602, 9615, 9683, 9705, 9722, 9736, 9755,
9757, 9758, 9815, 9874, 9875, 9876, 9890, 9903, 9920, 9921, 9940, 9941, 9942, 9943,
9947, 9948, 9949, 9952, 9953, 9979, 10 040, 10 048, 10 051, 10 162, 10 173, 10 276, 10 322,
10 324, 10 337, 10 338, 10 346, 10 347, 10 354, 10 388, 10 586, 10 592, 10 769, 10 773, 10 778,
10 803, 10 822, 10 827, 10 838, 10 867, 10 898, 10 903, 10 923, 10 937, 10 938, 10 940, 10 941,
10 942, 10 943, 10 951, 10 958, 10 961, 10 966, 10 969, 10 976, 10 982, 10 988, 10 991, 10 996,
10 999, 11 014, 11 015, 11 018, 11 023, 11 025, 11 026, 11 028, 11 029, 11 031, 11 033, 11 034,
11 035, 11 039, 11 042, 11 044, 11 046, 11 047, 11 049, 11 053, 11 056, 11 057, 11 058, 11 061,
11 068, 11 069, 11 086, 11 087, 11 111, 11 125, 11 141, 11 142, 11 143, 11 145, 11 155, 11 176,
11 177, 11 185, 11 187, 11 190, 11 191, 11 201, 11 212, 11 213, 11 214, 11 216, 11 218, 11 227,
11 236, 11 237, 11 240, 11 255, 11 263, 11 286, 11 290, 11 308, 11 357, 11 360, 11 361, 11 362,
11 363, 11 364, 11 365, 11 370, 11 371, 11 379, 11 381, 11 382, 11 384, 11 386, 11 387, 11 388,
11 389, 11 391, 11 393, 11 401, 11 404, 11 407, 11 412, 11 425, 11 445, 11 449, 11 481, 11 515,
11 517, 11 518, 11 521, 11 523, 11 525, 11 527, 11 528, 11 530, 11 532, 11 533, 11 534, 11 540,
11 546, 11 548, 11 550, 11 553, 11 555, 11 563, 11 564, 11 566, 11 567, 11 570, 11 574, 11 610,
11 625, 11 630, 11 633, 11 648, 11 658, 11 662, 11 665, 11 669, 11 671, 11 673, 11 675, 11 676,
11 692, 11 717, 11 725, 11 734, 11 736, 11 751, 11 765, 11 788, 11 790, 11 802, 11 810, 11 811,
11 813, 11 825, 11 828, 11 830, 11 841, 11 846, 11 858, 11 860, 11 869, 11 872, 11 881, 11 882,
11 890, 11 899, 11 903, 11 907, 11 919, 11 923, 11 925, 11 933, 11 934, 11 945, 11 948, 11 965,
11 975, 11 986, 11 991, 12 007, 12 011, 12 013, 12 020, 12 021, 12 037, 12 038, 12 045, 12 046,
12 052, 12 077, 12 112, 12 130, 12 131, 12 133, 12 138, 12 156, 12 157, 12 194, 12 205, 12 234,
12 242, 12 253, 12 267, 12 282, 12 295, 12 297, 12 303, 12 310, 12 318, 12 327, 12 341, 12 348,
12 381, 12 390, 12 413, 12 414, 12 415, 12 428, 12 432, 12 452, 12 458, 12 461, 12 468, 12 471,
12 473, 12 479, 12 481, 12 488, 12 495, 12 504, 12 513, 12 514, 12 519, 12 520, 12 598, 12 602,
12 614, 12 647, 12 653, 12 654, 12 656, 12 662, 12 665, 12 675, 12 677, 12 703, 12 711, 12 713,
12 718, 12 723, 12 740, 12 748, 12 760, 12 764, 12 766, 12 767, 12 768, 12 772, 12 773, 12 786,
12 787, 12 793, 12 794, 12 797, 12 798, 12 801, 12 802, 12 808, 12 814, 12 818, 12 822, 12 830,
12 831, 12 832, 12 833, 12 836, 12 851, 12 870, 12 878, 12 907, 12 944, 12 979, 12 992, 13 005,
13 008, 13 013, 13 016, 13 024, 13 027, 13 034, 13 042, 13 046, 13 059, 13 077, 13 079, 13 099,
13 110, 13 112, 13 125, 13 140, 13 142, 13 144, 13 146, 13 151, 13 152, 13 158, 13 165, 13 172,
13 175, 13 198, 13 199, 13 209, 13 224, 13 225, 13 226, 13 235, 13 243, 13 244, 13 247, 13 258,
13 259, 13 260, 13 263, 13 264, 13 272, 13 276, 13 297, 13 302, 13 344, 13 392, 13 396, 13 406,
13 417, 13 422, 13 429, 13 433, 13 434, 13 441, 13 445, 13 458, 13 461, 13 471, 13 494, 13 495,
13 499, 13 503, 13 521, 13 525, 13 526, 13 528, 13 529, 13 533, 13 537, 13 538, 13 542, 13 556,
13 559, 13 560, 13 562, 13 564, 13 568, 13 575, 13 577, 13 578, 13 582, 13 583, 13 600, 13 602,
13 605, 13 609, 13 622, 13 636, 13 637, 13 656, 13 665, 13 669, 13 677, 13 678, 13 680, 13 681,
13 682, 13 688, 13 693, 13 694, 13 695, 13 696, 13 697, 13 701, 13 706, 13 708, 13 709, 13 710,
13 713, 13 714, 13 716, 13 723, 13 727, 13 729, 13 733, 13 736, 13 737, 13 738, 13 741, 13 750,

```
    13 752, 13 756, 13 757, 13 758, 13 768, 13 777, 13 779, 13 782, 13 789, 13 791, 13 793, 13 804,
    13 805, 13 807, 13 808, 13 809, 13 810, 13 813, 13 817, 13 821, 13 855, 13 873, 13 876, 13 880,
    13 882, 13 895, 13 899, 13 913, 13 914, 13 934, 14 008, 14 014, 14 033, 14 037, 14 040, 14 041,
    14 043, 14 048, 14 060, 14 061, 14 070, 14 072, 14 073, 14 081, 14 084, 14 087, 14 089};

x = {300, 4700, 7000, 180, 3300, 1400, 19 700, 220, 6400, 20 890, 71 900, 71 900, 57 200, 3060, 2150,
    280, 370, 230, 6132, 1270, 6900, 650, 400, 310, 1600, 8140, 7160, 8140, 3720, 160, 2000,
    2600, 160, 2910, 8100, 300, 490, 900, 500, 1190, 290, 50, 340, 430, 660, 390, 500, 460,
    2500, 190, 460, 390, 1700, 1480, 1740, 4230, 1460, 1460, 4285, 655, 280, 1370, 830,
    4930, 400, 2400, 24 900, 24 900, 20 200, 15 000, 592, 16 400, 23 900, 21 120, 450, 430, 470,
    675, 570, 21 600, 770, 800, 375, 10 900, 10 900, 350, 170, 300, 12 500, 715, 9926, 6600,
    23 300, 1350, 1310, 395, 685, 3780, 870, 510, 8050, 9600, 19 900, 10 300, 11 000, 7300,
    12 500, 140, 70, 200, 610, 200, 400, 3400, 11 300, 11 900, 11 300, 8080, 400, 2910, 800,
    800, 400, 600, 7400, 9930, 15, 2600, 460, 1400, 1800, 250, 100, 600, 5900, 560, 290,
    5400, 1200, 190, 4600, 4400, 1400, 550, 770, 1100, 1200, 10 900, 10 300, 11 935, 11 935,
    120, 3320, 1360, 190, 720, 1340, 1800, 1150, 16 500, 16 500, 15 800, 4600, 3400, 725,
    1500, 5700, 3330, 440, 805, 830, 4200, 4400, 2500, 26 000, 11 550, 11 550, 880, 17 250,
    430, 360, 190, 930, 250, 5000, 10 600, 4458, 490, 3790, 3790, 560, 710, 1010, 1670, 470,
    930, 1980, 4900, 1080, 300, 1080, 270, 830, 3500, 880, 800, 90, 2200, 4600, 7200, 7600,
    1700, 5000, 1100, 780, 69 750, 69 750, 74 900, 9000, 15 800, 6600, 4700, 11 080, 9600,
    5800, 52 900, 49 700, 11 100, 9900, 9900, 34 600, 15 000, 8700, 4100, 1100, 7400, 7300,
    20 400, 17 960, 9740, 20 300, 10 300, 21 400, 1400, 4500, 27 100, 12 500, 10 100, 10 500,
    11 600, 5885, 37 700, 15 100, 6270, 6110, 14 500, 1870, 8300, 4000, 9500, 9500, 9500,
    9500, 9500, 39 700, 6800, 11 900, 11 900, 12 800, 50 600, 12 800, 7670, 10 200, 4300, 1200,
    9000, 9000, 18 700, 4875, 10 600, 10 600, 10 600, 10 600, 10 050, 7900, 7900, 80, 400, 950,
    460, 280, 175, 3400, 330, 340, 3300, 10 200, 310, 275, 10 900, 500, 15 200, 8800, 220,
    100, 1990, 390, 430, 260, 17 430, 430, 29, 320, 620, 2130, 5100, 15 300, 12 100, 21 300,
    23 100, 23 100, 520, 1100, 160, 5600, 1600, 2500, 322, 730, 850, 30 800, 15 300, 8900,
    10 600, 1400, 780, 3800, 2300, 3900, 8800, 22 550, 21 600, 9900, 290, 510, 820, 22 900,
    22 900, 20 730, 2900, 56 800, 5800, 6280, 2400, 160, 140, 1731, 3700, 680, 710, 680, 450,
    1000, 700, 11 600, 310, 1600, 6800, 2820, 390, 400, 490, 530, 680, 110, 925, 100, 8513,
    5120, 55, 2260, 2000, 2000, 2000, 1080, 290, 540, 440, 350, 540, 65, 1100, 1300, 500,
    100, 80, 200, 730, 100, 240, 1900, 80, 180, 210, 1100, 4600, 3550, 275, 1700, 1500,
    50, 210, 200, 13 850, 12 800, 7800, 1700, 1900, 6300, 17 200, 9000, 10 300, 5300, 22 200,
    25 250, 22 200, 25 250, 11 100, 17 300, 16 600, 12 200, 12 700, 14 850, 530, 12 700, 4242,
    320, 8220, 3600, 1050, 10 600, 33 300, 33 300, 33 300, 33 300, 16 000, 13 800, 3000, 2100,
    20, 20, 1200, 80, 40, 75, 1100, 2180, 80, 680, 40, 25, 25, 45, 50, 40, 40, 390, 425, 250,
    2000, 80, 80, 70, 4000, 3200, 40, 80, 280, 280, 500, 40, 40, 40, 80, 50, 50, 10, 60, 175,
    20, 150, 20, 20, 10, 20, 20, 20, 175, 120, 990, 10, 20, 40, 40, 1000, 640, 850, 210, 220,
    650, 180, 2160, 3000, 160, 150, 660, 170, 80, 670, 50, 50, 50, 50, 50, 100, 100, 50, 160,
    50, 80, 180, 50, 180, 250, 40, 200, 80, 175, 80, 300, 300, 160, 45, 20, 40, 350, 10, 260,
    10, 20, 2430, 40, 4475, 75, 27 950, 2100, 400, 4100, 20, 280, 270, 140, 100, 50, 80, 70,
    80, 75, 100, 100, 100, 750, 1100, 80, 100, 25, 50, 200, 171, 280, 120, 50, 25, 50, 1400,
```

```
2090, 300, 21, 50, 50, 50, 3600, 6500, 350, 5300, 500, 40, 50, 100, 100, 50, 50, 100,
50, 50, 200, 800, 700, 35, 75, 50, 50, 50, 760, 75, 530, 410, 100, 100, 75, 50, 150, 100,
530, 100, 50, 250, 580, 1500, 330, 2600, 100, 75, 140, 75, 27, 225, 35, 130, 20, 40, 80,
20, 230, 80, 6700, 20, 320, 40, 45, 250, 1800, 400, 400, 45, 80, 75, 5, 710, 500, 1500,
1000, 34, 14, 390, 270, 50, 1200, 750, 2500, 3600, 3800, 3800, 490, 40, 80, 80, 90, 40,
80, 80, 50, 610, 40, 640, 20, 175, 80, 40, 40, 10, 40, 25, 10 200, 1460, 1480, 1000, 40,
1500, 50, 1600, 5700, 9200, 3500, 6000, 6500, 8200, 6200, 170, 80, 230, 230, 230, 50,
210, 40, 50, 80, 40, 20, 350, 80, 10, 500, 2020, 230, 175, 25, 40, 420, 1100, 500, 370,
20, 60, 63, 800, 1900, 175, 75, 210, 30, 80, 75, 50, 50, 320, 50, 300, 740, 20, 125, 80,
1250, 252, 375, 1500, 350, 80, 500, 16 000, 2200, 10, 20, 150, 160, 80, 80, 70, 1, 450,
7800, 90, 20, 150, 490, 150, 610, 720, 130, 500, 75, 40, 190, 80, 450, 650, 50, 15, 175,
75, 215, 190, 50, 25, 280, 670, 20, 100, 40, 100, 100, 20, 1050, 230, 175, 175, 20, 40,
40, 80, 100, 50, 40, 360, 175, 300, 80, 500, 400, 20, 175, 50, 80, 80, 50, 25, 75, 75,
20, 80, 100, 400, 75, 20, 80, 20, 80, 30, 75, 175, 50, 40, 130, 20, 20, 100, 25, 20, 40,
40, 230, 200, 200, 310, 600, 200, 40, 190, 80, 40, 40, 30, 20, 40, 80, 80, 1070, 80, 20,
80, 480, 20, 260, 210, 200, 270, 2100, 220, 40, 40, 30, 40, 40, 50, 3400, 300, 80, 80,
13 000, 200, 1800, 590, 660, 20, 415, 800, 40, 40, 450, 660, 80, 40, 20, 40, 40, 450};

t = {34, 39, 57, 39, 62, 57, 55, 51, 48, 46, 44, 45, 45, 27, 72, 82, 83, 33, 42, 59, 55, 49, 48, 42,
40, 40, 40, 40, 40, 39, 39, 76, 29, 24, 73, 71, 51, 83, 51, 47, 35, 73, 62, 52, 40, 50, 25,
57, 80, 31, 25, 14, 72, 59, 60, 58, 57, 57, 55, 53, 54, 53, 53, 53, 52, 51, 50, 51, 48,
48, 48, 47, 50, 50, 49, 49, 49, 49, 49, 50, 47, 46, 47, 41, 42, 44, 44, 43, 40, 42, 41,
40, 23, 28, 71, 71, 71, 69, 83, 60, 47, 47, 48, 48, 47, 62, 33, 17, 59, 73, 76, 76, 57,
54, 46, 46, 46, 46, 35, 32, 74, 74, 71, 62, 37, 37, 74, 56, 48, 33, 30, 22, 62, 54, 61,
34, 47, 32, 51, 47, 32, 33, 74, 71, 62, 29, 46, 45, 45, 43, 43, 31, 46, 41, 23, 19, 69,
57, 47, 46, 46, 45, 72, 47, 74, 55, 81, 61, 58, 26, 64, 55, 57, 53, 43, 44, 44, 22, 15,
74, 88, 47, 20, 57, 28, 54, 52, 57, 56, 56, 56, 55, 54, 54, 54, 53, 52, 47, 51, 51, 50,
49, 47, 38, 32, 25, 73, 55, 40, 38, 34, 79, 44, 35, 74, 52, 51, 52, 51, 50, 49, 51, 49,
47, 48, 47, 47, 49, 49, 49, 46, 47, 43, 43, 43, 43, 43, 44, 46, 46, 46, 46, 46, 46, 44,
42, 42, 42, 42, 42, 7, 45, 42, 43, 43, 43, 43, 35, 35, 35, 36, 35, 35, 35, 36, 35, 35, 35,
39, 37, 34, 34, 23, 28, 27, 17, 17, 76, 47, 42, 42, 42, 42, 42, 42, 42, 81, 81, 37, 72,
73, 82, 51, 27, 63, 63, 49, 47, 76, 45, 39, 40, 29, 24, 24, 57, 52, 41, 31, 28, 26, 8, 25,
56, 54, 48, 41, 41, 42, 41, 41, 30, 39, 15, 62, 36, 74, 73, 72, 53, 52, 41, 42, 22, 56,
55, 49, 44, 46, 56, 51, 28, 27, 60, 51, 33, 52, 52, 52, 52, 38, 33, 28, 50, 32, 29, 27,
74, 71, 70, 69, 69, 38, 19, 15, 63, 61, 58, 52, 43, 71, 76, 87, 61, 54, 76, 46, 21, 32,
26, 26, 19, 19, 19, 80, 51, 49, 48, 46, 41, 20, 7, 62, 47, 28, 28, 26, 91, 55, 53, 41, 69,
69, 80, 62, 44, 43, 18, 27, 72, 32, 18, 57, 58, 58, 29, 58, 58, 82, 55, 48, 51, 51, 47,
46, 47, 46, 45, 47, 47, 46, 46, 41, 27, 28, 26, 61, 45, 73, 62, 59, 53, 53, 43, 43, 38,
33, 42, 75, 95, 85, 70, 76, 45, 76, 60, 80, 50, 44, 65, 88, 88, 86, 60, 60, 65, 56, 60,
50, 55, 85, 85, 93, 42, 46, 50, 55, 41, 93, 93, 65, 37, 95, 31, 83, 88, 84, 85, 80, 80,
34, 60, 42, 60, 60, 42, 39, 39, 36, 50, 30, 28, 90, 81, 80, 82, 75, 70, 70, 41, 76, 94,
76, 60, 63, 71, 90, 80, 56, 49, 60, 50, 36, 36, 96, 83, 81, 44, 82, 70, 86, 80, 74, 76,
84, 49, 81, 80, 80, 47, 81, 92, 40, 60, 80, 44, 49, 51, 44, 83, 96, 37, 60, 87, 85, 90,
```

```
    80, 46, 29, 71, 76, 37, 94, 85, 85, 93, 42, 93, 47, 71, 43, 41, 44, 97, 50, 76, 66, 70,
    76, 66, 42, 53, 81, 43, 39, 77, 80, 80, 83, 45, 35, 55, 42, 43, 45, 83, 91, 72, 50, 61,
    92, 101, 81, 76, 61, 87, 49, 49, 61, 76, 61, 91, 40, 77, 75, 41, 49, 74, 51, 71, 80, 73,
    80, 54, 69, 57, 49, 57, 53, 85, 81, 58, 57, 81, 80, 53, 83, 97, 47, 101, 50, 50, 70, 55,
    80, 72, 75, 62, 48, 82, 84, 61, 55, 80, 93, 81, 80, 80, 41, 87, 75, 72, 81, 88, 59, 47,
    38, 48, 38, 70, 52, 52, 52, 52, 81, 64, 74, 83, 81, 70, 69, 83, 46, 79, 81, 90, 73, 81,
    80, 64, 90, 49, 90, 26, 60, 60, 42, 75, 77, 54, 58, 53, 76, 77, 54, 52, 85, 55, 47, 77,
    70, 70, 77, 32, 65, 78, 82, 60, 51, 85, 47, 78, 90, 90, 80, 70, 70, 90, 70, 47, 55, 51,
    84, 97, 33, 93, 92, 48, 45, 87, 41, 90, 92, 30, 99, 45, 32, 51, 69, 92, 58, 80, 72, 103,
    90, 45, 43, 45, 37, 90, 74, 45, 84, 81, 60, 32, 31, 48, 92, 100, 123, 97, 73, 81, 33, 48,
    53, 54, 52, 46, 54, 51, 69, 41, 30, 48, 59, 80, 70, 51, 81, 78, 55, 72, 81, 30, 33, 74,
    88, 100, 90, 85, 83, 92, 81, 72, 51, 90, 90, 74, 72, 89, 91, 43, 64, 53, 68, 37, 83, 58,
    85, 80, 84, 65, 36, 51, 91, 80, 80, 70, 42, 98, 51, 85, 49, 46, 44, 89, 84, 42, 38, 82,
    80, 56, 42, 70, 60, 50, 49, 47, 48, 60, 36, 85, 50, 50, 56, 44, 47, 75, 41, 85, 77, 60,
    60, 35, 83, 77, 60, 30, 70, 77, 60, 47, 47, 47, 50, 37, 39, 50, 90, 60, 80, 83, 84, 36,
    48, 46, 81, 48, 89, 89, 58, 54, 90, 50, 43, 70, 31, 30, 37, 65, 60, 100, 41, 60, 46};

x = {7.6, 26.8, 8.2, 21.3, 15.2, 21.6, 13.7, 21.9, 27.7, 28, 21, 25.6, 24.4, 6.7, 7.5, 9.6, 9.6,
    18.2, 21.5, 6.7, 24.8, 23.6, 11, 32.9, 35.1, 16, 16, 16, 47.5, 45.6, 36.4, 15.2, 18.3,
    22.9, 12.6, 42.7, 25.9, 7.8, 12.8, 12.2, 10.1, 18.3, 23.2, 20.7, 14.5, 22.4, 7.6, 10.9,
    199, 12.2, 11.9, 13.1, 7.3, 12.6, 20.7, 22.9, 8.5, 5.9, 13, 18.3, 18.3, 15.7, 15.8, 9.9,
    23.7, 22.9, 15.2, 15.2, 12.8, 14.5, 21, 14.9, 13.1, 19.4, 20.7, 12.2, 6.4, 6.9, 18.3,
    8.2, 10.7, 8.4, 9.4, 17.4, 16.2, 9.8, 9.8, 10.7, 24.4, 18.1, 11, 16.8, 29.7, 8.5, 8.2,
    6.9, 7.9, 9.4, 12.2, 8.8, 18.3, 14.6, 15.2, 11.9, 11.9, 13.9, 36.4, 12.8, 21.3, 7.6, 7.5,
    7.4, 12.2, 10.7, 15.8, 18.6, 18.6, 20.7, 12.5, 13.7, 7.5, 11, 7.6, 20.4, 27.5, 22.2,
    12.6, 10.1, 7.5, 13.1, 16.5, 13.4, 18.3, 24.4, 7.6, 8.4, 17.4, 17.7, 13.7, 18, 18.1,
    18.1, 12.3, 13.1, 18.7, 13, 14.3, 18.7, 17.2, 21.6, 21.6, 14.6, 15.8, 16.6, 12.2, 13.4,
    9.6, 9.6, 20.7, 13.9, 16.2, 9.9, 14.2, 21, 18.7, 13.7, 8.1, 32, 12.8, 20.4, 12.6, 21.3,
    34.1, 12.2, 137.2, 16.8, 16.8, 11.9, 20.1, 8.6, 8.1, 11, 14.9, 19.4, 16.3, 29.5, 12.8,
    8.4, 9.4, 9.4, 15.5, 19.4, 13.4, 13.4, 6.4, 28, 9.4, 25.5, 13.7, 8.2, 9.5, 11, 14.3, 40.5,
    11.7, 15.2, 24.4, 8.8, 21, 40.2, 34.2, 9.6, 16.5, 16.9, 10.7, 14, 21.3, 17.2, 20.1, 22,
    22.1, 11, 24.8, 20.4, 19.4, 21.5, 15.8, 19.7, 20.7, 20.7, 13.4, 9.7, 19.5, 21.6, 23.7,
    29, 20.4, 13, 20.7, 20.7, 23.5, 25, 28.7, 11.9, 20, 33.8, 36.6, 35.5, 34, 35.4, 9.2,
    17.5, 13.7, 15.1, 15.1, 26.5, 13.4, 36.1, 19.4, 35.7, 35.4, 35.4, 35.7, 32.6, 35.4,
    24.4, 35.5, 31.9, 15.9, 13.4, 16.3, 18.3, 25.1, 116.9, 16.2, 9.4, 9.4, 20.7, 25.1, 23.2,
    23.2, 11.9, 11.9, 12.2, 15.8, 15.8, 8.4, 9.4, 39.6, 14.2, 14.2, 14.3, 15.8, 32, 17.6,
    7.9, 20.7, 12.8, 16, 29.1, 39.6, 25.3, 9.4, 12.8, 12.2, 11.9, 16.8, 12.2, 13.3, 39.6,
    13.7, 3.5, 12.8, 21.8, 14.9, 22.4, 25.5, 9.6, 23.9, 18.4, 18.4, 9.1, 16.9, 15.2, 24.4,
    23.1, 14.2, 17.9, 7.9, 9.5, 17.6, 23.2, 6.7, 33.6, 22.1, 18.3, 12.2, 17.1, 16.2, 7.6,
    25.8, 8.1, 15.2, 18.7, 12.2, 18.9, 12.2, 12.2, 12.2, 18.7, 21.3, 18.3, 14.6, 21.2, 13.7,
    12.2, 10.7, 6.2, 14.3, 14.6, 6.2, 7.3, 24.4, 9.8, 17.1, 9.3, 7.6, 8.2, 8.1, 11.4, 21.6,
    20.5, 7.3, 12.6, 14.2, 14.6, 10.7, 38.6, 16.2, 17.6, 16.5, 9.8, 8.5, 12.2, 6.9, 13.7,
    20.7, 9.4, 13.9, 11.3, 10.4, 14.3, 12.8, 15.8, 14, 11, 15.2, 11.3, 30.5, 12.8, 13.7, 8.4,
```

```
7.8, 11, 18.9, 32.3, 29.4, 13.4, 12.8, 18.4, 9, 16.8, 6.4, 9.6, 7.7, 12.2, 20.1, 20.1,
12.8, 11.6, 21.3, 21, 20.4, 14.5, 14.5, 12.6, 12.6, 29.7, 10.4, 10.4, 25.9, 26.5, 17.5,
17.1, 20.7, 7.6, 21.3, 13.9, 11.8, 7.7, 32.8, 14, 14, 29.6, 25.3, 33.5, 11, 21.5, 13.7,
8.1, 12.2, 9.4, 10.4, 5.8, 10.5, 6.3, 7.6, 7.2, 10.8, 7, 9.8, 9.4, 9.4, 7.2, 8.8, 8.8,
6.5, 14.3, 12.5, 7.3, 7.8, 7.8, 8.4, 6.6, 13.6, 7.4, 6.8, 7.3, 10.9, 9.4, 9.1, 7.1, 8.1,
7.2, 11.3, 7, 8.5, 7.3, 9.4, 9.4, 7.1, 9.8, 10.1, 3.2, 6.5, 4.7, 9.4, 6.9, 7.1, 7, 6.7,
6.4, 6.7, 11.9, 4.2, 9.1, 11.9, 14.6, 5.8, 8.9, 5.8, 28.4, 39.7, 9, 6.9, 8.7, 8.8, 6.6,
17.1, 14.9, 18, 18, 17.4, 11.3, 8.7, 21.3, 15, 9, 7.3, 41.5, 6.5, 6.9, 6.6, 6.6, 8.2, 6.8,
9, 8.2, 7.3, 23.9, 8.8, 5.8, 6.5, 7.3, 12, 9.1, 7.2, 9.8, 8.5, 6.9, 8.5, 9.5, 4.5, 6.6,
20.9, 11.9, 7.3, 10.8, 15.2, 9.7, 11.4, 22.3, 9.4, 5.2, 10.1, 13.5, 8.4, 14.2, 17.8, 9.9,
10.4, 6.6, 15.2, 9.4, 9.4, 7.3, 14.8, 4.5, 10.5, 8.8, 15.2, 5.9, 6.6, 15.5, 21.6, 9.4,
6.7, 18.3, 9.5, 7.8, 9, 12.6, 24.3, 18.8, 24.8, 18.9, 7.9, 9.5, 18.6, 11.7, 15.2, 7.2,
6.3, 30.5, 8.6, 8.7, 8.5, 6.3, 8.9, 7.6, 8, 6.5, 11, 15.5, 10.4, 9.8, 9.3, 11.9, 9.5,
12.6, 6.7, 12.1, 7, 6.9, 6, 6.9, 7.7, 7.7, 8.8, 6.9, 7.6, 18.2, 4.6, 15.7, 15.6, 8.5, 10,
7, 9.4, 11, 10.9, 7.3, 19.2, 9.4, 6.6, 15.2, 3.7, 6.2, 11.8, 9.6, 12.7, 12.8, 9.4, 9.4,
9.6, 6.9, 3.6, 8.1, 8.7, 8.2, 12, 11.7, 18, 9.1, 7.6, 9.4, 9.1, 11.3, 26.5, 8.5, 11.2,
9.6, 11.2, 7.6, 6.9, 11.1, 18.3, 8.9, 9.7, 11, 14.1, 7.1, 6.7, 9.5, 9.6, 9.4, 12.5, 7.9,
18.3, 12.4, 6.8, 9.4, 6.9, 39.3, 3.7, 3.9, 9.1, 12.6, 9.1, 8.4, 21.2, 14.6, 7.9, 11.9,
27.4, 27.7, 29.3, 15.7, 8.5, 9.3, 9, 11.9, 16, 15.7, 7.6, 7.7, 8.2, 8.9, 8.8, 10.9, 7.6,
12.6, 11.4, 7.4, 10.8, 11.7, 7.2, 9.4, 6.9, 8, 12.6, 8.7, 6.6, 8.8, 25.6, 9.7, 9.6, 6.4,
6.4, 9.4, 21.7, 9.6, 9.4, 9.4, 4.3, 8.1, 13.1, 7, 7.6, 10.7, 7.6, 9.4, 10.3, 7.5, 14.4,
13.8, 21.2, 21.2, 11.7, 9.8, 25, 6.9, 9.8, 8.2, 10.7, 6.9, 7, 9, 7.2, 36.6, 43.1, 17.4,
7.1, 7.1, 9.5, 15.5, 15.5, 11.9, 15.7, 10.7, 18.7, 6.6, 6.9, 7.7, 6.2, 7.5, 9, 11.9, 7.2,
9.1, 3.2, 6.8, 9.6, 10.4, 9.1, 9.1, 14.9, 17.9, 18, 10.7, 14.3, 12.6, 6.6, 8.4, 14.3,
14.2, 18.3, 8, 11.3, 7.8, 14.3, 14.3, 9.7, 9.6, 8.3, 12.8, 14.3, 6.7, 6.9, 6.6, 6.9, 6.9,
8.9, 7.8, 17.7, 14.9, 6.7, 6.7, 6.7, 7, 24.1, 8.5, 8.7, 8.1, 7.1, 7.1, 6.4, 7.4, 8.6,
8.9, 15.5, 8.2, 6.9, 7.6, 7, 8.8, 7.3, 7.6, 8.8, 8.5, 5.8, 6.7, 7.4, 7.1, 15, 13, 12.5,
20.1, 8.1, 4.4, 11.9, 7.2, 24.4, 8.9, 15.3, 19.7, 10.5, 16.5, 8.9, 7.7, 7, 4.4, 10.4,
7.1, 7.1, 7, 10.6, 16.3, 8.2, 6.8, 8.6, 3.4, 8.8, 8.4, 9.3, 17.7, 15.2, 9, 7.7, 7.9, 7.7,
6.9, 12.6, 8.5, 7, 13.4, 8.6, 6.7, 21.3, 7.3, 17.9, 17.7, 7.6, 8.8, 8.5, 9.3, 6.5};
```

```
f = Sum[(Log[Sech[a * (1 - (t[[i]] * Exp[c * x[[i]] + d * z[[i]]]) ^b *
        Coth[(t[[i]] * Exp[c * x[[i]] + d * z[[i]]]) ^b]) / b]] +
    Status[[i]] * Log[t[[i]] * (((a (- (t[[i]] * Exp[c * x[[i]] + d * z[[i]]])^(-1+b)
          Coth[(t[[i]] * Exp[c * x[[i]] + d * z[[i]]])^b] + (t[[i]] * Exp[
            c * x[[i]] + d * z[[i]]])^(-1+2b) Csch[(t[[i]] * Exp[c * x[[i]] + d * z[[i]]])^b]^2)

          Tanh[1/b (a (1 - (t[[i]] * Exp[c * x[[i]] + d * z[[i]]])^b Coth[(t[[i]] *
                  Exp[c * x[[i]] + d * z[[i]]])^b]))]))) * Exp[c * x[[i]] + d * z[[i]]]]), {i, 1, n}];
k = D[f, {{a, b, c, d}, 2}];
FindMaximum[{f, a > 0, b > 0}, {a, .0074913915321},
  {b, 2.98612131}, {c, 0}, {d, 0}, MaxIterations → 250]
```

$\{-340.613, \{a \to 0.000536888, b \to 2.09096, c \to 0.00337519, d \to 8.30868 \times 10^{-6}\}\}$

```
a = 0.000536888338570094 2`;
b = 2.0909646065681877`;
c = 0.003375190105352845 5`;
d = 8.308681939453002`*^-6;
Coef = {a, b, c, d};
VarianceCovariance = Inverse[-k]
SD = √Diagonal[VarianceCovariance]
(Coef / SD) ^2
```

$\{\{1.17243 \times 10^{-8}, -5.72841 \times 10^{-6}, -1.15961 \times 10^{-8}, -6.96188 \times 10^{-12}\},$
$\{-5.72841 \times 10^{-6}, 0.00286792, -2.87022 \times 10^{-6}, 1.58853 \times 10^{-9}\},$
$\{-1.15961 \times 10^{-8}, -2.87022 \times 10^{-6}, 4.55513 \times 10^{-7}, -1.13358 \times 10^{-10}\},$
$\{-6.96188 \times 10^{-12}, 1.58853 \times 10^{-9}, -1.13358 \times 10^{-10}, 1.24707 \times 10^{-12}\}\}$

$\{0.000108279, 0.053553, 0.000674917, 1.11672 \times 10^{-6}\}$

{24.5855, 1524.5, 25.009, 55.3571}

```
Needs["HypothesisTesting`"]
ChiSquarePValue[24.585523661348134`, 1]
ChiSquarePValue[1524.4960374385457`, 1]
ChiSquarePValue[25.008976519326783`, 1]
ChiSquarePValue[55.35707027914803`, 1]
```

$OneSidedPValue \to 7.10837 \times 10^{-7}$

$OneSidedPValue \to 1.862019086358366 \times 10^{-333}$

$OneSidedPValue \to 5.7064 \times 10^{-7}$

$OneSidedPValue \to 1.00508 \times 10^{-13}$