The Power of AND

3D Printing Herwig Hauser's Gallery of Algebraic Surfaces

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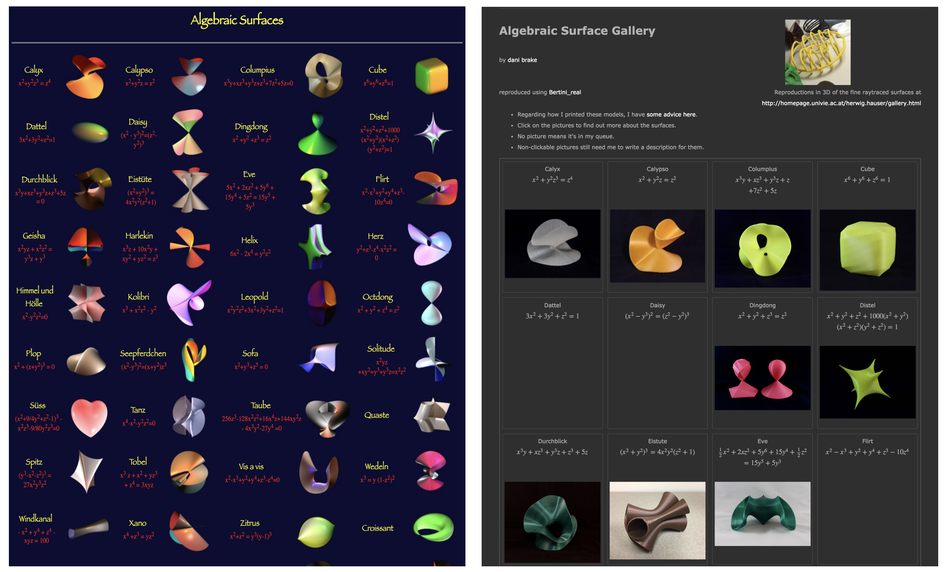
Research goals



The goal of our research was to extend the 3D printed gallery of Dr. Danielle Brake to mirror the gallery of Herwig Hauser's images of mathematical surfaces. Through 3D printing, we hoped to give a better visualization of the surfaces that we were computing.

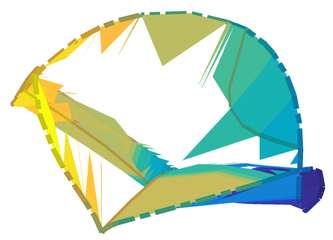
Challenges & future work

We face several challenges during the 3D printing process:

- Broken surface decomposition results in missing pieces of mathematical models.
- 3D printing failures can be caused by the lack of support structures or printer problems.
- Some 3D printed support structures can be difficult to remove.

Future work:

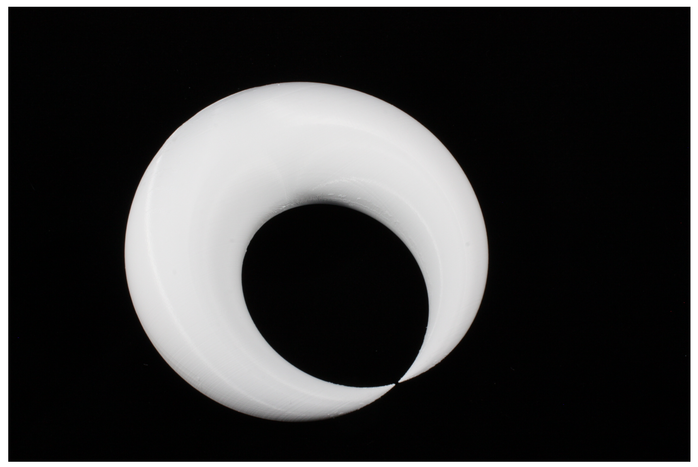
- Add an auto-complete sampler command
- Recreate bounding sphere to bounding box



Broken decomposition

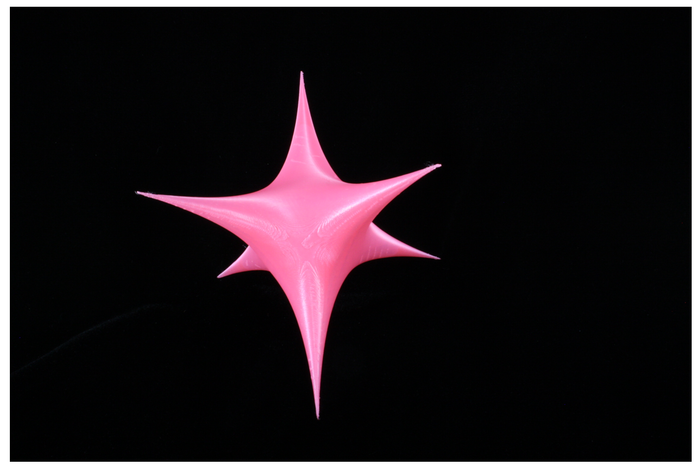
Printing failures

3D printing gallery



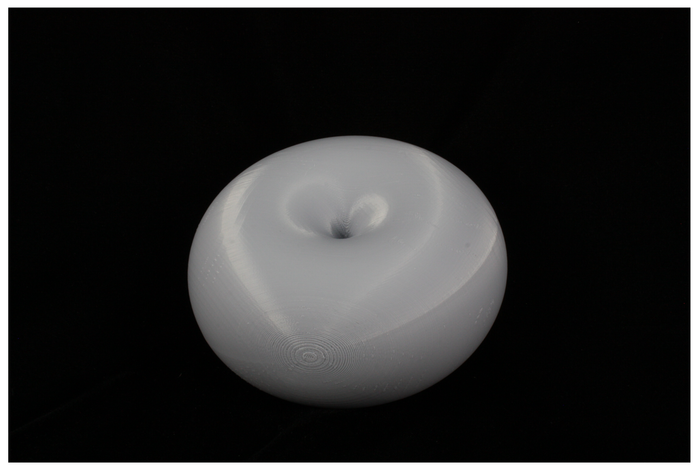
Croissant

$$(x^2+y^2+z^2+7\sqrt{5}/2-11/2)^2-((1+\sqrt{5})x$$
$$-7+3*\sqrt{5})^2-((1+\sqrt{5})^2y^2=0$$



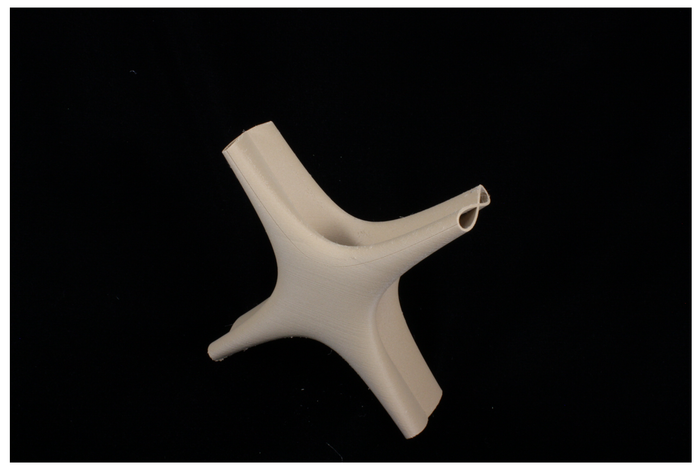
Stern

$$400(x^2y^2+y^2z^2+x^2z^2)+$$
$$(x^2+y^2+z^2-1)^3=0$$



Dullo

$$(x^2+y^2+z^2)^2-(x^2+y^2)=0$$



Helix

$$6x^2-2x^4=y^2z^2$$

Process

1. Run Bertini

```
Calculating traces for codimension 1.
Calculating 0 of 6
Using combinatorial trace test to
decompose codimension 1.
********* Witness Set Decomposition *********
| dimension | components | classified | unclassified
-----------------------------------------------------
|  2        |  1         |  6         |  0
-----------------------------------------------------
********** Decomposition by Degree **********
Dimension 2: 1 classified component
-----------------------------------------------------
   degree 6: 1 component
```

```
CONFIG
tracktype: 1;
tracktolbeforeeg: 1e-8;
END;

INPUT
variable_group x,y,z;
function f;
f = (x^3-1)^2 + (y^3-1)^2+ (z^2-1)^3;
END;
```

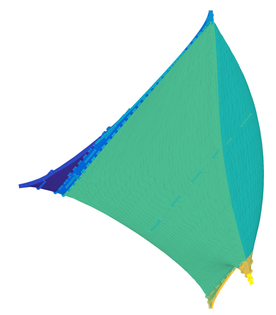
2. Run Bertini_Real

```
NOTE: You have requested to use adaptive path tracking.
Please make sure that you have setup the following
tolerances appropriately:
CoeffBound: 4.979639000000e+01,
DegreeBound: 9.000000000000e+00
AMPSafetyDigits1: 1, AMPSafetyDigits2: 1, AMPMaxPrec: 1024

Testing for a component of dimension 1.

Bertini verified that a witness point is a smooth
point on a 1 dimensional generically reduced component.

Isosingular deflation was successful!

Number of deflations: 2
Deflation sequence: 3, 2, 1, ...

Deflated system printed to 'input_singcurve_mult_3_0'.

testing points for deflation validity
multiplicity 3 0

computing critical points of the critical curve WRT pi_0
tracking path 0 of 108
tracking path 20 of 108
```

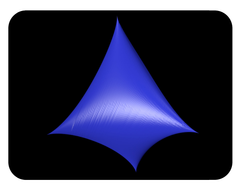
3. Run Sampler

```
sampling critical curve
sampling sphere curve
sampling mid slices
adaptively refining curve with 4 edges by
distance-movement method
adaptively refining curve with 8 edges by
distance-movement method
adaptively refining curve with 4 edges by
distance-movement method
adaptively refining curve with 8 edges by
distance-movement method
                    tracked 500 paths total.
adaptively refining curve with 4 edges by
 distance-movement method
Face 0 of 14
                    tracked 1000 paths total.
Face 1 of 14
Face 2 of 14
                    tracked 1500 paths total.
Face 3 of 14
```

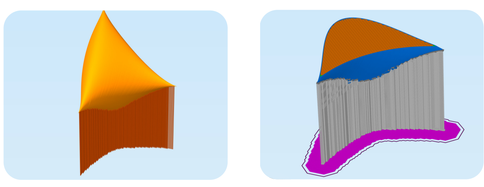
4. MATLAB



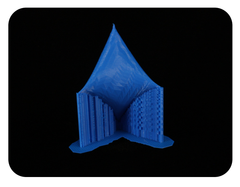
5. Blender



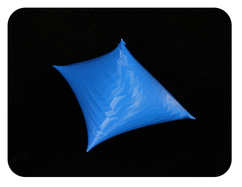
6. Simplify3D



7. 3D print



8. Remove Support



References and acknowledgements

[1] H. Hauser and J. Schicho, "Algebraic surfaces," 2014. `homepage.univie.ac.at/herwig.hauser/bildergalerie/gallery.html`

[2] D. A. Brake, D. J. Bates, W. Hao, J. D. Hauenstein, A. J. Sommese, and C. W. Wampler, "Algorithm 976: Bertini_real: Numerical decomposition of real algebraic curves and surfaces," *ACM Transactions on Mathematical Software (TOMS)*.

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